# Quasi-one-dimensional atomic gases across wide and narrow confinement-induced resonances

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# Quasi-1D atomic gases across wide and narrow confinement-induced-resonances

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We study quasi-one-dimensional atomic gases across wide and narrow confinement-induced-resonances (CIR). We show from Virial expansion that by tuning the magnetic field, the repulsive scattering branch initially prepared at low fields can continuously go across CIR without decay; instead, the decay occurs when approaching the non-interacting limit. The interaction properties essentially rely on the resonance width of CIR. Universal thermodynamics holds for scattering branch right at wide CIR, but is smeared out in narrow CIR due to strong energy-dependence of coupling strength. In wide and narrow CIR, the interaction energy of scattering branch shows different types of strong asymmetry when approaching the decay from opposite sides of magnetic field. Finally we discuss the stability of repulsive branch for a repulsively interacting Fermi gas in different trapped geometries at low temperatures.

## I. INTRODUCTION

Quasi-one-dimensional(1D) atomic gases across scattering resonances can be realized in laboratories by utilizing the confinement-induced-resonance(CIR)[1]. By initially preparing the system at high or low magnetic field and sweeping the field properly, the quasi-1D system can evolve on the attractive branch with molecules[2] or on the repulsive scattering branch that is free of molecules[3, 4]. Quasi-1D atomic gases have many fascinating properties that are very different from 3D gases. For instance, in quasi-1D a two-body bound state exists for an arbitrary s-wave scattering length $a_s$[1, 2]. The three-body recombination rate of 1D bosons is efficiently suppressed in the Tonks-Girardeau(TG) regime with strong repulsion[3, 5]. A long-lived metastable quantum phase, namely in the super-TG regime[6, 7], has been realized in the scattering branch of 1D bosonic system with strong attraction[4].

Apart from dimensionality, the resonance width is another important ingredient to affect many-body properties. Take (3D) Feshbach resonance(FR) for example. In wide FR, where the width is much larger than the typical energy scale $\mathcal{E}^*$[8], the system exhibits universal thermodynamics(UT) right at resonance where $a_s$ diverges[9, 10]. UT means that the thermodynamic potential is a universal function of the temperature and density, regardless of any detail of inter-particle interactions. However, in narrow FR, where the width is much smaller than $\mathcal{E}^*$, the universality is not evident due to considerable effective-range effect[11]. Another interesting property in narrow FR is that the interaction energy shows strong asymmetry when approaching resonance from different sides[12], as recently been observed in $^6$Li Fermi gas[13]. Considering the facts that quasi-1D is reduced from 3D geometry by confinements and 1D resonance is originated from 3D s-wave interaction, it is natural to expect the effective-range effect in 3D would also influence the interaction property of quasi-1D system.

In this work, from the analysis of two-body solutions and high-temperature Virial expansions, we study the scattering property and thermodynamics of quasi-1D atomic gases across CIR. We shall show that the quasi-1D geometry greatly modifies the stability of repulsive scattering branch compared to 3D case. Similar to FR, CIR can also be classified to wide and narrow counterpart according to the resonance width. We find drastically different thermodynamic properties between wide and narrow CIR due to the energy-dependence effect of coupling strength. The stability of repulsive branch in other trapped geometries, such as isotropic 3D trap or anisotropic quasi-low-dimensional trapped system, is also discussed in combination with recent developments on cold Fermi gases in the laboratories. We use $\hbar = k_B = 1$ throughout the paper.

The paper is organized as follows. In Sec. II, we introduce effective quasi-1D scattering, from which the wide and narrow CIR are defined and the corresponding thermodynamics are presented. In Sec. III we carry out high-T Virial expansion for effectively 1D system, and present an emphasized study on the stability and thermodynamic properties of the repulsive branch. An extensive discussion on the stability of repulsive branch for a cold Fermi gas in various trapped geometries is given in Sec. IV. Finally we summarize the paper in Sec. V.

## II. EFFECTIVE ONE-DIMENSIONAL SCATTERING AND THERMODYNAMICS

The Schrödinger equation for the relative motion of two atoms moving in quasi-1D is

$$H_0\psi(\mathbf{r}) + \frac{4\pi a_s(E)}{m}\delta(\mathbf{r})\frac{\partial}{\partial r}[r\psi(\mathbf{r})]|_{r\rightarrow 0} = E\psi(\mathbf{r}), \quad (1)$$

here the non-interacting part is $H_0 = -\nabla_{\mathbf{r}}^2/m + m\omega_{\perp}^2(x^2+y^2)/4$, $\mathbf{r} = (x,y,z)$ and $r = |\mathbf{r}|$; In the pseudopotential part, we use the energy-dependent s-wave scattering length obtained from a renormalization procedure[15, 16].

$$a_s(E) = a_{bg}(1 + \frac{W}{E/\delta\mu - (B - B_0)}). \quad (2)$$

$a_s(E)$ physically describe both wide and narrow FR, with background scattering length $a_{bg}$, magnetic field $B$, resonance position $B_0$, width $W$, and magnetic moment difference $\delta\mu$ between the atom and closed molecular state.

The reduced quasi-1D scattering from 3D s-wave interaction has been solved by Olshanii et al[1]. For low-energy scattering with $E=\omega_\perp+k^2/m$ and $k^2/m \ll 2\omega_\perp$, the wavefunction at large inter-particle distance is frozen at the lowest transverse mode, and its even-parity part is phase-shifted as is $\Psi_{\rm even}(\mathbf{r}) \sim \exp[-(x^2+y^2)/(2a_\perp^2)]\cos(k|z|+\delta_k)$, with $a_\perp=\sqrt{2/(m\omega_\perp)}$ and

$$\cot\delta_k = -\frac{ka_\perp}{2}[\frac{a_\perp}{a_s(E)} - C_0 + o(\frac{k^2a_\perp^2}{4})], \qquad (3)$$

where $C_0 = 1.4603$. Hereafter we neglect the small correction from last term in Eq.3. $\delta_k$ in turn determines an 1D energy-dependent coupling strength, $g(\bar{E}) = 2k\tan\delta_k/m$ with $\bar{E}=k^2/m$, as

$$g(\bar{E}) = g_{bg}(1+\frac{W_{1D}}{\bar{E}/\delta\mu-(B-B_{1D})}), \qquad (4)$$

where $g_{bg}=2\gamma\omega_\perp a_{bg}$, $W_{1D}=\gamma W$, $B_{1D}=B_0-(\gamma-1)W-\omega_\perp/\delta\mu$, with $\gamma=(1-C_0a_{bg}/a_\perp)^{-1}$(see also [16]). Eq.4 explicitly shows all realistic parameters describing CIR, namely, the background coupling $g_{bg}$, resonance position $B_{1D}$ and width $W_{1D}$. Near CIR ($B\sim B_{1D}$) and for $\bar{E} \ll \delta\mu W_{1D}$, one can construct an effective-range model to formulate 1D interaction,

$$\frac{1}{g(\bar{E})} = \frac{1}{g_{1D}} - \frac{m}{2\gamma^2\omega_\perp}r_0\bar{E}, \qquad (5)$$

with $g_{1D}$ the zero-energy coupling strength and $r_0 = -1/(ma_{bg}\delta\mu W)$ the effective range characterizing E-dependence in $a_s(E)$[17]. To this end, Eqs.(4,5) show the reduced effective-range effect(or E-dependence of coupling strength) from 3D to quasi-1D system.

In the tight transverse confinements and low atomic density($n$) limit, $na_\perp \ll 1$, we consider an effective 1D system with interaction given by Eq.4. Generally, the pressure takes the form

$$P = \mu(2m\mu)^{\frac{1}{2}}\mathcal{F}(\frac{T}{\mu},\{\frac{\delta\mu(B-B_{1D})}{\mu},\frac{\delta\mu W_{1D}}{\mu},\frac{E_{bg}}{\mu}\}), \qquad (6)$$

where $\mu$ is the chemical potential, $E_{bg}=mg_{bg}^2$, and $\mathcal{F}$ is a dimensionless function.

For wide CIR, $\delta\mu W_{1D} (\gg 2\omega_\perp) \gg n^2/m$, the E-dependence in Eq.4 and Eq.5 is negligible, and the interaction parameters in {...} of Eq.6 can be replaced by a single $g_{1D}$. The pressure is then reduced to

$$P = \mu(2m\mu)^{\frac{1}{2}}\mathcal{F}(\frac{T}{\mu},\frac{\mu}{mg_{1D}^2}). \qquad (7)$$

At wide CIR($g_{1D}=\infty$), $P$ is just a function of $T$ and $\mu$ (or $T$ and $n$) indicating UT for the scattering branch[18]. Particularly at $T=0$, UT can be established by noting that the bosons and spin-1/2 fermions with infinite repulsion are fully fermionalized, with the energy identical to that of an ideal single-species Fermi sea[7, 19]. However, at narrow CIR($B=B_{1D}$), Eq.6 still essentially relies on other interaction parameters $(W_{1D}, g_{bg})$ and thus UT is absent. More explicitly, UT can be identified by Virial expansions at high temperatures.

## III. HIGH-TEMPERATURE VIRIAL EXPANSION

At high temperatures, $n^2/m \ll T \ll 2\omega_\perp$, we carry out virial expansions to effectively 1D system[20]. The pressure can be expanded by small fugacity $z=e^{\mu/T}$ as $P=\alpha\frac{T}{\lambda}\sum_{n\geq1}b_nz^n$, where $\lambda=\sqrt{2\pi/(mT)}$ is the thermal wavelength, $\alpha$ is 1 for spinless boson and 2 for equal mixture of spin-1/2 fermions. Compared with non-interacting case(with superscript "0"),

$$P \ = \ P^{(0)} + \alpha\frac{T}{\lambda}\sum_{n\geq2}(b_n-b_n^{(0)})z^n. \qquad (8)$$

Here the difference, $b_n-b_n^{(0)}$, characterizes the interaction effect to the n-body cluster, which is generally a function of $\{\frac{\delta\mu(B-B_{1D})}{T},\frac{\delta\mu W_{1D}}{T},\frac{E_{bg}}{T}\}$. For wide CIR, $b_n-b_n^{(0)}$ only depends on one single parameter $1/(\lambda m g_{1D})$, which is free of parameter at $g_{1D}=\infty$ for any order of Virial expansion and leads to UT according to Eq.8. This also justifies us to examine UT within the second-order Virial expansion. The considering of higher-order expansions will not change the conclusion, except for a negligible correction (of higher order of $z$ or $n\lambda$) to the thermodynamic quantities.

Due to the interaction effect, the second Virial coefficient, $\Delta b_2 = (b_2-b_2^{(0)})/\sqrt{2}$, can be written as $\Delta b_2 = \sum_l[e^{-E_l/T}-e^{-E_l^{(0)}/T}]$ (here $l$ is the energy level for relative motion of two atoms). Given $P(T,\mu)$ in Eq.8, it is straightforward to obtain the density $n=\partial P/\partial\mu$ and entropy density $s=\partial P/\partial T$, and finally energy densities, $\mathcal{E}=\mu n+Ts-P$, for spinless bosons(b) and spin-1/2 fermions(f) as

$$\mathcal{E}^b \ = \ \frac{nT}{2}\Big[1+\frac{n\lambda}{2^{3/2}}(-1+2\epsilon_{int})+o((n\lambda)^2)\Big], \qquad (9)$$

$$\mathcal{E}^f \ = \ \frac{nT}{2}\Big[1+\frac{n\lambda}{2^{5/2}}(1+2\epsilon_{int})+o((n\lambda)^2)\Big], \qquad (10)$$

with dimensionless interaction energy

$$\epsilon_{int} = -\Delta b_2 + 2T\frac{\partial\Delta b_2}{\partial T}. \qquad (11)$$

In the following we derive $\Delta b_2$ in strictly 1D by enumerating the energy levels of two interacting particles in a tube($[-L/2, L/2]$). For simplicity, we first consider the scattering branch without inclusion of any bound state. The discretized wavevector($k>0$) is determined by boundary condition

$$k_l L/2+\delta_l=(l+1/2)\pi \quad (l=0,1,...). \tag{12}$$

By comparing to non-interacting $k_l^{(0)}$ where $\delta_l=0$, we obtain

$$\Delta b_2=\sum_l[\exp(-k_l^2/(mT))-\exp(-k_l^{(0)2}/(mT))], \tag{13}$$

which can be transformed to an integral as $2/(mT)\int_0^\infty dkk\delta_k e^{-k^2/(mT)}$ and further to

$$\Delta b_2^{sc} \;=\; -\frac{1}{2}+\frac{1}{\pi}\int_0^\infty dk e^{-\frac{k^2}{mT}}\frac{d\delta_k}{dk}. \tag{14}$$

Note that to obtain Eq.14 we extrapolate $\delta_{l=0}$ to $\delta_{k=0}$ in the thermodynamic limit, and set $\delta_{k=0}=-\pi/2$ considering $\delta_{l=0}<0$ as well as Eq.3. When considering a bound state (occupy $l=0$), the lowest available $l$ for scattering state should be $l=1$. This implies $\delta_{k=0}$ is up-shifted by $\pi$ as revealed by Levinson's theorem. In this case,

$$\Delta b_2^{bd} \;=\; e^{-|E_b|/T}-\frac{1}{2}+\frac{1}{\pi}\int_0^\infty dk e^{-\frac{k^2}{mT}}\frac{d\delta_k}{dk}. \tag{15}$$

To this end $\Delta b_2$ is obtained for both repulsive scattering branch (Eq.14) and attractive branch (Eq.15). Remarkably, comparing with 3D[9], $\Delta b_2$ in 1D has an additional term $(-1/2)$ resulted from zero-energy phase shift and the unique scattering property of 1D system. Eqs.(14,15) are consistent with results obtained from Bethe-ansatz solution[21] and analyses of real-space wave functions[22].

For quasi-1D system, the Virial expansions are carried out by setting $k^\Lambda=2/a_\perp$ as an upper limit of integrals in Eqs.(14,15). In the rest of this section, we shall mainly focus on the scattering branch(cf Eq.14) which might exhibit UT as discussed above.

### A. Wide CIR

With $\delta\mu W_{1D}(\gg 2\omega_\perp)\gg n^2/m$, we replace the E-dependent $g(E)$ by a constant $g_{1D}$. $g_{1D}$ is schematically plotted in Fig.1(A1), giving the phase shift ($\delta_k$) of scattering branch in Fig.1(B1). Here we have excluded the existence of bound state for any B-field, thus $\delta_k$ all start from $-\pi/2$ at $k=0$. By increasing B across CIR(from "a" to "e"), the amplitude of $\delta_k$ at finite $k$ gradually becomes enhanced, implying more repulsive energies in the system. Especially, $\delta_k$ is uniformly $-\pi/2$ for all $k$ right at CIR(between "b" and "c"), leading to universal values of $\Delta b_2^{sc}$ and $\epsilon_{int}^{sc}$ as shown below.

For strictly 1D system with constant $g_{1D}$, Eqs.(14, 15) can be analytically solved, for example,

$$\Delta b_2^{sc}=-\frac{1}{2}+\frac{sgn(g_{1D})}{2}\exp(\frac{1}{x^2})[1-erf(\frac{1}{x})], \tag{16}$$

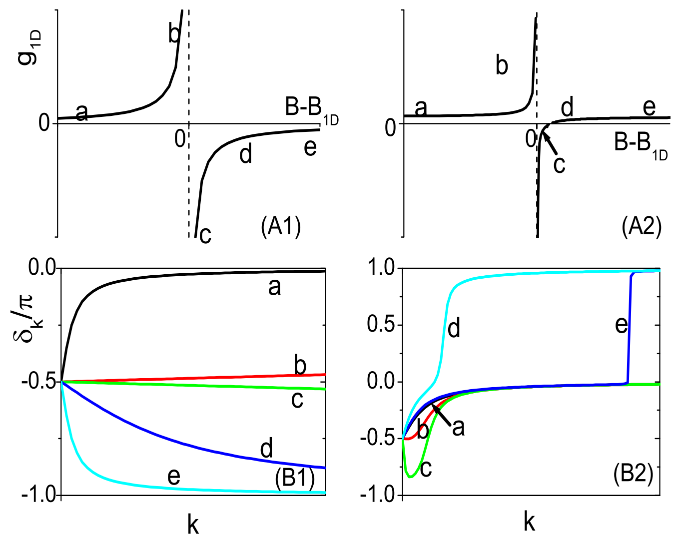


FIG. 1: (Color online). Upper panel: Schematic plot of zero-energy coupling strength $g_{1D}$ across wide (A1) and narrow (A2) CIR. The labels (a-e) correspond to $B\ll B_{1D}(a); B\rightarrow B_{1D}-0^+(b); B\rightarrow B_{1D}+0^+(c); B>B_{1D}(d); B\gg B_{1D}(e)$. Lower panel: Phase shift $\delta_k$ versus $k$ across wide (B1) and narrow (B2) 1D resonances, with each label (a,b,c,d,e) corresponding to specific $g_{1D}$ as marked in (A1) and (A2).

here $x=2\sqrt{2\pi}/(m|g_{1D}|\lambda)$; sgn() is sign function and erf() is error function. In the weak coupling limit ($x\rightarrow\infty$), we obtain

$$\Delta b_2^{sc}=-1/(\sqrt{\pi}x),\;\; \epsilon_{int}^{sc}=2/(\sqrt{\pi}x) \tag{17}$$

for scattering branch at $g_{1D}\rightarrow 0^+$ (corresponding to solid lines in small B-field in Fig.2); and

$$\Delta b_2^{bd}=1/(\sqrt{\pi}x),\;\; \epsilon_{int}^{bd}=-2/(\sqrt{\pi}x) \tag{18}$$

for attractive branch at $g_{1D}\rightarrow 0^-$[23] (dashed lines in large B-field in Fig.2). In the strong coupling limit ($x\rightarrow 0$), we obtain

$$\Delta b_2^{sc} \;=\; -\frac{1}{2}\pm\frac{1}{2\sqrt{\pi}}(x-\frac{x^3}{2}), \tag{19}$$

$$\epsilon_{int}^{sc} \;=\; \frac{1}{2}\mp\frac{x^3}{2\sqrt{\pi}}; \tag{20}$$

the universal values at $x=0$, $-\Delta b_2^{sc}=\epsilon_{int}^{sc}=1/2$, are direct consequences of $k-$independent phase shift $(-\pi/2)$ as mentioned above.

In Fig.2, we plot $\Delta b_2$ and $\epsilon_{int}$ for two-species $^6$Li fermions across wide CIR. For scattering branch, we see that all curves of $\Delta b_2^{sc}$(or $\epsilon_{int}^{sc}$) at different $T$ intersect at a single point in $\Delta b_2^{sc}$(or $\epsilon_{int}^{sc}$)-B plane[24], demonstrating the UT of scattering branch right at wide CIR. The scattering system at strongly repulsive side of CIR can smoothly evolve to strongly attractive side with even

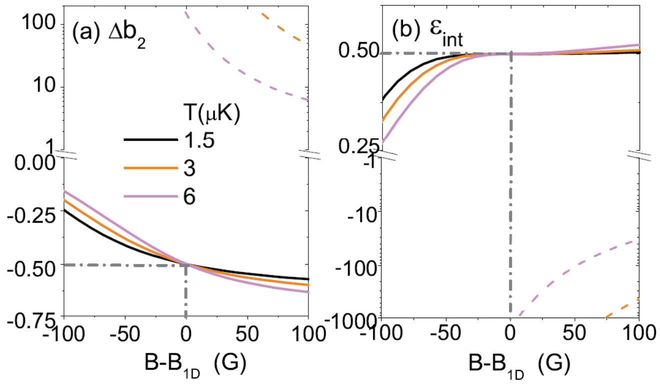


FIG. 2: (Color online). $\Delta b_2$(a) and $\epsilon_{int}$(b) for two-species $^6$Li fermions across wide CIR at $T(\mu K) = 1.5$(dark black), 3(medium orange), 6(light pink). We consider FR at $B_0 = 834.1G$ with width $W = -300G$[26]. Transverse confinements are generated by optical lattices with lattice space $a_L = 500nm$ and depth $V_0 = 25E_R$ ($E_R = \frac{1}{2m}(\frac{\pi}{a_L})^2$), giving $\omega_\perp = (2\pi)300$KHZ. CIR occurs at $B_{1D} = B_0 - 152.2G$ with $W_{1D} = -147.9G$, which satisfies $\delta\mu W_{1D} \gg 2\omega_\perp \gg T$. Solid and dashed lines are respectively for scattering(Eq.14) and attractive(Eq.15) branch. Dash-dot lines denote universal values $-\Delta b_2^{sc} = \epsilon_{int}^{sc} = 1/2$ at CIR.

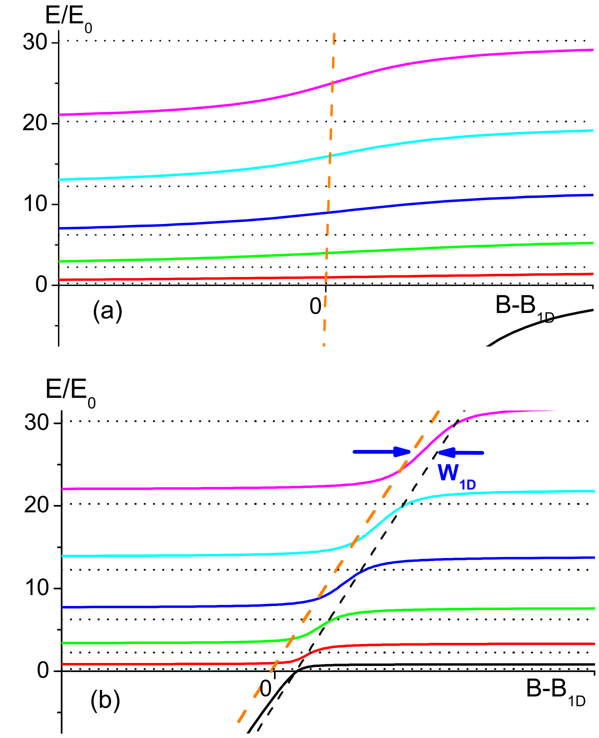


FIG. 3: (Color online). Two-body energy levels in the center-of-mass frame for quasi-1D system confined in a tube($[-L/2, L/2]$) across wide (a) and narrow (b) CIR. The orange and black dashed lines denote $\pi/2$ and 0 phase shift (corresponding to $g(E) = \infty$ and 0). $E_0 = (2\pi/L)^2/m$. The dotted lines denotes non-interacting energy levels with $E^{(0)}/E_0 = (l + \frac{1}{2})^2,\ l = 0, 1, 2....$

higher energy. This is consistent with previous theoretical predictions of sTG phase[6, 7] and its recent experimental realization in bosonic gas[4]. Virial expansion also shows that the scattering branch will achieve the strongest repulsion as $g_{1D} \to 0^-$ at large B-field, with $-\Delta b_2^{sc}, \epsilon_{int}^{sc} \to 1$. All above properties can be clearly seen by tracing any individual energy level of two scattering atoms in a tube, as shown in Fig.3(a).

Here we remark on the stability of scattering branch. In the framework of two-body clusters in Virial expansion, the decay of scattering branch manifest itself in the discontinuity of thermodynamic quantities, due to the relabeling of scattering states when the underlying bound state converts to the lowest scattering state. This is why in 3D the decay occurs right at FR where the bound state converts to scattering state at $a_s = \infty$[9]. In 1D, however, the conversion is at $g_{1D} = 0$ instead of at resonance, and therefore the scattering branch can extend far away from CIR until approaching zero coupling limit. A more comprehensive discussion of the stability of scattering branch in other trapped geometries will be given in Section IV.

At the end of this subsection we briefly discuss the second-order virial expansion in a 1D harmonic trap, which can be carried out given the two-body spectrum under coupling strength Eq.4. In particular, at wide CIR with $g_{1D} = +\infty$, the spectrum is $E_l = (2l + 3/2)\omega_z$ compared with $E_l^{(0)} = (2l + 1/2)\omega_z$; this gives $\Delta b_{2,trap}^{sc} = -1/(2\sqrt{2})$ compared with $-1/2$ in homogenous case. In fact, based on local density approximation(as used in 3D trapped system in Ref.[25]), we have

$$\Delta b_{n,trap}^{sc} = \frac{1}{\sqrt{n}} \Delta b_{n,hom}^{sc} \qquad (21)$$

for the scattering branch right at wide CIR. This shows a more rapid convergence of virial expansions in trapped 1D system than in homogeneous case.

### B. Narrow CIR

With $\delta\mu W_{1D} \ll n^2/m (\ll \omega_\perp)$, we take the full form of $g(\bar{E})$ (Eq.4) due to the strong E-dependence. Assume a positive background $a_{bg}$, we give the schematic plot of $g_{1D}$ in Fig.1(A2) and $\delta_k$ for scattering branch in Fig.1(B2). In Fig.4, we show $\Delta b_2$ and $\epsilon_{int}$ for $^{87}$Rb system across extremely narrow CIR.

Compared with wide CIR case, the scattering branch in narrow CIR shows many distinct properties.

First, $\delta_k$ is no longer universal at CIR; instead, it sensitively depends on $k$ and is quite small at finite-$k$ (see "b" and "c" in Fig.1(B2)). This can be attributed to the strong E-dependence in Eq.4 that $g(\bar{E})$ is far off resonance at finite $\bar{E}$ even its zero-energy value $g_{1D} \to \infty$.

Accordingly, as plotted in Fig.3(b) the two-body levels at finite energies are just shifted by a small amount although the lowest level is shifted half-way. As a result, there is no UT at narrow CIR, and the scattering branch is generally with very weak repulsion even close to CIR (see also Fig.4).

Second, shortly beyond CIR, the scattering branch goes through a decay at $B = B_{1D} + W_{1D}$ as manifested by discontinuous $\Delta b_2^{sc}$ and $\epsilon_{int}^{sc}$ there(see fig.4). This is exactly the place where $g_{1D}$ evolves from $0^-$ to $0^+$ and the bound state transforms to scattering state.

Third, after the decay, i.e., $B > B_{1D} + W_{1D}$ and $g_{1D} > 0$, we see from Fig. 1(B2) that $\delta_k$ will complete a continuous change from $-\pi/2$ to nearly $\pi$ within an energy window $\Delta\bar{E} \approx B - B_{1D}$. Due to the large $\pi$ shift for all energies larger than $\Delta\bar{E}$, we see an large and negative $\epsilon_{int}^{sc}$ in Fig. 4(b) despite of positive $g_{1D}$. Similarly to narrow FR in 3D[12], we expect the negative $\epsilon_{int}^{sc}$ in extremely narrow CIR will extend to much larger B-field, until $\delta\mu(B - B_{1D})$ approaches the typical energy scale of the system[8].

On a whole, the scattering branch in narrow CIR is with weak repulsion($\epsilon_{int}^{sc} \to 0^+$) or strong attraction($\epsilon_{int}^{sc} \to (-1)^+$) when B approaching the decay position ($B = B_{1D} + W_{1D}$) from the small or large field side. The asymmetry here differs from that in wide CIR, where $\epsilon_{int}^{sc}$ approach $1^-$ or $0^+$ respectively. It is also helpful to compare these features in quasi-1D with those in 3D system[9, 12]. For wide FR in 3D, the amplitudes of interaction energies are symmetric for the system evolving in different branches and approaching FR from different sides[9], which is in contrast with what we find in quasi-1D system across wide CIR. For narrow FR, the interaction effect are greatly suppressed for repulsive branch but greatly enhanced for attractive branch[12], which share the same feature as revealed above in quasi-1D system across narrow CIR. In cold atoms experiment, all these features in quasi-1D system can be detected using the technique of rf spectroscopy, as have been successfully applied to the quasi-2D Fermi gas[33] and 3D Fermi gas across narrow FR[13].

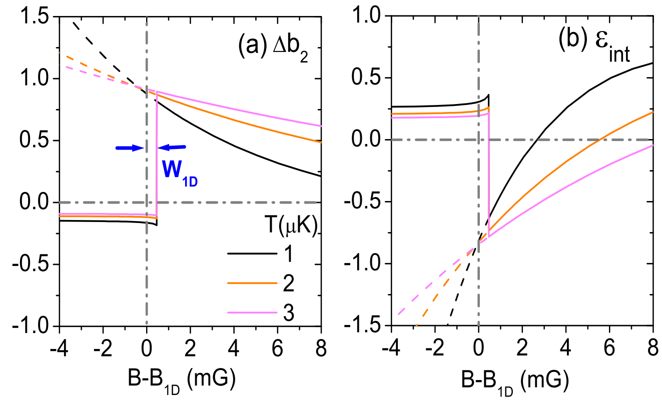


FIG. 4: (Color online). $\Delta b_2$(a) and $\epsilon_{int}$(b) for $^{87}$Rb bosons across narrow CIR at $T(\mu K) = 1$(dark black), 2(medium orange), 3(light pink). We consider FR at $B_0 = 406.2G$ with width $W = 0.4mG$[26]. The optical lattice is same as that for $^6$Li in Fig.2, giving $\omega_\perp = (2\pi)80$KHZ. CIR occurs at $B_{1D} = B_0 - 27.1mG$ with $W_{1D} = 0.5mG$, which satisfies $\delta\mu W_{1D} \ll T \ll 2\omega_\perp$. Decreasing $B$ across $B_{1D} + W_{1D}$, the scattering branch(solid lines) continuously evolves to attractive branch(dashed lines) with a bound state emerging at threshold.

## IV. STABILITY OF REPULSIVE FERMI GASES IN TRAPPED GEOMETRIES

Recently, the metastable repulsive branch of atomic gases has attracted lots of research interests, in the context of MIT experiment on itinerant ferromagnetism for a repulsively interacting Fermi gas[28]. The same group later claimed the absence of itinerant ferromagnetism from the measurement of spin susceptibility, and attributed this to the instability of repulsive branch against the molecule formation for a 3D Fermi gas near Feshbach resonance[29]. Similarly, the instability of a repulsive Fermi gas has also been observed in the quasi-2D Fermi gas[33], but at negative $a_s$ side.

There have also been quite a few theoretical studies on the problem why the repulsive Fermi gas in a 3D homogenous system is unstable close to Feshbach resonance[30–32]. For instance, the instability has been attributed to the shifted resonance in the background of a Fermi sea[30], the dominated pairing instability over ferromagnetism instability[31], or the vanishing zero-momentum molecule due to Pauli-blocking effect of Fermi-sea atoms[32]. All these studies can lead to the same conclusion at low temperatures, i.e., the 3D Fermi gas becomes unstable at the place where $a_s$ is comparable to the inter-particle distance ($1/k_F$), or equivalently, the two-body binding energy ($E_b \sim 1/ma_s^2$) is comparable to the Fermi energy ($E_F \sim k_F^2/m$). The scattering branch can only be stable when $E_b > E_F$, where the deep molecule can not be absorbed by the Fermi sea atoms[30–32]. In other words, in this parameter regime the existence of a deep two-body bound state effectively stabilizes a many-body system at the metastable repulsive branch. Since the physics behind this criterion does not depend on any detail of the dimension or trapped geometry, it should be equally applicable to other cases besides the homogeneous 3D system. In the following, we will use this criterion to study the stability of repulsive Fermi gas in various trapped geometries at low temperatures.

Typically we consider three different types of trapping potentials, namely, the isotropic or nearly isotropic 3D trap ($\omega_x \sim \omega_y \sim \omega_z$), the extremely anisotropic quasi-2D ($\omega_z \gg \omega_x, \omega_y$) and quasi-1D trap ($\omega_z \ll \omega_x, \omega_y$). To facilitate the discussion, we consider the system across wide resonance (with single interaction parameter), while the extension to narrow resonance should be straightforward.

In a trapped system, a two-body bound state is always supported no matter how weak the attractive interaction is[1, 14, 27]. For a 3D isotropic trap ($\omega_x \sim \omega_y \sim \omega_z \sim \omega$), however, it should be noted that the two-body binding energy $E_b$ at $a_s < 0$ side is less than the order of $\omega$, i.e., the level spacing of all scattering states[14]. As a result, in the thermodynamic limit with atom number $N \gg 1$, $E_b(< \omega)$ at $a_s < 0$ side is negligible compared with the Fermi energy $E_F \sim N\omega$. The system is then expected to behave similarly to the homogeneous case, in a sense that the repulsive branch is stable only with positive $a_s$ where the bound state is visibly deep ($E_b \sim N\omega$). This is consistent with what have been observed in MIT experiments[28, 29]. On the contrary, for anisotropic quasi-2D or quasi-1D trap, the energy spacing of scattering state is generally of the order of shallow trapping frequency, while the binding energy can be of order of tight frequency even at $a_s < 0$ side[1, 27]. For example, for a quasi-2D trapped system at $a_s = \infty$, $E_b \geq \omega_z \gg \omega_x, \omega_y$, and the existence of deep molecule would be possible to stabilize the repulsive branch at $a_s = \infty$ as long as $E_b > E_F \sim N\omega_{x,y}$. In this case, the stable scattering branch can even extend to negative $a_s$ side, as shown in the experiment of quasi-2D Fermi gas[33]. The same conclusion can be drawn to quasi-1D trapped case ($\omega_z \ll \omega_x, \omega_y$), which is also consistent with the high-temperature result presented in the last section.

In short summary of this section, at low temperatures, the stability of repulsive Fermi gas in a trapped geometry not only relies on the existence of two-body bound state, but more importantly, relies on the value of its binding energy compared with the typical energy scale of a many-body system. In other word, here the two-body physics should be evaluated in a many-body background. Generally, the repulsive branch in quasi-low-dimensional systems is expected to be more stable than that in an isotropic 3D system. Therefore the low-dimensional system provides us a more favorable platform to realize possible itinerant ferromagnetism in repulsively interacting Fermi gases.

## V. SUMMARY

In this paper, we have studied the quasi-1D atomic gases across wide and narrow CIR. Our main results are summarized as follows.

First, from high-temperature Virial expansions we obtain the following:

(1) By tuning the magnetic field across CIR, the repulsive scattering branch of quasi-1D system can evolve continuously across CIR, from $g_{1D} = +\infty$ to $g_{1D} = -\infty$ side.

(2) Universal thermodynamics are identified for the repulsive scattering branch right at wide CIR, but is found to be washed away at narrow CIR by the strong energy-dependence of coupling strength.

(3) The decay of quasi-1D repulsive branch occurs when $g_{1D} \to 0$. The interaction energy shows different types of strong asymmetry between wide and narrow CIR, when approaching the decay position from opposite sides of magnetic field.

Moreover, the second-order virial expansion presented in this paper also serves as a benchmark for testing future experiment on 1D atomic gases.

Second, we have discussed the stability of repulsive branch for a repulsively interacting Fermi gas at low temperatures in different trapped geometries. By evaluating the two-body bound state in the presence of a Fermi sea, we conclude that the system can generally be more stable in the quasi-low-dimensional trapped system than in a 3D isotropic trap. This should shed light on the current experiments to seek for ferromagnetism in more stable and strongly interacting Fermi gases in low dimensions.

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