Preparation of four-mode cluster states with distant atomic ensembles

Li-hui Sun and Gao-xiang Li

Phys. Rev. A **85**, 065801 — Published 14 June 2012

DOI: 10.1103/PhysRevA.85.065801

Preparation of four-mode cluster states with distant atomic ensembles

Li-hui Sun[1,2] and Gao-xiang Li[1*]

[1] *Department of Physics, Huazhong Normal University, Wuhan 430079, China*
[2] *College of Physical Science and Technology, Yangtze University, Jingzhou 434023, China*
(Dated: April 12, 2012)

We propose a scheme to generate four-mode cluster states with four atomic ensembles trapped in single-mode cavities connected by short fibres. With the aid of the cavity dissipation, we find that a four-mode cluster state can be unconditionally created by the simultaneous driving of the ensembles with laser pulses of suitably chosen Rabi frequencies and phases. The scheme could be easily extended to the case of N-mode cluster states.

PACS numbers: 42.50.Ct, 42.50.Dv, 42.60.Da

Over the last decade there has been a lot of interest in the generation of multi-mode continuous variable (CV) cluster states [1, 2]. The cluster states have been recognized as the basic building blocks for one-way quantum computation which describes a realization of quantum computation beyond the usual network picture [3–5]. Various theoretical and experimental schemes for the generation of CV cluster states have been proposed with most of them based on linear optics [6–8]. In particular, the generation of four-mode cluster states of the electromagnetic field modes has been demonstrated experimentally by utilizing two amplitude-quadrature and two phase-quadrature squeezed states [6]. Furusawa *et al.* [7] have proposed three different types of CV four-mode cluster states and demonstrated how these states could be experimentally constructed by applying squeezed light sources and a set of beam splitters.

In linear optics schemes squeezed light appears as a source of correlations necessary for the creation of entanglement between separate bosonic modes [9]. However, the schemes are extremely challenging due to practical problems of achieving a large efficiency in the coupling of squeezed light to the beam-splitters and the non-avoidable coupling of the beam-splitters to an environment. Therefore, alternative schemes have been proposed involving atomic ensembles [10–13] that have several advantages. Firstly, the external sources of squeezed light are not necessary in this case, as squeezed fields can be generated in the atomic ensembles by a suitable driving of the atomic transitions. Secondly, since the dynamics of the ensembles can be reduced to that of only the ground states of the atoms, the decoherence could be significantly reduced due to long atomic ground-state coherence lifetimes.

An another important issue is the implementation of the coupling between distant atoms or atomic ensembles. In most treatments, the atoms or atomic ensembles are located in separate cavities connected by a fibre that can lead to reliable transfer of a quantum state [14–18]. With this kind of coupling, highly reliable swap and entangling gates have been realized by trapping two-level atoms inside fibre-connected cavities [18]. A different type of the coupling between distant atomic ensembles have been proposed by Krauter *et al.* [19] who demonstrated, both theoretically and experimentally, that an entanglement between two separated macroscopic atomic ensembles can be created by the dissipative process of spontaneous emission.

So far, schemes considered for multi-mode entanglement have involved atomic ensembles placed inside a single- or two-mode cavity [12, 20]. Some of the common problems of these schemes include separate steps of the preparation of each atomic ensemble in a squeezed state that may take a relatively long time. In this paper, we propose a procedure that may lead to the generation of four-mode cluster states. The scheme involves four atomic ensembles placed inside separate cavities connected by short fibres and driven by a set of laser pulses. In practice, this scheme could perform better than the linear optic schemes in that the coherence could be easily controlled by adjusting Rabi frequencies and phases of the external driving laser fields. Moreover, the fast performance of the procedure could avoid the decoherence process of the coherence between the ground states of the atoms. In addition, the procedure could be easily extended to the case of massive size cluster states of N atomic ensembles placed inside N separate cavities.

The system we consider consists of four cavities each containing an atomic ensemble. The cavities are coupled to each other by short optical fibres, as illustrated in Fig. 1(a). In the interaction picture, the interaction Hamiltonian between the four cavity modes is given by

$$H_1 = \eta\left[\left(a_1 a_2^\dagger + a_2 a_3^\dagger + a_3 a_4^\dagger\right) + \text{H.c.}\right], \tag{1}$$

where a_n and $a_n^\dagger$ are the annihilation and creation operators for the nth cavity mode, and η is the coupling strength between the cavity modes.

The atomic ensembles contain the same number of N identical four-level atoms. As illustrated in Fig. 1(b), each atom has two stable ground states $|0_{jn}\rangle, |1_{jn}\rangle$ and two excited states $|\mu_{jn}\rangle$ and $|s_{jn}\rangle$. Such a scheme might be realized in practice, e.g., by employing alkali-metal atoms, with $|0_{jn}\rangle$ and $|1_{jn}\rangle$ chosen as different ground-state sublevels. The ground state $|0_{jn}\rangle$ of energy $E_{0_{jn}} = 0$ is coupled to the excited state $|s_{jn}\rangle$ by a laser field of the Rabi frequency Ω_{kn}^s and frequency $\omega_{L_{sn}}$ that is detuned from the atomic transition frequency by $\Delta_{sn} = \omega_{sn} - \omega_{L_{sn}}$, where $\omega_{sn} = E_{sn}/\hbar$ and E_{sn} is the energy of the state $|s_{jn}\rangle$. Similarly, the ground state

* Electronic mail:gaox@phy.ccnu.edu.cn

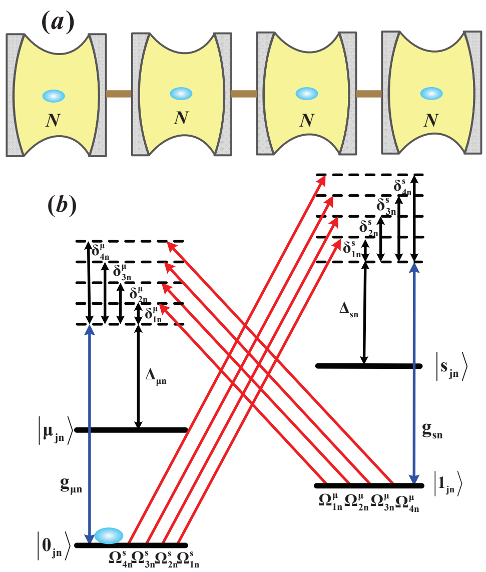

FIG. 1. (Color online) (a) A scheme for creation of four-mode cluster states. Four cavities each containing an atomic ensemble are connected by short fibres. (b) Atomic level configuration.

$|1_{jn}\rangle$ of energy $E_{1_{jn}} = \hbar\omega_{1_{jn}}$ is coupled to the excited state $|\mu_{jn}\rangle$ by a laser field of the Rabi frequency Ω^{μ}_{kn} and frequency $\omega_{L_{\mu n}}$ that is detuned from the atomic transition frequency by $\Delta_{\mu n} = \omega_{\mu n} - \omega_{1_{jn}} - \omega_{L_{\mu n}}$, where $\omega_{\mu n} = E_{\mu n}/\hbar$ and $E_{\mu n}$ is the energy of the state $|\mu_{jn}\rangle$. Here we assume that the cavity-mode detunings $\Delta_{\mu n} = \Delta_{sn} = \Delta$, i.e. the modes of all the cavities are coupled to the atomic transitions with the same strengths and they are also equally detuned from the atomic transition frequencies.

In our model we assume that the atomic transitions $|1_{jn}\rangle \to |\mu_{jn}\rangle$ and $|0_{jn}\rangle \to |s_{jn}\rangle$ are driven by a set of laser fields each composed of four distinct frequency components detuned from each other by δ^m_{kn}, with $k, n = 1, 2, 3, 4$ and $m = \mu, s$. For simplicity, we assume that δ^m_{kn} are independent of m and n, so that $\delta^m_{kn} = \delta_k$. In addition, we assume that the Rabi frequencies of the components, Ω^{μ}_{kn} and Ω^{s}_{kn} are complex numbers that could have different magnitudes $|\Omega^m_{kn}|$ and phases ϕ^m_{kn} $(m = \mu, s)$. The cavity modes of frequency ω_c are coupled to the transitions $|0_{jn}\rangle \to |\mu_{jn}\rangle$ and $|1_{jn}\rangle \to |s_{jn}\rangle$ with the coupling strengths $g_{\mu n}$ and g_{sn} $(g_{\mu n} = g_{sn} \equiv g)$.

In the interaction picture, the Hamiltonian describing the interaction between the atoms and the fields is

$$H_2 = \sum_{k=1}^{4}\sum_{n=1}^{4}\sum_{j=1}^{N} \Big[g a_n \left(|\mu_{jn}\rangle\langle 0_{jn}| + |s_{jn}\rangle\langle 1_{jn}|\right) e^{-i\Delta t} + (\Omega^{\mu}_{kn}|\mu_{jn}\rangle\langle 1_{jn}| + \Omega^{s}_{kn}|s_{jn}\rangle\langle 0_{jn}|)\, e^{-i(\Delta+\delta_k)t} + \text{H.c.}\Big], \quad (2)$$

When the cavity damping is included, the dynamics of the system are then determined by the density operator which satisfies the following master equation

$$\dot{\rho} = -i[H_1 + H_2, \rho] + L_a\rho, \quad (3)$$

where $L_a\rho = \sum_{n=1}^{4} \kappa\left(2a_n\rho a_n^\dagger - a_n^\dagger a_n\rho - \rho a_n^\dagger a_n\right)$. Here, κ is the damping rate of the cavity modes, which is assumed to be the same for all cavities.

Since the cavity modes are coupled to each other, it is found convenient to introduce new bosonic operators that are linear combinations of the cavity field operators

$$c_{1,4} = \frac{1}{\sqrt{2(\lambda_1^2+1)}}\left[\lambda_1(a_2 \mp a_3) \mp (a_1 \mp a_4)\right],$$
$$c_{2,3} = \pm\frac{1}{\sqrt{2(\lambda_1^2+1)}}\left[\lambda_1(a_1 \pm a_4) \mp (a_2 \pm a_3)\right]. \quad (4)$$

Expressed in terms of the new operators, the Hamiltonian H_1 in Eq. (1) takes a diagonal form

$$H_1 = \eta\left[\lambda_1(c_4^\dagger c_4 - c_1^\dagger c_1) + \lambda_2(c_3^\dagger c_3 - c_2^\dagger c_2) + \text{H.c.}\right], \quad (5)$$

where $\lambda_1 = 1/\lambda_2 = (\sqrt{5}+1)/2$.

The diagonal form of the Hamiltonian H_1 prompts us to make the transformation $H_2'(t) = \exp(iH_1t)H_2\exp(-iH_1t)$ to get the atom-field coupling Hamiltonian $H_2'(t)$ in the interaction picture. Also we assume that the cavity modes and the driving lasers are significantly detuned from the atomic transition frequencies, $|\Delta + \delta_k| \gg g, |\Omega^m_{kn}|$. Under this large detuning condition, only Raman two-photon take place between the levels $|1_{jn}\rangle$ and $|0_{jn}\rangle$, so the interaction of the atoms with the fields occurs in a highly nonresonant dispersive manner. We then perform the standard adiabatic elimination [22, 23] of the atomic excited states and obtain an effective Hamiltonian $H_{e_1} = -iH_2'(t)\int H_2'(t')dt'$, which describes the dynamics of the atomic system confined to only the ground states determined by collective spin operators $S_{zn} = \frac{1}{2}\sum_{j=1}^{N}(|0_{jn}\rangle\langle 0_{jn}| - |1_{jn}\rangle\langle 1_{jn}|)$, $S_n^\dagger = \sum_{j=1}^{N}|1_{jn}\rangle\langle 0_{jn}|$.

In the current experiments with atomic ensembles of $N \simeq 10^5$ atoms trapped inside a cavity [21, 24], the coupling strengths of the atomic transitions to the cavity modes are of order of $g_{\mu n} = g_{sn} = g/2\pi = 10$ kHz, the coupling strength between the cavity modes $\eta/2\pi = 40$ kHz. The cavity decay rate $\kappa/2\pi = 20$ kHz are achieved. Putting $\Delta_{\mu n} = \Delta_{sn} = \Delta/2\pi = 2\times 10^4$ kHz, $|\Omega^m_{kn}| = 10g$, $\delta_k = 0.06\Delta$, then $|\Delta + \delta_k|$ occurs to be much larger than the coupling strengths g and $|\Omega^m_{kn}|$, so that the condition of $|\Omega^m_{kn}|/|\Delta + \delta_k| \ll 1$ can be achieved. Hence the approximation of the effective Hamiltonian H_{e_1} assumed here appears to be practical. Moreover, with this choice of the parameters, the spontaneous emission rate due to off-resonant excitation of the atomic excited states is estimated at $\frac{1}{4}(\gamma/2\pi)(|\Omega^m_{kn}|/\Delta)^2 < 40$ Hz, where $\gamma/2\pi = 6$ MHz has been assumed. Thus, at that parameter choice the spontaneous emission is negligible.

Since we have taken into account large detunings of the driving fields, we expect that the excitation of the atoms is much smaller than the number of atoms, i.e., $\langle S_{zn}\rangle \ll N$. In this case, we can express the collective spin operators in terms of bosonic operators by using the Holstein-Primakoff representation [25], $S_{zn} = -N/2$ and $S_n^\dagger = \sqrt{N}d_n^\dagger$, where d_n and $d_n^\dagger$ obey the standard bosonic commutation relation, $[d_n, d_n^\dagger] = 1$. In this representation, we regard the collective atomic operators d_n and $d_n^\dagger$ as the bosonic operators.

After that we make the further transformation $\exp(iH_{e_0}t)H_{e_1}\exp(-iH_{e_0}t) - H_{e_0}$, with $H_{e_0} = [(\lambda_1\eta + \delta_1)c_1^\dagger c_1 + (\lambda_2\eta + \delta_2)c_2^\dagger c_2 - (\lambda_2\eta - \delta_3)c_3^\dagger c_3 - (\lambda_1\eta - \delta_4)c_4^\dagger c_4 + \text{H.c.}]$ to obtain the new effective atom-field interaction Hamiltonian

$$\begin{aligned}
H_e = &- \sum_{n=1,4} \beta_n \Big[\Omega^\mu_{n1} d_1^\dagger + \lambda_1 \Omega^\mu_{n3} d_3^\dagger + (-1)^n \Big(\lambda_1 \Omega^\mu_{n2} d_2^\dagger + \Omega^\mu_{n4} d_4^\dagger \Big) \\
&+ \Omega^s_{n1} d_1 + \lambda_1 \Omega^s_{n3} d_3 + (-1)^n (\lambda_1 \Omega^s_{n2} d_2 + \Omega^s_{n4} d_4) \Big] c_n \\
&+ \sum_{m=2,3} \beta_m \Big[\lambda_1 \Omega^\mu_{m1} d_1^\dagger - \Omega^\mu_{m3} d_3^\dagger - (-1)^m \Big(\Omega^\mu_{m2} d_2^\dagger - \lambda_1 \Omega^\mu_{m4} d_4^\dagger \Big) \\
&+ \lambda_1 \Omega^s_{m1} d_1 - \Omega^s_{m3} d_3 - (-1)^m (\Omega^s_{m2} d_2 - \lambda_1 \Omega^s_{m4} d_4) \Big] c_m + \text{H.c.},
\end{aligned} \tag{6}$$

with

$$\beta_{1,4} = \frac{\sqrt{N}g}{\Delta \mp \eta\lambda_1}, \quad \beta_{2,3} = \frac{\sqrt{N}g}{\Delta \mp \eta\lambda_2}. \tag{7}$$

where we have discard the constant energy terms and choose

$$\begin{aligned}
\delta_{1,4} &= \frac{\sqrt{2(1+\lambda_1^2)}Ng^2}{\Delta \mp \eta\lambda_1} \mp \lambda_1\eta, \\
\delta_{2,3} &= \frac{\sqrt{2(1+\lambda_1^2)}Ng^2}{\Delta \mp \eta\lambda_2} \mp \lambda_2\eta,
\end{aligned} \tag{8}$$

to compensate the Stark shifts. It is seen from Eq. (6) that the dispersive interaction gives rise to nonlinear, squeezing type couplings between the collective and cavity modes.

In this case, the master equation of Eq. (3) reduces to

$$\dot{\rho} = -i[H_e, \rho] + L_c\rho, \tag{9}$$

where the dissipative part of the master equation is of the form $L_c\rho = \sum_{n=1}^4 \kappa\left(2c_n\rho c_n^\dagger - c_n^\dagger c_n\rho - \rho c_n^\dagger c_n\right)$, which is of the same form as $L_a\rho$ after we introduce new bosonic operators c_n as Eq. (4).

Now we demonstrate how to apply the effective Hamiltonian (6) to generate a four-mode cluster state. We set a pure linear-type cluster state as an example, which is of the following form

$$\begin{aligned}
|\psi_L\rangle = \exp\Big\{ &-\frac{\xi}{10} \big[d_1^2 - d_2^2 - d_3^2 + d_4^2 - 8i(d_1d_2 + d_3d_4) \\
&- 4\,(d_1 + id_2)(d_3 - id_4) \big] - \text{H.c.} \Big\} |0_d\rangle,
\end{aligned} \tag{10}$$

where ξ is a squeezing parameter, and $|0_d\rangle = |0_{d_1}, 0_{d_2}, 0_{d_3}, 0_{d_4}\rangle$ represents the initial state with zero photons in each of the collective atomic modes d_n.

To demonstrate that the state $|\psi_L\rangle$ is an analog of a linear-type cluster state, we introduce the variances of linear combinations of the quadrature components $q_n = (d_n + d_n^\dagger)/\sqrt{2}$ and $p_n = -i(d_n - d_n^\dagger)/\sqrt{2}$. It is not difficult to show that in the state $|\psi_L\rangle$, the quadrature components are

$$\begin{aligned}
&\Delta(p_1 - q_2) = \Delta(p_4 - q_3) = \mathrm{e}^{-2\xi}, \\
&\Delta\,(p_2 - q_1 - q_3) = \Delta\,(p_3 - q_2 - q_4) = \frac{3}{2}\mathrm{e}^{-2\xi}.
\end{aligned} \tag{11}$$

We see that the variances tend to zero when the squeezing parameter $\xi \to \infty$. Hence, according to the definition of cluster state [7], the state $|\psi_L\rangle$ is an analog of a linear-type four-mode cluster state.

We now show how to unconditionally prepare the linear-type cluster state (10) among four atomic ensembles located in four separate single-mode cavities coupled by short fibres. Firstly, we make a unitary transformation $d_{L_n} = T d_n T^\dagger$ as

$$\begin{aligned}
d_{L_1} &= -(id_1 + d_2)/\sqrt{2}, \\
d_{L_2} &= -(id_1 - d_2 - 2id_3 - 2d_4)/\sqrt{10}, \\
d_{L_3} &= -(d_3 + id_4)/\sqrt{2}, \\
d_{L_4} &= (2d_1 + 2id_2 + d_3 - id_4)/\sqrt{10}.
\end{aligned} \tag{12}$$

It is easy to check that the modes d_{L_n} are orthogonal to each other, so the state $|\psi_L\rangle$ can be written as

$$|\psi_L\rangle = T^\dagger \exp\left[\frac{\xi}{2}\sum_{n=1}^4 (d_{L_n}^2 - d_{L_n}^{\dagger 2})\right] |0_{d_{L_n}}\rangle, \tag{13}$$

where $|0_{d_{L_n}}\rangle = |0_{d_{L_1}}, 0_{d_{L_2}}, 0_{d_{L_3}}, 0_{d_{L_4}}\rangle$. Thus, each mode might be prepared separately in a desired state.

Secondly, we show that the modes can be simultaneously prepared in squeezed states by a suitable choice of the Rabi frequencies and phases of the driving lasers. It is not difficult to find that the choice of the Rabi frequencies $\Omega^\alpha_{ij} = |\Omega^\alpha_{ij}|\exp(i\phi^\alpha_{ij})$ $(\alpha = \mu, s,\ i, j = 1, 2, 3, 4)$ with

$$\begin{aligned}
&|\Omega^\mu_{11}| = |\Omega^\mu_{33}| = |\Omega^s_{11}|/r = |\Omega^s_{33}|/r = \Omega, \\
&|\Omega^\mu_{12}| = |\Omega^\mu_{34}| = |\Omega^s_{12}|/r = |\Omega^s_{34}|/r = \Omega/\lambda_1, \\
&|\Omega^\mu_{21}| = |\Omega^\mu_{43}| = |\Omega^s_{21}|/r = |\Omega^s_{43}|/r = \Omega/(\sqrt{5}\lambda_1), \\
&|\Omega^\mu_{22}| = |\Omega^\mu_{44}| = |\Omega^s_{22}|/r = |\Omega^s_{44}|/r = \Omega/\sqrt{5}, \\
&|\Omega^\mu_{23}| = |\Omega^\mu_{41}| = |\Omega^s_{23}|/r = |\Omega^s_{41}|/r = 2\Omega/\sqrt{5}, \\
&|\Omega^\mu_{24}| = |\Omega^\mu_{42}| = |\Omega^s_{24}|/r = |\Omega^s_{42}|/r = 2\Omega/(\sqrt{5}\lambda_1),
\end{aligned} \tag{14}$$

and the phases

$$\begin{aligned}
&\phi^\mu_{12} = \phi^\mu_{22} = \phi^\mu_{41} = \phi^\mu_{43} = \phi^s_{12} = \phi^s_{22} = \phi^s_{41} = \phi^s_{43} = 0, \\
&\phi^\mu_{21} = \phi^\mu_{23} = \phi^\mu_{42} = \phi^s_{11} = \phi^s_{34} = \phi^s_{44} = \pi/2, \\
&\phi^\mu_{24} = \phi^\mu_{33} = \phi^s_{24} = \phi^s_{33} = \pi, \\
&\phi^\mu_{11} = \phi^\mu_{34} = \phi^\mu_{44} = \phi^s_{21} = \phi^s_{23} = \phi^s_{42} = 3\pi/2,
\end{aligned} \tag{15}$$

results in the effective Hamiltonian of the form

$$TH_eT^\dagger = \sum_{n=1}^4 \beta'_n \left(c_n d_{L_n}^\dagger + r c_n d_{L_n} + \text{H.c.} \right), \tag{16}$$

with $\beta'_n = \Omega\beta_n/\sqrt{2}$. The term $c_n d_{L_n}^\dagger$ describes a linear-mixing process between the cavity and d_{L_n} modes. It can lead to the possibility of exchanging the quantum information between the cavity field and the mode d_{L_n}. On the other hand, the term $c_n d_{L_n}$ describes a nondegenerate parametric amplification process. This term is responsible for the generation of continuous variable entanglement between the cavity and d_{L_n} modes.

Under the interaction with the laser pulses, the system to decay to a stationary state with the cavity damping. To see it clear, we make a squeezing transformation of the density operator $\tilde{\rho} = S^{\dagger}(\xi)T\rho T^{\dagger}S(\xi)$ with $S(\xi) = \exp[(\xi/2)\sum_{n=1}^{4}(d_{L_n}^{2} - d_{L_n}^{\dagger 2})]$, and find that the master equation of the transformed density operator can be written as

$$\frac{d}{dt}\tilde{\rho} = -i\sum_{n=1}^{4}\beta_n' \left[c_n d_{L_n}^{\dagger} + c_n^{\dagger} d_{L_n}, \tilde{\rho}\right] + L_c\tilde{\rho}, \quad (17)$$

where $\xi = \frac{1}{2}\ln(\frac{1+r}{1-r})$ with $0 < r < 1$. It is seen that the master equation (17) represents dynamics of linearly coupled modes with the cavity modes damped with the rate κ. In order to ensure that the system decays to a stable steady state, we calculate the eigenvalues of Eq. (17) and find $\eta_{\pm} = -\frac{\kappa}{2} \pm \left[\left(\frac{\kappa}{2}\right)^2 - (\beta_n')^2\right]^{\frac{1}{2}}$. As long as the effective Rabi frequency overcomes the cavity damping, $|\beta_n'| > \kappa/2$, both eigenvalues have negative real parts. Under this condition and for a sufficiently long duration of the driving laser pulses, $t \gg 1/\kappa$, the cavity dissipative relaxation will drive the system to a stationary state, in which all the modes c_n will be found in their vacuum states $|0_{c_n}\rangle$, while the modes d_{L_n} will be found in squeezed vacuum states. We then take the inverse unitary transformation and find that the system is in a pure state described by the density operator

$$\rho = T^{\dagger}S(\xi)\tilde{\rho}S^{\dagger}(\xi)T = |\psi_L\rangle\langle\psi_L| \otimes |0_{c_n}\rangle\langle 0_{c_n}|, \quad (18)$$

where $|\psi_L\rangle = T^{\dagger}S(\xi)|0_{d_{L_n}}\rangle$ is an example of the the pure linear-type CV quadripartite cluster state, and the variances of Eq. (11) tend to zero when $\xi \to \infty$.

There have already been many applications of linear cluster states. For example, a deterministically controlled-X operation has been designed with cluster states created in a linear optics scheme [26, 27]. The unconditional CV one-way quantum computation and a controlled-phase gate have also been demonstrated experimentally a linear optics scheme by use of a linear CV cluster state with four entangled optical modes [28, 29]. Following the scheme demonstrated experimentally by Furusawa *et al.* [28], the four-mode linear cluster state created in atomic ensembles may have the potential applications in quantum computation based on the atomic ensembles.

Although we have discussed here only of how to generate a linear four-mode cluster state, the proposed procedure can be easily extended to the case of different shapes of cluster state, such as T or square shapes. Moreover, the procedure can also be extended the case of N-mode cluster states involving an arbitrary number of atomic ensembles trapped in independent cavities connected by short fibres. By applying a set of lasers, we can construct N independent single-mode squeezed vacuum states in the combined bosonic representation similar to Eq. (12). Then, by the appropriate choice of the Rabi frequencies and the phases of the laser pulses, we can find that the cavity modes decay to the vacuum state whereas the collective atomic modes decay to squeezed vacuum states. The resulting stationary state of the collective modes would be in an N-mode cluster state.

In summary, we have proposed a procedure which generates four-mode cluster states in a fast single step of the preparation. We have shown that this could be done only by a proper driving of atomic ensembles composed of four-level atoms located in four distant cavities connected by an optical fibre. What is required is the simultaneous driving of the atomic ensembles with laser pulses of suitably chosen Rabi frequencies and phases. Moreover, the procedure can be easily applied to generate other type of cluster states, such as square or T-shape cluster states [7]. It could be easily extended to the generation of N-mode cluster states of N atomic ensembles only by a suitable change of the Rabi frequencies and phases of the driving lasers.

This work is supported by the National Natural Science Foundation of China (Grant No. 11074087), the Natural Science Foundation of Hubei Province (Grant No. 2010CDA075), the Nature Science Foundation of Wuhan City (Grant No. 201150530149), the National Basic Research Program of China (Grant No. 2012CB921602), and the Technology Creative Project of Excellent Middle & Young Team of Hubei Province(Grant No. T201204).

[1] H. J. Briegel, and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
[2] J. Zhang and S. L. Braunstein, Phys. Rev. A **73**, 032318 (2006).
[3] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[4] N. C. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. C. Ralph, and M. A. Nielsen, Phys. Rev. Lett. **97**, 110501 (2006).
[5] N. C. Menicucci, S. T. Flammia, and O. Pfister, Phys. Rev. Lett. **101**, 130501 (2008).
[6] X. Su, A. Tan, X. Jia, J. Zhang, C. Xie, and K. Peng, Phys. Rev. Lett. **98**, 070502 (2007).
[7] M. Yukawa, R. Ukai, P. van Loock, and A. Furusawa, Phys. Rev. A **78**, 012301 (2008).
[8] J. Song, Y. Xia, and H. S. Song, Appl. Phys. Lett. **96**, 071102 (2010).
[9] A. Rai, S. Das, and G. S. Agarwal, Opt. Express **18**, 6241 (2010).
[10] L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[11] C. W. Chou, H. de Riedmatten, D. Felinto, S. V. Polyakov, S. J. van Enk, and H. J. Kimble, Nature (London) **438**, 828 (2005).
[12] A. S. Parkins, E. Solano, and J. I. Cirac, Phys. Rev. Lett. **96**, 053602 (2006).
[13] K. Hammerer, A. S. Sørensen, and E. S. Polzik, Rev. Mod. Phys. **82**, 1041 (2010).
[14] T. Pellizzari, Phys. Rev. Lett. **79**, 5242 (1997).
[15] P. Peng and F. L. Li, Phys. Rev. A **75**, 062320 (2007).
[16] S. B. Zheng, Appl. Phys. Lett. **94**, 154101 (2009).
[17] S. B. Zheng, Z. B. Yang, and Y. Xia, Phys. Rev. A **81**, 015804 (2010).

[18] A. Serafini, S. Mancini, and S. Bose, Phys. Rev. Lett. **96**, 010503 (2006).
[19] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, Phys. Rev. Lett. **107**, 080503 (2011).
[20] G. X. Li, S. S. Ke, and Z. Ficek, Phys. Rev. A **79**, 033827 (2009).
[21] F. Dimer, B. Estienne, A. S. Parkins, and H. J. Carmichael, Phys. Rev. A **75**, 013804 (2007).
[22] F. O. Prado, N. G. de Almeida, M. H. Y. Moussa, and C. J. Villas-Bôas, Phys. Rev. A **73**, 043803 (2006).
[23] D. F. V. James and J. Jerke, Can. J. Phys. **85**, 625 (2007).
[24] J. Cho, D. G. Angelakis, and S. Bose, Phys. Rev. **78**, 022323 (2008).
[25] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).
[26] A. Tan, C. Xie, and K. Peng, Phys. Rev. A **79**, 042338 (2009).
[27] Y. Wang, X. Su, H. Shen, A. Tan, C. Xie, and K. Peng, Phys. Rev. A **81**, 022311 (2010).
[28] R. Ukai, N. Iwata, Y. Shimokawa, S. C. Armstrong, A. Politi, J. I. Yoshikawa, P. van Loock, and A. Furusawa, Phys. Rev. Lett. **106**, 240504 (2011).
[29] R. Ukai, S. Yokoyama, J. I. Yoshikawa, P. van Loock, and A. Furusawa, Phys. Rev. Lett. **107**, 250501 (2011).