Stability of higher-charged vortex solitons in defocusing Kerr media with an imprinted radial lattice

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Stability of higher-charged vortex solitons in defocusing Kerr media with an imprinted radial lattice

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Abstract

We demonstrate the existence and stability properties of fundamental and ring-profile vortex solitons in a defocusing Kerr medium with an imprinted radially symmetric lattice with a lower-index core covering several lattice rings. The decrease of energy flow with the growth of topological charge is explained using the law of conservation of energy. In contrast to the vortices in bulk media with competing nonlinearities, vortex solitons in radial lattices with defects are stable at lower or moderate energy flow. In particular, we reveal that vortex solitons with different charges share a substantially collective stability area. Higher-charged vortices at higher energy flow suffer a weak oscillatory instability, which allows them surviving very long propagation distances without visible distortions.

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I. INTRODUCTION

Vortices are fundamental objects which appear in many branches of physics [1]. In nonlinear optics, vortex solitons are associated with the phase dislocations (or phase singularities) carried by the nondiffracting optical beams [2], and share many common properties with the vortices observed in other systems, e.g., superfluids and Bose-Einstein condensates [3, 4]. Optical vortex solitons have been intensively studied in diverse schemes, including bulk [5–10], lattice-modulated [11, 12], nonlocal [13] media, and so on.

In bulk media, stable vortex solitons are known to exist in models with competing cubic-quintic or quadratic-cubic nonlinearities [7, 9]. Yet, the experimental realization of vortex solitons in such media is hard, as the requirement of very high energy flow of light usually excites other higher-order nonlinearities, which may be dominant and suppress the occurrence of competing nonlinearities. Successful alternatives are confined systems, such as graded-index optical fibers [14], nonlinear photonic crystals with defects [15], linear and nonlinear optical lattices [11, 16, 17], or optical lattices with defects [18], where the azimuthal instability of vortices can be suppressed by the corresponding confining potentials. Different types of vortex solitons, such as discrete vortices [19], vortex-ring "discrete" solitons [20], and second-band vortices [12] were observed in experiments.

Vortex solitons in harmonic, triangular, and hexagonal lattices exhibit discrete intensity distributions [11, 12, 20, 21]. The potentials induced by the periodic lattices may prevent or weaken the angular rotation of the vortex beams during propagation. Kartashov *et al.* suggested another novel type of optically induced lattice, i.e., radial Bessel lattice, which can be used to stabilize vortex solitons with continuous intensity distributions around the phase dislocations [16].

Defects and defect states exist in a variety of linear and nonlinear systems, including solid state physics, photonic crystals, and Bose-Einstein condensates. When lights propagate in an optical lattice with a local defect, the band-gap guidance results in the formation of linear or nonlinear defect modes [22, 23]. Recently, defect guiding phenomena of light in diverse settings, such as photonic crystals [15], fabricated waveguide arrays [24, 25], and optically induced photonic lattices [26–31], have been predicted theoretically and observed experimentally. Ye and his coworkers proposed that stable nonlinear modes can be trapped in a lower-index defect sandwiched between two optical lattices, or in the cylindrical core of a radial lattice [32]. The variation of defect scales, depths and shapes can be used to stabilize and reshape the fundamental, dipole and vortex solitons [18].

Thus far, vortex solitons and their stabilities in lattices with defects are still poorly understood. Dynamics of vortices in a radially symmetric lattice with a defect covering several lattice rings have not yet been explored. Main efforts in local or lattice-modulated nonlinear media were devoted to the analysis of vortex solitons with charges less than or equal to two. Stable localized ring-profile vortex solitons with charges higher than two were only predicted in defocusing media modulated by Bessel lattices [33] and mixed linear-nonlinear circular arrays [17]. Moreover, the stability domain of vortex solitons shrinks rapidly with the growth of topological charge with the only exceptions in mixed linear-nonlinear circular arrays [17] and in azimuthally modulated Bessel lattices [34]. We also should point out that, to realize the stable vortex solitons with different charges, one usually needs to adjust the lattice parameters and input beams simultaneously, which increases the experimental difficulty. Thus, the proposal of a simple model with fixed system parameters admitting stable vortex solitons with different charges is an important issue.

In this paper, we reveal that the defocusing media with an imprinted radially symmetric lattice with a lower-index defect covering several lattice rings can support stable vortex solitons with higher charges under appropriate conditions. In contrast to the cases in competing media, vortex solitons can propagate stably at lower or moderate energy flow. In lattices with fixed depth and defect scale, vortex solitons are completely stable provided that the propagation constant exceeds a critical value. In particular, we illustrate that the variation of topological charges slightly influences the existence and stability domains of vortex solitons.

II. MODEL

We consider light propagation along the z axis of a defocusing Kerr medium with an imprinted transverse modulation of the refractive index. Dynamics of the beam can be described by the nonlinear Schrödinger equation for the dimensionless complex field amplitude A:

$$i\frac{\partial A}{\partial z} = -\frac{1}{2}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) + |A|^2 A - pR(x, y)A. \tag{1}$$

Here, the longitudinal z and transverse x, y coordinates are scaled in the terms of diffraction length and beam width, respectively; p denotes the lattice depth; the refractive-index profile is given by $R(x, y) = \cos^2(\Omega r)$ for $r \geq (2N-1)\pi/(2\Omega)$ and $R(x, y) = 0$ otherwise, where $r = (x^2+y^2)^{1/2}$ is the radial distance, Ω is the frequency, and $N = 1, 2...$ is the number of rings removed from the lattice and characterizes the defect scale. Thus, the transverse modulation of refractive index features a lower-index guiding core. By comparing the defocusing bulk media without external potentials, the radial lattices with defects can confine the beams in a local region. An example of such refractive-index landscapes is shown in Fig. 1(a).

Although there are defects in radial lattices, the wings of nonlinear modes still penetrate into the bulk of lattices. Thus, the existence of nonlinear modes strongly depends on the transverse lattices. Since the term $1/rd/dr$ in Laplacian can be neglected at $r \to \infty$, the band-gap structure of a radially symmetric lattice is slightly different from that of 1D periodic lattice [32]. Thus, it is convenient to use the band-gap structure of 1D periodic lattice to approximately analyze the existence of solitons. Due to the fact that nonlinear modes in defocusing Kerr media can only be found in the finite gaps, we are interested in the solitons residing in the first finite gap. Equation (1) conserves several quantities, including the energy flow U and the Hamiltonian H:

$$\begin{aligned} U &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(x,y)|^2 dxdy \\ H &= \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\left|\frac{\partial A}{\partial x}\right|^2 + \left|\frac{\partial A}{\partial y}\right|^2 - 2pR|A|^2 + |A|^4 \right) dxdy. \end{aligned} \tag{2}$$

We search for stationary solutions of Eq. (1) by assuming $A(x, y, z) = q(r)\exp(ibz+im\phi)$, where q is a r-dependent real function depicting the profile of stationary solution, b is a propagation constant associating with the energy flow, and m is an integer known as the topological charge of vortex soliton. The nonlinear mode degenerates to a fundamental radially symmetric mode when $m = 0$. The substitution of the light field into Eq. (1) yields:

$$\frac{d^2q}{dr^2} + \frac{1}{r}\frac{dq}{dr} - \frac{m^2}{r^2}q - 2bq - 2q^3 + 2pRq = 0, \tag{3}$$

which can be solved numerically by means of a Newton iterative method. Mathematically, various families of stationary solutions are determined by the propagation constant b, lattice depth p, modulation frequency Ω and defect scale N. We vary b, p, N and fix $\Omega \equiv 2$ in following discussions.

The stability of solitons can be analyzed by considering the perturbed solution in the form of: $A(x, y, z) = [q(r) + u(r)\exp(\lambda z + in\phi) + v^*(r)\exp(\lambda^* z - in\phi)]\exp(ibz + im\phi)$, here the perturbation components u, v could grow with a complex rate λ during propagation, and n is an integer representing the angle dependence of the perturbation and is termed as an azimuthal index. The substitution of the perturbed solution into Eq. (1) results in a system of eigenvalue equations:

$$\begin{aligned} i\lambda u &= -\frac{1}{2}(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{(m+n)^2}{r^2})u + bu + (v + 2u)q^2 - pRu \\ -i\lambda v &= -\frac{1}{2}(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{(m-n)^2}{r^2})v + bv + (u + 2v)q^2 - pRv. \end{aligned} \quad (4)$$

The coupled equations can be solved by a finite-difference method. In Cartesian coordinates, the square of the above linearization operator is self-adjoint if the stationary solutions are angle independent (fundamental solitons). Thus, the discrete eigenvalue is either purely real or purely imaginary. The instability growth rates with purely real parts correspond to the Vakhitov-Kolokolov (V-K) instability [35]. When the stationary solutions are angle dependent (vortex solitons), the eigenvalues may have both real and imaginary parts associating with an oscillatory instability. Stationary solutions are completely stable provided that all real parts of eigenvalues equal zero.

III. NUMERICAL RESULTS

Firstly, we discuss the properties of fundamental radially symmetric solitons residing in the first band gap of the undefected lattice. For nonlinear modes with $m = 0$, it follows from Eq. (3) that $q(r \to 0) = \sqrt{-b}$, which means that the solitons can be found only at $b < 0$. In deep lattice, the energy flow is a monotonically decreasing function of the propagation constant. When the lattice is shallow, there exists a narrow region close to the upper cutoffs of propagation constant, where the energy flow dependence changes its slope, i.e., $dU/db > 0$ [Fig. 1(b)]. Such solitons feature a plateau inside the defect region and pronounced decaying oscillations in the bulk of lattice [Figs. 1(c) and 1(d)]. According to the relation $q(r \to 0) = \sqrt{-b}$, the height of the plateau decreases with the growth of propagation constant. Similar behavior occurs for the decaying swings in the bulk of lattice. By comparing the profiles of solitons in lattices with defect scales $N = 3$ and $N = 10$, it is easily found that the width of plateau is solely determined by the defect scale. This property

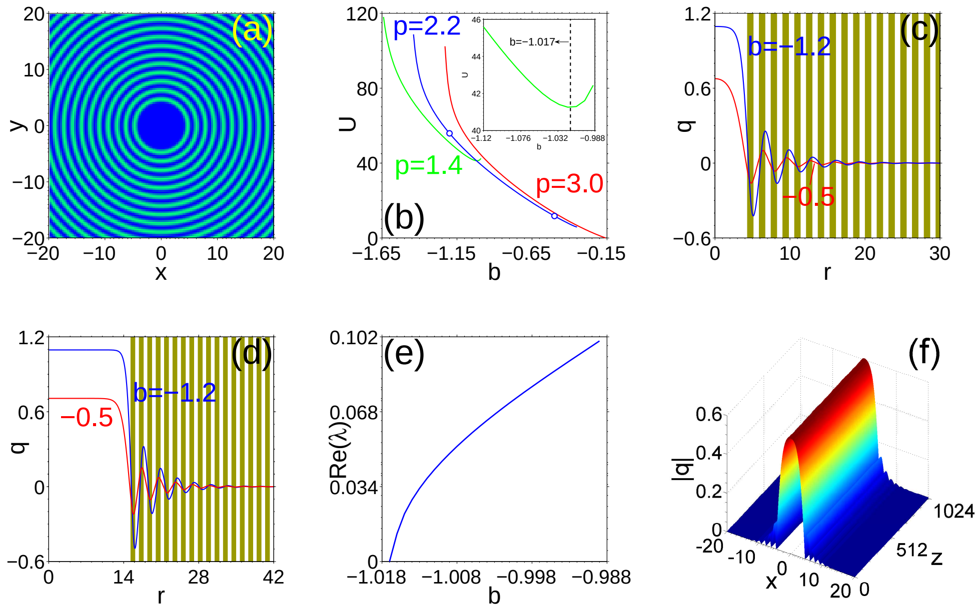

FIG. 1. (Color online) (a) Radial lattice with defect. (b) Energy flow of fundamental solitons vs b. The inset zooms in on the graph near the upper cutoff with $p = 1.4$. (c, d) Profiles of solitons at $p = 2.2$, $N = 3$ and 10, respectively. (e) Perturbation growth rate vs b at $p = 1.4$. (f) Stable propagation of soliton at $p = 2.2, b = -0.5$. Cut of intensity distribution at $y = 0$ is shown. Bar areas in (c, d) correspond to $R(x) \geq 1/2$. $N = 3$ except for (d). All quantities are plotted in dimensionless units.

allows one to realize flat-topped solitons with different core sizes by changing the defect scale N. Linear-stability analysis on the stationary solutions reveals that fundamental solitons in shallow lattices are stable in a substantial part of their existence domain, except for a narrow region ($dU/db > 0$) near the lower cutoffs of propagation constant, where V-K instability may take place [Fig. 1(e)]. The V-K instability vanishes and the solitons are completely stable in their whole existence domain when the lattices are deeply modulated. A stable propagation example is shown in Fig. 1(f).

Now, we consider vortex solitons with unit charge supported by the defocusing Kerr media with an imprinted radial lattice with a defect. Without loss of generality, we set the defect scale $N = 10$ in the following discussions. In contrast to the fundamental solitons, the energy flow U of vortices is always a monotonically decreasing function of propagation constant b [Fig. 2(a)]. Vortex soliton only exists when the lattice depth exceeds a critical value. For example, as shown in Fig. 2(b), the threshold value of lattice depth for the appearances of

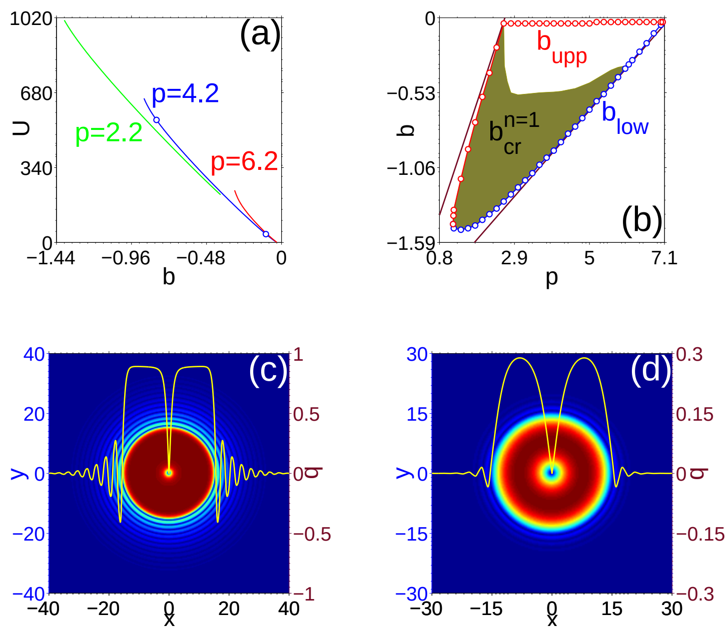

FIG. 2. (Color online) (a) Energy flow of vortex solitons with $m = 1$ vs b for different p. (b) Areas of existence and instability (patched) on the (p, b) plane. Solid lines denote the edges of the first gap of 1D periodic lattice. (c, d) Contour and amplitude profiles of vortices at $p = 4.2, b = -0.8, U = 556.2$ and $p = 4.2, b = -0.1, U = 37.9$, respectively. In all cases $N = 10$. All quantities are plotted in dimensionless units.

vortices with unit charge is $p_{th} \approx 1.18$, below which no localized vortex solutions can be found. For $p \leq 2.57$, the existence domain expands with the lattice depth. It shrinks with the growth of lattice depth if $p \in (2.57, 7.05]$, due to the restriction of the ascending lower edge of the first band gap. It is the restriction of the first gap which accounts for the hoofed existence domain [Fig. 2(b)].

Figures 2(c) and 2(d) display two typical profiles of vortex solitons with unit charge at different energy flow. The vortex at higher energy flow looks like a flat-topped beam embedded with a dark core at which the amplitude is zero and the phase is undefined. Vortices become more localized with the growth of propagation constant. Vortices at higher energy flow penetrate deeply into the bulk of lattice, which leads to the multi-ring structures of beam intensity distributions. By comparing with the dark vortex solitons (with non-vanishing amplitudes at infinity) in uniform defocusing media, the vortices in our model are localized. In other words, the radial lattice with a defect plays a role of confining the vortex into a local region.

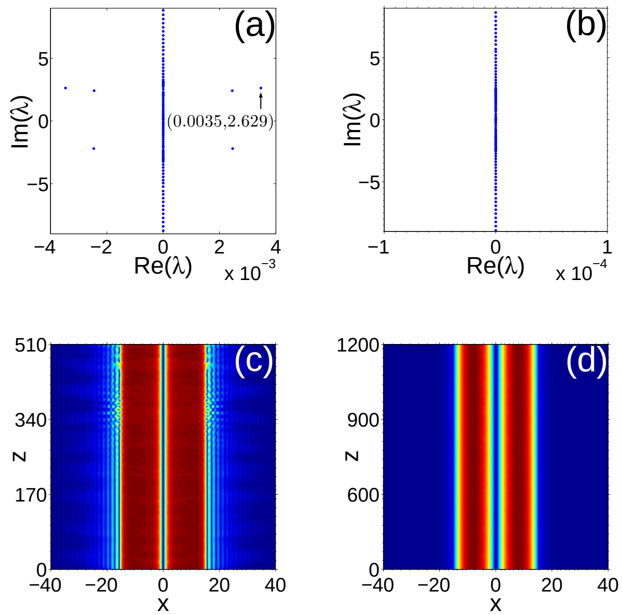

FIG. 3. (Color online) Spectra of the linearization operator (a, b) and unstable and stable propagations (c, d) of vortex solitons shown in Figs. 2(c) and 2(d). All quantities are plotted in dimensionless units.

To examine the stability of vortex solitons with unit charge, we conduct linear-stability analysis on the stationary solutions according to Eqs. (4). Typical spectra of the linearization operator for vortex solitons at $b = -0.8$ and $b = -0.1$ in lattice with defect scale $N = 10$ at $p = 4.2$ are shown in Figs. 3(a) and 3(b), respectively. Vortices at higher energy flow suffer an oscillatory instability with complex growth rates [Re(λ) $\ll$Im(λ)], while vortices at lower or moderate energy flow are completely stable [Re(λ) = 0]. To confirm the stability analysis results, we numerically integrate Eq. (1) with a standard beam propagation method code, using the stationary solutions as the initial inputs. Representative unstable and stable propagation examples are illustrated in Figs. 3(c) and 3(d). Obviously, unstable vortex solitons can survive large distances (hundreds of diffraction lengths), greatly exceeding the present experimentally feasible sample lengths.

We summarize the linear-stability analysis results in Fig. 2(b). We show the critical value of propagation constant $b_{cr}^{n=1}$ above which no perturbations with the azimuthal index n and nonzero real part of growth rate were found. Vortex solitons are dynamically stable in a broad region near the upper cutoffs of propagation constant. It is the combination of defocusing nonlinearity and confining potential who affords the stability of vortex solitons. The precise structure of instability regions (patched) is rather complicated. There may exist multiple narrow stability windows.

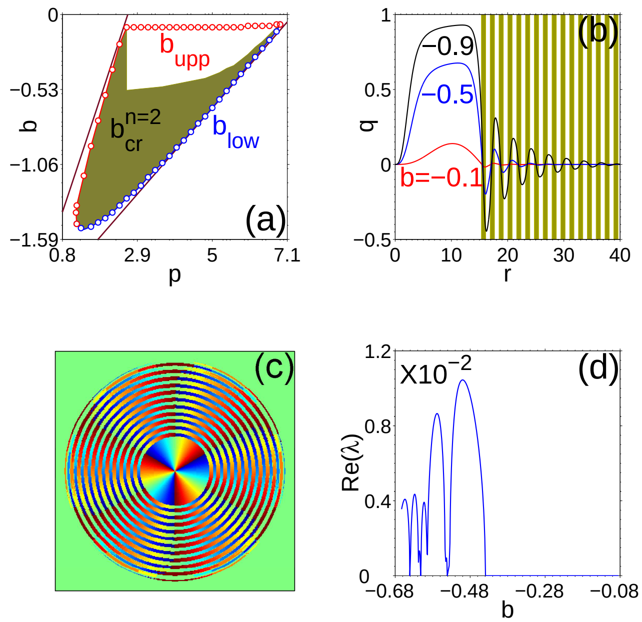

FIG. 4. (Color online) (a) Areas of existence and instability (patched) of vortex solitons with $m = 3$ on the (p, b) plane. Solid lines denote the edges of the first gap of 1D periodic lattice. (b) Profiles of vortices at $p = 4.2$. (c) Phase distribution at $b = -0.5, p = 4.2$. (d) Real part of instability growth rate associating with $n = 2$ vs b at $p = 5$. In all cases $N = 10$. All quantities are plotted in dimensionless units.

Next, we focus on the vortex solitons with higher topological charges in a radially lattice with a defect imprinted in a defocusing Kerr medium. Figure 4(a) shows the hoofed existence domain of vortex solitons with $m = 3$. Vortex solitons can be found in lattices with $p \in [1.18, 6.97]$. By comparing the existence domains of vortex solitons with $m = 1$ [Fig. 2(b)] and $m = 3$, one can find that the upper cutoff of propagation constant drops from ~ -0.04 to ~ -0.08, which leads to the decrease of the upper threshold value of lattice depth p (from 7.05 to 6.97) and thus the shrinkage of the existence domain. Yet, the existence area of vortex solitons with $m = 3$ still occupies almost the whole of the first gap of undefected lattice (for $b < 0$).

The energy flow of vortex solitons with $m = 3$ also decreases with the propagation constant. The pronounced decaying oscillations of such modes in the bulk of lattice become stronger with the decrease of propagation constant [Fig. 4(b)]. The maxima of intensity distribution around the phase dislocation move towards the center of the defect core with the growth of propagation constant. Figure 4(c) displays an example of screw-type phase distribution of vortex soliton with $m = 3$.

The instability of vortex solitons with higher charges usually depends on the azimuthal index n [10, 16, 33]. Linear-stability analysis results reveal that for vortices with $m = 3$, the instability area associating with $n = 2$ is always dominant. For vortex solitons with $m = 3$ in a lattice with $p = 5$, the widths of instability windows associating with azimuthal indices $n = 1, 2$ and 3 occupy $\approx$ 26.77%, $\approx$ 38.69% and $\approx$ 16.93% of the width of the whole existence domain, respectively. An example of instability growth rate corresponding to azimuthal index $n = 2$ versus propagation constant is illustrated in Fig. 4(d). It indicates that vortex solitons suffer a weak azimuthal instability, which allows them to propagate without obvious shape distortion over large propagation distances. Vortex soliton will be completely stable provided that its propagation constant exceeds a critical value.

By comparing the stability areas of vortex solitons with $m = 1$ [Fig. 2(b)] and $m = 3$ [Fig. 4(a)], one finds that the stability area of vortices with $m = 3$ is slightly narrower than that of vortices with $m = 1$, which constitutes one of our central results. That is to say, the stability area is slightly affected by the growth of topological charge, which allows one to realize stable vortex solitons with even higher charges. Since vortices with different charges share a collective stability area, one can input beams with different charges to excite vortex solitons with corresponding charges in certain parameter windows without changing the lattice depth, defect scale, modulation frequency etc. It should be noted again that vortex solitons with different charges can propagate stably at lower or moderate energy flow, which is in sharp contrast to the cases in competing media, where very high energy flow is needed to stabilize the vortices [7, 9]. Thus, in addition to the Bessel lattice [16], the radial lattice with defect is another effective alternative for the realization of stable vortex solitons at lower or moderate energy flow, especially for vortices with higher charges.

To confirm the above conclusions, we investigate the dynamics of vortices with $m = 4, 5...10$. The existence and stability domains shrink slowly with the topological charge due to the slow decrease of the upper cutoff of propagation constant. The energy flow decreases with the growth of topological charge when the lattice parameters are fixed [Fig. 5(a)]. This can be explained by the law of conservation of energy. For linearly polarized vortex beam, the

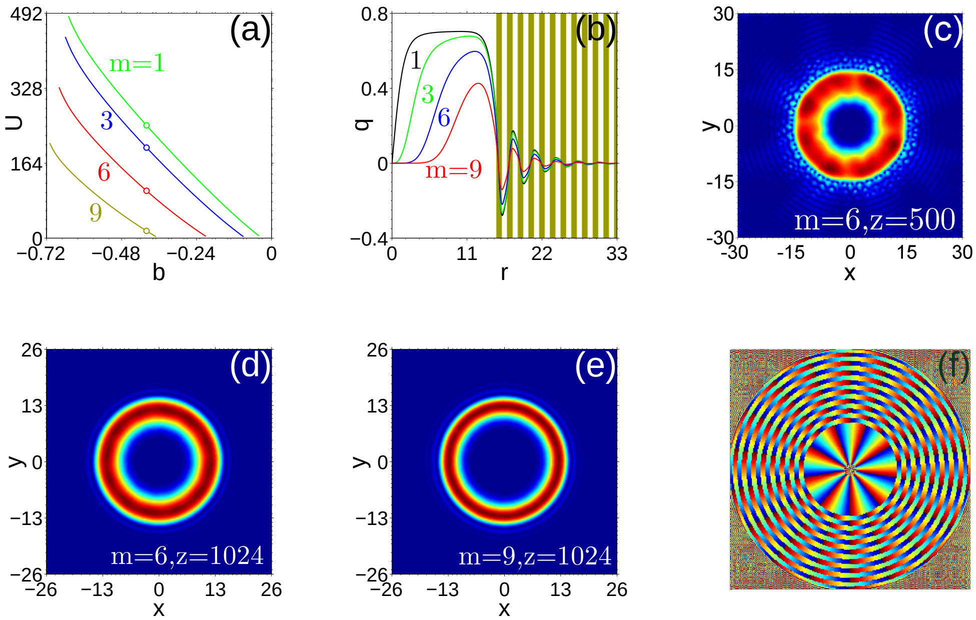

FIG. 5. (Color online) (a) Energy flow of vortex solitons with different m vs b. (b) Profiles of vortex solitons with different m at $b = -0.4$. (c, d) Unstable and stable propagations of vortex solitons with $m = 6$ at $b = -0.67$ and $b = -0.4$, respectively. (e, f) Field modulus and phase distribution of vortex soliton with $m = 9$ at $b = -0.4, z = 1024$. In all panels, $p = 5$. All quantities are plotted in dimensionless units.

total energy includes two parts. The first part is the energy carried by the photons, and the second part is the rotational energy associating with the orbital angular momentum. From the definition of the energy flow of vortex solitons [Eqs. (2)], one finds that the expression of energy flow only defines the energy carried by the photons. Yet, the rotational energy of vortex solitons is proportion to the square of topological charge and effectively rotational radius. Given that the allowed energy of different modes in a fixed system is a constant, the energy flow carried by photons decreases with the increase of rotational energy. The conclusion may be generalized to vortex solitons with continuous intensity distributions in other models.

The above discussions can also explain the decrease of the thickness of vortex solitons shown in Fig. 5(b). With the growth of topological charge, the decrease of effective mass of the beam is in companion with the increase of effectively rotational radius and angular velocity. For fixed propagation constant, the delocalization of vortex soliton weakens with the growth of topological charge. A representative propagation example of unstable vortex solitons is illustrated in Fig. 5(c). The vortex can propagate without visible shape distortion over hundreds of diffraction lengths. Figures 5(d) and 5(e) show two instances of stable propagations of vortex solitons at $b = -0.4$ with topological charges $m = 6$ and 9, respectively. The phase structure of vortex soliton at $z = 1024$ with $m = 9$ is displayed in Fig. 5(f).

We also solve the vortex solutions with a Neumann boundary condition (rather than zero boundary condition) and found there exist vortex solutions with non-vanishing amplitudes when the lattice depth is small. It indicates that there exists a transition between the delocalized and localized vortices. That is to say, the localized vortex soliton is a continuum of the delocalized dark vortex in the vanishing lattice case, and it belongs to the family of dark solitons. The discussion of dynamics of dark vortex solitons with non-vanishing amplitudes is beyond the scope of this paper.

By comparing with the conventional dark vortex solitons in bulk defocusing media, the higher-charged dark vortex solitons in our model exhibit some new features, though they can bifurcate from the conventional dark vortex solitons. First, due to the confining effect of the optical potential, the dark vortices are localized while the amplitudes of conventional dark vortex solitons do not vanish at infinity. Second, the unstable vortex soliton in this paper suffers an oscillatory instability with complex growth rates, which is different from the conventional dark vortex solitons, which usually break up into several single-charged vortices built in an infinite ground. They are also different from the unstable bright vortices in focusing media, where vortex solitons may break up into several fragments associating with the topological charge of vortices [10]. Third, the properties of vortex solitons in the present model can be controlled by adjusting the lattice or defect parameters, which may be useful in their practical applications.

Finally, we briefly discuss the influence of lattice parameters on the existence of vortex solitons. Localized vortex solutions cannot be found in radial lattices without defects. The existence domain expands with the growth of defect scale and approaches an ultimate at $N = 4$. It shrinks with the increase of modulation frequency Ω. The existence domain shrinks slowly with the topological charge if other parameters are fixed. No matter what topological charge or lattice parameters are, the stable area always occupies a region near the upper cutoffs of propagation constants. We stress that although the vortex solitons residing in the patched areas shown in Figs. 2(b) and 4(a) are unstable, they can survive large propagation distances. Unstable vortex solitons with higher charges exhibit a similar behavior. Thus, we expect that all vortices in radial lattices with defects can be observed in experiments.

IV. CONCLUSIONS

In conclusion, we addressed the dynamics of fundamental and vortex solitons in defocusing kerr media with an imprinted radial lattice featuring a lower-index defect covering several lattice rings. The defect scale can be utilized to control the energy flow of both types of solitons. Vortex solitons with various charges are stable in a region near the upper cutoffs of propagation constant. Although higher-charged vortices at higher energy flow suffer oscillatory instability, they can survive very long distances without visible distortions. Vortex solitons at lower or moderate energy flow are completely stable under appropriate conditions. Especially, we revealed that the variation of topological charges slightly influences the existence and stability domains of vortex solitons.

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