Designs of biased Stark guides for polar molecules

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Overcoming Meek's Theorem: Design of biased Stark guides for polar molecules

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Abstract

Here we consider three possible biased electrostatic guides of polar molecules. The design of these guides is motivated by their possible uses in a precision measurement of the electron's electric dipole moment. These guides may also have applications in the alignment-preserving transportation of ultracold molecules.

I. INTRODUCTION

Experimentalists have been using static electric fields to manipulate neutral polar molecules for well over half a century[1–15]. Early work was motivated by a desire to spatially focus a single (or a few) quantum states of a molecule in order to carry out state-selected spectroscopic, photodissociation, and reaction dynamics studies[1–3, 7, 8, 12]. More recent work has been motivated by a desire to produce a steady state source of cold molecules and more specifically, to select slowly moving particles from a thermal effusive source[4–6, 8–11, 13]. These efforts have produced sources of ND_3[4, 8, 10, 11], NO[5], H_2O and its isotopologs[9], and CH_3CN[13]. Typical translational temperatures at the output of these electrostatic skimmers range from 0.5K to 10K.

This work is motivated by a new application of the Stark guide: Namely precision measurement searches for the electric dipole moment of the electron (e-EDM). The importance of the e-EDM to fundamental theories of Physics and, in particular, the matter-antimatter asymmetry of the Universe has been reviewed elsewhere[16, 17]. The e-EDM experiment we consider here is similar to many studies currently underway[18–22]: A heavy paramagnetic molecule is initially prepared in a coherent superposition of two states and allowed to evolve in an electric field. These two states differ only by the sign of the projection M of the molecule's total angular momentum on the electric field axis. For the case of zero magnetic field and a uniform electric field, one expects these two states to be degenerate. If the electron possesses an e-EDM, this degeneracy is lifted. The strategy for searching for an e-EDM is to attempt to measure this energy difference. For a typical e-EDM sensitive molecule, this energy difference ranges from roughly 10 mHz (for an e-EDM near the current limit of $1.2 \times 10^{-27} e{\cdot}$cm[18]) to 0.1 pHz (for an e-EDM at the $10^{-38} e{\cdot}$cm prediction of the Standard Model[23].) Whereas the Standard Model's 0.1 pHz measurement is out of the range of current experiments, alternative models predict a large e-EDM comparable to the current limit. This fact makes the current generation of molecular e-EDM measurements important searches for Physics beyond the Standard Model.

The Stark-guided e-EDM experiments envisioned here are shown schematically in Fig. 1. In brief, a beam of polar molecules is polarized in a region of uniform electric field using optical and/or microwave radiation. This polarization process creates a coherent superposition of states that differ only by the sign of the projection M of total angular momentum of

the molecule onto an electric field axis. The molecules are then allowed to evolve for a time τ in the electric field of a long Stark guide. This guide is biased to have a minimum value of electric field that is strong enough to align the dipole moment of the molecule with respect to the electric field. After polarization and guiding, the molecules enter a second region of uniform electric field and are probed with optical/microwave radiation to determine the accumulated quantum phase between the two states. For a properly designed experiment, reversal of the electric field in the system will result in a difference in phase that is proportional to the e-EDM. In the language of beam resonance experiments, the biased Stark guide becomes the Ramsey cavity. By guiding the particles tens or even hundreds of meters, the coherence time of the experiment (and hence the sensitivity to an e-EDM) can be enhanced by several orders of magnitude.

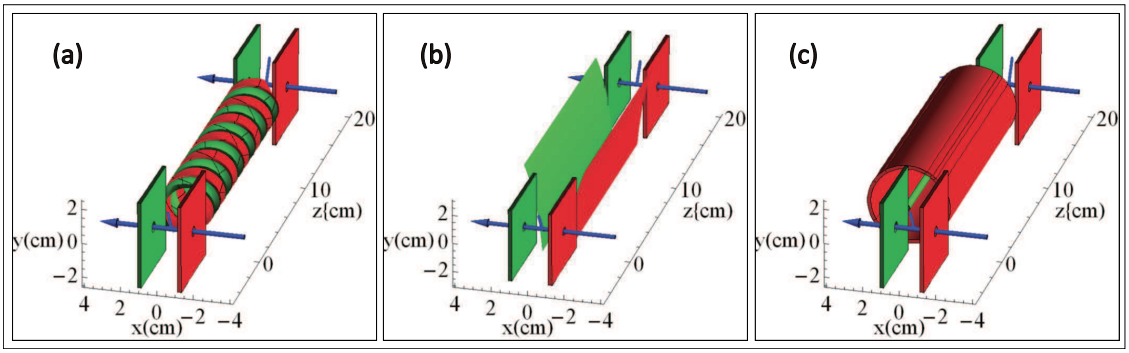

FIG. 1. (Color online) Two uniform field regions connected by various guides. In the envisioned experiment, a beam of molecules is polarized by laser radiation, guided by the field within the electrodes, and then probed with laser radiation. The guides shown are only 15 cm long, but guides exceeding 100 meters may be considered.

Complementary to previous experimental studies of Stark guides there are several theoretical studies that investigate the focussing and guiding properties of various two-dimensional Stark potentials[11, 24–29]. Each of these two-dimensional potentials have a minimum in the magnitude of electric field at a location in space where the electric field vanishes. This fact is not by any special design. Rather it is a result of a theorem conceived and proven by Samuel Meek[30]. This theorem starts with Maxwell's equations in a vacuum to prove that, *all extrema in the magnitude of a two-dimensional electric field occur at zero electric field.* (It is interesting to note that this theory does not extend to three dimensions where several biased trap configurations have been demonstrated[31].) This presents a problem for both

the e-EDM measurement we envision and for any application of a Stark guide for which one wishes to guide polar molecules while conserving the M state alignment of the system. In this paper we present three possible ways one might still guide a beam of molecules over long distances in a biased electrostatic guide.

II. CASE 1: THE HELICAL GUIDE.

As stated in the introduction, we are interested in creating Stark guide of molecules for which the minimum electric field magnitude is non-zero. For this reason, a purely two dimensional guide, such as a modified hexapole or quadrupole guide is inappropriate: By Meek's theorem, the minimum electric field magnitude in such a guide will occur at a point in space for which $E = 0$.

The first possible solution to this problem we present in this section is a *helical Stark guide* that employs curved plates spiraling around a central guide region in order to guide molecules in the z direction (Fig. 1a). When opposite voltages $\pm V$ are applied to the two electrodes of the guide, the potential

$$\Phi = -E_o \frac{2I_1(kr)}{k} \cos(kz - \phi) \tag{1}$$

is created. Here I_1 is the modified Bessel function of the first kind and $k = 2\pi/\lambda$ is a parameter that determines the pitch of the helix. Along axis of this guide, the electric field is of constant magnitude E_o and rotates with the z dimension:

$$\vec{E}(\vec{r} = \vec{0}) = E_o(\cos kz\hat{\imath} + \sin kz\hat{\jmath}) \tag{2}$$

The electric field magnitude E throughout the guide can be shown to be given by

$$E = E_o\sqrt{[1 + (kr)^2 f_p(kr)]^2 \cos^2(kz - \phi) + [1 + (kr)^2][1 + (kr)^2 f_a(kr)]^2 \sin^2(kz - \phi)}. \tag{3}$$

Here we have defined the two even functions $f_a(\alpha)$ and $f_p(\alpha)$:

$$f_a(\alpha) = \frac{2}{\alpha^3} I_1(\alpha) - \frac{1}{\alpha^2} = \sum_{n=0}^{\infty} \frac{1}{4(n+1)!(n+2)!} (\frac{\alpha}{2})^{2n} \tag{4}$$

$$f_p(\alpha) = \frac{2}{\alpha^2} \frac{dI_1(\alpha)}{d\alpha} - \frac{1}{\alpha^2} = \sum_{n=0}^{\infty} \frac{2n+3}{4(n+1)!(n+2)!} (\frac{\alpha}{2})^{2n} \tag{5}$$

We note that $f_a(0) = 1/8$, $f_p(0) = 3/8$, and both functions increase rapidly as α goes from zero to positive infinity. Thus the minimum field magnitude everywhere in the twisted guide is E_o, with the field rapidly increasing as a function of distance from the center of the guide.

The electrodes of the helical Stark guide shown in Fig. 1a are equipotential surfaces of the potential given by Eq. 1. The topology of these surfaces suggests that the guide could easily be made from two intertwined wires twisted with a pitch-length $\lambda = 2\pi/k$ to inner-diameter d ratio determined by the voltage $\pm V_o$ on the electrodes:

$$\frac{1}{\pi} I_1(\pi \frac{d}{\lambda}) = \frac{V_o}{\lambda E_o}. \tag{6}$$

A numerical solution to this transcendental equation is given in Fig. 2.

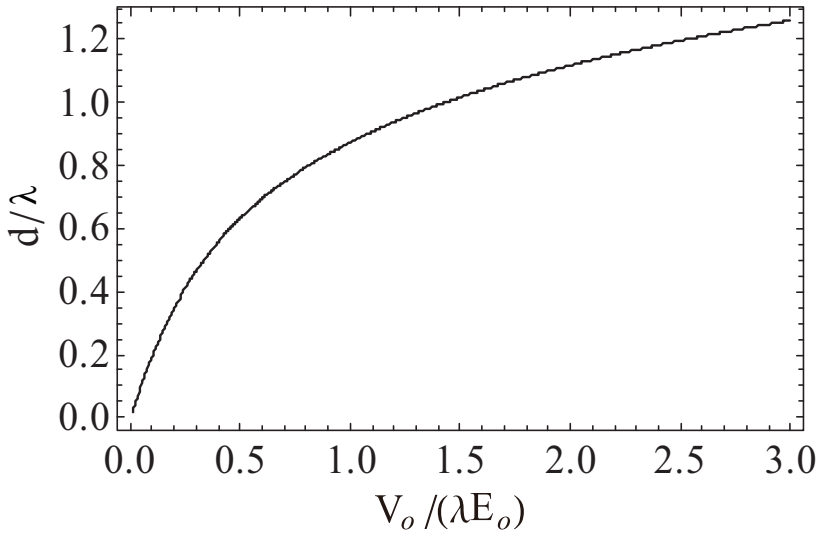

FIG. 2. Relationship between the d/λ and $V_o/(\lambda E_o)$ for the helical guide. Here voltages $\pm V_o$ are placed on the electrodes, d is the inner diameter of the guide, $\lambda = 2\pi/k$ is the pitch length, and E_o is the bias field of the guide.

To test the performance of the helical guide, we performed classical trajectory calculations. Here we assume a force given by

$$\vec{F} = -\frac{\partial U(E)}{\partial E} \vec{\nabla} E + m\vec{g}, \tag{7}$$

where $U(E)$ is the Stark potential of the guided state, m the mass of the particle, and $\vec{g}$ the acceleration due to gravity. For illustrative purposes we assume $U(E)$ is that of the $J = 1/2$, $F = 1, |M| = 1$, Ω_+ rotational state of the $X_1(v = 0)$ ground state of the e-EDM sensitive ^{208}Pb^{19}F molecule. The Stark energy is determined by diagonalizing a spin-rotational Hamiltonian determined from detailed microwave and optical spectroscopy[32].

We have carried out this analysis and fit this potential to a quadratic polynomial with the result

$$U(E) = \mu\text{eV}[23.056(\frac{E}{10\text{kv/cm}})^2 - 31.358(\frac{E}{10\text{ kv/cm}})^3 + 10.004(\frac{E}{10\text{kv/cm}})^4], \qquad (8)$$

where the fit is valid in the range $0 < E < 10$ kV/cm. A plot of this potential is given in Fig. 3.

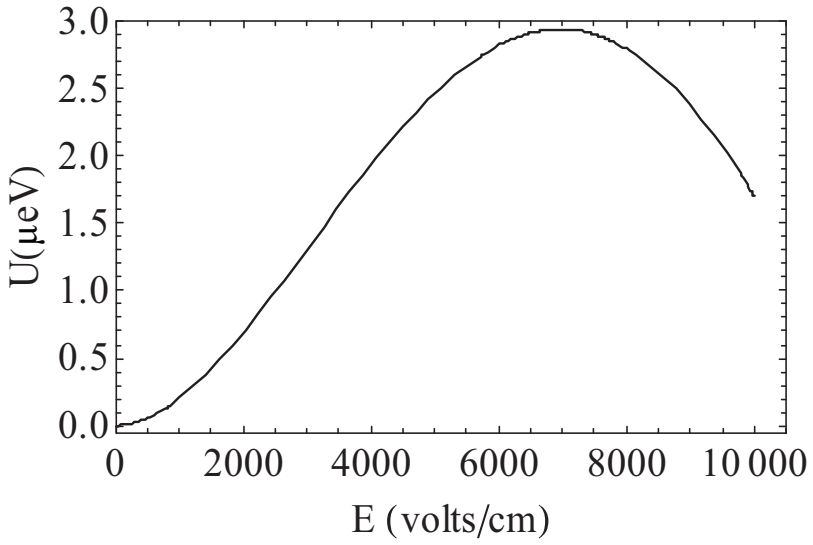

FIG. 3. Stark energy of the low-field seeking $X_1\ ^2\Pi_{1/2}(v = 0,\ F = 1,\ |M| = 1,\ J = 1/2,\ \Omega_+)$ state of $^{208}Pb^{19}F$.

Fig. 4a gives the transverse $(x-y)$ trajectory of a typical molecule with the Stark potential of Eq. 8 as it travels a distance $z = 10,000$ cm down a helical guide with $\lambda = 2\pi/k = 2.5$ cm. This electric field is chosen in such a way that it won't extend to the high field range to become high-field seeking, and it also needs to be large enough to fully polarize the PbF molecules. From Fig. 3, we can therefore choose a moderate field $E_o = 5$ kv/cm, which is subject to the experimental test. The initial conditions of this trajectory are given by $\vec{r} = (0.10\ \hat{\imath} - 0.09\ \hat{\jmath} + 0.00\ \hat{k})$ cm and $\vec{v} = (-6.5\ \hat{\imath} - 46.2\ \hat{\jmath} + 20,000\ \hat{k})$ cm/s. It is notable that the molecules are guided 10,000 cm (4000 twists) without being lost. It is not immediately obvious that this three dimensional guide will lead to stable trajectories as the energy in axial motion could, in principle, be coupled to the transverse motion. For the guide shown and a beam velocity of 200 m/s, particles with transverse-kinetic plus potential energy less than or equal to 85% of the barrier height of 2.94 μeV were guided the entire 100 m length of the guide. This corresponds to an acceptance of approximately 2×10^{-5} steradian.

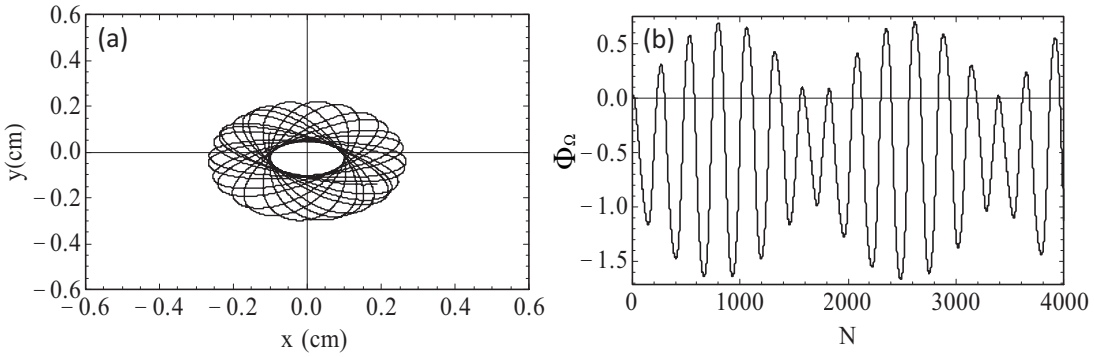

FIG. 4. (a) Simulated x-y trajectory of a particle as it moves $N = 4000 = 100$ m through the helical guide described in the text. (b) Accumulated geometric phase after a particle with the initial conditions described in the text travels from a uniform field region, through a helical guide of Nturns, and exits into a second region of uniform field region.

Although the helical guide is effective at guiding polar molecules over long distances, it is not likely to be suited to an e-EDM measurement. Whereas the helical guide will conserve the $|M|$ state population, it can not be expected to conserve any phase coherence between two states that differ only by the sign of M. This is because, as the electric field direction is allowed to vary in three dimensions, a geometric phase ϕ_Ω is accumulated which leads to rapid decoherence of the molecules in the beam. To quantify this geometric phase effect, we imagine an experiment in which molecules are polarized in a uniform electric field, travel the length of a helical guide, and finally probed with laser radiation (Fig. 1). The expected rate of molecular detection is expected to be given by

$$\Gamma = \Gamma_N \left(\frac{2-c}{2} - \frac{c}{2} \cos(\phi_{EDM} + \phi_B + \phi_L + \phi_\Omega) \right). \tag{9}$$

Here Γ_N is the rate of detection of an unpolarized beam of molecules and c is an experimentally determined contrast with $0 < c < 1$. The angle $\phi_{EDM} = 2p_e E_{eff}\tau/\hbar$ is the effect of the e-EDM with p_e the electric dipole moment of the electron, τ the time of flight of the molecule as it travels from the polarization to probe region, and E_{eff} the effective internal field of the molecule. For heavy diatomic radicals, E_{eff} is of the order of 10-100 GV/cm[33]. The angle $\phi_B = 2g\mu_B\tau/\hbar$ is the contribution of background magnetic fields. For the purpose of this discussion, we assume that magnetic fields are shielded so ϕ_B is insignificantly small. The angle ϕ_Ω is due to the geometric phase effect. Lastly, ϕ_L is the angle between the initial and final laser polarization and is modulated to gain sensitivity to $\phi_{EDM} + \phi_\Omega$. Final de-

termination of the e-EDM would be determined by reversal of the electric field which would change the sign of ϕ_{EDM}, but not ϕ_Ω.

In the adiabatic approximation, the phase ϕ_Ω may predicted by performing the integration

$$\phi_\Omega = \int -\frac{2|M|}{e^2}\left(\frac{d\vec{e}}{dt}\times\vec{e}\right)\cdot\hat{z}\;dt \tag{10}$$

over the trajectory[34]. Here $\vec{e} = \hat{E} + \hat{z}$ where $\hat{E}$ is the direction of the electric field and $\hat{z}$ is the direction of a laboratory-fixed quantization axis, which we take to be along the axis of the guide. A plot of ϕ_Ω modulus 2π verses N is given for the trajectory of Fig. 4b. Here N is the number of twists of the helical guide and the total guide length is (2.5 cm) $\times$ N. The large-amplitude oscillations in this phase as a function of distance down the guide is very sensitive to the initial conditions of the trajectory and hence very difficult to control. For this reason, the almost random final geometric phase of each molecule will to lead to rapid decoherence of the experimental measurement and, as a result, loss of sensitivity to the e-EDM.

Although the helical guide is most likely not of use to an e-EDM experiment, it might be of use in other applications which require the transport of molecules from one region of space to another without loss due to mixing of $|M|$ states. We also note that the combined rotating radial field and oscillating axial field seen by a polar molecule is similar to the field seen by a trapped ion in an envisioned e-EDM experiment. However, in the case of this ion trap experiment, the axial field is orders of magnitude smaller than the radial field and, as a result, the geometric phase is not a major concern[22].

III. CASE 2: THE STARK GRAVITATIONAL GUIDE

Here we show that Meek's theorem can be overcome by creating a *Stark gravitational guide* that uses two slightly distorted field plates (see Fig. 1b). This distortion leads to an increasing Stark energy when a polar molecule moves in the down ($-\hat{y}$), left ($-\hat{x}$) or right ($+\hat{x}$) direction. The fact that the Stark energy decreases as the molecule moves in the up ($+\hat{y}$) direction, assures compliance with Meek's theorem. However, this decrease is not enough to overcome gravity. Thus, in this manner, one constructs a trough for molecular flow without dispersion. The distorted plates of Fig. 1b are equipotential surfaces of the following electric potential, created from a series of increasingly high order terms odd in x

each with zero Laplacian:

$$\Phi = -E_o x + \beta_1\ xy + \beta_3(xy^2 - \frac{1}{3}x^3) + \beta_5(x^3y^2 - \frac{1}{2}xy^4 - \frac{1}{10}x^5). \tag{11}$$

The Stark gravitational potential is given by

$$U_{sg}(x,y) = mgy + U_i(E) \tag{12}$$

Here $\vec{E} = -\vec{\nabla}\Phi$, and mg is the weight of the molecule. For the purpose of example, we continue with the example system of the last section, namely the Stark energy $U_i(E)$ of the $X_1\ ^2\Pi_{1/2}$ $(v = 0, F = 1, |M| = 1, J = 1/2, \Omega_+)$ state of $^{208}Pb^{19}F$. Rewriting Eq. 8 as

$$U_i(E) = C_2E^2 + C_3E^3 + C_4E^4 \tag{13}$$

$U_{sg}(x,y)$ can be written in terms of a series expansion in x and y. The potential parameters β_1, β_3, and β_5 may be taken to zero in the terms of order y, x^2y, and x^4y. The result is the Stark gravitational potential of the form

$$U_{sg}(x,y) = U_i(E_o) + \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + O[y^3] + O[x^4]. \tag{14}$$

with

$$\begin{aligned} k_x &= \frac{(mg)^2}{2C_2E_o^2 - 4C_4E_o^4} \\ &= 0.0195\ \mu\text{eV/cm}^2 = 23.4\mu\text{K/cm}^2 \end{aligned} \tag{15}$$

$$\begin{aligned} k_y &= \left[\frac{4C_2^2 + 6C_2C_3E_o - 9C_3^2E_o^2 - 60C_3C_4E_o^3 - 80C_4^2E_o^4}{(2C_2 + 3C_3E_o + 4C_4E_o^2)^2}\right] k_x \\ &= 0.0233\ \mu\text{eV/cm}^2 = 28.0\mu\text{K/cm}^2 \end{aligned} \tag{16}$$

Here the values of k_x and k_y are taken by assuming coefficients C_i that model the $J = 1/2$, Ω_+, $F = 1, |M| = 1$ quantum state of interest in this experiment and a trap field bias field $E_o = 2500$ volts/cm. The simple harmonic potential described by the k_x and k_y terms alone does a very good job of modeling the exact Stark gravitational potential $U_{sg}(x,y)$ everywhere inside a guide created by plates separated by 2 cm.

Two factors make the Stark gravitational guide somewhat miserable to work with. The first is that the trap force parameters (constants k_x and k_y) rapidly decrease with increasing value of E_o, making a central field much greater than 2500 volts/cm impossible. This limits the polarization field, and hence e-EDM sensitivity of the state of the PbF molecule we

considered in the last section to 8.2 mHz/$10^{-27}e$·cm, roughly 65% of its sensitivity when fully polarized. A second is the fact that the trap depth is frustratingly small, stopping only those particles that begin their journey from the center of the trap with a transverse velocity of 9 cm/s or less. For a beam velocity of 200 m/s, this corresponds to an acceptance of only $6. \times 10^{-7}$ steradian. However, the guide has an overwhelmingly positive feature that may make its disadvantages worth coping with: Because the trap is two-dimensional, there is no accumulation of geometric phase as a polarized molecule travels down the beam. This implies the length of the guide is limited only by one's ability to control vacuum, black-body radiation, and background magnetic fields. Indeed, one can imagine an 1 km machine with five second coherence time that fits within one's abilities to control these factors.

IV. CASE 3: THE CYLINDRICAL GUIDE

The guides of the previous sections collimate a beam of low-field seeking states. Here we consider a guide of high field seeking states. Because it is not possible to create a local maximum in the magnitude of electric field, this guide must be a dynamic guide for which the translational angular momentum of the molecules keeps them from colliding into a central electrode.

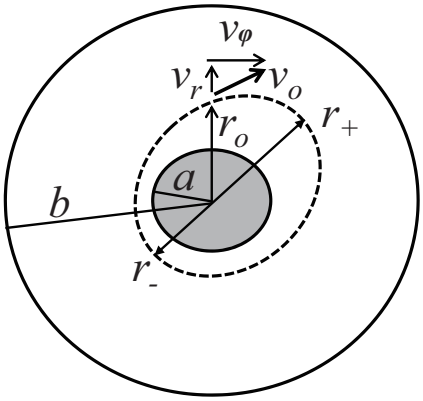

FIG. 5. Cross section of the cylindrical guide.

The electrodes of this guide (Fig. 1c and Fig. 5) are a central rod of radius $a = 0.2$ cm surrounded by a coaxial cylindrical electrode of inner radius $b = 1.0$ cm. For the case that voltages $V = \pm 8050$ volts are applied to the electrodes, the field magnitude varies inversely with r from $50,000$ volts/cm to $10,000$ volts/cm. The strong electric field strength can not

only keep the molecular beam of the high-field seeking state from diverging, but also fully polarize the PbF molecule to gain enough sensitivity to the e-EDM measurement. This field strength is manageable in the lab environment, but the determination of the appropriate values is still dependent of the design of the electrodes and the ability of controlling vacuum. For the case of a molecule interacting with a linear Stark interaction $U_S = -\bar{d}\,E$, the energy of a molecule in the guide is (ignoring gravity) that of Kepler motion:

$$\frac{U}{m} = -\frac{k}{r} + \frac{1}{2}v^2 \tag{17}$$

with

$$k = \frac{\Delta V}{\ln(b/a)m}\bar{d}. \tag{18}$$

Here $\Delta V = 16100$ volts is the potential difference between the electrodes and m the mass of the molecule. For strongly mixed $J = 1/2$, $J = 3/2$ states of a $^2\Sigma_{1/2}$ or $^2\Pi_{1/2}$ molecule, the approximation $\bar{d} \approx \frac{2}{3}d$ where d is the dipole moment of the ground state of the molecule may be used. Over the range of fields in the cylindrical guide described here, this value of $\bar{d}$ does a good job modeling the Stark interaction of the e-EDM sensitive low field seeking states of YbF($d = 3.58$ Debye[35]), PbF($d = 3.40$ Debye[32]), and HgF($d = 2.18$ Debye[36]).

We now consider the fraction of a beam of molecules entering the guide a distance r_o away from the center that will be sent into orbits that do touch the electrodes (*i.e.*, for which $a < r_-$ and $b > r_+$, in Fig. 5). We make approximation of Keplerian motion to allow us to proceed analytically. Because of the dispersion of beam velocities, we expect each molecule to enter with a unique initial transverse velocity $\vec{v_o} = v_r\hat{r} + v_\phi\hat{\phi}$ as shown in Fig. 5. If a molecule enters with $v_r = 0$ and $v_o = v_c = \sqrt{k/r_o}$, then the molecule will enter a circular orbit of constant r_o and therefore avoid the electrodes. By analyzing other Kepler orbits, one finds the range of input velocities that will lead to stable trajectories:

$$v_c\sqrt{\frac{2a}{r_o+a}} < v_\phi < v_c\sqrt{\frac{2ab}{(a+b)r_o}} \text{ and } |v_r| \le \sqrt{\frac{2(r_o-a)}{a}\left(\frac{r_o+a}{2a}v_\phi^2 - v_c^2\right)} \tag{19}$$

or

$$v_c\sqrt{\frac{2ab}{(a+b)r_o}} < v_\phi < v_c\sqrt{\frac{2b}{r_o+b}} \text{ and } |v_r| \le \sqrt{\frac{2(b-r_o)}{b}\left(v_c^2 - \frac{r_o+b}{2b}v_\phi^2\right)} \tag{20}$$

This region of velocity space for the case that $r_o = (a+b)/2 = 0.6$ cm, is given in Fig. 6 for the molecules ^{174}Yb^{19}F, ^{208}Pb^{19}F, and ^{198}Hg^{19}F.

To test the approximations that, for the case of ^{208}Pb^{19}F, gravity can be ignored and $U \approx -\frac{2}{3}dE$, we carried out a Monte Carlo calculation. This calculation incorporates both gravity and a potential energy $U(E)$ taken from a detailed calculation of the Stark interaction using known spectroscopic parameters of the high-field seeking $X_1(v = 0,\ F = 1,\ |M| = 1, J = \frac{1}{2},\ \Omega_-)$ state of ^{208}Pb^{19}F. In this study, values of v_ϕ and v_r are chosen randomly and trajectories are evolved from the point $x = 0$, $y = 0.6$ cm. For each trajectory that evolves for 50 ms, the initial velocities v_ϕ and v_r are recorded and plotted on Fig. 6. From this calculation we see that the Kepler approximation slightly overestimates the acceptance of the guide.

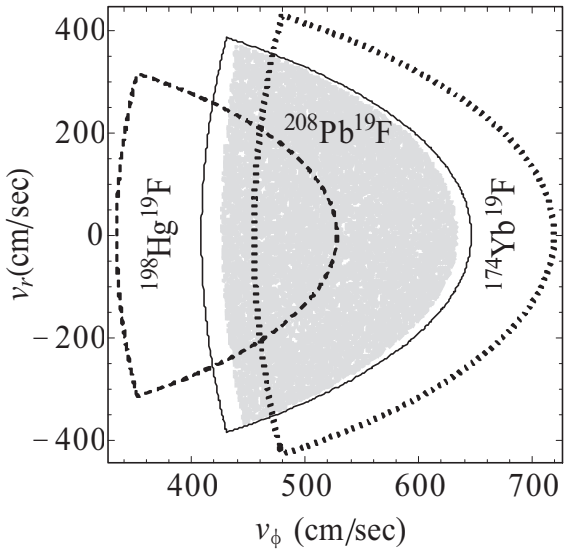

FIG. 6. Kepler-motion prediction of acceptance velocities of the cylindrical guide for the ground state of ^{198}Hg^{19}F (dashed line), ^{208}Pb^{19}F(solid line), and ^{174}Yb^{19}F(dotted line.) The shaded area indicates initial conditions of stable trajectories in a Monte Carlo Simulation assuming motion of high-field seeking ground-state ^{208}Pb^{19}F$(v = 0,\ F = 1,\ |M| = 1,\ J = \frac{1}{2},\ \Omega_-)$ molecules governed by Eq. 7 and a potential energy $U(E)$ taken from a detailed calculation of the Stark interaction using known spectroscopic parameters.

Like the Stark gravitational guide, the electric field in this guide is constrained to two dimensions and, as a result, the geometric phase effect is not expected to be of concern. This guide has many advantages over the Stark gravitational guide. One is that, for $^2\Sigma_{1/2}$ states with small spin rotational constants, the high field seeking ground state is more sensitive to an e-EDM then low field seeking ground states. For example, The most sensitive low-field

seeking state of $^{174}Yb^{19}F$, is 4 to 5 times less sensitive to an e-EDM than is the most sensitive high-field seeking state. For $^{198}Hg^{19}F$ this low-field seeking state is approximately 2 times less sensitive. In addition, while both the low- and high- field seeking states of the ground $^2\Pi_{1/2}$ state of $^{208}Pb^{19}F$ exhibit similar sensitivities to an e-EDM, the Stark gravitational guide only functions at low fields for which the PbF molecule is only partially polarized, leading to an approximately 35% reduction in sensitivity. Thus for each of these three important e-EDM molecules, sensitivity to an e-EDM is substantially greater in the cylindrical guide.

A second advantage of the cylindrical guide is its tremendous acceptance. This acceptance is given by the area of the region of trapped velocities divided by the beam velocity squared. For a cylindrical guide of a 200 m/s beam of PbF , this acceptance is 3×10^{-4} steradian, a factor of 500 times greater than the 6×10^{-7} steradian acceptance of the Stark gravitational guide. For a statistics limited experiment, this increased acceptance could, in principle, lead to a factor of 20 improvement in sensitivity to an e-EDM. Many factors may offset this advantage. One problem with the cylindrical guide is that the electric field at the entrance and exit is complex and likely to cause a substantial spatially-dependent geometric phase shift that restricts the probe region to a small volume in space. A second problem is the beam must be loaded with a substantial translational angular momentum. This angular momentum may couple with distortions in the electric field to create false e-EDM signals. If these problems can be overcome, then the cylindrical guide may prove to be the most promising candidate for use as a Ramsey cavity in an e-EDM experiment.

V. SUMMARY

We have introduced three guides of polar molecules for possible use as the Ramsey cavity in an optical double resonance measurement of the electron's electric dipole moment (e-EDM). Each of these guides must overcome Meek's theorem that states that, in two dimensions, all extrema in electric field magnitude occur at zero electric field magnitude.

The first guide we consider is a helical guide formed from two twisted wires (Fig. 1a). This guide is likely to be useful for manipulating low field seeking states without loss of alignment, but is unlikely to be useful in an e-EDM measurement: Molecules traversing the beam will accumulate a large trajectory-dependent geometric phase. This phase would cause rapid loss of coherence in an optically polarized beam transversing the guide.

The second guide is a Stark gravitational guide formed from two slightly distorted plates (Fig. 1b). This guide offers simplicity of construction. More importantly, because the trap is created from fields restricted to two dimensions, no geometric phase will accumulate within the guide. However trap acceptance is limited by the size of the gravitational acceleration g, with an acceptance of 6×10^{-7} steradian for a 200 m/s beam of ground state of $^{208}Pb^{19}F$ molecules.

The final guide we present is the cylindrical guide (Fig. 1c). This guide is a dynamical guide of high-field seeking states. The trap is also created by a two-dimensional field, so decoherence due to the accumulation of geometric phase within the guide is not expected. In addition, the effective guide depth is very deep, leading to a acceptance of 3×10^{-4} steradian for a 200 m/s beam of ground state $^{208}Pb^{19}F$ molecules. If difficulties associated with its precise construction can be overcome, this guide may prove the best candidate for a long coherence time guided e-EDM experiment.

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