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Shear viscosity and damping of collective modes in a two-dimensional Fermi gas

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Abstract

We compute the shear viscosity of a two dimensional Fermi gas interacting via a short range potential with scattering length a_{2d} in kinetic theory. We find that classical kinetic theory predicts that the shear viscosity to entropy density ratio of a strongly interacting two dimensional gas is comparable to that of the three dimensional unitary gas. We apply our results to the damping of collective modes of a trapped Fermi gas, and compare to experimental data recently obtained in E. Vogt et al., arXiv:1111.1173.

I. INTRODUCTION

The study of transport properties of strongly interacting, scale invariant or approximately scale invariant fluids has led to many recent discoveries that connect the physics of cold atomic gases, properties of the quark gluon plasma, and quantum gravity [1]. Nearly ideal hydrodynamic flow in cold atomic gases was observed in the expansion of a dilute Fermi at unitarity [2], and similar results were observed in heavy ion collisions at the relativistic heavy ion collider (RHIC) [3]. Recent analyses show that both the quark gluon plasma and the dilute Fermi gas at unitarity are characterized by a shear viscosity to entropy density ratio $\eta/s \lesssim 0.5\,\hbar/k_B$ [4–7]. This result is close to the value $\eta/s = \hbar/(4\pi k_B)$ which was found in the strong coupling limit of a large class of field theories that can be analyzed using the AdS/CFT correspondence [8, 9].

The AdS/CFT result is independent of the dimensionality of the fluid, and it is interesting to study whether nearly perfect fluidity can be observed in two-dimensional fluids. It was suggested, for example, that electrons in graphene might behave as a nearly perfect fluid [10]. Recently, a group at the Cavendish Laboratory investigated the damping of collective modes in a cold atomic Fermi gas tightly confined in one direction [11]. Vogt et al. determined the damping constant as a function of T/T_F in the range $T/T_F = (0.3 - 0.8)$, and for different interaction strengths $\log(k_F a_{2d}) = (2.7 - 42)$. Here, T/T_F is the temperature in units of the Fermi temperature, k_F is the Fermi momentum, and a_{2d} is the two-dimensional scattering length. In the present work we compare these results with the predictions of kinetic theory. Formally, kinetic theory is reliable in the limit of high temperature, $T \gg T_F$, or in the case of weak interactions, $K_F a_{2d} \gg 1$. In the case of the three dimensional Fermi gas at unitarity it was observed that the range of applicability of kinetic theory is larger than one might expect, extending down to $T \sim 0.4\,T_F$ [12–14].

II. KINETIC THEORY

The viscous stress tensor in hydrodynamics is given by $\delta\Pi_{ij} = -\eta\sigma_{ij} - \zeta\delta_{ij}\langle\sigma\rangle$ with

$$\sigma_{ij} = \partial_i v_j + \partial_j v_i - \frac{2}{d}\,\delta_{ij}\partial_k v_k\,, \tag{1}$$

and $\langle\sigma\rangle = \partial_k v_k$. Here, v_i is the flow velocity and $d = 2, 3, \ldots$ is the number of spatial dimensions. We will determine η by matching the hydrodynamic result to kinetic theory.

The stress tensor in kinetic theory is given by

$$\delta\Pi_{ij} = \nu \int d\Gamma_p \, \frac{p_i p_j}{m} \, \delta f_p \,, \tag{2}$$

where ν is the number of degrees of freedom ($\nu = 2$ for a two-component Fermi gas), $d\Gamma_p = \frac{d^dp}{(2\pi)^d}$ is the volume element in momentum space, and δf_p is the off-equilibrium correction to the distribution function. We will use the ansatz

$$\delta f_p = f_p^0 - \frac{f_p^0}{T}\, \chi_{ij}(p)\sigma_{ij} \,, \qquad \chi_{ij}(p) = p_{ij}\chi\left(\frac{p^2}{2mT}\right) \,, \tag{3}$$

where f_p^0 is the classical equilibrium distribution function and $p_{ij} = p_i p_j - \frac{1}{d}\delta_{ij}p^2$. We will study the role of quantum statistics below. We compute δf_p by solving the Boltzmann equation for fermions with dispersion relation $E_p = \frac{p^2}{2m}$ subject to elastic two-body scattering. At this level of approximation the bulk viscosity vanishes. This is the correct result for 3d fermions at unitarity [15–17], but the bulk viscosity is expected to be non-zero for 3d fermions away from unitarity, and for 2d fermions at any value of the scattering length. The dependence of the integral $\int d\omega\, \zeta(\omega)$ on $(k_F a)^{-1}$ is constrained by sum rules [18–20], but the bulk viscosity at zero frequency has not been determined. Vogt et al. measured the damping of a 2-d quadrupole mode [11], which is not sensitive to bulk viscosity.

Matching the kinetic theory expression for $\delta\Pi_{ij}$ to hydrodynamics we get

$$\eta = \frac{2\nu}{(d-1)(d+2)}\, \frac{1}{mT}\, \langle p_{ij}|\chi_{ij}\rangle \,, \tag{4}$$

where we have defined the inner product $\langle a|b\rangle = \int d\Gamma_p\, f_p^0 a(p)b(p)$. The function $\chi_{ij}(p)$ is determined by the linearized Boltzmann equation

$$\frac{1}{2m}\, |p_{ij}\rangle = C|\chi_{ij}\rangle \,. \tag{5}$$

Here C is the linearized collision operator $C|\chi_{ij}\rangle = |C[\chi_{ij}]\rangle$ with

$$\begin{aligned} C[\chi_{ij}(p_1)] \;=\; & \prod_{i=2}^{4}\left(\int d\Gamma_i\right) f^0(p_2)(2\pi)^{d+1}\delta^d(P-P')\delta(E-E')\,|\mathcal{T}|^2 \\ & \cdot\left[\chi_{ij}(p_1)+\chi_{ij}(p_2)-\chi_{ij}(p_3)-\chi_{ij}(p_4)\right] \,, \end{aligned} \tag{6}$$

where $\mathcal{T}$ is the T-matrix for elastic two-body scattering $12 \to 34$. We have also defined $p_{1,2} = \frac{P}{2} \pm q$, $p_{3,4} = \frac{P'}{2} \pm q'$, $E = E_{p_1} + E_{p_2}$ and $E' = E_{p_3} + E_{p_4}$. Given the T-matrix we can determine χ_{ij} from equ. (5) and then compute the shear viscosity using equ. (4). In practice it is useful to reformulate the calculation as a variational problem. The shear viscosity can be written as

$$\eta = \frac{\nu}{(d-1)(d+2)} \frac{1}{m^2 T} \frac{\langle \chi_{ij} | p_{ij} \rangle^2}{\langle \chi_{ij} | C | \chi_{ij} \rangle} . \tag{7}$$

The equivalence of this result and the previous expression given in equ. (4) follows from the linearized Boltzmann equation. The result is variational in the sense that for a trial function χ_{ij}^{var} equ. (7) provides a lower bound on the shear viscosity. The exact solution of the linearized Boltzmann equation can be found by maximizing equ. (7). In the three dimensional case it is known that the quadratic ansatz $\chi_{ij} = p_{ij}$ is an excellent solution, providing results for the shear viscosity that are accurate to 2% [21]. We will see that despite the different structure of the scattering amplitudes in two and three dimensions the matrix elements of the collision operator are very similar. We will therefore use the trial function $\chi_{ij} = p_{ij}$.

In two dimensions the scattering matrix for elastic scattering mediated by a short range potential is given by [22]

$$\mathcal{T} = \frac{4\pi}{m} \frac{1}{-\log(q^2 a_{2d}^2) + i\pi} , \tag{8}$$

where a_{2d} is the two-dimensional scattering length. The cross section is $\frac{d\sigma}{d\Omega} = \frac{m^2}{4q} |\mathcal{T}|^2$. The matrix element of the linearized collision operator can be reduced to a one-dimensional integral. We find

$$\langle \chi_{ij} | C | \chi_{ij} \rangle = 4T(mT)^3 \int_0^\infty dx \, \frac{x^5 e^{-x^2}}{\log^2(x^2 T / T_{a,2d}) + \pi^2} , \tag{9}$$

where we have defined $T_{a,2d} = 1/(m a_{2d}^2)$. The integral in equ. (9) can be computed using the saddle point approximating. This amounts to replacing the term x^2 in the denominator by 5/2. The final result for the shear viscosity is

$$\eta_{2d} = \frac{mT}{2\pi^2} \left(\left[\log\left(\frac{5T}{2T_{a,2d}} \right) \right]^2 + \pi^2 \right) , \tag{10}$$

where we have set ν, the number of spin states, equal to two. We can use this results to compute the dimensionless quantities η/n and η/s. We find

$$\frac{\eta_{2d}}{n} = \frac{\pi}{2} \left(\frac{T}{T_F^{loc}} \right) \left(1 + \frac{1}{\pi^2} \left[\log\left(\frac{5T}{2T_{a,2d}} \right) \right]^2 \right) , \tag{11}$$

where $T_F^{loc} = (k_F^{loc})^2/(2m)$ is a function of the local Fermi momentum, $k_F^{loc} = (2\pi n)^{1/2}$. The entropy per particle is $s/n = \log(T/T_F^{loc}) + 2$.

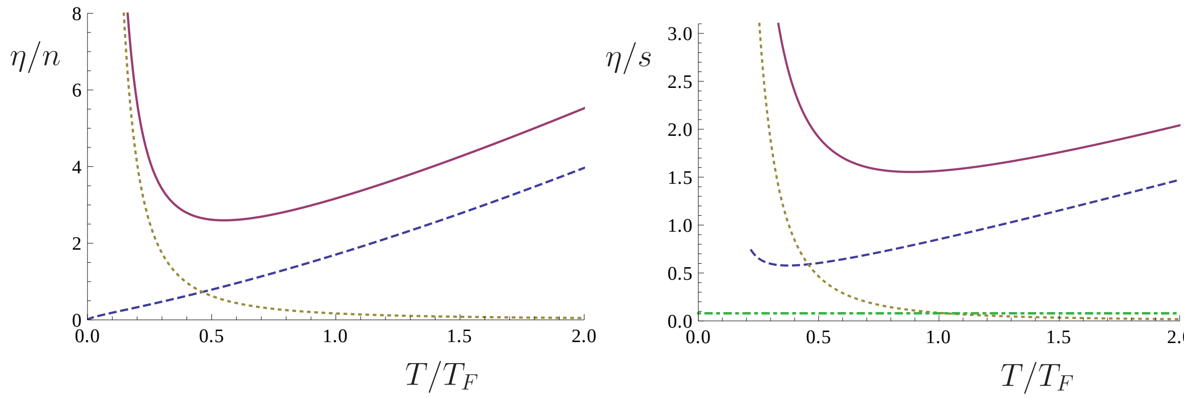

FIG. 1: (Color online) The left panel shows the viscosity to density ratio η/n as a function of T/T_F for a two-dimensional Fermi gas with $(k_F a)^2 = 2$. Here, $T_F = k_F^2/(2m)$ and $k_F = (2\pi n)^{1/2}$ characterize the homogeneous Fermi gas. The solid line includes the effects of quantum statistics, the dashed line shows the high temperature limit given in equ. (11), and the dotted line shows the low temperature limit. The right panel displays the shear viscosity to entropy density ratio. The dash-dotted line shows the proposed bound $\eta/s = 1/(4\pi)$.

It is instructive to compare these expressions to the analogous formulas in three dimensions. The T-matrix is

$$\mathcal{T} = \frac{4\pi}{m}\frac{1}{-a_{3d}^{-1} + iq}\,, \tag{12}$$

and the cross section is $\frac{d\sigma}{d\Omega} = \frac{m^2}{16\pi^2}|\mathcal{T}|^2$. The collision integral is

$$\langle \chi_{ij}|C|\chi_{ij}\rangle = \frac{16 m^{7/2} T^{9/2}}{3\pi^{5/2}} \int_0^\infty dx\, \frac{x^5 e^{-x^2}}{1 + T_{a,3d}/(x^2 T)}\,, \tag{13}$$

where $T_{a,3d} = 1/(m a_{3d}^2)$. At unitarity, $T_{3d} \to \infty$, the integrand differs from the result in two dimensions only by logarithmic terms. The shear viscosity at unitarity is

$$\eta_{3d} = \frac{15}{32\sqrt{\pi}}\,(mT)^{3/2}\,. \tag{14}$$

In the limit $T_{a,3d}/T \gg 1$ we find $\eta_{3d} = 5(mT)^{1/2}/(32\sqrt{\pi}a^2)$. The three dimensions the density is $n = (k_F^{loc})^3/(3\pi^2)$, and the shear viscosity to density ratio is

$$\frac{\eta_{3d}}{n} = \frac{45\pi^{3/2}}{64\sqrt{2}}\left(\frac{T}{T_F^{loc}}\right)^{3/2}\,. \tag{15}$$

Finally, the entropy per particle is $s/n = \frac{3}{2}\log(\pi T/T_F^{loc}) + \log(3/4) + 5/2$.

The results in two dimensions are plotted as the blue dashed lines in Fig. 1. We have chosen $(k_F a_{2d})^2 = 2$, which means that the two body binding energy $E_B = 1/(ma_{2d}^2)$ is equal to the Fermi energy. This corresponds to the BEC/BCS crossover regime. We observe that for $T/T_F^{loc} \lesssim 0.5$ the shear viscosity to entropy density ratio reaches $\eta_{2d}/s \simeq 0.5$, comparable to the result for the three dimensional Fermi gas at unitarity. In this regime kinetic theory is not reliable – effects due to quantum statistics, correlations and fluctuations are likely to play a role. Quantum statistics can be included straightforwardly in the kinetic theory calculation by including appropriate statistical factors in equ. (2,3) and (6). The result is shown as the solid line in Fig. 1. Pauli blocking suppresses the scattering matrix element and leads to $\eta_{2d}/n \sim (T_F^{loc}/T)^2[\log(T_F^{loc}/T)]^2$ as $T \to 0$. This result is expected from Landau Fermi liquid theory [23]. We observe that in two dimension the effect of Pauli blocking is quite large, but we also emphasize that at strong coupling the inclusion of quantum statistics is not necessarily an improvement over the classical calculation. In the case of thermodynamic quantities, like the second Virial coefficient, it is well known that effects of quantum statistics appear at the same order in T/T_F as higher order interaction terms. A similar effect is seen in the many body T-matrix calculation of the shear viscosity of the three dimensional gas at unitarity by Enss et al. [16]. These authors include pairing correlations and vertex corrections in addition to the effects of quantum statistics. They find that the shear viscosity to entropy density ratio remains very close to the classical result even in the very degenerate regime $T \sim (0.2 - 0.5)\, T_F$.

III. DAMPING OF COLLECTIVE MODES IN A TRAPPED GAS

In hydrodynamics the damping of collective modes is governed by the rate of energy dissipation

$$\dot{E} = -\frac{1}{2}\int d^3x\, \eta(x)\, (\sigma_{ij})^2\ , \tag{16}$$

where we have neglected bulk viscosity and assumed that the system remains isothermal (so that heat conductivity can be neglected). For simple modes like the quadrupole oscillation studied by Vogt et al. the velocity field is linear in the coordinates and the stress tensor is spatially constant. In this case the decay rate is sensitive to the spatial integral of $\eta(x)$. On dimensional ground we can write the viscosity of the homogeneous system as $\eta = n\alpha_n(T/T_F^{loc}, k_F^{loc}a)$. The spatial integral over $\eta(x)$ can then be written as $N\langle\alpha_n\rangle$, where

N is the total number of particles and $\langle\alpha_n\rangle$ is the value of α_n averaged over the density distribution of the cloud. In the hydrodynamic regime measurements of the damping constant of collective modes can therefore be interpreted as measurements of $\langle\alpha_n\rangle$.

The difficulty with this approach is that in kinetic theory $\eta = n\alpha_n$ is independent of the density and the spatial average $\langle\alpha_n\rangle$ is formally infinite. Physically, this problem is related to the fact that for any finite collective mode frequency hydrodynamics cannot be applicable in the dilute corona of the cloud, so that the integral in equ. (16) has to be cut off at low density [14]. In kinetic theory this can be done by taking into account the frequency dependence of the shear viscosity

$$\eta(\omega) = \frac{\eta(0)}{1+\tau_R^2\omega^2}\,, \tag{17}$$

where τ_R is the viscous relaxation time, which is the time it takes for the stress tensor to relax to the Navier-Stokes form $\delta\Pi_{ij} = -\eta(0)\sigma_{ij}$. We will see that the relaxation time is inversely proportional to the density, and that the spatial integral over $\eta(\omega)$ is therefore finite [13, 14].

The relaxation time can be determined in various ways, for example by solving the linearized Boltzmann equation in a time-dependent velocity field [24, 25], by computing the viscosity spectral function [26], or by evaluating the relaxation time in second order hydrodynamics [27]. The relaxation time is also constrained by viscosity sum rules [18–20]. Using the methods described in [26] we can show that in kinetic theory $\eta(\omega)$ satisfies the sum rule

$$\frac{1}{\pi}\int d\omega\,\eta(\omega) = \frac{P}{2}\,, \tag{18}$$

where P is the pressure. This sum rule is valid in both two and three dimensions. Combining equ. (17) with the viscosity sum rule equ. (18) we get $\tau_R = \eta/P \simeq \eta/(nT)$.

We note that the sum rule in equ. (18) follows from the definition of the stress tensor in kinetic theory, see equ. (2). If the stress tensor is defined as an operator in the quantum theory one finds that the spectral function in two dimensions has a $1/\omega$ tail at high frequency [20]. The corresponding behavior in three dimension is $\rho(\omega) \sim 1/\sqrt{\omega}$. This tail does not appear in kinetic theory because kinetic theory is an effective theory for energies $\omega \lesssim T$. In the quantum mechanical sum rule the high frequency has to be subtracted. In the high temperature regime, $T \gtrsim T_F$, the conclusion is the same as before: the high frequency tail does not contribute to the sum rule, and the width of the transport peak is controlled by the relaxation time $\tau_R = \eta/(nT)$ [16].

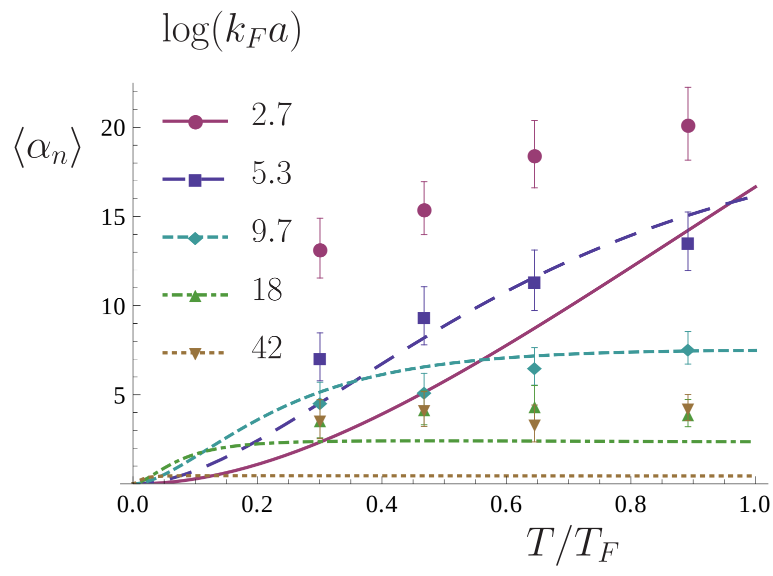

FIG. 2: (Color online) This figure shows the trap average of the shear viscosity to density ratio α_n as a function of T/T_F for different values of $\log(k_F a_{2d})$, where log is the natural logarithm. $T_F \equiv \omega_\perp N^{1/2}$ is the Fermi temperature and $k_F = (2mT_F)^{1/2}$ is the Fermi momentum in the trap. The scale is set by $\omega_\perp$, the transverse (two dimensional) confinement frequency. We have used $N = 4 \cdot 10^3$. We compare our results to the data from Vogt et al. [11].

We can now compute the trap average of $\eta(\omega)$. We will use the high temperature approximation for the cloud density. This is consistent with the classical kinetic calculation of η, and is expected to be a good approximation in the regime $T/T_F \geq 0.3$ studied by Vogt et al. In this limit the density profile of a 2-dimensional cloud is

$$n(x) = \frac{mT}{2\pi}\left(\frac{T_F}{T}\right)^2 \exp\left(-\frac{m\omega_\perp^2 x^2}{2T}\right) \tag{19}$$

where $T_F = \omega_\perp N^{1/2}$ is the Fermi temperature of the trapped gas. For the 2-dimensional quadrupole mode the frequency is given by $\omega = \sqrt{2}\omega_\perp$ [28–30]. We note that the quadrupole mode is volume conserving, and the frequency is independent of the equation of state. We get

$$\langle\alpha_n\rangle = \frac{1}{2\pi} R\left(\frac{T}{T_F}\right)^2 \log\left[1 + \frac{N\pi^2}{2R^2}\left(\frac{T_F}{T}\right)^2\right], \qquad R = \left[\log\left(\frac{5T}{2T_{a,2d}}\right)\right]^2 + \pi^2 . \tag{20}$$

This result is plotted in Fig. 2. We observe that for small values of $\log(k_F a_{2d})$ and T/T_F the trap average $\langle\alpha_n\rangle$ grows approximately as T^2. This power law can be understood as one factor of T arising from the temperature scaling of η, and one factor of T from the inverse density at the center of the trap. For larger values of $\log(k_F a_{2d})$ the growth of the relaxation time compensates the growth in η and the trap average $\langle\alpha_n\rangle$ is only weakly temperature dependent.

In Fig. 2 we also compare our results to the data obtained by Vogt et al. [11]. We observe that the predicted dependence of $\langle\alpha_n\rangle$ on T/T_F and $\log(k_F a_{2d})$ is in qualitatively agreement with the data. The theoretical predictions are in quantitative agreement with the data for $\log(k_F a_{2d}) = 5.3$ and 9.7. The disagreement between theory and data for $\log(k_F a_{2d}) = 2.7$ is somewhat puzzling, because this value of $\log(k_F a_{2d})$ corresponds to a more strongly interacting fluid, and we would expect hydrodynamics to work better. Of course, the kinetic theory calculation of the shear viscosity might break down at strong coupling and $T/T_F \lesssim 1$. Another possible issue is that the experimental analysis used a free Fermi gas model to estimate the energy of the mode. At strong coupling this approach will tend to overestimate the energy, and the extracted trap average $\langle\alpha_n\rangle$ is too large. The theory also under-predicts the data for large values of $\log(k_F a_{2d})$. This is less surprising, because hydrodynamics is expected to break down in this regime.

IV. OUTLOOK

The observed qualitative agreement between experiment and the predictions of kinetic theory suggests that the shear viscosity of the two dimensional Fermi gas can be extracted from measurements of the damping of collective modes. In order to do this quantitatively a number of effects will have to be studied more carefully. We observe, in particular, that for $\log(k_F a_{2d}) \gtrsim 5$ the measured collective mode frequencies are not close to the hydrodynamic predictions. This implies that dissipative effects are not accurately described by the hydrodynamic expression given in equ. (16). A more appropriate approach is to treat the collective mode itself in kinetic theory. This calculation will also provide an indication whether the observed damping at large $\log(k_F a_{2d})$ is related to collisions, or other effects that are not taken into account in a kinetic or hydrodynamic treatment.

We note that even though the observed trap averaged values of $\langle\alpha_n\rangle$ are on the order of 1 or larger the corresponding value of η/s at the center of the trap could be quite small, on the order of $\eta/s \sim 0.5$, see Fig. 1. In the interesting regime $T \lesssim 0.5\,T_F$ classical kinetic theory is not reliable. In two dimensions, in particular, correlations and fluctuations are likely to play an important role. An important example of a correlation effect is the pseudo-gap phenomenon which was argued to play an important role in the transport behavior of the three dimensional gas [31]. A pseudo-gap has been observed in the two dimensional gas in the regime $\log(k_F a_{2d}) \lesssim 1$ [32]. The phase transition in two dimensions is of Berezinsky-Kosterlitz-Thouless (BKT) type, and the two dimensional Fermi gas may provide a very clean system to study transport properties near the BKT transition. It is also known that in two dimensions hydrodynamic fluctuations lead to a slow, logarithmic, divergence of the shear viscosity with the system size [33].

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