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Phys. Rev. A **85**, 023621 — Published 14 February 2012

DOI: 10.1103/PhysRevA.85.023621

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Abstract

We study non-equilibrium dynamics of ultracold two-component Fermi gases in low-dimensional geometries after the interactions are quenched from weakly interacting to strongly interacting regime. We develop a T-matrix formalism that takes into account the interplay between Pauli blocking and tight confinement in low-dimensional geometries. We employ our formalism to study the formation of molecules in quasi-two-dimensional Fermi gases near Feshbach resonance and show that the rate at which molecules form depends strongly on the transverse confinement. Furthermore, Pauli blocking gives rise to a sizable correction to the binding energy of molecules.

PACS numbers: 03.65.Nk, 03.75.Ss, 05.30.Fk, 34.50.-s

I. INTRODUCTION

Theoretical prediction and experimental observation of magnetic-field induced Feshbach resonances in ultracold atoms [1] paved the way for many exciting discoveries including demonstration of fermionic superfluidity [2], observation of Efimov trimers and Fermi polarons [3–7], and creation of quantum degenerate gases of polar molecules [8]. Surprisingly, additional Feshbach resonances can be found in systems with reduced dimensionality. Earlier theoretical work on two particle scattering in systems confined to one dimensional tubes [9] and two dimensional pancakes [10] suggested a possibility realizing "confinement-induced resonances" (CIR), i.e., special scattering resonances made possible by restricting the transverse motion of atoms. Such resonances have been observed in both one [11] and two dimensional [12] systems. In optical lattices, Feshbach resonances can give rise to nontrivial manifestations of mixing of higher Bloch bands [13, 14].

Most of the earlier work has focused on the interplay of dimensional confinement and resonant interactions in two-body problems. Very few extensions to many-body systems have been considered so far. On the other hand, the primary motivation for studying low-dimensional systems is to understand the surprising properties of low dimensional many-body systems (see Refs. [15–17] for a review). Moreover, experiments are always performed in systems with a finite density and in many cases it may not be easy to disentangle many-body effects from two-particle scattering. For example, confinement-induced molecules are relatively large on the BCS side of resonance [10]. Already for a modest density of fermions, distances between particles may become comparable to the size of bound pairs, and Pauli principle can have a strong effect on the collisional properties of atoms and, as a result, on the properties of CIRs.

In this paper we provide a theoretical analysis of a many-body system composed of two-component fermions confined in two-dimensional (2D) geometries in the vicinity of a Feshbach resonance. We focus on quench type experiments, where a non-interacting mixture is rapidly taken to the regime of strong interactions [18, 19]. We analyze many-body corrections to the energies of confinement-induced molecules and calculate the rate at which they are formed out of unbound atoms.

One of the intriguing questions raised by recent experiments concerns the possibility of using fermionic systems close to Feshbach resonance for exploring many-body phenomena associated with strong repulsive interactions. For example, positive scattering length on the BEC side of the

Feshbach resonance has been suggested as a route to observe the Stoner instability [20]. While the first experiments by Jo *et al*. have been interpreted using a simple mean-field picture of such transition [20, 21], subsequent measurements showed that the system is strongly dominated by fast molecule formation [22], as predicted theoretically in Refs. [19, 23]. In this paper, we demonstrate that dimensional confinement can have a dramatic effect on the dynamics of molecule formation. We find that the peak in the molecule formation rate should be shifted from the BEC to the BCS side of the resonance with increasing transverse confinement. Testing our predictions in experiments will help to distinguish between different models of molecule formation [19, 23].

Another conceptually intriguing aspect of the system we study is that one cannot use separation of energy scales to simplify the analysis. Typically when many-body systems of ultracold atoms are studied, it is assumed that one can start by solving a two-body problem to obtain the strength of contact interaction and then work with this contact interaction when analyzing the many-body problem. In our system the effective two particle scattering can be strongly modified by the presence of other particles [19]. Hence, an accurate analysis of our system requires understanding the interplay of few-body and many-body phenomena.

II. VACUUM T-MATRIX

Traditionally two-body problems in low-dimensional geometries have been analyzed using the Schrödinger equation which can be simplified into two decoupled single particle problems corresponding to the relative and the center-of-mass (COM) motion [9, 10]. This approach is, however, difficult to generalize to the many-body case. In the presence of a filled Fermi sea, the COM momentum of the scattering pair relative to Fermi sea is important and cannot be taken into account by a simple momentum boost. Therefore we re-examine the two-body problem in quasi-2D geometries by recasting the results of Ref. [10] to the form of a T-matrix in vacuum. For a discussion regarding Feshbach resonances in low-dimensional systems, see Ref. [24]. We take the gas to be homogeneous in a 2D plane and assume a strong harmonic confinement in the transverse direction.

We start from the full 3D scattering problem and use a contact interaction $V_{\rm int}(\boldsymbol{r}-\boldsymbol{r}') = V_0\delta(\boldsymbol{r}-\boldsymbol{r}')$ to describe the inter-particle interactions. In order to make the connection to the many-body problem we do not separate relative and center-of-mass motion from the outset. This gives rise to a T-matrix which depends on energy $\hbar\omega$ as well as on the harmonic oscillator quantum numbers $\vec{n} = (n_1, n_2)$ and $\vec{n}' = (n_1', n_2')$ corresponding to incoming and outgoing particles. For

the contact interaction the Lippmann-Schwinger equation takes a simple form

$$T^0_{\vec{n},\vec{n}'}(\omega) = V_{\vec{n},\vec{n}'} + \sum_{\vec{n}''} V_{\vec{n},\vec{n}''} \Pi^{(0)}_{\vec{n}''}(\omega)\, T^0_{\vec{n}'',\vec{n}'}(\omega), \tag{1}$$

where the polarization operator is given by

$$\Pi^{(0)}_{\vec{n}}(\omega) = \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\hbar\omega - 2\varepsilon_{\boldsymbol{k}} - \hbar\omega_z(n_1+n_2) + i0^+}, \tag{2}$$

and we have denoted the trap frequency in transverse direction by ω_z. The polarization operator in vacuum has the property $\Pi^{(0)}_{\vec{n}}(\omega) = \Pi^{(0)}_{n_1+n_2}(\omega)$ for $\vec{n} = (n_1, n_2)$. We will utilize these two notations interchangeably when discussing the properties of the many-body T-matrix. The dispersion is given by $\varepsilon_{\boldsymbol{k}} = \hbar^2 k^2/2m$, and we measure energies and frequencies with respect to the zero-point energy $\hbar\omega_z$. Thus, a confined particle in the lowest vibrational state with no in-plane momentum is assumed to have zero energy.

Since $V_{\rm int}$ depends only on the relative motion of scattering particles, we write the matrix elements $V_{\vec{n},\vec{n}'}$ in terms of the quantum numbers corresponding to relative (n_r) and center-of-mass motion (N)

$$V_{\vec{n},\vec{n}'} = V_0 \sum_{N,n_r,n_r'} \mathcal{C}^{\vec{n}\,*}_{N,n_r} \mathcal{C}^{\vec{n}'}_{N,n_r'} \varphi^*_{n_r}(0)\, \varphi_{n_r'}(0). \tag{3}$$

Here φ_{n_r} is a harmonic oscillator eigenfunction corresponding to relative motion and the harmonic oscillator length in the transverse direction is denoted by $\ell_z = \sqrt{\hbar/m\omega_z}$. The Clebsch-Gordan coefficients arising from the change of basis are defined as $\mathcal{C}^{\vec{n}}_{N,n_r} = \langle N, n_r | n_1, n_2\rangle$. Quantum numbers n_1, n_2, n_r, and N are non-negative integers and energy conservation imposes condition $n_1 + n_2 = N + n_r$ for the non-zero elements $\mathcal{C}^{\vec{n}}_{N,n_r}$.

The form of matrix elements $V_{\vec{n},\vec{n}'}$ suggests we look for a solution in the basis of relative and COM quantum numbers and then go back to the original basis. We find that (see Appendix A)

$$T^0_{\vec{n},\vec{n}'}(\omega) = \sqrt{2\pi}\,\ell_z \sum_{N,n_r,n_r'} \mathcal{C}^{\vec{n}\,*}_{N,n_r} \mathcal{C}^{\vec{n}'}_{N,n_r'}\, \varphi^*_{n_r}(0)\varphi_{n_r'}(0) \times \mathcal{T}_0(\omega - N\omega_z). \tag{4}$$

The structure of $T^0_{\vec{n},\vec{n}'}$ shows explicitly the decoupling of relative and COM motion. Furthermore, since the interaction potential depends only on the relative motion, the COM quantum number does not change in the scattering and contributes only as shift to the energy of scattering particles. When the bare interaction V_0 is eliminated, $\mathcal{T}_0$ is given by [10, 25] (for details, see Appendix A)

$$\frac{1}{\mathcal{T}_0(\omega)} = \frac{m}{4\pi\hbar^2}\left[\sqrt{2\pi}\,\ell_z/a_{3D} + w(\omega/\omega_z + i0^+)\right], \tag{5}$$

where function $w(z)$ is defined as

$$w(z) = \lim_{n\to\infty}\left[2\sqrt{\frac{n}{\pi}}\ln\frac{n}{e^2} - \sum_{\ell=0}^{n}\frac{(2\ell-1)!!}{(2\ell)!!}\ln(\ell - z/2)\right]. \tag{6}$$

The double factorial is given by $n!! \equiv n\cdot(n-2)\cdot(n-4)...$, and by definition $(-1)!! = 0!! = 1$. The two-body T-matrix has a series of poles corresponding to different values of the center-of-mass quantum number N. In particular, there is a bound state corresponding to $N = 0$ which exists for all a_{3D} and coincides with the Feshbach molecule deeply on the BEC side. Deeply on the BCS side of resonance ($|a_{3D}| \ll \ell_z$), the energy of the confinement-induced two-body bound state has a simple expression $\varepsilon_b = -\frac{B}{\pi}\hbar\omega_z\, e^{-\sqrt{2\pi}\,\ell_z/|a_{3D}|}$, where $B = 0.905$ [1]. In general the pole has to be computed numerically from Eq. (5).

III. MANY-BODY T-MATRIX AND COOPERON

Let us next discuss the many-body effects in the formation of confinement-induced molecules in quasi-2D geometries. The system is described by a many-body Hamiltonian

$$\begin{aligned} H = &\sum_{\boldsymbol{k},n,\sigma}\xi_{\boldsymbol{k},n,\sigma}\, c^\dagger_{\boldsymbol{k},n,\sigma}c_{\boldsymbol{k},n,\sigma} \\ &+ \frac{1}{\mathcal{V}}\sum_{\boldsymbol{k},\boldsymbol{q},\boldsymbol{p}}\sum_{\vec{n},\vec{n}'} V_{\vec{n},\vec{n}'}\, c^\dagger_{\boldsymbol{k}+\boldsymbol{q},n_1,\uparrow}c^\dagger_{\boldsymbol{p}-\boldsymbol{q},n_2,\downarrow}c_{\boldsymbol{p},n_2',\downarrow}c_{\boldsymbol{k},n_1',\uparrow}, \end{aligned} \tag{7}$$

where $\xi_{\boldsymbol{k},n_i,\sigma} = \varepsilon_{\boldsymbol{k}} - \varepsilon_{F,\sigma} + \hbar\omega_z n_i$ and particles carry $2D$ momentum $\boldsymbol{k}$ as well as harmonic oscillator quantum number n_i. We have also allowed a possible imbalance between the two fermion species.

To incorporate the Pauli blocking to our analysis, we derive a T-matrix in the presence of Fermi sea (Cooperon). We approximate the full Bethe-Salpeter equation by taking into account the ladder diagrams and obtain

$$T^{\mathrm{MB}}_{\vec{n},\vec{n}'}(\omega,\boldsymbol{q}) = V_{\vec{n},\vec{n}'} + \sum_{\vec{n}''} V_{\vec{n},\vec{n}''}\Pi_{\vec{n}''}(\omega,\boldsymbol{q})\, T^{\mathrm{MB}}_{\vec{n}'',\vec{n}'}(\omega,\boldsymbol{q}), \tag{8}$$

where assume that the scattering particles can have finite COM momentum $\boldsymbol{q}$ in the 2D plane. The full polarization operator $\Pi_{\vec{n}}(\omega,\boldsymbol{q})$ is of the form

$$\Pi_{\vec{n}}(\omega,\boldsymbol{q}) = \int \frac{d\boldsymbol{k}}{(2\pi)^2}\, \frac{1-n_F(\xi_{\boldsymbol{k}+\boldsymbol{q},n_1,\uparrow})-n_F(\xi_{\boldsymbol{k},n_2,\downarrow})}{\hbar\omega(1+i0^+)-\xi_{\boldsymbol{k}+\boldsymbol{q},n_1,\uparrow}-\xi_{\boldsymbol{k},n_2,\downarrow}}. \tag{9}$$

Although COM and relative motion become coupled in the presence of Fermi sea, we utilize insights from the two-body problem and look for a solution where COM and relative motion are at least partially decoupled. We find that the solution of Bethe-Salpeter equation can be written in terms of a T-matrix depending only on the COM quantum numbers

$$T^{\mathrm{MB}}_{\vec{n},\vec{n}'} = \sqrt{2\pi}\,\ell_z \sum_{\substack{N,N'\\ n_r,n_r'}} \mathcal{C}^{\vec{n}\,*}_{N,n_r}\,\mathcal{C}^{\vec{n}'}_{N',n_r'}\,\varphi^*_{n_r}(0)\varphi_{n_r'}(0)\,\mathcal{T}_{N,N'}. \tag{10}$$

We use the two-body T-matrix $\mathcal{T}_0$ to renormalize the UV divergence associated with polarization operator (9) and obtain (for details, see Appendix B)

$$\mathcal{T}^{-1}_{N,N'}(\omega,\boldsymbol{q}) = \mathcal{T}_0^{-1}(\omega - N\omega_z - \omega_{\boldsymbol{q}})\delta_{N,N'} - \mathcal{D}_{N,N'}(\omega,\boldsymbol{q}), \tag{11}$$

where the renormalized polarization operator is given by

$$\begin{aligned}\mathcal{D}_{N,N'} = \sum_{\vec{n},n_r} u_{n_r,N+n_r-K}\,\mathcal{C}^{\vec{n}\,*}_{K,N+n_r-K}\,\mathcal{C}^{\vec{n}}_{N,n_r} \\ \times\,[\Pi_{\vec{n}}(\omega,\boldsymbol{q}) - \Pi^{(0)}_{\vec{n}}(\omega-\omega_{\boldsymbol{q}})].\end{aligned} \tag{12}$$

The coefficients $u_{n,m}$ are related to the zeros of the harmonic oscillator eigenfunctions [see Eq. (A4)] and they are given by

$$u_{n,m} = \frac{(-1)^{(n+m)/2}(n-1)!!\,(m-1)!!}{\sqrt{n!\,m!}} \tag{13}$$

for even and non-negative n and m. Otherwise $u_{n,m}$ is zero. We have also defined $\hbar\omega_{\boldsymbol{q}} = \frac{1}{2}\varepsilon_{\boldsymbol{q}} - \varepsilon_{F,\uparrow} - \varepsilon_{F,\downarrow}$. In order to correctly renormalize the UV divergence associated with the 2D momentum integral in Eq. (9), we have to evaluate the two-body T-matrix such that the Fermi surface and finite COM momentum are taken into account. This shifts the argument of $\mathcal{T}_0$ by $\omega_{\boldsymbol{q}}$ in Eq. (11).

Conservation of energy and parity impose selection rules for the allowed scattering processes and render matrix $V_{\vec{n},\vec{n}'}$ non-invertible. Since both T^0 and T^{MB} share the same structure as $V_{\vec{n},\vec{n}'}$, they also lack well-defined inverses and Eqs. (4) and (10) have to be solved in terms of matrices $\mathcal{T}_0$ and $\mathcal{T}$ which are both regular. The full solution retains all discrete energy levels in the transverse direction and although the most interesting 2D limit does involve real processes via higher bands, virtual scattering processes become important near the Feshbach resonance. The general solution

based on Eqs. (10)–(12) enables a systematic analysis of pairing instabilities from the strictly 2D regime at zero temperature to the confinement dominated 3D regime where temperature and Fermi energy become comparable with $\hbar\omega_z$.

IV. MOLECULE FORMATION

To analyze the possible pairing instabilities, we assume that the system is initially spin balanced and weakly interacting. In the spirit of Ref. [18], we consider an instantaneous quench where interactions are rapidly modified utilizing a 3D Feshbach resonance. The molecule formation is associated with the appearance of poles $\hbar\omega = \Omega_{\boldsymbol{q}} + i\Delta_{\boldsymbol{q}}$ in the many-body T-matrix $T^{\mathrm{MB}}_{\vec{n},\vec{n}'}(\omega, \boldsymbol{q})$ [26]. We identify the real part $\Omega_{\boldsymbol{q}}$ as the binding energy of the molecule and the imaginary part $\Delta_{\boldsymbol{q}}$ as the growth rate of the instability towards formation of molecules [19].

Similarly to the 3D case [19], we find that the system exhibits an instability towards molecule formation via two-body processes as long as the Fermi sea can absorb the binding energy of the molecules. This results in a sharp cutoff in the growth rate, see Fig. 1(a). For a fixed ℓ_z/a_{3D}, the binding energy of molecules depends strongly on the ratio $\varepsilon_F/\hbar\omega_z$ and Fig. 1(b) shows that the binding energy increases with increasing strength of the transverse confinement. The location of the peak value for the growth rate of instability can be varied by adjusting the ratio ℓ_z/a_{3D} and, in particular, tight enough transverse confinement can move the pairing instability completely to the BCS side. On the other hand, when $\varepsilon_F/\hbar\omega_z \simeq 0.1$ as in Refs. [12, 27], the pairing instability extends to the BEC side and fast two-body processes dominate the three-body processes. When the molecule formation via two-body processes is no longer possible, the leading instability is a three-body recombination which is suppressed for Fermi gases due to low densities and Pauli principle.

The binding energy in vacuum is compared to the binding energy at finite densities in Fig. 2. The relation between vacuum and finite density binding energies depends again on $\varepsilon_F/\hbar\omega_z$, and when the system becomes more three-dimensional (i.e. when $\varepsilon_F/\hbar\omega_z$ increases), Pauli blocking by the Fermi sea can result in a stronger binding of molecules. The crossover takes place roughly at $\varepsilon_F/\hbar\omega_z = 0.5$. In 3D gases many-body corrections always result in stronger binding [19] and Pauli blocking induced weaker binding is a manifestation of 2D physics. On the other hand, the binding energy of molecules is larger than the binding energy of the Feshbach molecules existing on the BEC side of resonance.

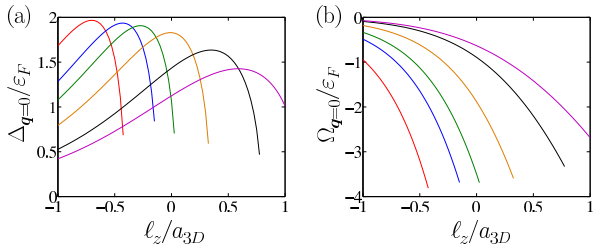

Figure 1. (Color online) Growth rate of the pairing instability (a) and the binding energy of molecules (b) at zero temperature as a function of ℓ_z/a_{3D}. The values of $\varepsilon_F/\hbar\omega_z$ are (from left to right) $\varepsilon_F/\hbar\omega_z = 0.0175,\ 0.025,\ 0.0375,\ 0.075,\ 0.2$, and 0.4.

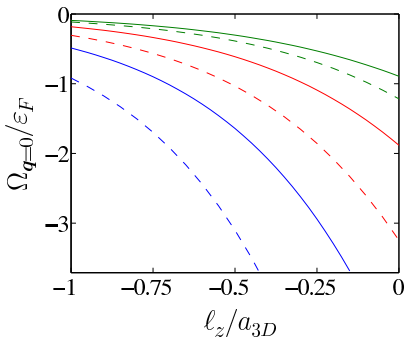

Figure 2. (Color online) Binding energy of molecules at zero temperature for $\varepsilon_F/\hbar\omega_z = 0.025,\ 0.075$, and 0.2 (from bottom to top). The binding energy in the presence of Fermi sea (solid lines) is always smaller than the vacuum binding energy (dashed line).

Finite temperature suppresses strongly the growth rate of pairing instability whereas the binding energy decreases more slowly with increasing temperature. In Fig. 3, the growth rate is shown at different temperatures for $\varepsilon_F/\hbar\omega_z = 0.1$ corresponding to the experimental parameters of Refs. [12, 27]. Pairing instability at the BCS side of the resonance is sensitive to the temperature since thermal fluctuations can easily break molecules at small binding energies. At high enough temperatures the pairing instability can become completely suppressed for weak attractive interactions. On the other hand, the cutoff in the growth rate at $\ell_z/a_{3D} \approx 0.5$ does not in general depend strongly on the temperature. We note that although the pairing instability can persist to quite high temperatures, the critical temperature for the superfluid transition is typically much lower near the Feshbach resonance or deeply in the BEC regime [28, 29].

The vacuum T-matrix has several poles on the BEC side of the Feshbach resonance corresponding to the different COM quantum numbers. This can in principle give rise to several pairing

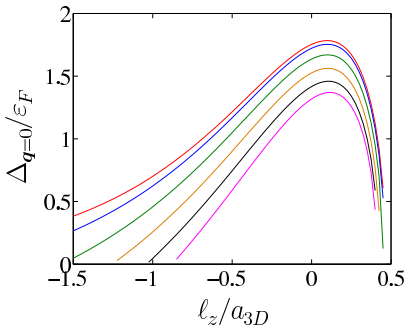

Figure 3. (Color online) The growth rate of the pairing instability as a function of ℓ_z/a_{3D} for $\varepsilon_F/\hbar\omega_z = 0.1$. The temperature is (from top to bottom) $T/T_F = 0,\ 0.1,\ 0.2,\ 0.3,\ 0.4$, and 0.5.

instabilities which show up as distinct poles in the Cooperon. For small $\varepsilon_F/\hbar\omega_z$ these poles do not coexist for given ℓ_z/a_{3D} since the Fermi sea is unable to absorb large binding energies. When $\varepsilon_F/\hbar\omega_z$ increases, the poles start to overlap and additional poles with non-zero $\Delta_{\boldsymbol{q}}$ become discernible. However, these additional instabilities remain weak compared to the primary instability corresponding to the pole of the vacuum T-matrix with $N = 0$.

So far we have analyzed the pairing instability in the case of zero COM momentum $\boldsymbol{q}$. We find that the results remain qualitatively the same for finite $\boldsymbol{q}$ and in the spin balanced case the most unstable mode is always at $\boldsymbol{q} = 0$. However, the growth rate of instability decreases slowly as a function of $|\boldsymbol{q}|$ and in a realistic quench experiment it is likely that molecules with a wide distribution of momenta are created. We find that finite $\boldsymbol{q}$ reduces the binding energy due to the smaller number of low-energy states that are available for scattering [30]. In spin imbalanced systems the lowest energy state can shift to finite momentum [31, 32].

V. DISCUSSION

We have studied pairing instabilities in spin balanced quasi-2D Fermi gases when interactions are dynamically quenched to the regime of strong interactions using 3D Feshbach resonances. We found that the pairing instability can be shifted to the BCS side of resonance by adjusting the axial confinement with respect to Fermi energy. Pauli blocking was found to renormalize significantly the binding energies, resulting in weaker binding in the 2D limit than warranted by the two-body description.

The growth rate of pairing instability can be measured by monitoring the atom loss [18] and

the binding energy can be probed using rf spectroscopy [12, 33, 34]. In a related work [35], we argue that the recent experiment [12] probing the properties of 2D Fermi gases can be interpreted in terms of dynamically created polarons. Another recent experiment [34] measures directly the binding energies of the molecules and finds agreement with a theoretical prediction for the two-body bound states in 1D optical lattices [36]. On the other hand, our calculation (Fig. 2) predicts that the two-body bound state energy should be significantly renormalized by the presence of the Fermi sea. The discrepancy could stem from the fact that our calculation probes an unpaired gas which is rapidly quench to the strongly interacting regime, whereas in Ref. [34] the system corresponds to a strongly interacting gas in equilibrium with a large number of paired atoms.

The T-matrix approach presented here can be used to probe the competition between polaron and molecule [31, 32, 37, 38] in quasi-2D systems and to investigate dimensional crossover from 2D to 3D [29, 34, 39, 40]. Our formalism is also useful for studies of pair formation in other low-dimensional geometries and Bose gases. In particular, it can be used to investigate the effective three-body collisions induced by virtual excitations of the transverse modes [41–43].

ACKNOWLEDGMENTS

We would like to thank L. Radzihovsky, M. Köhl, and W. Zwerger for insightful discussions. This work was supported by the Academy of Finland (VP), the Army Research Office with funding from the DARPA OLE program, Harvard-MIT CUA, NSF Grant No. DMR-07-05472, AFOSR Quantum Simulation MURI, AFOSR MURI on Ultracold Molecules, and the ARO-MURI on Atomtronics.

Appendix A: Vacuum T-matrix

We shortly discuss the technical details regarding the calculation of the vacuum T-matrix as well as the many-body T-matrix. For simplicity, we set $\hbar = 1$ in Appendices A and B.

The form of the Matrix elements in Eq. (3) suggest the following ansatz for the vacuum T-matrix

$$T^0_{\vec{n},\vec{n}'}(\omega) = \sqrt{2\pi}\ell_z \sum_{N,n_r,n_r'} C^{\vec{n}\,*}_{N,n_r} C^{\vec{n}'}_{N,n_r'} \varphi^*_{n_r}(0)\varphi_{n_r'}(0)\mathcal{T}_N(\omega). \tag{A1}$$

Substituting Eq. (A1) to Eq. (1) we obtain

$$\mathcal{T}_N(\omega) = \frac{V_0}{\sqrt{2\pi}\ell_z} + \sum_{n_r} \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{\sqrt{2\pi}\ell_z |\varphi_{n_r}(0)|^2 V_0}{\omega - \frac{k^2}{m} - \omega_z(N+n_r) + i0^+} \mathcal{T}_N(\omega).$$

We observe that the COM index comes only through the shift of energy. Thus we can take $T_N(\omega) = T_0(\omega - N\omega_z)$ and for $T_0(\omega)$ we obtain

$$\frac{1}{T_0(\omega)} = \frac{\sqrt{2\pi}\ell_z}{V_0} - \sum_{n_r} \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{\sqrt{2\pi}\ell_z |\varphi_{n_r}(0)|^2}{\omega - \frac{k^2}{m} - \omega_z n_r + i0^+}. \tag{A2}$$

We can calculate the integral in Eq. (A2) using the identity

$$\frac{1}{A} = -\int_0^{\mp\infty} d\tau\, e^{A\tau}, \tag{A3}$$

where $\mp = \mathrm{sgn}(\mathrm{Re}\, A)$. For Eq. (A2) we have two cases: (a) $\mathrm{Re}\,\omega < 0$ and (b) $\mathrm{Re}\,\omega \geq 0$. We discuss case (a) and case (b) follows from an analogous calculation. We note that the harmonic oscillator eigenfunctions in Eq. (A2) satisfy

$$\sqrt{2\pi}\ell_z |\varphi_n(0)|^2 = \begin{cases} \frac{(n-1)!!}{n!!}, & \text{for even } n \\ 0, & \text{for odd } n. \end{cases} \tag{A4}$$

Using the identity in Eq. (A3), we obtain

$$\begin{aligned} I &= \sum_{n_r} \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{\sqrt{2\pi}\ell_z |\varphi_{n_r}(0)|^2}{\omega - \frac{k^2}{m} - \omega_z n_r + i0} \\ &= -\int_0^\infty d\tau \sum_{n=0}^\infty \frac{(2n-1)!!}{(2n)!!} e^{\tau(\omega - 2n\omega_z + i0^+)} \left(\frac{m}{4\pi\tau}\right) \\ &= -\int_0^\infty d\tau\, e^{\tau(\omega+i0^+)} \left(\frac{e^{\omega_z\tau}}{2\sinh\omega_z\tau}\right)^{1/2} \left(\frac{m}{4\pi\tau}\right). \end{aligned}$$

The COM part of the quasi-2D T-matrix satisfies therefore an equation

$$\frac{1}{T_0(\omega)} = \frac{\sqrt{2\pi}\ell_z}{V_0} + \int_0^\infty d\tau\, e^{\tau(\omega+i0^+)} \sqrt{\frac{e^{\omega_z\tau}}{2\sinh\omega_z\tau}} \left(\frac{m}{4\pi\tau}\right). \tag{A5}$$

The UV divergence associated with the original contact interaction is manifested as a singularity of the integrand in the limit $\tau \to 0$. We regularize this divergence using the 3D T-matrix which is given by an analogous equation

$$\frac{1}{T_{3D}(\omega)} = \frac{1}{V_0} + \int_0^\infty d\tau\, e^{\tau(\omega+i0^+)} \left(\frac{m}{4\pi\tau}\right)^{3/2}. \tag{A6}$$

We take $\omega \to 0$ of Eq. (A6) to obtain

$$\frac{m}{4\pi a_{3D}} = \frac{1}{V_0} + \int_0^\infty d\tau \, \left(\frac{m}{4\pi\tau}\right)^{3/2}. \tag{A7}$$

Using this identity, we eliminate the bare interaction V_0 from Eq. (A5). This gives us an $T_0(\omega)$ which is manifestly free from UV divergences

$$\frac{1}{T_0(\omega)} = \frac{m}{4\pi}\left\{\frac{\sqrt{2\pi}\ell_z}{a_{3D}} + \int_0^\infty dx\, \frac{1}{x}\left[e^{x(\omega/\omega_z+i0^+)}\left(\frac{e^x}{2\sinh x}\right)^{1/2} - \left(\frac{1}{2x}\right)^{1/2}\right]\right\}. \tag{A8}$$

The latter term in Eq. (A8) is the integral representation of the function $w(\omega/\omega_z + i0^+)$ defined in Eq. (6).

Appendix B: Many-body T-matrix

To solve the Bethe-Salpeter equation (8) we generalize the ansatz in Eq. (A1) and assume that the many-body T-matrix is of the form

$$T^{\rm MB}_{\vec{n},\vec{n}'} = \sqrt{2\pi}\,\ell_z \sum_{\substack{N,N'\\ n_r,n_r'}} \mathcal{C}^{\vec{n}\,*}_{N,n_r}\,\mathcal{C}^{\vec{n}'}_{N',n_r'}\,\varphi^*_{n_r}(0)\varphi_{n_r'}(0)\,\mathcal{T}_{N,N'}, \tag{B1}$$

where we have temporarily suppressed the frequency and momentum arguments. The polarization operator satisfies the following useful identity

$$\begin{aligned}\sum_{\vec{n}\,\vec{n}'} \mathcal{C}^{\vec{n}'}_{N,n_r}\mathcal{C}^{\vec{n}\,*}_{N',n_r'}\Pi_{\vec{n},\vec{n}'}(\omega,\boldsymbol{k}) &= \delta_{N,N'}\delta_{n_r,n_r'}\,\Pi^{(0)}_{N+n_r}(\omega-\omega_{\boldsymbol{k}}) + \\ &\sum_{\vec{n}\,\vec{n}'} C^{\vec{n}'}_{N,n_r}C^{\vec{n}\,*}_{N',n_r'}\big[\Pi_{\vec{n},\vec{n}'}(\omega,\boldsymbol{k}) - \delta_{\vec{n},\vec{n}'}\Pi^{(0)}_{\vec{n}}(\omega-\omega_{\boldsymbol{k}})\big],\end{aligned} \tag{B2}$$

where $\omega_{\boldsymbol{k}} = \frac{1}{2}\varepsilon_{\boldsymbol{k}} - \varepsilon_{F,\uparrow} - \varepsilon_{F,\downarrow}$. Substituting the ansatz (B1) to the Bethe-Salpeter equation (8) and using the above identity, we obtain an equation for the COM part

$$\begin{aligned}\mathcal{T}_{N,N'} = \frac{V_0}{\sqrt{2\pi}\ell_z} + \frac{V_0}{\sqrt{2\pi}\ell_z}\sum_K (\mathcal{D}_0)_{N,K}\mathcal{T}_{K,N'} + \\ \frac{V_0}{\sqrt{2\pi}\ell_z}\sum_K (\mathcal{D})_{N,K}\mathcal{T}_{K,N'},\end{aligned} \tag{B3}$$

where matrices $\mathcal{D}_0$ and $\mathcal{D}$ given by

$$(\mathcal{D}_0)_{N,K} = \delta_{N,K}\sum_{n_r} u_{n_r,n_r}\Pi^{(0)}_{K+n_r}(\omega-\omega_{\boldsymbol{k}}), \tag{B4}$$

and Eq. (12), respectively. Coefficients u_{n_r,n_r} are given by Eq. (13) in the main text. Equation (B3) for $\mathcal{T} = (\mathcal{T}_{N,N'})$ can be written in a matrix form

$$\mathcal{T}^{-1} = \frac{\sqrt{2\pi}\ell_z}{V_0} - \mathcal{D}_0 - \mathcal{D}. \tag{B5}$$

Denoting $\mathcal{T}^{(0)} = \mathrm{diag}(\mathcal{T}_N)$, where $\mathcal{T}_N(\omega) = \mathcal{T}_0(\omega - N\omega)$, we obtain

$$\mathcal{T}^{(0)^{-1}} = \frac{\sqrt{2\pi}\ell_z}{V_0} - \mathcal{D}_0. \tag{B6}$$

This gives us an equation for the many-body T-matrix such that the UV divergence associated with $\mathcal{D}_0$ is renormalized

$$\mathcal{T}^{-1} = \mathcal{T}^{(0)^{-1}} - \mathcal{D}. \tag{B7}$$

Equation (B7) is illustrated in more detail in Eqs. (11) and (12) of the main text.

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