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Effect of the third level on time evolution of the spontaneous upper level decay due to counter rotating terms

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Abstract

The third level has great influence on the time evolution of the spontaneous decay from the excited state, when the counter rotating terms are taken into account. The influence in the cascade-type and V-type three-level systems is investigated for two initial states, excited from the ground states of the whole system (atom plus vacuum modes) and from the bare atom. The third level results in the additional virtual photon processes, emitting a photon from one level and reabsorbing the same photon to another level and vice versus. The main influence of the third level is to accelerate the decay, which leads to or enhances the anti-Zeno effect, especially for the initial state excited from the ground state of the whole system. Therefore, the third level could not be neglected, if the counter rotating terms are taken into account.

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(I) Introduction

The time evolution of an atomic spontaneous decay in a vacuum reservoir has attracted a lot of attention in recent years, where the Zeno and anti-Zeno effects can be realized [1-19]. For the time evolution, the rotating wave approximation (RWA) cannot be applied, because the counter rotating terms (CRTs) have great influence. It was found that the initial time evolution is quite different for different initial states [20]. It was also found that in the very short time period, the effective decay rate of the initial state is much slower than the exponential decay of the long time limit, which is the so-called quantum Zeno effect (QZE) [1-9]. Further study finds that the effective decay can be larger than the exponential decay after the QZE period, which is the so-called quantum anti-Zeno effect (QAZE) [10-16]. The QAZE was first discussed in a two-level system under the RWA. It was found that there is no QAZE if the CRTs are included for the initial state [15-17] excited from the ground state of the whole system (the atom plus vacuum reservoir) [15,20], while there is QAZE in a two-level atom if the initial state is the excited state of the bare atom (atom alone) [21]. These studies tell us that the QZE and QAZE heavily depend on the initial states and the CRTs. It is well known that the two-level atom is not a good model for the spontaneous decay when the CRTs are included, because the difference of the energy between other levels and the upper level and the energy of a photon, $\omega_{i1}-\omega_k$ ($\omega_{i1}\equiv\omega_i-\omega_1$), could be larger than sum of the transition energy and the energy of a photon, $\omega_{12}+\omega_k$. It is nature to ask what influence of additional levels on the time evolution of the spontaneous decay from upper level to the lower level is, and how the influence depends on the initial states.

In this paper, we investigate the influence of the third level on the time evolution of the spontaneous decay from the upper level in a cascade-type and a V-type three-level atom for two different initial states without the RWA. The two initial states are the excited states from the ground state of the bare atom and from the ground state of the whole system, respectively. The state excited from whole system ground state is more realistic, as the atom is always in the reservoir. Our study shows that both the initial states and the additional third level have great influence on the time evolution of the atom in the initial period. The third level usually results in the acceleration of the decay, that is to say, the enhancement of the QAZE.

This paper is prepared as follows: In Sec. II, we give the unitary transformation of the Hamiltonian of the system composed of a multilevel atom and vacuum reservoir. We obtain the effective Hamiltonian with considering the counter-rotating terms. In Sec. III, we introduce two kinds of ground and initial excited states in two pictures. In Sec. IV, we discuss the survival probabilities and decay rates in a cascade-type and a V-type three-level atom for the two initial states without the RWA, and Sec. V is a summary. The general formulae of the dynamic evolution can be found in Appendix.

(II) Effective Hamiltonian without RWA

The interaction between a multi-level atom and the vacuum reservoir can be described by the Hamiltonian (setting $\hbar=1$) [22],

$$H=H_0+H_1=\sum_i\omega_i\left|i\right\rangle\left\langle i\right|+\sum_{\mathbf{k}}\omega_k b_{\mathbf{k}}^+b_{\mathbf{k}}+\sum_{i,j\neq i,\mathbf{k}}g_{\mathbf{k},ij}(b_{\mathbf{k}}^++b_{\mathbf{k}})\left|i\right\rangle\left\langle j\right|,\qquad(1)$$

where ω_i is the energy of the $\left|i\right\rangle$ state, $b_{\mathbf{k}}^+$ ($b_{\mathbf{k}}$) is the creation (annihilation) operator of the **k**th mode vacuum field with frequency ω_k, and $g_{\mathbf{k},ij}$ is the coupling constant between the atomic transition ($\left|i\right\rangle\leftrightarrow\left|j\right\rangle$) and the **k**th mode vacuum field. The interaction, H_1, contains the rotating and the counter-rotating terms. Using the unitary transform [15,16] e^{iS} with $S=\sum_{i,j\neq i,\mathbf{k}}\frac{-ig_{\mathbf{k},ij}}{|\omega_{ij}|+\omega_k}(b_{\mathbf{k}}^+-b_{\mathbf{k}})\left|i\right\rangle\left\langle j\right|$, and neglecting the terms higher than the second order of $g_{\mathbf{k},ij}$ (this approximation is used throughout the paper), we obtain the transformed Hamiltonian $H^S=H_0+H_1^S+H_2^S$ with

$$H_1^S = \sum_{i,j>i,\mathbf{k}} V_{\mathbf{k},ji} \left(b_{\mathbf{k}}^{+} |i\rangle\langle j| + b_{\mathbf{k}} |j\rangle\langle i| \right), \tag{2}$$

$$H_2^S = -\sum_{i,q,j,\mathbf{k}}^{i\neq q,q\neq j} g_{\mathbf{k},iq} g_{\mathbf{k},qj} \frac{2|\omega_{qj}| - \omega_{qj} + 2|\omega_{iq}| - \omega_{qi} + 2\omega_k}{2(|\omega_{iq}| + \omega_k)(|\omega_{qj}| + \omega_k)} |i\rangle\langle j|, \tag{3}$$

where $V_{\mathbf{k},ji} = \dfrac{2g_{\mathbf{k},ji}\omega_{ji}}{\omega_{ji} + \omega_k}$. Note that H_2^S in Eq. (3) includes the non-diagonal terms with ($i \neq j$), which represents the indirect transition (virtual photon processes), emitting a photon from $|i\rangle$ (or $|j\rangle$) and reabsorbing the photon to $|j\rangle$ (or $|i\rangle$) and were neglected in [15,16]. The self-energy of the free electron [22,23] is due to its exchange of virtual photons with the vacuum, and has the following form [18-23]

$$\begin{aligned} E_{se} &= -\frac{1}{4\pi\varepsilon_0} \frac{2e^2}{3\pi m^2 c^3} \int_0^{\omega_c} \mathbf{p}^2 d\omega = -\frac{1}{4\pi\varepsilon_0} \frac{2e^2}{3\pi m^2 c^3} \int_0^{\omega_c} \sum_{i,j} \langle i|\mathbf{p}^2|j\rangle |i\rangle\langle j| d\omega \\ &= -\sum_{i,q,j,\mathbf{k}}^{q\neq i,q\neq j} \frac{g_{\mathbf{k},iq} g_{\mathbf{k},jq}}{\omega_k} |i\rangle\langle j|, \end{aligned} \tag{4}$$

with ω_c being the cutoff frequency. The self-energy should be subtracted from the Hamiltonian, as it can't be observed. With subtracting the self-energy, the Hamiltonian can be written as

$$\tilde{H}^S = \sum_i \omega_i' |i\rangle\langle i| + \sum_{\mathbf{k}} \omega_k b_{\mathbf{k}}^{+} b_{\mathbf{k}} + \sum_{i,j,\mathbf{k}}^{i<j} V_{\mathbf{k},ji} \left(b_{\mathbf{k}}^{+} |i\rangle\langle j| + b_{\mathbf{k}} |j\rangle\langle i| \right) + \sum_{i,j}^{i\neq j} \eta |i\rangle\langle j|, \tag{5}$$

where $\omega_i' = \omega_i + \Delta E_{nd}^{(i)}$ with $\Delta E_{nd}^{(i)} = \sum_{i,q\neq i,\mathbf{k}} \dfrac{g_{\mathbf{k},iq}^2}{\omega_k} \dfrac{\omega_{qi}(\omega_{qi} + \omega_k)}{(\omega_k + |\omega_{iq}|)^2}$ denoting the nondynamic shift [15-19] independent of the atomic decay process and $\eta = \sum_{q\neq i,j,\mathbf{k}} \dfrac{g_{\mathbf{k},iq} g_{\mathbf{k},qj}}{\omega_k} \dfrac{2|\omega_{iq}\omega_{qj}| + \omega_{qj}\omega_k + \omega_{qi}\omega_k}{2(|\omega_{iq}| + \omega_k)(|\omega_{qj}| + \omega_k)} |i\rangle\langle j|$ is a parameter arising from the virtual photon processes $|i\rangle \leftrightarrow |q\rangle \leftrightarrow |j\rangle$.

(III) Ground and initial excited states

When we make the unitary transform, not only the operators (e.g. the Hamiltonian H) but also the states (e.g. the initial state) are transformed. In the two pictures (before and after the unitary transformation), which are called H-picture and S-picture, respectively, they are related

$$|\varphi(t)\rangle^S = e^{iS} |\varphi(t)\rangle^H, \quad |\varphi(t)\rangle^H = e^{-iS} |\varphi(t)\rangle^S, \tag{6}$$

$$A^S = e^{iS} A^H e^{-iS}, \quad A^H = e^{-iS} A^S e^{iS}, \tag{7}$$

where the superscript H represents the picture before the transformation, and the superscript S represents the picture after the transformation. Here, we use the direct product of the atomic eigen-states ($|i\rangle$) and the modes of the reservoir ($|\{n_{\mathbf{k}}\}\rangle$) as the basis of the whole system. Let us consider the expressions of the ground states and the excited states in the two pictures.

A. The ground state of the free Hamiltonian (H_0)

The ground state of the free Hamiltonian (H_0) in H-picture is

$$\left|g^H\right\rangle = \left|g,\{0_{\mathbf{k}}\}\right\rangle. \tag{8}$$

By the unitary transformation, the expression of the above state in the S-picture, $\left|g^S\right\rangle$, is

$$\begin{aligned}\left|g^S\right\rangle &= e^{iS}\left|g^H\right\rangle = e^{iS}\left|g,\{0_{\mathbf{k}}\}\right\rangle \\ &\approx \left[1-\frac{1}{2}\sum_{i\neq g,\mathbf{k}}\frac{g^2_{\mathbf{k},ig}}{(\omega_{ig}+\omega_k)^2}\right]\left|g,\{0_{\mathbf{k}}\}\right\rangle + \sum_{i\neq g,\mathbf{k}}\frac{g_{\mathbf{k},ig}}{\omega_{ig}+\omega_k}\left|i,1_{\mathbf{k}}\right\rangle,\end{aligned} \tag{9}$$

Note that the energy of this state is much higher than that of the ground state of the whole system.

B. The ground state of the whole system (H).

The ground state of the whole system (H) in the S-picture, $\left|G^S\right\rangle$, is [15]

$$\left|G^S\right\rangle = \left|g,\{0_{\mathbf{k}}\}\right\rangle. \tag{10}$$

By the inverse unitary transformation, one can obtain the expression of the above state in the H-picture [20], $\left|G^H\right\rangle$,

$$\begin{aligned}\left|G^H\right\rangle &= e^{-iS}\left|G^S\right\rangle = e^{-iS}\left|g,\{0_{\mathbf{k}}\}\right\rangle \\ &\approx \left[1-\frac{1}{2}\sum_{i\neq g,\mathbf{k}}\frac{g^2_{\mathbf{k},ig}}{(\omega_{ig}+\omega_k)^2}\right]\left|g,\{0_{\mathbf{k}}\}\right\rangle - \sum_{i\neq g,\mathbf{k}}\frac{g_{\mathbf{k},ig}}{\omega_{ig}+\omega_k}\left|i,1_{\mathbf{k}}\right\rangle.\end{aligned} \tag{11}$$

C. Excited states from the ground states of H_0 and H.

Two excited states are used for the initial states in our calculations, which are generated by the Hermitian operator $\left(|g\rangle\langle e|+|e\rangle\langle g|+\sum_{j\neq e,g}|j\rangle\langle j|\right)$ acting on the ground states of H_0 and H, respectively. We use $\left|e^H\right\rangle$ and $\left|e^S\right\rangle$ (note little letter) as the initial state excited from the ground state of H_0 in H-picture (superscript H) and S-picture (superscript S), respectively. We use $\left|E^H\right\rangle$ and $\left|E^S\right\rangle$ (note capital letter) as the initial state excited from the ground state of H in H-picture and S-picture, respectively. Please note in the whole paper, we use little letter (*e*) for the state excited from the ground state of the bare atom (H_0) and capital letter (*E*) for that of the whole system (H), while the superscripts, H and S, are for H- and S-pictures.

In the H-picture, the excited state (using little letter *e*) from the ground state of H_0 is,

$$\left|e^H\right\rangle = \left(|g\rangle\langle e| + |e\rangle\langle g| + \sum_{j\neq e,g}|j\rangle\langle j|\right)\left|g^H\right\rangle = \left(|g\rangle\langle e| + |e\rangle\langle g|\right)\left|g,\{0_{\mathbf{k}}\}\right\rangle = \left|e,\{0_{\mathbf{k}}\}\right\rangle, \quad (12)$$

and the excited state (using capital letter *E*) from that of H is,

$$\begin{aligned}\left|E^H\right\rangle &= \left(|g\rangle\langle e| + |e\rangle\langle g| + \sum_{j\neq e,g}|j\rangle\langle j|\right)\left|G^H\right\rangle \\ &\approx \left[1 - \frac{1}{2}\sum_{\mathbf{k}}\frac{g^2_{\mathbf{k},eg}}{(\omega_{eg}+\omega_k)^2} - \frac{1}{2}\sum_{j\neq e,g,\mathbf{k}}\frac{g^2_{\mathbf{k},jg}}{(\omega_{jg}+\omega_k)^2}\right]\left|e,\{0_{\mathbf{k}}\}\right\rangle \\ &\quad - \sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}}{\omega_{eg}+\omega_k}\left|g,1_{\mathbf{k}}\right\rangle - \sum_{j\neq e,g,\mathbf{k}}\frac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k}\left|j,1_{\mathbf{k}}\right\rangle.\end{aligned} \quad (13)$$

In the S-picture, the corresponding states of Eqs. (12)-(13) are,

$$\begin{aligned}\left|e^S\right\rangle &= e^{iS}\left|e^H\right\rangle = e^{iS}\left|e,\{0_{\mathbf{k}}\}\right\rangle = \left[1 + iS + \frac{1}{2}(iS)^2 + \cdots\right]\left|e,\{0_{\mathbf{k}}\}\right\rangle \\ &\approx \left[1 - \frac{1}{2}\sum_{\mathbf{k}}\frac{g^2_{\mathbf{k},eg}}{(\omega_{eg}+\omega_k)^2} - \frac{1}{2}\sum_{j\neq e,g,\mathbf{k}}\frac{g^2_{\mathbf{k},ej}}{(|\omega_{ej}|+\omega_k)^2}\right]\left|e,\{0_{\mathbf{k}}\}\right\rangle \\ &\quad + \sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}}{\omega_{eg}+\omega_k}\left|g,1_{\mathbf{k}}\right\rangle + \sum_{j\neq e,g,\mathbf{k}}\frac{g_{\mathbf{k},ej}}{|\omega_{ej}|+\omega_k}\left|j,1_{\mathbf{k}}\right\rangle,\end{aligned} \quad (14)$$

and

$$\begin{aligned}\left|E^S\right\rangle = e^{iS}\left|E^H\right\rangle &\approx \left[1 - \frac{1}{2}\sum_{j\neq e,g,\mathbf{k}}\left(\frac{g_{\mathbf{k},ej}}{|\omega_{ej}|+\omega_k} - \frac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k}\right)^2\right]\left|e,\{0_{\mathbf{k}}\}\right\rangle \\ &\quad + \sum_{j\neq e,g,\mathbf{k}}\left(\frac{g_{\mathbf{k},ej}}{|\omega_{ej}|+\omega_k} - \frac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k}\right)\left|j,1_{\mathbf{k}}\right\rangle.\end{aligned} \quad (15)$$

Because the atom is always in the vacuum, the initial state excited from the ground state of the whole system, $\left|E^{H,S}\right\rangle$ is easy to obtain. The initial state excited from the ground state of the bare atom, $\left|e^{H,S}\right\rangle$ is difficult to be generated, because we don't know how to have the ground state of the bare atom when it is always in the cavity or we need a difficult method of injecting the excited atoms into the cavity. Note the energy of $\left|e^{H,S}\right\rangle$ is much higher than the energy of

$\left|E^{H,S}\right\rangle$.

(IV) Time evolution of survival probability and time-dependent decay rate

Next we investigate the dynamic evolution of the multi-level atom. We consider the survival probabilities $P^{(E)}(t)$ and $P^{(e)}(t)$, and the corresponding time-dependent effective decay rates $\gamma^{(E)}(t)$ and $\gamma^{(e)}(t)$ for the two initial states. Here the superscripts (*E*) and (*e*) indicate the initial states $\left|E^{S}\right\rangle$ and $\left|e^{S}\right\rangle$ in the S-picture, respectively. Note the probabilities and effective decay rates are the same in S- and H-pictures for the same initial state. The wave function at time t is determined by the Schrödinger equation, which is

$$\begin{aligned}\left|\varphi(t)\right\rangle^{S} &= \alpha(t)e^{-i\omega'_{eg}t}\left|e,\{0_{\mathbf{k}}\}\right\rangle + \beta(t)\left|g,\{0_{\mathbf{k}}\}\right\rangle \\ &+ \sum_{j\neq e,g} C_j(t)e^{-i\omega'_{jg}t}\left|j,\{0_{\mathbf{k}}\}\right\rangle + \sum_{\mathbf{k}}\alpha_{\mathbf{k}}(t)e^{-i(\omega'_{eg}+\omega_k)t}\left|e,1_{\mathbf{k}}\right\rangle. \\ &+ \sum_{\mathbf{k}}\beta_{\mathbf{k}}(t)e^{-i\omega_k t}\left|g,1_{\mathbf{k}}\right\rangle + \sum_{j\neq e,g,\mathbf{k}} C_{j,\mathbf{k}}(t)e^{-i(\omega'_{jg}+\omega_k)t}\left|j,1_{\mathbf{k}}\right\rangle\end{aligned} \tag{16}$$

The detailed derivation is shown in the Appendix.

The survival probability of the initial state $\left|E^{H}\right\rangle$ ($\left|E^{S}\right\rangle$) is

$$\begin{aligned}P^{(E)}(t) &= \left|\left\langle\varphi^{H}(0)\middle|\varphi^{H}(t)\right\rangle\right|^2 = \left|\left\langle\varphi^{S}(0)\middle|\varphi^{S}(t)\right\rangle\right|^2 = \exp[-\gamma^{(E)}(t)t] \\ &\approx \frac{1}{A}\left\{\left|\alpha(0)\right|^2\left|\alpha(t)\right|^2 + \sum_{j\neq e,g,\mathbf{k}} 2\,\mathrm{Re}\left[\alpha(0)\alpha^*(t)C^*_{j,\mathbf{k}}(0)C_{j,\mathbf{k}}(t)e^{i(\omega'_{ej}-\omega_k)t}\right]\right\},\end{aligned} \tag{17}$$

where A is the normalization factor (note the superscript *E* for the whole system). Note the second term in the last line of Eq. (17) originates from the correlation between $\left|e,\{0_{\mathbf{k}}\}\right\rangle$ and $\left|j,1_{\mathbf{k}}\right\rangle$ and the superscript *e* for the bare atom. The survival probability of the initial state $\left|e^{H}\right\rangle$ ($\left|e^{S}\right\rangle$) is,

$$\begin{aligned}P^{(e)}(t) &= \exp[-\gamma^{(e)}(t)t] \\ &\approx \frac{1}{A}\left\{\left|\alpha(0)\right|^2\left|\alpha(t)\right|^2 + \sum_{j\neq e,g,\mathbf{k}} 2\,\mathrm{Re}\left[\alpha(0)\alpha^*(t)C^*_{j,\mathbf{k}}(0)C_{j,\mathbf{k}}(t)e^{i(\omega'_{ej}-\omega_k)t}\right]\right. \\ &\qquad \left. + \sum_{\mathbf{k}} 2\,\mathrm{Re}\left[\alpha(0)\alpha^*(t)\beta^*_{\mathbf{k}}(0)\beta_{\mathbf{k}}(t)e^{i(\omega'_{eg}-\omega_k)t}\right]\right\}.\end{aligned} \tag{18}$$

In above, the j summation is over all levels except e and g. In the following, we consider the influence of the additional levels, for simplicity, with two configurations.

A. Cascade-type configuration ($E_g < E_e < E_j$)

In this case, the two electric dipole allowed transitions are $e \leftrightarrow g$ and $e \leftrightarrow j$, while the transition between j and g is forbidden ($g_{\mathbf{k},jg} = 0$), see Fig. 1.

(A1) The evolution and effective decay rate for the excited state $\left|E^S\right\rangle$ ($\left|E^H\right\rangle$)

In the cascade-type, the initial state can be written, from Eq. (15), as

$$\left|E^S\right\rangle = \left[1 - \frac{1}{2}\sum_{\mathbf{k}} \frac{g_{\mathbf{k},je}^2}{(\omega_{je}+\omega_k)^2}\right]\left|e,\{0_{\mathbf{k}}\}\right\rangle + \sum_{\mathbf{k}} \frac{g_{\mathbf{k},je}}{\omega_{je}+\omega_k}\left|j,1_{\mathbf{k}}\right\rangle. \tag{19}$$

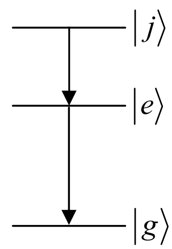

Fig.1.Cascade configuration

The coefficients of the state evolution in Eq. (16) can be obtained as

$$\alpha(t) = \alpha(0)\exp[-\gamma_e^S(t)t/2 - i\Delta E_{dyn}^{S(e)}(t)t], \tag{20}$$

and $C_{j,\mathbf{k}}(t) = C_{j,\mathbf{k}}(0)$ with the initial values

$$\alpha(0) = 1 - \frac{1}{2}\sum_{\mathbf{k}} \frac{g_{\mathbf{k},je}^2}{(\omega_{je}+\omega_k)^2}, \tag{21a}$$

$$C_{j,\mathbf{k}}(0) = \frac{g_{\mathbf{k},je}}{\omega_{je}+\omega_k}. \tag{21b}$$

In Eq. (20) $\gamma_e^S(t)$ and $\Delta E_{dyn}^{S(e)}(t)$ are the time dependent decay rate from the state $\left|e\right\rangle$ to the state $\left|g\right\rangle$ and the time dependent shift of the state $\left|e\right\rangle$ without the third level [19,20], respectively,

$$\gamma_e^S(t) = 2\pi\sum_{\mathbf{k}} \frac{4\omega_{eg}^2 g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2} \frac{2\sin^2\left(\frac{\omega_{eg}-\omega_k}{2}t\right)}{\pi(\omega_{eg}-\omega_k)^2 t} \tag{22a}$$

$$\Delta E_{dyn}^{S(e)}(t)=\sum_{\mathbf{k}}\frac{4\omega_{eg}^2 g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}\left\{\frac{1}{\omega_{eg}-\omega_k}-\frac{\sin[(\omega_{eg}-\omega_k)t]}{(\omega_{eg}-\omega_k)^2 t}\right\},. \tag{22b}$$

The detailed derivation of Eqs. (20)-(22) can be found in the Appendix, see Eqs. (A9)-(A12). Substituting Eqs. (20)-(22) into Eq. (17), for a very short time we have $\exp[-\gamma(t)t]\approx 1-\gamma(t)t$, and the survival probability of the initial state,

$$\begin{aligned}P^{(E)}(t)&=\exp[-\gamma^{(E)}(t)t]\\&\approx\frac{1}{A}|\alpha(0)|^4\exp\left\{-\gamma_e^S(t)t+\sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\cos[(\omega_k+\omega'_{je})t]\right\},\end{aligned} \tag{23}$$

where $A=|\alpha(0)|^4\exp\left[\sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\right]$. Accordingly, the effective decay rate of the initial state $\left|E^H\right\rangle$ ($\left|E^S\right\rangle$) in the cascade configuration is,

$$\begin{aligned}\gamma^{(E)}(t)&=\gamma_e^S(t)+\sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\frac{1-\cos[(\omega_k+\omega_{je})t]}{t}\\&\approx\gamma_e^S(t)+2\pi\sum_{\mathbf{k}}g_{\mathbf{k},je}^2\frac{2\sin^2\left(\frac{\omega_k+\omega_{je}}{2}t\right)}{\pi(\omega_{je}+\omega_k)^2 t}\\&\approx 2\pi\int_0^\infty G'_{eg}(\omega)F(\omega-\omega_{eg},t)d\omega+2\pi\int_0^\infty G_{je}(\omega)F(\omega+\omega_{je},t)d\omega.\end{aligned} \tag{24}$$

Here the approximation of $\omega'_{eg(je)}\approx\omega_{eg(je)}$ (always in the paper) has been used as the difference between them is proportional to the square of the coupling constant, and

$$F(\omega,t)=\frac{2\sin^2(\omega t/2)}{\pi\omega^2 t}, \tag{25}$$

$$G_{je}(\omega)=\sum_{\mathbf{k}}g_{\mathbf{k},je}^2\delta(\omega-\omega_k), \tag{26}$$

$$G'_{eg}(\omega)=\sum_{\mathbf{k}}\frac{4\omega_{eg}^2}{(\omega_{eg}+\omega_k)^2}g_{\mathbf{k},eg}^2\delta(\omega-\omega_k), \tag{27}$$

In Fig. 2, we plot $\gamma^{(E)}(t)$ in the red dashed curve, where we see both the QZE and QAZE. The first term in Eq. (24), $\gamma_e^S(t)$, is just the effective decay rate without the third level, which results in only the QZE [20]. Therefore, the QAZE comes from the second term in Eq. (24) due to the third level, which is dependent on the initial condition $|C_{j,\mathbf{k}}(0)|^2/|\alpha(0)|^2$ and the frequencies ω_k and ω_{je}. In order to identify the effect of level $\left|j\right\rangle$, we plot the decay rates for different coupling strength of γ_{je} in Fig. 3 (see the solid curves), which tells that the larger γ_{je} is the stronger the influence of level $|j\rangle$ will be. It is clear that level $|j\rangle$ leads to quite different dynamic evolution. The great influence of the third level cannot be neglected for the dynamic evolution of the atom in the short time region.

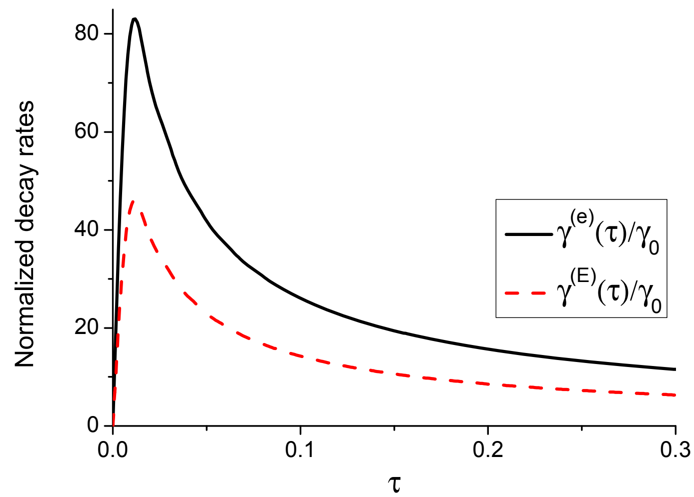

Fig.2 The normalized decay rates vs time $\tau=\omega_{eg}t$ for different initial states in the cascade configuration with $\gamma_{je} = 8\times10^{-7}\omega_{eg}$, $\gamma_{eg} = 6.4\times10^{-7}\omega_{eg}$, $\gamma_{jg} = 0$, $\omega_{je} = 0.3\omega_{eg}$. Here γ_0 is the long time decay rate in the free vacuum for a two-level atom. Note $\omega_{eg} = 1.55\times10^{16}$ rad/s and $\gamma_0 = 6.26\times10^{8}$ rad/s for the 2p-1s transition of the hydrogen atom.

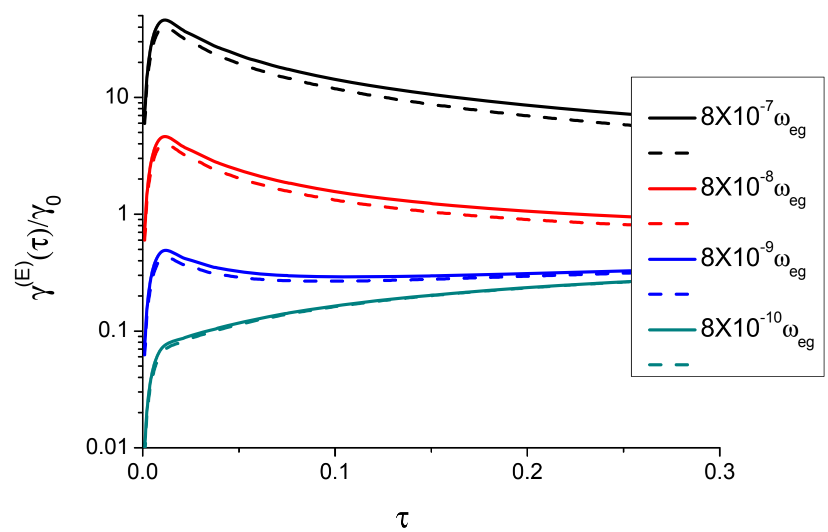

Fig.3. The normalized decay rates $\gamma^{(E)}(\tau)/\gamma_0$ vs time τ with $\gamma_{eg} = 6.4\times10^{-7}\omega_{eg}$ in the cascade type (solid lines) with $\omega_{je} = 0.3\omega_{eg}$ for different coupling

strength $\gamma_{je} = 8\times(10^{-10}–10^{-7})\omega_{eg}$, and in the V-type (dashed curves) with $\omega_{jg} = 0.7\omega_{eg}$, $\omega_{ej} = 0.3\omega_{eg}$ for different coupling strength $\gamma_{jg} = 8\times(10^{-10}–10^{-7})\omega_{eg}$.

(A2) The initial state $|e^S\rangle$ ($|e^H\rangle$) (excited from the ground state of the bare atom)

In this case, the initial state, Eq. (14), becomes

$$|e^S\rangle = \left[1-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},je}^2}{(\omega_{je}+\omega_k)^2}\right]|e,\{0_{\mathbf{k}}\}\rangle + \sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}}{\omega_{eg}+\omega_k}|g,1_{\mathbf{k}}\rangle+\sum_{\mathbf{k}}\frac{g_{\mathbf{k},je}}{\omega_{je}+\omega_k}|j,1_{\mathbf{k}}\rangle. \tag{28}$$

The survival probability of this initial state is determined by Eq. (18). Although $\alpha(t)$ in Eq. (18) has the same form as in Eq. (20), $\alpha(t)=\alpha(0)\exp[-\gamma_e^S(t)t/2-i\Delta E_{dyn}^{S(e)}(t)t]$ (see Eq. (A12)), its initial value $\alpha(0)$ is different,

$$\alpha(0)=1-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},je}^2}{(\omega_{je}+\omega_k)^2}, \tag{29}$$

and $\gamma_e^S(t)$ and $\Delta E_{dyn}^{S(e)}(t)$ are

$$\gamma_e^S(t)\approx 2\pi\sum_{\mathbf{k}}\frac{2\omega_{eg}g_{\mathbf{k},eg}^2}{\omega_{eg}+\omega_k}\frac{2\sin^2\left(\frac{\omega_{eg}-\omega_k}{2}t\right)}{\pi(\omega_{eg}-\omega_k)^2 t}, \tag{30a}$$

$$\Delta E_{dyn}^{S(e)}(t)=\sum_{\mathbf{k}}\frac{4\omega_{eg}^2 g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}\left\{\frac{1}{\omega_{eg}-\omega_k}-\left(1-\frac{\omega_{eg}-\omega_k}{2\omega_{eg}}\right)\frac{\sin[(\omega_{eg}-\omega_k)t]}{(\omega'_{eg}-\omega_k)^2 t}\right\}. \tag{30b}$$

which are different from that in Eqs. (22a) and (22b) because of the different initial state $|E^S\rangle$ in Eq. (19). The $\beta_{\mathbf{k}}(t)$ in Eq. (18) is (see Eq. (A13)),

$$\beta_{\mathbf{k}}(t)\approx\beta_{\mathbf{k}}(0)-\alpha(0)V_{\mathbf{k},eg}\frac{1-e^{-i(\omega_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)}, \tag{31}$$

with the initial value $\beta_{\mathbf{k}}(0)=\frac{g_{\mathbf{k},eg}}{\omega_{eg}+\omega_k}$. The survival probability of the initial state $|e^H\rangle$ ($|e^S\rangle$) is

$$
\begin{aligned}
P^{(e)}(t) &= \exp[-\gamma^{(e)}(t)t] \\
&\approx \frac{|\alpha(0)|^4}{A}\exp\left\{-\gamma_e^S(t)t + \sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\cos[(\omega_k+\omega_{je})t]\right. \\
&\left.\quad + \sum_{\mathbf{k}}\frac{2|\beta_{\mathbf{k}}(0)|^2}{|\alpha(0)|^2} - \sum_{\mathbf{k}} 2\frac{\beta_{\mathbf{k}}^*(0)}{\alpha^*(0)}\left[\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} - \frac{V_{\mathbf{k},eg}}{\omega'_{eg}-\omega_k}\right]\{1-\cos[(\omega_{eg}-\omega_k)t]\}\right\}
\end{aligned} \tag{32}
$$

where $A = |\alpha(0)|^4 \exp\left[\sum_{\mathbf{k}}\frac{2|\beta_{\mathbf{k}}(0)|^2}{|\alpha(0)|^2} + \sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\right]$. The corresponding time-dependent decay rate is

$$
\begin{aligned}
\gamma^{(e)}(t) &= \gamma_e^S(t) + \sum_{\mathbf{k}}\frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\frac{1-\cos[(\omega_k+\omega_{je})t]}{t} \\
&\quad + \sum_{\mathbf{k}} 2\frac{\beta_{\mathbf{k}}^*(0)}{\alpha^*(0)}\left[\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} - \frac{V_{\mathbf{k},eg}}{\omega'_{eg}-\omega_k}\right]\frac{1-\cos[(\omega_{eg}-\omega_k)t]}{t} \\
&\approx 2\pi\sum_{\mathbf{k}} g_{\mathbf{k},eg}^2 \frac{2\sin^2\left(\frac{\omega_{eg}-\omega_k}{2}t\right)}{\pi(\omega_{eg}-\omega_k)^2 t} + 2\pi\sum_{\mathbf{k}} g_{\mathbf{k},je}^2 \frac{2\sin^2\left(\frac{\omega_{je}+\omega_k}{2}t\right)}{\pi(\omega_{je}+\omega_k)^2 t} \\
&= 2\pi\int_0^\infty G_{eg}(\omega)F(\omega-\omega_{eg},t)d\omega + 2\pi\int_0^\infty G_{je}(\omega)F(\omega+\omega_{je},t)d\omega,
\end{aligned} \tag{33}
$$

where $F(\omega,t)$ and $G_{je}(\omega)$ are defined by Eq. (25) and Eq. (26), and

$$
G_{eg}(\omega) = \sum_{\mathbf{k}} g_{\mathbf{k},eg}^2 \delta(\omega-\omega_k). \tag{34}
$$

In Fig. 2, $\gamma^{(e)}(t)$ is plotted in the black solid curve. The first term in the last equality of Eq. (33) is the decay rate of a two-level atom with RWA [20, 23-25], which results in the QAZE. The second term in Eq. (33) is the same as the second term in Eq. (24), which enhances the QAZE as shown in the black curve. We would like to mention that for the lambda configuration ($|j\rangle$ below $|e\rangle$), its dynamic evolution is similar to the cascade one discussed above.

B. V-type configuration ($E_g < E_j, E_e$)

In this case, the two allowed electric dipole transitions are $e \leftrightarrow g$ and $j \leftrightarrow g$, and the transition between e and j is forbidden ($g_{\mathbf{k},ej} = 0$), see Fig. 4.

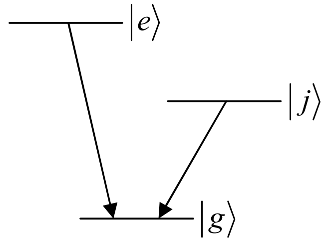

Fig.4. V-configuration

(B1) The initial state $|E^S\rangle$ ($|E^H\rangle$) (excited from the ground state of the whole system)

In this V-type, the initial state van be written from Eq. (15) as

$$|E^S\rangle = \left[1 - \frac{1}{2}\sum_{\mathbf{k}} \frac{g_{\mathbf{k},jg}^2}{(\omega_{jg}+\omega_k)^2}\right]|e,\{0_{\mathbf{k}}\}\rangle - \sum_{\mathbf{k}} \frac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k}|j,1_{\mathbf{k}}\rangle. \tag{35}$$

The survival probability of the initial state is determined by Eq. (17), where $\alpha(t)$ has the same form as Eq. (20), $\alpha(t)=\alpha(0)\exp[-\gamma_e^S(t)t/2 - i\Delta E_{dyn}^{S(e)}(t)t]$ with $\gamma_e^S(t)$ and $\Delta E_{dyn}^{S(e)}(t)$ determined by Eq. (22a) and (22b), and $\alpha(0)$

$$\alpha(0) = 1 - \frac{1}{2}\sum_{\mathbf{k}} \frac{g_{\mathbf{k},jg}^2}{(\omega_{jg}+\omega_k)^2}, \tag{36}$$

and $C_{j,\mathbf{k}}(t) = C_{j,\mathbf{k}}(0) = -\dfrac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k}$. See Eqs. (A21)-(A28) in the Appendix for details. The survival probability of the initial state $|E^H\rangle$ ($|E^S\rangle$) is

$$\begin{aligned} P^{(E)}(t) &= \exp[-\gamma^{(E)}(t)t] \\ &\approx \frac{|\alpha(0)|^4}{A}\exp\left\{-\gamma_e^S(t)t + \sum_{\mathbf{k}} \frac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\cos[(\omega_{ej}-\omega_k)t]\right\}, \end{aligned} \tag{37}$$

where $A = |\alpha(0)|^4 \exp\left[\sum_{\mathbf{k}} \dfrac{2|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\right]$. Accordingly, the effective time-dependent decay rate of the initial state $|E^H\rangle$ ($|E^S\rangle$) is

$$\gamma^{(E)}(t)=\gamma_e^S(t)+\sum_{\mathbf{k}}\frac{2\,|C_{j,\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\frac{1-\cos[(\omega_{ej}-\omega_k)t]}{t}$$
$$\approx 2\pi\sum_{\mathbf{k}}\frac{4\omega_{eg}^2 g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}\frac{2\sin^2\left(\frac{\omega_{eg}-\omega_k}{2}t\right)}{\pi(\omega_{eg}-\omega_k)^2 t}$$
$$+2\pi\sum_{\mathbf{k}}\frac{(\omega_{ej}-\omega_k)^2}{(\omega_{jg}+\omega_k)^2}g_{\mathbf{k},jg}^2\frac{2\sin^2\left(\frac{\omega_{ej}-\omega_k}{2}t\right)}{\pi(\omega_{ej}-\omega_k)^2 t} \tag{38}$$
$$\approx 2\pi\int_0^\infty G'_{eg}(\omega)F(\omega-\omega_{eg},t)d\omega+2\pi\int_0^\infty G'_{jg}(\omega)F(\omega-\omega_{ej},t)d\omega,$$

where $F(\omega,t)$ and $G'_{eg}(\omega)$ are the same as Eq. (25) and Eq. (27), and

$$G'_{jg}(\omega)=\sum_{\mathbf{k}}\frac{(\omega_{ej}-\omega_k)^2}{(\omega_{jg}+\omega_k)^2}g_{\mathbf{k},jg}^2\delta(\omega-\omega_k). \tag{39}$$

In Fig. 5, $\gamma^{(E)}(t)$ is plotted in the red dashed curve, where we have QAZE. The first term in Eq. (38), $\gamma_e^S(t)$, results in the QZE. Therefore, the QAZE arises from the second term in Eq. (38), that is to say from level $|j\rangle$.

(B2) The initial state $|e^S\rangle$ ($|e^H\rangle$) excited from the bare atom ground state.

In this case, the initial state, Eq. (14), becomes

$$|e^S\rangle=\left[1-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}\right]|e,\{0_{\mathbf{k}}\}\rangle+\sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}}{\omega_{eg}+\omega_k}|g,1_{\mathbf{k}}\rangle. \tag{40}$$

The survival probability of the initial state is determined by Eq. (18), with $\gamma_e^S(t)$ and $\Delta E_{dyn}^{S(e)}(t)$ determined by Eq. (30a) and (30b), $\beta_{\mathbf{k}}(t)$ is the same as Eq. (31) (see Eq. (A31)), and

$$\alpha(0)=1-\frac{1}{2}\sum_{\mathbf{k}}\frac{g_{\mathbf{k},eg}^2}{(\omega_{eg}+\omega_k)^2}, \tag{41}$$

and $C_{j,\mathbf{k}}(t)=C_{j,\mathbf{k}}(0)=0$. Therefore, the survival probability of the initial state $|e^H\rangle$ ($|e^S\rangle$) is

$$P^{(e)}(t) = \exp[-\gamma^{(e)}(t)t]$$

$$\approx \frac{|\alpha(0)|^4}{A}\exp\left\{-\gamma_e^S(t)t + \sum_{\mathbf{k}} 2\frac{|\beta_{\mathbf{k}}(0)|^2}{|\alpha(0)|^2} - \sum_{\mathbf{k}} 2\frac{\beta_{\mathbf{k}}^*(0)}{\alpha^*(0)}\left[\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} - \frac{V_{\mathbf{k},eg}}{\omega'_{eg}-\omega_k}\right]\{1-\cos[(\omega'_{eg}-\omega_k)t]\}\right\}, \tag{42}$$

where $A = |\alpha(0)|^4 \exp\left[\sum_{\mathbf{k}} 2\frac{|\beta_{\mathbf{k}}(0)|^2}{|\alpha(0)|^2}\right]$. The corresponding time-dependent decay rate is

$$\gamma^{(e)}(t) = \gamma_e^S(t) + \sum_{\mathbf{k}} 2\frac{\beta_{\mathbf{k}}^*(0)}{\alpha^*(0)}\left[\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} - \frac{V_{\mathbf{k},eg}}{\omega'_{eg}-\omega_k}\right]\frac{1-\cos[(\omega'_{eg}-\omega_k)t]}{t}$$

$$\approx 2\pi\sum_{\mathbf{k}} g_{\mathbf{k},eg}^2 \frac{2\sin^2\left(\frac{\omega_{eg}-\omega_k}{2}t\right)}{\pi(\omega_{eg}-\omega_k)^2 t} \tag{43}$$

$$= 2\pi\int_0^\infty G_{eg}(\omega)F(\omega-\omega_{eg},t)d\omega.$$

where $F(\omega,t)$ and $G_{eg}(\omega)$ are defined by Eq. (25) and Eq. (34).

In Fig. 5, $\gamma^{(e)}(t)$ is plotted in the black curve. It is seen from Eq. (43) that $\gamma^{(e)}(t)$ is independent of $|j\rangle$ and is the same as that in a two-level system under the RWA, which is known to lead to the QAZE [20,23-25].

The influence of the third level on the dynamic evolution is plotted in Fig. 3 with the dashed curves. It is clear that influence of the third level is great in the initial short period. For long time all the curves in Fig. 3 will approach one.

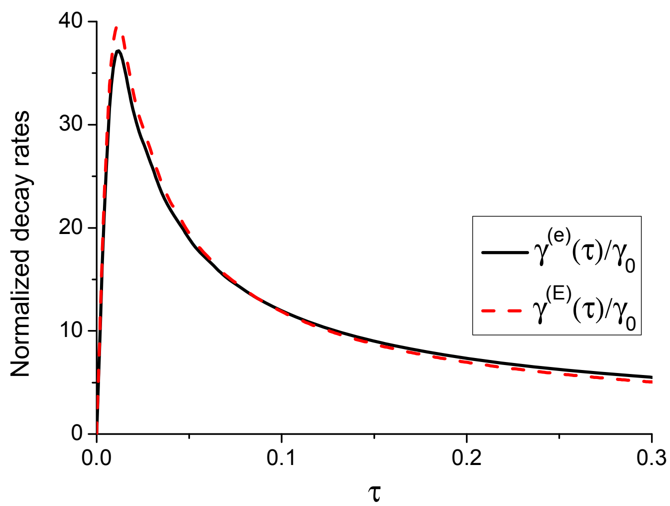

Fig. 5 The normalized decay rates vs time τ for different initial states in V

configuration with γ_{eg} = 6.4×$10^{-7}\omega_{eg}$, γ_{jg} =8×$10^{-7}\omega_{eg}$, ω_{ej} = 0.3ω_{eg}, ω_{jg} = 0.7ω_{eg}.

(V). Conclusion

We have studied the important influence of the third level on the time evolution of the spontaneous decay in the cascade-type or V-type atom with two different initial states, when the counter-rotating terms and the self-energy are taken into account in the Hamiltonian. The third level results in the acceleration of the effective decay in the short-time regime, which leads to or enhances the anti-Zeno effect, because of the re-absorption of the photon by one level emitted from another level (virtual photon processes). This influence is sensitive to the third level, but not sensitive to the initial states discussed. The third level could not be neglected, if the counter rotating terms are taken into account.

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APPENDIX: DYNAMIC EVOLUTION AND THE SOLUTIONS

In the interaction picture, the interaction Hamiltonian in Eq. (5) reads as

$$\tilde{H}_I^S = \sum_{i,j,\mathbf{k}}^{i<j} V_{\mathbf{k},ji} e^{i(\omega'_{ji}-\omega_k)t} b_{\mathbf{k}} |j\rangle\langle i| + \sum_{i,j,\mathbf{k}}^{i<j} V_{\mathbf{k},ji} e^{-i(\omega'_{ji}-\omega_k)t} b_{\mathbf{k}}^{+} |i\rangle\langle j| + \sum_{i,j}^{i\neq j} \eta e^{i\omega'_{ij}t} |i\rangle\langle j|, \quad \text{(A1)}$$

where $\eta = \sum_{q\neq i,j,\mathbf{k}} \frac{g_{\mathbf{k},iq} g_{\mathbf{k},qj}}{\omega_k} \frac{2|\omega_{iq}\omega_{qj}| + \omega_{qj}\omega_k + \omega_{qi}\omega_k}{2(|\omega_{iq}|+\omega_k)(|\omega_{qj}|+\omega_k)} |i\rangle\langle j|$. The time evolution of the wave function,

$$\begin{aligned} |\varphi(t)\rangle_I^S &= \alpha(t)|e,\{0_{\mathbf{k}}\}\rangle + \beta(t)|g,\{0_{\mathbf{k}}\}\rangle + \sum_{j\neq e,g} C_j(t)|j,\{0_{\mathbf{k}}\}\rangle \\ &\quad + \sum_{\mathbf{k}} \alpha_{\mathbf{k}}(t)|e,1_{\mathbf{k}}\rangle + \sum_{\mathbf{k}} \beta_{\mathbf{k}}(t)|g,1_{\mathbf{k}}\rangle + \sum_{j\neq e,g,\mathbf{k}} C_{j,\mathbf{k}}(t)|j,1_{\mathbf{k}}\rangle \end{aligned} \quad \text{(A2)}$$

is governed by the Hamiltonian $\tilde{H}_I^S$. Note that the relation between the Schrödinger picture and the interaction picture is $|\varphi(t)\rangle^S = \exp(-iH_0^S t)|\varphi(t)\rangle_I^S$.

1) Cascade configuration ($E_g < E_e < E_j$)

The equations of motion are

$$\dot{\alpha}(t) = -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(t) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t}, \tag{A3}$$

$$\dot{\beta}(t) = -i\sum_{j>e} C_j(t)\eta e^{-i\omega'_{jg}t}, \tag{A4}$$

$$\dot{C}_j(t) = -i\sum_{\mathbf{k}} \alpha_{\mathbf{k}}(t) V_{\mathbf{k},je} e^{i(\omega'_{je}-\omega_k)t} - i\beta(t)\eta e^{i\omega'_{jg}t}, \tag{A5}$$

$$\dot{\alpha}_{\mathbf{k}}(t) = -i\sum_{j>e} C_j(t) V_{\mathbf{k},je} e^{-i(\omega'_{je}-\omega_k)t}, \tag{A6}$$

$$\dot{\beta}_{\mathbf{k}}(t) = -i\alpha(t) V_{\mathbf{k},eg} e^{-i(\omega'_{eg}-\omega_k)t}, \tag{A7}$$

$$\dot{C}_{j,\mathbf{k}}(t) = 0. \tag{A8}$$

From Eq. (A8) we can get,

$$C_{j,\mathbf{k}}(t) = C_{j,\mathbf{k}}(0). \tag{A9}$$

Integrating Eq. (A7),

$$\beta_{\mathbf{k}}(t) = \beta_{\mathbf{k}}(0) - i\int_0^t \alpha(t') V_{\mathbf{k},eg} e^{-i(\omega'_{eg}-\omega_k)t'} dt', \tag{A10}$$

and then inserting into Eq. (A3), we get

$$\dot{\alpha}(t) = -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t} - \sum_{\mathbf{k}} \int_0^t dt' \alpha(t') V_{\mathbf{k},eg}^2 e^{i(\omega'_{eg}-\omega_k)(t-t')}. \tag{A11}$$

For a sufficient short time t, we can replace $\alpha(t')$ by $\alpha(0)$ [11,21] and

$$\begin{aligned}
\alpha(t) &\approx \alpha(0) - \alpha(0)\int_0^t dt' \int_0^{t'} dt'' \sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 e^{i(\omega'_{eg}-\omega_k)(t'-t'')} - i\int_0^t dt' \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t'} \\
&= \alpha(0) - \alpha(0)\int_0^t dt'(t-t') \sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 e^{i(\omega'_{eg}-\omega_k)t'} - i\int_0^t dt' \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t'} \\
&\approx \alpha(0)\exp\left\{-t\left\{-\frac{1}{t}\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left\{\left[1-\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right]\frac{e^{i(\omega'_{eg}-\omega_k)t}-1}{(\omega'_{eg}-\omega_k)^2} - \frac{it}{\omega'_{eg}-\omega_k}\right\}\right\}\right. \\
&= \alpha(0)\exp\left\{-t\left\{\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left[1-\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right]\frac{2\sin^2\left(\frac{\omega'_{eg}-\omega_k}{2}t\right)}{(\omega'_{eg}-\omega_k)^2 t}\right.\right. \\
&\qquad \left.\left. + i\left\{\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left\{\frac{1}{\omega'_{eg}-\omega_k} - \left[1-\frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right]\frac{\sin[(\omega'_{eg}-\omega_k)t]}{(\omega'_{eg}-\omega_k)^2 t}\right\}\right\}\right\} \\
&= \alpha(0)\exp\left\{-t[\gamma_e^S(t)/2 + i\Delta E_{dyn}^{S(e)}(t)]\right\}.
\end{aligned} \tag{A12}$$

The solutions of $\beta_{\mathbf{k}}(t)$ is obtained by replacing $\alpha(t')$ with $\alpha(0)$ in Eq. (A10) [21],

$$\beta_{\mathbf{k}}(t) \approx \beta_{\mathbf{k}}(0) - \alpha(0) V_{\mathbf{k},eg} \frac{1-e^{-i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)}. \tag{A13}$$

Integrating Eqs. (A4) and (A6), we have

$$\beta(t) = -i\int_0^t dt' \sum_{j>e} C_j(t')\eta e^{-i\omega'_{jg}t'}, \tag{A14}$$

$$\alpha_{\mathbf{k}}(t) = -i\int_0^t dt' \sum_{j>e} C_j(t') V_{\mathbf{k},je} e^{-i(\omega'_{je}-\omega_k)t'}, \tag{A15}$$

where $\beta(0) \approx 0$, $\alpha_{\mathbf{k}}(0) \approx 0$ have been used. By substituting Eqs. (A14) and (A15) into Eq. (A5), one can easily prove with neglecting the higher order terms

$$\beta(t) = \alpha_{\mathbf{k}}(t) = C_j(t) \approx 0. \tag{A16}$$

2) V configuration ($E_g < E_j, E_e$)

$$\dot{\alpha}(t) = -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(t) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t} - i\sum_{j\neq e,g} C_j(t)\eta e^{i\omega'_{ej}t}, \tag{A17}$$

$$\dot{C}_j(t) = -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(t) V_{\mathbf{k},jg} e^{i(\omega'_{jg}-\omega_k)t} - i\alpha(t)\eta e^{i\omega'_{je}t}, \tag{A18}$$

$$\dot{\beta}_{\mathbf{k}}(t) = -i\alpha(t) V_{\mathbf{k},eg} e^{-i(\omega'_{eg}-\omega_k)t} - i\sum_{j\neq e,g} C_j(t) V_{\mathbf{k},jg} e^{-i(\omega'_{jg}-\omega_k)t}, \tag{A19}$$

$$\dot{\beta}(t) = 0,\ \dot{\alpha}_{\mathbf{k}}(t) = 0,\ \text{ and }\ \dot{C}_{j,\mathbf{k}}(t) = 0. \tag{A20}$$

From Eq. (A20), we can get

$$\beta(t) = \alpha_{\mathbf{k}}(t) = 0,\ \text{and}\ C_{j,\mathbf{k}}(t) = C_{j,\mathbf{k}}(0) = -\frac{g_{\mathbf{k},jg}}{\omega_{jg}+\omega_k} \tag{A21}$$

Integrating Eq. (A20),

$$\beta_{\mathbf{k}}(t) = \beta_{\mathbf{k}}(0) - i\int_0^t dt' \alpha(t') V_{\mathbf{k},eg} e^{-i(\omega'_{eg}-\omega_k)t'} - i\sum_{j\neq e,g}\int_0^t dt' C_j(t') V_{\mathbf{k},jg} e^{-i(\omega'_{jg}-\omega_k)t'}, \tag{A22}$$

then substituting into Eqs. (A17) and (A18), one has

$$\begin{aligned}\dot{\alpha}(t) = &-i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t} - \sum_{\mathbf{k}}\int_0^t dt' \alpha(t') V_{\mathbf{k},eg}^2 e^{i(\omega'_{eg}-\omega_k)(t-t')} \\ &- \sum_{j\neq e,g,\mathbf{k}}\int_0^t dt' C_j(t') V_{\mathbf{k},eg} V_{\mathbf{k},jg} e^{i\omega'_{ej}t} e^{i(\omega'_{jg}-\omega_k)(t-t')} - i\sum_{j\neq e,g} C_j(t)\eta e^{i\omega'_{ej}t},\end{aligned} \tag{A23}$$

$$\begin{aligned}\dot{C}_j(t) = &-i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},jg} e^{i(\omega'_{jg}-\omega_k)t} - \sum_{\mathbf{k}}\int_0^t dt' \alpha(t') V_{\mathbf{k},eg} V_{\mathbf{k},jg} e^{-i\omega'_{ej}t} e^{i(\omega'_{eg}-\omega_k)(t-t')} \\ &- \sum_{\mathbf{k}}\int_0^t dt' C_j(t') V_{\mathbf{k},jg}^2 e^{i(\omega'_{jg}-\omega_k)(t-t')} - i\alpha(t)\eta e^{-i\omega'_{ej}t}.\end{aligned} \tag{A24}$$

Replacing $\alpha(t')$, $C_{j,\mathbf{k}}(t')$ by $\alpha(0)$, $C_{j,\mathbf{k}}(0)$ [11,21], we have

$$\dot{\alpha}(t) \approx -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t} - \alpha(0)\sum_{\mathbf{k}} \int_0^t dt' V_{\mathbf{k},eg}^2 e^{i(\omega'_{eg}-\omega_k)(t-t')}$$
$$- \sum_{j\neq e,g,\mathbf{k}} C_j(0) \int_0^t dt' V_{\mathbf{k},eg} V_{\mathbf{k},jg} e^{i\omega'_{ej}t} e^{i(\omega'_{jg}-\omega_k)(t-t')} - i\sum_{j\neq e,g} C_j(t)\eta e^{i\omega'_{ej}t}, \tag{A25}$$

$$\dot{C}_j(t) \approx -i\sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},jg} e^{i(\omega'_{jg}-\omega_k)t} - \alpha(0)\sum_{\mathbf{k}} \int_0^t dt' V_{\mathbf{k},eg} V_{\mathbf{k},jg} e^{-i\omega'_{ej}t} e^{i(\omega'_{eg}-\omega_k)(t-t')}$$
$$- C_j(0)\sum_{\mathbf{k}} \int_0^t dt' V_{\mathbf{k},jg}^2 e^{i(\omega'_{jg}-\omega_k)(t-t')} - i\alpha(t)\eta e^{-i\omega'_{ej}t}. \tag{A26}$$

Integrating Eq. (26) and substituting into Eq. (25) and replace $\alpha(t')$ by $\alpha(0)$ [11,21], we have

$$\alpha(t) = \alpha(0) - i\alpha(0)\int_0^t dt' \sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \frac{1-e^{i(\omega'_{eg}-\omega_k)t'}}{(\omega'_{eg}-\omega_k)} - i\int_0^t dt' \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} e^{i(\omega'_{eg}-\omega_k)t'}$$
$$- i\int_0^t dt' \sum_{j\neq e,g,\mathbf{k}} C_j(0) V_{\mathbf{k},eg} V_{\mathbf{k},jg} \frac{e^{i\omega'_{ej}t'} - e^{i(\omega'_{eg}-\omega_k)t'}}{(\omega'_{jg}-\omega_k)} - i\int_0^t dt' \sum_{j\neq e,g} C_j(0)\eta e^{i\omega'_{ej}t'}$$
$$= \alpha(0)\left\{1 - \sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left[\frac{1-e^{i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)^2} + \frac{it}{(\omega'_{eg}-\omega_k)}\right]\right\} + \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},eg} \frac{1-e^{i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)}$$
$$+ \sum_{j\neq e,g,\mathbf{k}} C_j(0) \frac{V_{\mathbf{k},eg} V_{\mathbf{k},jg}}{\omega'_{jg}-\omega_k} \left[\frac{1-e^{i\omega'_{ej}t}}{\omega'_{ej}} - \frac{1-e^{i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)}\right] + \sum_{j\neq e,g} C_j(0)\eta \frac{1-e^{i\omega'_{ej}t}}{\omega'_{ej}}. \tag{A27}$$

Due to $C_j(0) \approx 0$, Eq. (A27) becomes

$$\alpha(t) \approx \alpha(0)\left\{1 - \sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left\{\left[1 - \frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right] \frac{1-e^{i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)^2} + \frac{it}{\omega'_{eg}-\omega_k}\right\}\right\}$$
$$\approx \alpha(0)\exp\left\{-\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left\{\left[1 - \frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right] \frac{1-e^{i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)^2} + \frac{it}{\omega'_{eg}-\omega_k}\right\}\right\}$$
$$= \alpha(0)\exp\left\{-t\left\{\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left[1 - \frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right] \frac{2\sin^2\left(\frac{\omega'_{eg}-\omega_k}{2}t\right)}{(\omega'_{eg}-\omega_k)^2 t}\right.\right.$$
$$\left.\left. + i\left\{\sum_{\mathbf{k}} V_{\mathbf{k},eg}^2 \left\{\frac{1}{\omega'_{eg}-\omega_k} - \left[1 - \frac{\beta_{\mathbf{k}}(0)}{\alpha(0)} V_{\mathbf{k},eg}^{-1}(\omega'_{eg}-\omega_k)\right] \frac{\sin[(\omega'_{eg}-\omega_k)t]}{(\omega'_{eg}-\omega_k)^2 t}\right\}\right\}\right\}\right\}$$
$$= \alpha(0)\exp\{-t[\gamma_e^S(t)/2 + i\Delta E_{dyn}^{S(e)}(t)]\}. \tag{A28}$$

Integrating Eq. (A25) and substituting into Eq. (A26) and replace $C_{j,\mathbf{k}}(t')$ by $C_{j,\mathbf{k}}(0)$, we have

$$
\begin{aligned}
C_j(t) \approx\ & C_j(0) - iC_j(0)\int_0^t dt' \sum_{\mathbf{k}} V_{\mathbf{k},jg}^2 \frac{1-e^{i(\omega'_{jg}-\omega_k)t'}}{(\omega'_{jg}-\omega_k)} - i\int_0^t dt' \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},jg} e^{i(\omega'_{jg}-\omega_k)t'} \\
& - i\alpha(0)\int_0^t dt' \sum_{\mathbf{k}} V_{\mathbf{k},eg} V_{\mathbf{k},jg} \frac{e^{-i\omega'_{ej}t'} - e^{i(\omega'_{jg}-\omega_k)t'}}{(\omega'_{eg}-\omega_k)} - i\alpha(0)\eta\int_0^t dt' e^{-i\omega'_{ej}t'} \\
=\ & C_j(0)\left\{1 - \sum_{\mathbf{k}} V_{\mathbf{k},jg}^2 \left[\frac{1-e^{i(\omega'_{jg}-\omega_k)t}}{(\omega'_{jg}-\omega_k)^2} + \frac{it}{(\omega'_{jg}-\omega_k)}\right]\right\} + \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},jg} \frac{1-e^{i(\omega'_{jg}-\omega_k)t}}{(\omega'_{jg}-\omega_k)} \\
& - \alpha(0)\sum_{\mathbf{k}} \frac{V_{\mathbf{k},eg} V_{\mathbf{k},jg}}{\omega'_{eg}-\omega_k}\left[\frac{1-e^{-i\omega'_{ej}t}}{\omega'_{ej}} + \frac{1-e^{i(\omega'_{jg}-\omega_k)t}}{(\omega'_{jg}-\omega_k)}\right] - \alpha(0)\eta\frac{1-e^{-i\omega'_{ej}t}}{\omega'_{ej}}.
\end{aligned}
\tag{A29}
$$

With $C_j(0) \approx 0$, we obtain,

$$
\begin{aligned}
C_j(t) \approx & \sum_{\mathbf{k}} \beta_{\mathbf{k}}(0) V_{\mathbf{k},jg} \frac{1-e^{i(\omega'_{jg}-\omega_k)t}}{(\omega'_{jg}-\omega_k)} - \alpha(0)\eta\frac{1-e^{-i\omega'_{ej}t}}{\omega'_{ej}} \\
& - \alpha(0)\sum_{\mathbf{k}} \frac{V_{\mathbf{k},eg} V_{\mathbf{k},jg}}{\omega'_{eg}-\omega_k}\left[\frac{1-e^{-i\omega'_{ej}t}}{\omega'_{ej}} + \frac{1-e^{i(\omega'_{jg}-\omega_k)t}}{(\omega'_{jg}-\omega_k)}\right].
\end{aligned}
\tag{A30}
$$

and from Eq. (A22) and replacing $\alpha(t')$ by $\alpha(0)$, we have [21],

$$
\beta_{\mathbf{k}}(t) \approx \beta_{\mathbf{k}}(0) - \alpha(0) V_{\mathbf{k},eg} \frac{1-e^{-i(\omega'_{eg}-\omega_k)t}}{(\omega'_{eg}-\omega_k)}. \tag{A31}
$$

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