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Coherent control of quantum tunneling in an open double-well system

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We investigate how to apply a high-frequency driving field to the quantum control of a single particle in an open double-well system. The linear stability analysis points out that the stability depends on the external-field parameters and the loss (or gain) coefficients of the system, and the instability leads to transition of the Floquet quasi-energy from real to complex values and results in the decaying probabilities of the particle in the double-well. Combining the analytical solutions in the high-frequency approximation with the numerical calculations based on the accurate model, we exhibit quantum-dynamical behaviors of the particle such as the Floquet oscillation, coherent destruction of tunneling, quasi-noon-state population, partial tunneling of one-particle, and the decaying behavior of the probabilities, which are due to the competition and balance between the quantum coherence and the loss (or gain) effect. The results propose an experimental method for testing the quantum motions of the open system by adjusting the driving field.

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I. INTRODUCTION

A periodically driven double-well system has focused much attention during the last few years [1, 2]. The main research interest was motivated by showing the coherent control of quantum tunneling through such a system [3–6]. For applicability purposes, open quantum systems have recently become the subject of extensive studies [7]. The non-Hermiticity was due to the presence of various gain or loss mechanisms in the open systems [8–11], which brought many new contents to the quantum control [12–15]. For a single particle the non-Hermitian double-well system (or the mathematically equivalent two-level system) is a basic system for researching the coherent control [16–18]. It's not only a simple extension of the corresponding Hermitian one-body system, but also can be used to simulate the non-Hermitian many-body system without interaction [19], while the zero interaction strength can be realized by the Feshbach resonance technique [20]. However, most of the previous works on the non-Hermitian double-well system took into account of the effect of a static field on the stationary states [16–18] and the incoherent control of quantum tunneling [21], only a few works concerning the coherent control via a periodic external field are reported [22].

In this paper, we study a different non-Hermitian system with a single particle held in an open and high-frequency-driven double-well, and seek the analytical solutions and their boundedness conditions. By applying the coherent-control method of the Hermitian system [23, 24] to the non-Hermitian system, we explore the competition and balance between the coherent enhancement or suppression of tunneling and the loss (dissipation) or gain from the environment, and further apply them to manipulate the stable quantum motions. Un-

der the high-frequency approximation, our analytical results reveal the effects of the external-field parameters and loss- or gain-coefficient on the system's stability, and display that the loss of stability leads to the transition of the Floquet quasi-energy spectrum from real (exact phase) to complex (broken phase) values [8] and the corresponding decay of particle's occupied-probabilities [21, 25]. Due to the competition and balance between the quantum coherence and loss or gain, the quantum effects such as the Floquet oscillations of the quantum states with real energies, coherent destruction of tunneling (CDT), quasi-noon-state population [26], partial tunneling of one-particle, Schrödinger cat-like states, and the decaying probabilities, are shown. The numerical computations from the accurate model confirm agreement with the analytical results. Based on the capacity of the current setups [5, 27, 28], we expect that the quantum motions of the open system can be experimentally tested by adjusting the driving parameters.

II. GENERAL ANALYTICAL SOLUTION IN THE HIGH-FREQUENCY APPROXIMATION

We consider a single particle held in an open double-well, whose quantum dynamics is dominated by the \mathcal{PT} -symmetric non-Hermitian Hamiltonian [8]

$$\begin{aligned} H(t) &= \varepsilon_1(t)a_1^\dagger a_1 - \varepsilon_2(t)a_2^\dagger a_2 + \nu(a_1^\dagger a_2 + a_2^\dagger a_1), \\ \varepsilon_j(t) &= \alpha \cos(\omega t) - i\beta_j, \quad j = 1, 2, \end{aligned} \quad (1)$$

where $a_j(a_j^\dagger)$ are annihilation (creation) operators for the atom in j -th well with $j = 1, 2$; ν is the coupling parameter which presents tunneling rate between the two wells; $\varepsilon_j(t)$ contains the driving field with amplitude α and frequency ω and the j -th well's loss coefficient for $\beta_j > 0$ and/or gain one for $\beta_j < 0$. To simplify, Eq. (1) has been treated as a dimensionless equation in which the reference frequency $\omega_0 \sim 10^2 \text{Hz}$ and $\hbar = 1$ are set

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such that the parameters α , β_j and ν are in units of ω_0 and time is normalized in units of ω_0^{-1} . Obviously, when $\beta_j = 0$ for $j = 1, 2$ are taken, system (1) becomes the familiar Hermitian system of a single particle in the double-well [23, 24]. Particularly, when $\omega = 0$, $\alpha = \varepsilon$, $\beta_i = 0, \beta_j \neq 0, i \neq j$ or $\omega = 0$, $\beta_1 = \beta_2 = \gamma$ are selected, we arrive at the non-Hermitian many-particle Hamiltonian without interaction [19].

Using the localized states $|1\rangle$ and $|2\rangle$ as the basis, the quantum state $|\psi\rangle$ of system (1) can be expanded as [23, 24]

$$|\psi\rangle = C_1(t)|1\rangle + C_2(t)|2\rangle, \quad (2)$$

where C_i for $i = 1, 2$ denote the time-dependent probability amplitudes in the two wells. Inserting Eqs. (1) and (2) into the Schrödinger equation $i\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle$ produces the coupled equations

$$\begin{aligned} i\dot{C}_1(t) &= \varepsilon_1(t)C_1(t) + \nu C_2(t), \\ i\dot{C}_2(t) &= -\varepsilon_2(t)C_2(t) + \nu C_1(t) \end{aligned} \quad (3)$$

of the probability amplitudes. Although Eq. (3) is very simple, no analytical solution in a finite form exists, because of the presence of periodic functions $\varepsilon_j(t)$. However, in the case of high-frequency driving with $\omega \gg \nu, \beta_j$, we can get the approximate analytical solution. To do this, we introduce the slowly varying functions $d_i(t)$ for $i = 1, 2$ and make the function transformations [23, 24]

$$\begin{aligned} C_1(t) &= \exp\left[-i\frac{\alpha}{\omega}\sin(\omega t)\right]d_1(t), \\ C_2(t) &= \exp\left[i\frac{\alpha}{\omega}\sin(\omega t)\right]d_2(t), \end{aligned} \quad (4)$$

which transform Eq. (3) into the coupled equations between the slowly varying functions,

$$\begin{aligned} id_1(t) &= -i\beta_1 d_1(t) + \nu \exp\left[i\frac{2\alpha}{\omega}\sin(\omega t)\right]d_2(t), \\ id_2(t) &= i\beta_2 d_2(t) + \nu \exp\left[-i\frac{2\alpha}{\omega}\sin(\omega t)\right]d_1(t). \end{aligned} \quad (5)$$

In the high frequency limit, the slowly varying functions $d_i(t)$ can be treated approximately as the constants during a short period $\frac{2\pi}{\omega}$. Thus the rapidly varying exponential functions of Eq. (5) can be replaced by their time-average [29] so that Eq. (5) is simplified as [23]

$$\begin{aligned} id_1(t) &= -i\beta_1 d_1(t) + Jd_2(t) \\ id_2(t) &= i\beta_2 d_2(t) + Jd_1(t), \end{aligned} \quad (6)$$

where $J = \nu J_0(\frac{2\alpha}{\omega})$ is the effective or renormalized tunneling rate with $J_0(\frac{2\alpha}{\omega})$ being the zero-order Bessel function of $\frac{2\alpha}{\omega}$, which depends on the field parameters through the ratio of driving strength and frequency. We here consider the strong-field case [5] in which the ratio of field parameters obeys $0.45 \leq \frac{\alpha}{\omega} < 2.6$. This gives the field amplitude α being in the order of ω . In the high-frequency regime, such a driving strength means that the

used driving field is a strong field. From the first equation of Eqs. (6), we directly arrive at

$$Jd_2 = i[\dot{d}_1(t) + \beta_1 d_1(t)]. \quad (7)$$

Combining Eq. (6) with Eq. (7), the former equation is decoupled to the form

$$\ddot{d}_1(t) + (\beta_1 - \beta_2)\dot{d}_1(t) + (J^2 - \beta_1\beta_2)d_1(t) = 0. \quad (8)$$

Clearly, Eq. (8) is a second-order linear equation with the two constant coefficients, the smaller ‘‘damping factor’’ $\Gamma = \beta_1 - \beta_2$ and the lower ‘‘quadratic frequency’’ $\omega_l^2 = J^2 - \beta_1\beta_2$ for the slowly varying functions $d_1(t)$. The physical bounded solutions of Eq. (8) exist only under the boundedness conditions $\Gamma \geq 0$ and $\omega_l^2 \geq 0$, where the equal-signs mean some balances between the driving and damping. The former condition is fixed by the external environment, and the latter is controlled by the effective tunneling rate. The constant ω_l describes oscillation frequency of the slowly varying functions $d_i(t)$ and satisfies the inequality $\omega_l \ll \omega$.

The general solution of Eq. (8) is mathematically well-known,

$$d_1 = F_1 \exp(\lambda_1 t) + F_2 \exp(\lambda_2 t), \quad (9)$$

where λ_1 and λ_2 are the characteristic values associated with the linear equations (6), F_1 and F_2 are undetermined constants determined by the initial conditions and normalization. The substitution of Eq. (9) into Eq. (7) gives another slowly varying function

$$d_2 = \frac{i}{J}[F_1(\lambda_1 + \beta_1) \exp(\lambda_1 t) + F_2(\lambda_2 + \beta_1) \exp(\lambda_2 t)]. \quad (10)$$

Inserting Eqs. (9) and (10) into Eq. (6) and setting $F_1 = 0$ or $F_2 = 0$, respectively, we obtain the characteristic values

$$\lambda_{1,2} = \frac{1}{2}[(\beta_2 - \beta_1) \pm \sqrt{(\beta_1 - \beta_2)^2 - 4(J^2 - \beta_1\beta_2)}] \quad (11)$$

with λ_1 and λ_2 corresponding to the positive and negative signs, respectively. Given the general solutions (9) and (10), we can easily exhibit the time evolutions of probability $P_i = |d_i|^2$ for the particle localized in i -th well.

The stability analysis [30] on the linear equations (6) and (7) reveals that real parts of the characteristic values $\lambda_{1,2}$ are related to stability of the linear system. Writing $\lambda_{1,2}$ in the form $\lambda_{1,2} = \text{Re}(\lambda_{1,2}) + i\text{Im}(\lambda_{1,2})$, we know that two special situations of $\lambda_{1,2}$ are associated to the particular properties of the solutions. Firstly, if the boundedness conditions $\Gamma \geq 0$ and $\omega_l^2 \geq 0$ cannot be satisfied, the characteristic values with $\text{Re}(\lambda_n) > 0$ ($n = 1$ and/or 2) appear such that the probability of particle in j -th well tends to infinity, $\lim_{t \rightarrow \infty} |d_j(t)|^2 \rightarrow \infty$. This means that the system loses its stability in the sense of Lyapunov [30]. The instability causes that the Floquet quasi-energy transits from real to complex values.

The corresponding solutions of Eqs. (9) and (10) do not satisfy the requirement of the probability interpretation in quantum mechanics, and should be dropped thereby. Secondly, when the real parts $Re(\lambda_{1,2})$ are equal to zero under the boundedness conditions, the imaginary parts correspond to the Floquet quasi-energies [31, 32], $E_{1,2} = -Im(\lambda_{1,2})$ in the case $F_1 = 0$ and case $F_2 = 0$ respectively. Inserting d_j with $F_1 = 0$ or $F_2 = 0$ into Eqs. (2) and (4) gives the corresponding Floquet state $|\psi_1\rangle$ or $|\psi_2\rangle$. Therefore, the quantum state in Eq. (2) with the general solutions (9) and (10) may be a coherent superposition of the two Floquet states, $|\psi\rangle = D_1|\psi_1\rangle + D_2|\psi_2\rangle$ with D_1, D_2 being constants adjusted by the initial conditions and normalization. *The superposition states imply quantum interferences and may cause the coherent enhancement or suppression of tunneling, whose degree is described by the value of the effective tunneling rate J .*

Generally, for the appropriate environment with $\Gamma \geq 0$, we can obtain the physically meaningful solutions of Eq. (2) from the general solutions (9) and (10), by adjusting the field parameters to obey the boundedness conditions $\omega_l^2 \geq 0$. We divide the physical solutions into the three cases as follows.

Case A, CDT and quasi-noon-state populations under the dissipation balance. By the dissipation balance we mean that the loss (gain) coefficients of the two wells take the same values, $\beta_1 = \beta_2$. Such a balance could be established by making the two wells in the same environment. For such a case the adjustment to the field parameters or equivalent J in Eq. (11) could lead to $Re(\lambda_n) = 0$, ($n = 1$ and 2) such that the probability amplitudes $d_j(t)$ ($j = 1$ and 2) in Eqs. (9) and (10) are the periodic functions [$Im(\lambda_n) \neq 0$] or constants [$Im(\lambda_n) = 0$]. The corresponding quantum states in Eq. (2) become the Floquet states with real quasi-energy spectrum or zero-energy, which describe the particle's quasi-stationary states whose populations don't change and the same occupied-probability in any well is kept as a constant. Such quasi-stationary states are called the atomic quasi-noon-states, compared to the noon-state as a stationary state with the same occupied-probability in each well [26]. The invariable populations mean that CDT results in stable populations of particle in the initially occupied states.

Case B, Stable populations without the dissipation balance. The unbalanced dissipation infers $\beta_1 \neq \beta_2$ and the part loss of the stability, which will cause changes of the Floquet quasi-energy from real to complex values [8]. By adjusting the effective tunneling rate J , we can get $Re(\lambda_n) < 0$, $\lambda_{n'} = 0$, ($n \neq n'$) which are associated with that the probabilities $|d_j(t)|^2$ for $j = 1$ and 2 decrease and increase from the initial values to the different final values, respectively. For any well the probability difference between the initial and final states indicates the single-direction tunneling with different degrees. The populations may tend to a stable Schrödinger cat-like state with total probability finding the particle in the double-well is less than one. But a special ratio between the two loss

parameters could make the total probability tending to one.

Case C, Instability and decaying probabilities. For the unbalanced dissipations and under the boundedness conditions, regulations to the field parameters can make $Re(\lambda_n) < 0$ for $n = 1$ and 2 so that $|d_j|^2$ for $j = 1$ and 2 decrease exponentially fast. The latter means loss of the specific stability of quantum mechanics [33]. Thus the survival probability [21, 25] of the particle in an initial state and the total probability finding the particle in the double-well decay to zero rapidly.

In the next section, we will show that some balance conditions between the coherent enhancement or suppression of tunneling and the loss or gain from the environment can be realized by adjusting the field parameters, and such adjustments can control the particle's instability. Therefore, we can arrive at the physical solutions in the above-mentioned three cases and can manipulate the corresponding quantum motions of the open system via the periodic field.

III. COHERENT CONTROL OF QUANTUM TUNNELING VIA EXTERNAL FIELD

From Eqs.(9), (10) and (11) we know that the populations of the particle in the open double-well depend not only on the effective tunneling rate J determined by the amplitude α and frequency ω of external field, but also on the loss or gain coefficients from the environment, β_1 and β_2 . The “quadratic frequency” $\omega_l^2 = J^2 - \beta_1\beta_2$ reflects the competition between the coherent enhancement or suppression of tunneling and the loss or gain from environment. The condition $\omega_l^2 = 0$, ($J^2 = \beta_1\beta_2$) means the corresponding balance which differs from the above-mentioned dissipation balance. For a fixed environment with the given loss or gain coefficient, characteristic values $\lambda_{1,2}$ are adjusted only by the effective tunneling rate J . Therefore, in order to produce a required quantum state, we can select a higher driving frequency ω and then regulate the driving strength α to change the value of ω_l^2 and to control the competition. Because the different states are associated with different atomic populations, the quantum state (2) with the analytical solutions (9) and (10) are referable for experimentally researching the quantum tunneling and localization of the particle in the open double-well. We will enter into details of the above-mentioned three situations, respectively.

A. CDT and quasi-noon-state populations under the dissipation balance

At first, we consider the general situation of *case A* in section II: $Re(\lambda_n) = 0$ and $Im(\lambda_n) \neq 0$. Because of the dissipation balance, we can set $\beta_1 = \beta_2 = \beta$ and apply this to Eq. (11) to yield

$$\lambda_{1,2} = \pm \sqrt{-(J^2 - \beta^2)}. \quad (12)$$

For a weak loss or gain with small β value, we can adjust the field parameter $\frac{2\alpha}{\omega}$ to satisfy the competition condition $J^2 - \beta^2 > 0$ between J and β , such that Eq. (12) becomes

$$\lambda_{1,2} = \pm i\sqrt{J^2 - \beta^2} = \pm i\omega_l, \quad (13)$$

where ω_l is the lower frequency of slow-varying function $d_j(t)$. The high-frequency condition implies $\omega_l \ll \omega$. Inserting Eq. (13) into Eqs. (10) and (11) gives the periodic solutions

$$\begin{aligned} d_1 &= F_1 e^{i\omega_l t} + F_2 e^{-i\omega_l t}, \\ d_2 &= \frac{i}{J} [F_1 (i\omega_l + \beta) e^{i\omega_l t} + F_2 (-i\omega_l + \beta) e^{-i\omega_l t}]. \end{aligned} \quad (14)$$

We already know that this solution-pair is stable according to the linear stability analysis in the previous section. Using the normalization condition

$$\begin{aligned} |d|^2 &= |d_1|^2 + |d_2|^2 \\ &= 2(|F_1|^2 + |F_2|^2) + \frac{4\beta F_1 F_2}{J^2} [\beta \cos(2\omega_l t) - \omega_l \sin(2\omega_l t)] \\ &= 1, \end{aligned} \quad (15)$$

we establish the relationships between constants F_1 and F_2 as

$$2(|F_1|^2 + |F_2|^2) = 1, \quad 4\beta F_1 F_2 = 0. \quad (16)$$

Given Eq. (16), we assert that for a Hermitian system without dissipation ($\beta = 0$), constants F_1 and F_2 are constrained only by the first equation of Eq. (16), and the corresponding solution-pair (14) contains more selections for the constants. The solutions without dissipation have been discussed previously and will not be considered here. For our non-Hermitian system, $\beta \neq 0$, the second equation of Eq. (16) needs $F_i = 0$ for $i = 1$ or 2 , and the first equation of Eq. (16) gives $F_i = 0, F_j = e^{i\phi}/\sqrt{2}$ for $i, j = 1, 2$ and $i \neq j$ with ϕ being a constant. Neglecting the immaterial phase factor $e^{i\phi}$, we get two sets of solutions as follows. The first set of solutions from Eq. (14) with $F_1 = 0$ reads as

$$d_1 = \frac{1}{\sqrt{2}} e^{-i\omega_l t}, \quad d_2 = \frac{(\omega_l + i\beta)}{\sqrt{2}J} e^{-i\omega_l t}, \quad (17)$$

which periodically change in time with frequency ω_l obeying $\omega_l \ll \omega$. Inserting Eq. (17) into Eqs. (4) and (2) yields the quantum state

$$|\psi_1\rangle = \frac{e^{-i\omega_l t}}{\sqrt{2}} \left[e^{-i\frac{\alpha}{\omega} \sin(\omega t)} |1\rangle + \frac{(i\beta + \omega_l)}{J} e^{i\frac{\alpha}{\omega} \sin(\omega t)} |2\rangle \right]. \quad (18)$$

Similarly, the second set of solutions from Eq. (14) with $F_2 = 0$ leads to the quantum state

$$|\psi_2\rangle = \frac{e^{i\omega_l t}}{\sqrt{2}} \left[e^{-i\frac{\alpha}{\omega} \sin(\omega t)} |1\rangle + \frac{(i\beta - \omega_l)}{J} e^{i\frac{\alpha}{\omega} \sin(\omega t)} |2\rangle \right]. \quad (19)$$

Equations (18) and (19) denote a pair of the oscillating Floquet states whose phases change periodically and the corresponding Floquet quasi-energies read $E_{\pm} = \pm\omega_l$, respectively. From Eq. (13) we know $J^2 = \beta^2 + \omega_l^2$. Making use of this to Eq. (17) produces the constant probabilities $|d_j|^2 = \frac{1}{2}$ for $j = 1$ and 2 . They describe the quasi-noon-state population with the same occupied probability of the particle in each well [26]. The initially occupied probabilities are kept and no quantum tunneling happens, this just is the well-known CDT. It is interesting to compare our CDT condition $J^2 = \beta^2 + \omega_l^2$ with the CDT condition $J = 0$ for a Hermitian double-well system [23, 24]. The former condition means the balance between the quantum coherence and the environment damping.

If the field parameter $\frac{2\alpha}{\omega}$ is regulated to reach the balance condition $J^2 - \beta^2 = \omega_l^2 = 0$ ($J = \beta$), from Eq. (13) we arrive at the special situation of *case A*, $Re(\lambda_n) = 0$ and $Im(\lambda_n) = \pm\omega_l = 0$. Inserting this into Eqs. (18) and (19) yields the quasi-stationary state with zero Floquet quasi-energy and invariable population,

$$\begin{aligned} |\psi\rangle &= |\psi_1\rangle = |\psi_2\rangle \\ &= \frac{1}{\sqrt{2}} e^{-i\frac{\alpha}{\omega} \sin(\omega t)} (|1\rangle + e^{i[\pi/2 + (\frac{2\alpha}{\omega}) \sin(\omega t)]} |2\rangle), \end{aligned} \quad (20)$$

which describes the different quasi-noon-state.

The quasi-noon-states (18)-(20) with periodic phases are the standard single-particle noon-states at any fixed time [26]. They can be regarded as the Schrödinger cat-like states of a single particle in two mode entanglement, which supply a new approach for investigating the many-body entanglement and single-particle cat state [26]. Our results reveal existence of the quasi-noon-state, and provide a theoretical reference for experimentally preparing a quasi-noon-state in the open system.

B. Stable populations without dissipation balance

When the system is in the *case B* of the second section, instability causes transition of the Floquet quasi-energy from real to complex values [8]. By setting the field parameters, from Eq. (11) we can get $Re(\lambda_n) < 0$ and $\lambda_{n'} = 0$ for $n \neq n'$. Applying such characteristic values to the general solutions (9) and (10), it is expectable that the particle evolves from a given initial state to a stationary final state with a certain probability. The single-direction tunneling and decaying can occur simultaneously that lead to the another kind of Schrödinger cat-like states with total probability finding the particle in the double-well being less than one. However, the special ratio $\beta_1/\beta_2 = 3$ could make the total probability tending to one. The tunneling of particle in the open double-well depends on the competition between the coherent enhancement or suppression of tunneling and the loss or gain from the environment, so it is an interesting phenomenon differing from that of the corresponding isolated system.

According Eq. (11), for the dissipation coefficients obeying $\beta_1 - \beta_2 > 0$ (or $\beta_1/\beta_2 > 1$) we can take $\lambda_1 = 0$ and $\lambda_2 = \beta_2 - \beta_1 < 0$, by adjusting the field parameter $\frac{2\alpha}{\omega}$ to get the new balance between the quantum coherence and loss, $\omega_l^2 = J^2 - \beta_1\beta_2 = 0$. Thus the general solutions (9) and (10) become

$$\begin{aligned} d_1 &= F_1 + F_2 \exp[(\beta_2 - \beta_1)t], \\ d_2 &= \frac{i}{J}(F_1\beta_1 + F_2\beta_2 \exp[(\beta_2 - \beta_1)t]). \end{aligned} \quad (21)$$

When the particle is initially located in the first well, we have the initial conditions $|d_1(0)| = 1$, $|d_2(0)| = 0$. Inserting them into Eq. (21) gets the undetermined constants F_1 and F_2 in the forms,

$$F_1 = \frac{\beta_2}{\beta_2 - \beta_1}, \quad F_2 = \frac{\beta_1}{\beta_2 - \beta_1}. \quad (22)$$

Combining Eq. (22) with Eq. (21) results in the total probability finding the particle in the two wells,

$$\begin{aligned} P &= |d|^2 = |d_1|^2 + |d_2|^2 \\ &= \frac{(\beta_1 + \beta_2)[\beta_2 + \beta_1 e^{2(\beta_2 - \beta_1)t}] - 4\beta_1\beta_2 e^{(\beta_2 - \beta_1)t}}{(\beta_2 - \beta_1)^2}. \end{aligned} \quad (23)$$

It is well-known that the probability interpretation of quantum mechanics requires the total probability to be less than or equal to one. For an open system, the survival probability of particle maybe decay [21, 25], and the initial normalized total probability may be decreased in time. Therefore, it is necessary to confine the maximal value of P to $P_{max} \leq 1$.

From $dP/dt|_{t=t_j} = 0$ we find that the total probability given in Eq. (23) has the three extreme-value points of time, $t_1 = 0$, $t_2 = \frac{1}{\beta_2 - \beta_1} \ln\left(\frac{2\beta_2}{\beta_1 + \beta_2}\right)$, $t_3 = \infty$. At $t_1 = 0$, Eq. (23) gives $P_{max}(0) = |d(0)|^2 = 1$ which agrees with the initial condition. When $t = t_2$ is reached, the total probability satisfies $P_{max}(t_2) = \frac{\beta_2}{\beta_1 + \beta_2} < 1$. As time increasing to $t \rightarrow \infty$, Eq. (23) denotes the total probability of the final state, $P_{max}(t_3) = |d(\infty)|^2 = \frac{\beta_2(\beta_1 + \beta_2)}{(\beta_1 - \beta_2)^2}$. The physical requirement $P_{max}(t_3) \leq 1$ means the corresponding parameter range, $\beta_1/\beta_2 \geq 3$, which agrees with the previous confining condition $\beta_1/\beta_2 > 1$ for Eq. (21). When $\beta_1/\beta_2 = 3$ is set, the biggest final-state probability reads $|d(\infty)|^2 = 1$. If β_j are limited in the range $\beta_1/\beta_2 > 3$, the total probability finding the particle is less than 1, but the particle still can be confined in the double-well with a certain probability. As increasing the value of β_1/β_2 , Eq. (23) exhibits that the total probability $|P(\infty)|$ of final state will decrease its value.

Now let us numerically illustrate the above results. We take three sets of the parameters (ω , $\frac{2\alpha}{\omega}$, ν , β_1 , β_2) to satisfy $J^2 - \beta_1\beta_2 = 0$ and $\beta_1/\beta_2 = 3, 4, 9$, respectively, and from Eqs. (21), (22) and (23) plot the time-evolution figures of the probabilities $P_j = |d_j(t)|^2$ for the particle in j th-well and the total probability $P = |d(t)|^2$, as labeled by the circular points in Figs. 1(a), 1(b) and 1(c).

In the same parameter conditions and based on the accurate model (3), we numerically make the time-evolution figures of $P_j = |d_j(t)|^2$ and $P = |d(t)|^2$, as shown by the curves in Figs. 1(a), 1(b) and 1(c). Obviously, in the high-frequency regime, the analytical and numerical solutions are in good agreement. Hereafter, an immaterial difference between the both is that for the considered driving-frequency $\omega = 80(\omega_0) = 8 \times 10^3 \text{Hz}$ the numerical solutions oscillate around the analytical solutions with the high-frequency and some small-amplitudes.

In Fig. 1(a) with the special ratio $\beta_1/\beta_2 = 3$, we can see that after a transient decay, the total probability monotonically tends to the biggest value $|P(\infty)| = 1$, and the particle will be confined in the double-well stably. The probability $P_1 = |d_1(t)|^2$ of the initially occupied state decays quickly for a short time, then increases slowly to approach the final value 0.3. The probability $P_2 = |d_2(t)|^2$ of the particle in the second well monotonically increases and will near 0.7 for a time large enough. This means that the particle will tunnel partly from the first well to the second well with probability 0.7. For the parameter ratio $\beta_1/\beta_2 = 4$, the comparison between Fig. 1(b) and 1(a) displays that the final value of any one of the probabilities P_j and P has a little decrease. The final total probability is less than 1, and the tunneling probability from the first well to second well is less than 0.5. If $\beta_1/\beta_2 = 9$ is set, in Fig. 1(c) we show that the probabilities for every well and the total probability further reduce. The probability P_1 in the initial state is finally closed to zero, and the tunnel probability to the second well tends to 0.2, which approximately equates the total probability.

Similarly, if the particle is initially localized in the second well, $|d_1(0)| = 0$, $|d_2(0)| = 1$, the total probability is

$$P = \frac{(\beta_1 + \beta_2)[\beta_1 + \beta_2 e^{2(\beta_2 - \beta_1)t}] - 4\beta_1\beta_2 e^{(\beta_2 - \beta_1)t}}{(\beta_2 - \beta_1)^2}. \quad (24)$$

It's easy to prove that under the match condition $\beta_1/\beta_2 > 1$ of Eq. (21), the total probability in Eq. (24) may be greater than 1, which is physically unallowable and must be dropped. Therefore, for the field parameters satisfying $J^2 - \beta_1\beta_2 = 0$, the particle initially occupying well 2 cannot be stably trapped in the double-well.

It is worth noting that under the balance condition $J^2 = \beta_1\beta_2$, the change of β_j from $\beta_1 > \beta_2$ to $\beta_1 < \beta_2$ will lead to the non-physical case in which one of the $\text{Re}(\lambda_n)$ is greater than 0 and values of the two d_j tend to infinity. The physical requirement $\beta_1 > \beta_2$ implies that the external damping of the left-well 2 is weaker, compared to that of the right well 1. Note that direction of the external field is toward the right.

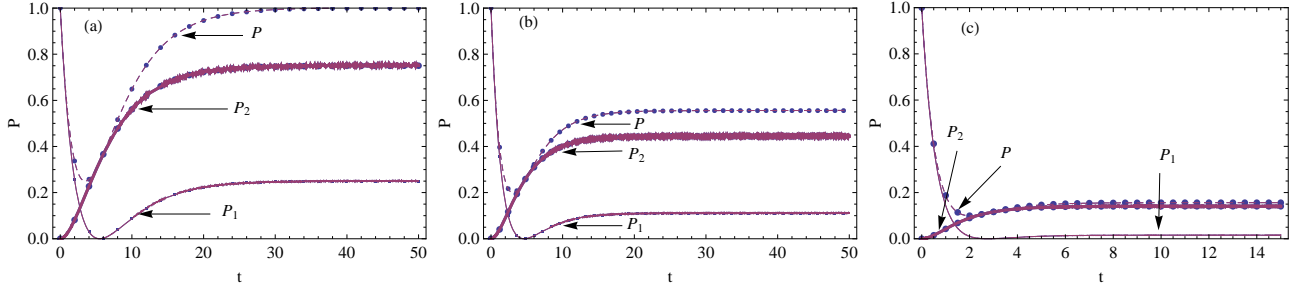


FIG. 1: (Color online) Time evolution curves of the probabilities $P_j = |d_j(t)|^2$ for the particle in j th-well and the total probability $P = P_1 + P_2$, under the balance condition $J^2 - \beta_1\beta_2 = 0$. The circular points label the analytical solutions and the curves represent the numerical results for the high frequency $\omega = 80$ and the parameters: (a) $\beta_1 = 0.3$, $\beta_2 = 0.1$, $\nu = 1$, $\frac{\alpha}{\omega} = 2.50669$ and $\beta_1/\beta_2 = 3$; (b) $\beta_1 = 0.4$, $\beta_2 = 0.1$, $\nu = 1$, $\frac{\alpha}{\omega} = 2.46537$ and $\beta_1/\beta_2 = 4$; (c) $\beta_1 = 0.9$, $\beta_2 = 0.1$, $\nu = 1$, $\frac{\alpha}{\omega} = 2.29242$ and $\beta_1/\beta_2 = 9$. The probabilities are dimensionless and time is in units of $\omega_0^{-1} = 0.01$ s.

C. Instability and decaying probabilities

When the system is in the *case C* of the second section, $\text{Re}(\lambda_i) < 0$, ($i = 1$ and 2), the total probability finding particle in the double-well will exponentially decay to zero. Thus the system loses the special stability of quantum mechanics, which is an important case in the open system [33].

The above case corresponds to the loss or gain coefficient obeying $\beta_1 - \beta_2 > 0$ and the field parameter $\frac{2\alpha}{\omega}$ obeying the new competition condition $4(J^2 - \beta_1\beta_2) > (\beta_1 - \beta_2)^2$ such that from Eq. (11) we have

$$\lambda_{1,2} = \frac{1}{2}[(\beta_2 - \beta_1) \pm i\varpi] \quad (25)$$

with $\varpi = \sqrt{4J^2 - (\beta_1 + \beta_2)^2}$. For such a case, Eq. (25) gives the real part of λ_j to be always less than zero. According to the stability analysis, we know that for any initial conditions, the system will lose its stability of quantum mechanics and the total probability finding the particle in the system tends to zero. To simplify, we take the initial conditions $|d_1(0)| = 0$, $|d_2(0)| = 1$ as an example. Inserting the initial conditions into Eqs. (9) and (10), and using the normalization conditions produce the undetermined constants

$$F_1 = \pm \frac{J}{\varpi}, \quad F_2 = -F_1. \quad (26)$$

Then applying the F_1 and F_2 to Eqs. (9) and (10) yields the probabilities of the particle in the two wells,

$$|d_1|^2 = e^{-(\beta_1 - \beta_2)t} \left\{ \frac{2J^2[1 - \cos(\varpi t)]}{\varpi} \right\},$$

$$|d_2|^2 = e^{-(\beta_1 - \beta_2)t} \left[\frac{\varpi \cos(\frac{1}{2}\varpi t) + (\beta_1 - \beta_2) \sin(\frac{1}{2}\varpi t)}{\varpi} \right]^2. \quad (27)$$

As an example, we take the parameter set $(\omega, \frac{2\alpha}{\omega}, \nu, \beta_1, \beta_2)$ to match the condition $4(J^2 - \beta_1\beta_2) > (\beta_1 - \beta_2)^2$. Adopting such parameters we illustrate the time evolutions of the probabilities $P_j = |d_j(t)|^2$ and $P = P_1 + P_2$ from Eq. (27), as labeled by the circular points

in Fig. 2. In the same conditions, we numerically solve the exact model (3), which are shown by the different curves of Fig. 2. Obviously, the analytical and numerical solutions are in good agreement. From Fig. 2 we can see

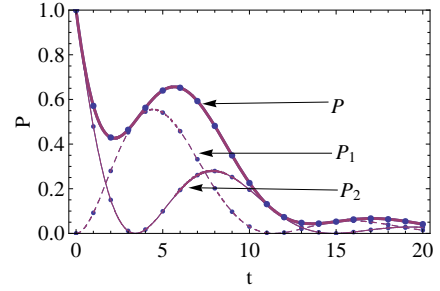


FIG. 2: (Color online) Time-evolution curves showing the decaying probabilities of the particle in the double-well. The circular points describe the analytical results from Eq. (27) and the curves indicate the numerical solutions based on Eq. (3). The system parameters and the initial conditions are the same for the analytical and numerical solutions. The probabilities are dimensionless and time is in units of $\omega_0^{-1} = 0.01$ s.

that after some transitory oscillations, the probability of the particle in any well and the total probability finally tend to zero. So in this case, the particle cannot exist in the double-well for a long time.

In order to conveniently describe the above behaviors, people employ the conception of survival probability in an initial state [21, 25] to investigate the population of particle in an open system. The survival probability of particle in the initial state $|\psi(0)\rangle$ is defined as [21, 25]

$$P_{\text{surv}}(t) = |\langle \psi(0) | \psi(t) \rangle|^2. \quad (28)$$

In the initial conditions $|d_1(0)|^2 = 0$, $|d_2(0)|^2 = 1$, combining Eqs. (2), (4) with Eq. (27), from Eq. (28) we obtain $P_{\text{surv}}(t) = |\langle d_2(0) | d_2(t) \rangle|^2 = P_2$, which has been exhibited in Fig. 2. Clearly, after a transitory oscillation the survival probability of initial state $|\psi(0)\rangle = |2\rangle$ decays and tends to zero.

IV. CONCLUSION AND DISCUSSION

We have considered a single particle held in an open and high-frequency-driven double-well. The coherent-control method of quantum tunneling for the Hermitian system [23, 24] is applied to the non-Hermitian system, which leads to the analytical solutions and their boundedness conditions under the high-frequency limit. By using the analytical results we show the effects of the field parameters and loss- or gain-coefficient on the system's stability and exhibit how to manipulate the stable quantum motions. We have revealed that the loss of stability leads to the transition of the Floquet quasi-spectrum from real to complex values [8] and the corresponding decay of particle's probabilities [21, 25]. The competition and balance between the coherent enhancement or suppression of tunneling and the loss (dissipation) or gain from the environment are found, and the quantum effects such as the Floquet oscillations of the quantum states with real quasi-energies, coherent destruction of tunneling for the new balance conditions, quasi-noon-

state population [26], partial tunneling of one-particle, Schrödinger cat-like states, and the decaying probabilities, are shown. By comparing the analytical solutions with the numerical computations from the accurate model, we find good agreement between them, which emphasizes the correctness of the conclusions from the different methods and the suitability of the high-frequency approximation method for the open system. Based on the capacity of the current setups [5, 27, 28], it is expectable to experimentally test the quantum motions of the open system via a high-frequency driving field.

Acknowledgments

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