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Vortex structures of rotating spin-orbit coupled Bose-Einstein condensates

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We consider the quasi-2D two-component Bose-Einstein condensates with Rashba spin-orbit (SO) coupling in a rotating trap. The rotation angular velocity couples to the mechanical angular momentum which contains a non-canonical part arising from SO coupling. The effects of an external Zeeman term favoring spin polarization along the radial direction is also considered, which has the same form as the non-canonical part of the mechanical angular momentum. The rotating condensate exhibits a variety of rich structures as varying the strengths of the trapping potential and interaction. With a strong trapping potential, the condensate exhibits a half-quantum vortex-lattice configuration. Such a configuration is driven to the normal one by introducing the external radial Zeeman field. In the case of a weak trap potential, the condensate exhibits a multi-domain pattern of plane-wave states under the external radial Zeeman field.

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I. INTRODUCTION

Spin-orbit (SO) coupling plays an important role in various aspects in condensed matter systems including spintronics [1] and topological insulators [2, 3]. However, SO effects in bosonic systems has not been attracted much attention until recently. For example, ^4He atoms are spinless and ultracold spinful bosons are too heavy to exhibit relativistic SO coupling. This situation is significantly changed by the recent experimental progress in both semiconductor exciton systems and cold atom systems with synthetic gauge fields. Excitons are composite bosons of electrons and holes. Their effective masses are light enough to exhibit relativistic SO coupling. Exotic SO coupled condensates with stripe and skyrmion types spin texture configurations was theoretically predicted by Wu and Mondragon-Shem [4]. Excitingly, spin textures have been observed in the SO coupled exciton condensates by High *et al* [5]. On the other hand, many theoretical schemes have been proposed in ultracold atomic systems to create artificial non-Abelian gauge fields by using laser-atom interactions [6–17], which generate effective SO coupling without special relativity.

It has been shown that bosons with SO coupling support exotic ground states beyond the “no-node” theorem [18–21]. This theorem states that the ground state wavefunctions of bosons under very general conditions are positive definite, which is essentially a direct result of the Perron-Frobenius theorem of matrix analysis [22]. However, the linear coupling to momentum in the SO coupling invalidates the proof of the “no-node” theorem. For example, spontaneous time-reveal symmetry breaking states exhibiting spin-density wave ordering [4, 19, 28–31] and spontaneous half-quantum vortex configuration [4, 19] have been studied. Both of them exhibit either nodal or complex-valued condensate wavefunctions, and thus are beyond the “no-node” theorem. Especially, the realization of SO coupled Bose-Einstein condensations (BEC) of ^{87}Rb [32, 33] provides a valuable

opportunity to investigate this type of exotic physics, experimentally. Another way to bypass “no-node” theorem is to employ the meta-stable excited states, in which “no-node” theorem does not apply either. For example, cold alkali bosons have been pumped into the high orbitals in optical lattices [34, 35]. It was shown that interactions among p -orbital bosons obey an “orbital Hund’s rule”, which generates a class of orbital superfluid states with complex-valued wave functions breaking TR symmetry spontaneously [23–27].

On the other hand, vortex properties in rotating BECs are a characteristic topological feature of superfluidity including ^4He and ultra-cold bosons, which have been studied extensively both experimentally and theoretically [37]. For spinor BECs and spinful Cooper pairing superfluidity (e.g. superfluid ^3He A and B-phases), exotic spin textures and fractional quantized vortices can form under rotation [38]. However, to our knowledge, the vortex properties of rotation SO coupled BECs have not been thoroughly investigated before.

In this article, we investigate the rotating SO coupled condensate in a quasi-2D harmonic trap with the angular velocity along the z -axis. The angular velocity couples to the mechanical angular momentum whose non-canonical part behaves like a Zeeman term polarizing spin in the radial direction. We also consider the effect from an external Zeeman term with the same form. The single particle ground states in the absence of interaction can have non-zero vortex numbers, which differ by one in the spin-up and down components as a result of SO coupling. With many-body interactions, the rotating condensate exhibit a variety of configurations depending on the strengths of the trapping potential and interaction. If the trapping potential is strong and interaction is relatively weak, a half-quantum vortex lattice is formed under rotation. Its spin configuration is a lattice of skyrmions. The condensate of the spin up component breaks into disconnected density peaks, which overlap the vortex cores of the spin-down condensate. The presence of the external Zeeman

field drives the system from a half quantum vortex lattice state to a normal quantum vortex lattice state. In the case of a weak trap potential, the condensate favors a plane-wave state or a two-plane-wave state with twist phase profiles under rotation. With the external Zeeman field, the condensate develops multi-domain configuration of plane-wave states. The configuration of wavevectors can be clockwise or counter-clockwise depending on the direction of the field.

The rest part of the paper is organized as follows. The model Hamiltonian of the rotating Rashba coupled BEC is introduced in Sect. II. The solution of the single particle wavefunction is presented in Sect. III. The rich structures of the vortex configurations with spin textures are given in Sect. IV. conclusions are given in Sect. V.

II. THE MODEL HAMILTONIAN

We consider the quasi-2D two-component BECs with Rashba SO coupling in the xy -plane subject to a rotation angular velocity Ω_z along the z -direction. The free part of the Hamiltonian of Rashba SO coupling under rotation is defined through the standard minimal coupling as

$$H_0 = \int d^3\vec{r} \psi_\mu^\dagger(\vec{r}) \left[\frac{1}{2M} (-i\hbar\vec{\nabla} + M\lambda\hat{z} \times \vec{\sigma} - \vec{A})^2 - \mu + V_{ext}(\vec{r}) - \frac{1}{2}M\Omega_z^2(x^2 + y^2) \right]_{\mu\nu} \psi_\nu(\vec{r}), \quad (1)$$

where $\vec{\sigma} = \sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z}$ with $\sigma_{x,y,z}$ the usual Pauli matrices; λ is the Rashba SO coupling strength with the unit of velocity; μ, ν take values of \uparrow, \downarrow as pseudospin indices; $\vec{A} = (-M\Omega_z y, M\Omega_z x, 0)$ is the vector potential from Coriolis force; $V_{ext}(\vec{r}) = \frac{1}{2}M\omega_T(x^2 + y^2)$ is the external harmonic trapping potential; the last term in Eq. 1 is the centrifugal force. The interaction part H_{int} is defined as

$$H_{int} = \frac{g_{\mu\nu}}{2} \int d^3\vec{r} \psi_\mu^\dagger(\vec{r}) \psi_\nu^\dagger(\vec{r}) \psi_\nu(\vec{r}) \psi_\mu(\vec{r}). \quad (2)$$

We assume the equal intra-component interactions as $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g$, and inter-component interaction $g_{\uparrow\downarrow} = gc$ with c a constant coefficient.

Due to the presence of SO coupling, Ω_z couples to the mechanical angular momentum L^{mech} rather than the canonical one L_z . We extract this coupling from Eq. 1 as

$$H_{rot} = -\Omega_z \int d^3\vec{r} \psi_\mu^\dagger(\vec{r}) \left[L^{mech} \right]_{\mu\nu} \psi_\nu(\vec{r}), \quad (3)$$

where

$$L^{mech} = L_z + M\lambda(x\sigma_x + y\sigma_y). \quad (4)$$

Therefore, rotation in the presence of SO coupling induces an effective magnetic field distribution $\vec{B}_R(\vec{r}) = \Omega_z M\lambda(x, y, 0)$ in the xy -plane. As we will see below, this

non-canonical part in L^{mech} plays a crucial role during the understanding the single-particle ground state properties.

For the later convenience, we also introduce an external spatially dependent Zeeman term as

$$H_B = - \int d^3r \psi_\mu^\dagger(\vec{r}) (B_{ex,x}\sigma_x + B_{ex,y}\sigma_y)_{\mu\nu} \psi_\nu(\vec{r}), \quad (5)$$

where $\vec{B}_{ex}(\vec{r}) = (B_0x, B_0y, 0)$ varies linearly in the xy -plane. Such a term can tune the strength of the non-canonical part of the mechanical momentum, which renders the model adjustable in a wider range of the parameter space.

Many efforts have been made to implement the above Hamiltonian in ultra-cold atomic gases. Several schemes have been proposed to generate Rashba SO coupling [6, 16, 17] with tunable SO coupling strength. In particular, proposals in Ref. [16, 17] have the advantage to overcome the drawback of the spontaneous emission in the tripod scheme. The spatially dependent Zeeman term H_B can be generated through coupling two spin components using two standing waves in the x and y directions with a phase difference of $\pi/2$. The resulting Rabi coupling is written as

$$-\Omega [\sin(k_L x) + i \sin(k_L y)] \psi_\downarrow^\dagger(\vec{r}) \psi_\uparrow(\vec{r}) + h.c. \quad (6)$$

In the region of $x, y \ll 2\pi/k_L$, it reduces to the desired form of Eq. 5 with $B_0 = \Omega k_L$.

III. THE SINGLE PARTICLE SPECTRA

We start with the non-interacting Hamiltonian $H_0 + H_B$, to gain some intuition. The confining trap is characterized by the length scale $l = \sqrt{\hbar/M\omega}$. We define another length scale $l_{so} = \hbar/(M\lambda)$ from SO coupling. The ratio between them $\alpha = l/l_{so}$ is dimensionless parameter to describe the strength of SO coupling. For the typical setup used in the NIST group [33], $\alpha \sim 10$. Below we vary the values of α from 0 \sim 10. Experimentally, the regime of small α can be reached by using a deeper trap potential.

If without the confining potential and rotation, the single-particle eigenstates is of the form

$$\psi_{\pm, \vec{k}} = e^{i\vec{k} \cdot \vec{r}} |\pm, \vec{k}\rangle, \quad (7)$$

where $|\pm, \vec{k}\rangle = \frac{1}{\sqrt{2}} (1, \mp e^{i\theta_{\vec{k}}})^T$, and $\theta_{\vec{k}}$ is the azimuthal angle of \vec{k} . Since the condensate is uniform along the \hat{z} direction, we always have $k_z = 0$ for the ground state. The corresponding dispersion relations come into two branches $\epsilon_{\pm} = \hbar^2(k^2 \pm 2k_0k)/(2M)$ with $k_0 = 1/l_{so}$. Therefore, the single particle ground states are infinitely degenerate along a ring in momentum space with radius k_0 .

The external harmonic potential has an important effect which lifts the degeneracy along the Rashba ring as

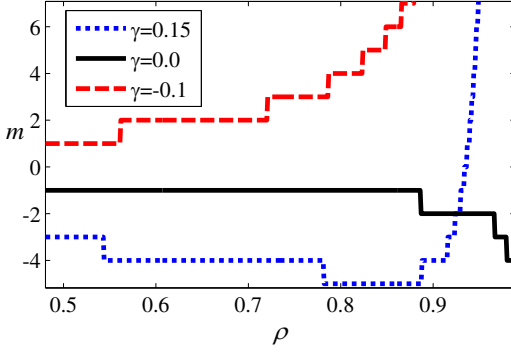


FIG. 1: (Color online) The canonical angular momenta m of the single particle ground states described in Eq. 10 *v.s.* ρ for $\gamma = -0.1, 0.0$, and 0.15 , respectively.

pointed out in Ref. [4]. In the momentum representation, the harmonic potential becomes $\frac{1}{2}M\omega^2(i\hbar\nabla_{\vec{k}} - \vec{A}')^2$ in the lower branch and couples different plane wave states around the Rashba ring, where $\vec{A}'(\vec{k}) = i\langle\psi_{-, \vec{k}}|\nabla_{\vec{k}}|\psi_{-, \vec{k}}\rangle$ corresponding to a π -flux at the origin. Therefore the motion along the Rashba ring is quantized and maintains time-reversal (TR) invariance. The single particle spectra exhibit the fermion-type Kramer degeneracy with $T^2 = -1$. The lowest single particle eigenstates carry $j_z = \pm\frac{1}{2}$. As shown in Ref. [4], the angular quantization gives rise to the dispersion on j_z as

$$\frac{1}{\alpha^2}|j_z|^2\hbar\omega_T. \quad (8)$$

On the other hand, the radial quantization is the same as in the ordinary harmonic trap, which is at the order of $\hbar\omega_T$ [4]. In the strong SO coupling limit, *i.e.*, $\alpha \gg 1$, the dispersion over j_z is nearly flat. Thus the radial quantum number can be viewed as band index, and the quantum number j_z marks each state in the band.

To be more precise, we define two independent annihilation operators as $\hat{a}_d = \frac{1}{2}(\bar{z} + 2\partial_z)$ and $\hat{a}_g = \frac{1}{2}(z + 2\partial_{\bar{z}})$ where $z = (x \pm iy)/l$ and \bar{z} is the complex conjugate of z [37, 39]. The single-particle Hamiltonian can be rewritten in the unit $\hbar\omega$ as

$$H_0 + H_B = (1 - \rho)\hat{N}_d + (1 + \rho)\hat{N}_g + 1 + \alpha \{[(1 - \kappa)\hat{a}_d - (1 + \kappa)\hat{a}_g^\dagger]\sigma^+ + h.c.\}, \quad (9)$$

where

$$\rho = \Omega_z/\omega, \quad \hat{N}_d = \hat{a}_d^\dagger \hat{a}_d, \quad \hat{N}_g = \hat{a}_g^\dagger \hat{a}_g, \quad \sigma^+ = \frac{1}{2}(\sigma_x + i\sigma_y),$$

and $\kappa = \gamma + \rho$ with $\gamma = B_0/(M\omega\lambda)$. The corresponding canonical angular momentum reads $L_z = \hbar l_z = \hbar(\hat{N}_d - \hat{N}_g)$. The κ -term represents the combined effect from the non-canonical part of H_{rot} and the Zeeman term H_B .

We diagonalize Eq. 9 to obtain the single particle spectra, and present the solutions in the coordinate represen-

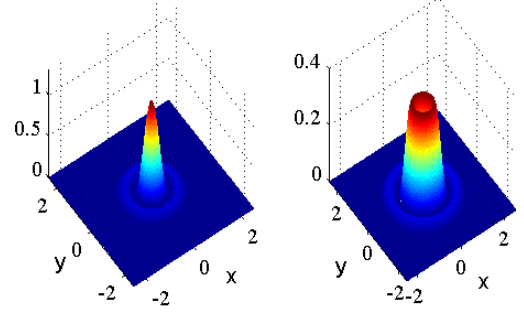


FIG. 2: (Color online) Density profiles of spin-up and down components for the single-particle ground state $\phi_{j_z=\frac{1}{2}}$ with the parameter values of $\alpha = 4$, and $\rho = \gamma = 0$. Only the spin-down component carries a vortex. The corresponding density profile for $\phi_{j_z=-\frac{1}{2}}$ is obtained by interchanging the spin indices. Here we use the length unit defined by the harmonic trap.

tation, in which the ground state wavefunction reads as

$$e^{im\phi} \begin{pmatrix} f(r) \\ g(r)e^{i\phi} \end{pmatrix}. \quad (10)$$

The total canonical angular momentum $j_z = l_z + \frac{1}{2}\sigma_z = m + \frac{1}{2}$ remains a conserved quantity, thus the canonical orbital angular momenta in the two spin components differ by one due to SO coupling. Fig. 1 shows m as a function of the rotational angular velocity ρ for different external magnetic field \vec{B}_{ex} at $\alpha = 4$. In the absence of \vec{B}_{ex} , the total angular momentum $j_z = -\frac{1}{2}$ for small ρ and decreases when $\rho \rightarrow 1$. Introducing the field \vec{B}_{ex} changes the ground state dramatically. If \vec{B}_{ex} is parallel to the induced magnetic field \vec{B}_R , *i.e.*, $\gamma > 0$, j_z first decreases then increases with the rotational angular velocity ρ . However, for $\gamma < 0$, j_z increases with ρ monotonically.

The above results can be understood as follows. In the case of $\Omega_z = 0$, the two states $\phi_{j_z=\pm\frac{1}{2}}$ are degenerated due to TR symmetry. Since only one of the two spin components carries a vortex, the ground state can be viewed as a half quantum vortex state with the density profiles of two spin components shown in Fig. 2. The spin density distributions exhibit skyrmion-type texture configurations, as depicted in Fig. 3 A and B. Intuitively, one might expect that an infinitesimal Ω_z selects the $\phi_{j_z=\frac{1}{2}}$ state since it has lower rotational energy $-\Omega_z\langle L_z \rangle$. However, the presence of the induced magnetic field \vec{B}_R contributes another term to the total rotational energy of the system as

$$\langle H_{rot} \rangle = -\Omega_z\langle L_z \rangle - \vec{B}_R \cdot \langle \vec{\sigma} \rangle. \quad (11)$$

The spin pattern $\langle \vec{\sigma} \rangle$ for $\phi_{j_z=\frac{1}{2}}$ in the xy -plane is antiparallel to \vec{B}_R near the trap center (see Fig. 3(A)), which is energetically unfavorable. Therefore, when $-\vec{B}_R \cdot \langle \vec{\sigma} \rangle$

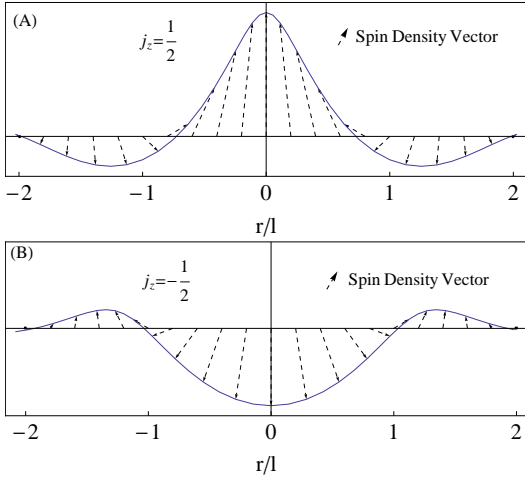


FIG. 3: (Color online) The spin density vector distributions along the x -axis lie in the xz -plane as shown in (A) $j_z = \frac{1}{2}$ and (B) $j_z = -\frac{1}{2}$ with $\alpha = 4$, and $\rho = \gamma = 0$. They are time-reversal counterparts to each other, and both exhibit the skyrmion-type texture configuration.

dominates, j_z of the ground state can be $-\frac{1}{2}$ for a rotating trap. As increasing Ω_z , the condensates expand, which also favors the magnetic energy term. The total angular momentum j_z can decrease even when ρ increases. Such a counter-intuitive effect for the ground state constitutes a characteristic feature of SO coupled BECs in a rotating trap. Introducing the external magnetic field \vec{B}_{ex} strengthen or weaken this effect induced by \vec{B}_R depending on its direction, which explains the different behaviors of m with ρ for $\gamma > 0$ and $\gamma < 0$, as shown in Fig. 1.

IV. VORTEX CONFIGURATIONS OF ROTATING SO COUPLED BEC

Interaction effects in the absence of rotation have been investigated extensively in the literature, which are summarized below. In the case of a strong trapping potential and weak interaction, the single-particle energy dominates. The condensate maintains rotational symmetry but spontaneously breaks TR symmetry [4, 19]. One spin-component carry one vortex, and the other is non-rotating, thus the condensate possesses a half-quantum vortex. The total angular momentum of each particle is $|j_z| = \frac{1}{2}$. In momentum space, this kind of ground state distributes uniformly around the Rashba ring. On the contrary, if the trapping potential is weak and interaction is strong, the condensate breaks rotational symmetry. The condensate is approximately superposition of plane-wave states modified by the cylindrical boundary condition. Results based on the Gross-Pitaevskii (G-P) equation show that the spin-spiral condensate with two counter-propagating plane-waves is favored at $c > 1$, while a single plane-wave is favored at $c < 1$ [28–31]. These two different condensates are degenerate for the

spin-independent interactions, *i.e.*, $c = 1$. However, calculations including quantum fluctuations of the zero-point energy show that the spin-spiral state wins at $c = 1$, and thus shift the phase boundary to a smaller value of c [19].

In this section, we study the vortex configurations of SO coupled BECs in both cases. The results of strong trapping potentials and weak interactions are presented in Sect. IV A, and those of the opposite limit are presented in Sect. IV B.

A. Vortex lattice configurations with a strong trapping potential

In this subsection, we turn on rotation and consider a strong trapping potential with a small value of α . The ground state condensate is obtained by numerically solving the following SO coupled GP equation. We assume that the condensate is uniform along the z -axis, and define the normalized condensate wavefunction $(\tilde{\psi}_\uparrow, \tilde{\psi}_\downarrow)^T$ satisfying $\int d^2\vec{r}(|\tilde{\psi}_\uparrow|^2 + |\tilde{\psi}_\downarrow|^2) = 1$. The dimensionless version of the G-P equation can then be written as

$$\frac{\mu}{\hbar\omega} \tilde{\psi}_\uparrow = \hat{T}_\uparrow \tilde{\psi}_\uparrow + \beta(|\tilde{\psi}_\uparrow|^2 + c|\tilde{\psi}_\downarrow|^2) \tilde{\psi}_\uparrow, \quad (12a)$$

$$\frac{\mu}{\hbar\omega} \tilde{\psi}_\downarrow = \hat{T}_\downarrow \tilde{\psi}_\downarrow + \beta(|\tilde{\psi}_\downarrow|^2 + c|\tilde{\psi}_\uparrow|^2) \tilde{\psi}_\downarrow, \quad (12b)$$

where

$$\begin{aligned} \hat{T} = & -\frac{1}{2}l^2(\partial_x^2 + \partial_y^2) + \alpha l(-i\partial_y\sigma_x + i\partial_x\sigma_y) \\ & + \frac{1}{2l^2}(x^2 + y^2) - \rho(-ix\partial_y + iy\partial_x) \\ & - \frac{\alpha\kappa}{l}(x\sigma_x + y\sigma_y), \end{aligned} \quad (13)$$

where μ is the chemical potential; the interaction parameter $\beta = gN/(\hbar\omega l_z)$; N is the particle number in the condensate; l_z is the system size along the z -direction.

The density and phase configurations at various parameters are shown in Fig. 4 (a) - (g), which exhibit rich structures of vortex-lattice. We look at Fig. 4 (d) in the absence of \vec{B}_{ex} , *i.e.* $\gamma = 0$. The density distribution of the spin-up component is composed of several disconnected density peaks near the trap center. On the other hand, the low density region is connected in contrast to the usual vortex lattice structure in which the low density region of vortex cores is disconnected. Nevertheless, we identify the locations of the singular points of the phase distribution pattern around which the phase winds with an integer number. These singular points are squeezed out to the edge of the condensate. On the other hand, the spin-down component exhibits the regular vortex-lattice structure, whose vortex cores overlap with the density peaks of the spin-up component. Around each vortex core, the two spin components show a half-quantum vortex configuration as those depicted in Fig. 2. Therefore, the condensates of two components together exhibit a

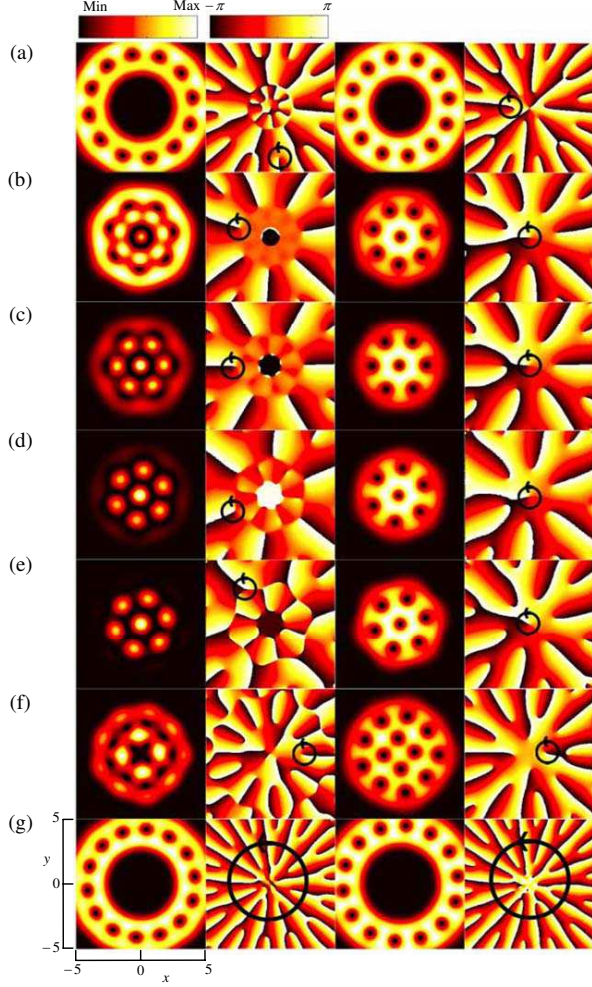


FIG. 4: (Color online) From left to right: the density and phase profiles of spin-up and down components with parameter values of $\alpha = 0.5$, $\beta = 10$, $\rho = 0.97$, and $c = 1$. From (a)-(g), γ is taken as 0.5, 0.25, 0.1, 0.0, -0.1, -0.25, and -0.5, respectively. At small values of $|\gamma|$ in (c) - (e), a half quantum vortex lattice is formed near the trap center. The spin-up component breaks into several density peaks, and the low density region is connected. As increasing the magnitude of $|\gamma|$ (b) and (f), the half-quantum vortex lattice evolves to the normal vortex lattice. For the large value of $|\gamma| = 0.5$ (a) and (g), the condensates show a lattice configuration around a ring. The black circle with an arrow indicates the the direction of the circulation around the vortex core. The unit of length for the figures is l .

lattice of half-quantum vortices. The corresponding spin density vector $\langle \vec{\sigma} \rangle$ shows a skyrmion-lattice structure, as shown in Fig. 5.

Now we turn on the external Zeeman term Eq. 5. For both cases of $\gamma > 0$ and $\gamma < 0$, at small values of $|\gamma|$, the half-quantum vortex lattice still forms, which is similar to that at $\gamma = 0$ as depicted in Fig. 4 (b, c, e). As increasing the strength of \vec{B}_{ex} , i.e., $|\gamma|$, more vortices appear as depicted in Fig. 4 (b,f). The condensates of the spin-up component gradually evolves to the usual

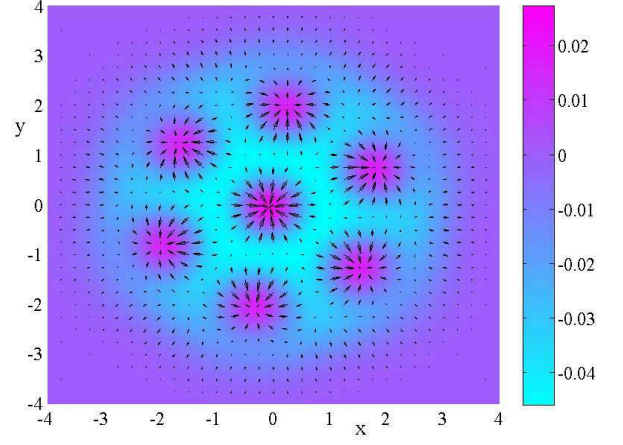


FIG. 5: (Color online) The ground-state spin density vector of Fig. 4 (d) with parameter values of $\alpha = 0.5$, $\beta = 10$, $\rho = 0.97$, $c = 1$, and $\gamma = 0$. The projection of $\langle \vec{\sigma} \rangle$ in the xy -plane is shown as black vectors. A color map is used to illustrate the $\langle \sigma_z \rangle$ component. The unit of length for the figure is l .

vortex-lattice configuration. The high density region becomes connected, while the density minima become disconnected vortex cores. On the other hand, the condensates of the spin-down component remains the usual vortex lattice configuration. For even larger values of $|\gamma|$, the ring-shaped vortex lattice with a giant vortex core is observed as shown in Fig. 4 (a) and (g). This is because the combined effect of the harmonic trap $V_{ext}(\vec{r})$ and the additional Zeeman term H_B shifts the potential minimum to a ring in real space with the radius of $r = \alpha\gamma l = |B_0|/(M\omega^2)$. The condensates of both spin up and down components distribute around this ring and from a giant vortex configuration. Additionally, the Zeeman term grows linearly as increasing r and favors in-plane polarization of \vec{S} . As a result, the vortex cores of the spin up and down components overlap with each other.

We stress that in all cases in Fig. 4 (a-g), the vortex numbers in the spin-up and down components differ by one, which is a characteristic feature brought by SO coupling. As shown in Eq. 10, for the eigenstate of the single-particle Hamiltonian with $j_z = m + \frac{1}{2}$, the two spin components carry different canonical orbital angular momenta m and $m+1$, respectively. In the presence of interaction, the giant vortex splits into a lattice of single-quantum vortices in each spin component. Nevertheless, the total vortex number in each component remains unchanged and differs by one.

B. Weak trapping potential

In this subsection, we study the rotating SO coupled BEC with a weak trapping potential and strong interactions.

Fig. 6 shows the density and phase profiles of each

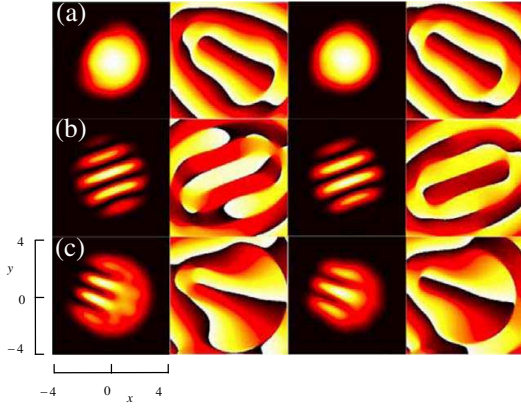


FIG. 6: (Color online) From left to right: the density and phase profiles of spin up and down components with the parameter values of $\alpha = 2.5$, $\beta = 40$, and $\gamma = 0$. (a) $c = 0.6$ and $\rho = 0.2$, a plane-wave-like state is obtained with a distorted phase pattern; (b) $c = 1.2$ and $\rho = 0.2$, the spin-spiral condensate is favored; (c) $c = 1.2$ and $\rho = 0.5$, the condensate exhibits an intermediate configuration between those of (a) and (b). The color scales for the density and phase distributions are the same as those in Fig. 4. The unit of length for the figures is l .

spin component in the absence of external magnetic field \vec{B} , i.e., $\gamma = 0$. In Fig. 6 (a) with $c < 1$, the condensate is a twisted plane-wave state subject to the cylindrical boundary condition. The spin polarization mainly lies in the xy -plane. In the representation eigen-basis of s_z , the spin up and down components show nearly the same distributions of density and phase profiles. Nevertheless, the phase distribution is distorted from the exact plane-wave state. On the other hand, as depicted in Fig. 6 (b), at $c > 1$ the spin-spiral-like condensate with two counter-propagating plane-waves is still favored with twisted phase profiles. As shown in Fig. 6 (c), increasing the angular velocity ρ gives rise to an intermediate configuration between the distorted spin-spiral and the single-plane wave states. In all the patterns, vortices locate either on the edge of the condensate or the density minima of each component.

Next, we consider the case of $\gamma \neq 0$. Introducing H_B significantly enriches the structures of the rotating SO coupled condensates. We only consider a small angular velocity at $\rho = 0.1$ for the reason of numerical convergence, but vary the values of γ from $0.5 \sim -0.7$ as presented in Fig. 7 (a)-(h), respectively. With small and intermediate values of $|\gamma|$ (e.g. Fig. 7 (b)-(f)), the condensate breaks into several domains. Inside each domain, the condensate can be approximated as a single plane-wave state. Vortices center around the local density minima. The local wavevectors are configured such that the local spin polarization $\langle \vec{S} \rangle$ align along the local Zeeman field of $\vec{B}_{ex}(\vec{r})$. If $\gamma > 0$ at which the external Zeeman field enhances the rotation induced ones, we obtain a clockwise configuration of wavevectors. There is

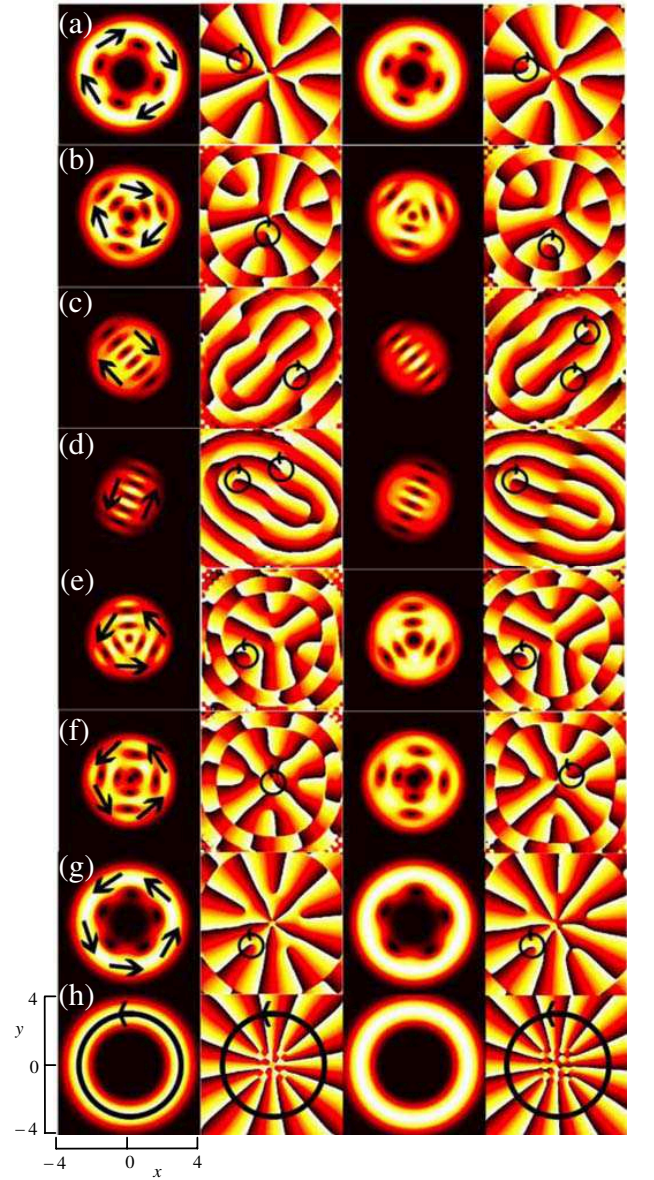


FIG. 7: (Color online) From left to right: the density and phase profiles of spin up and down components with parameter values of $\alpha = 4$, $\beta = 20$, $c = 1$, and $\rho = 0.1$. From (a)-(h), γ is taken as 0.5 , 0.3 , 0.1 , -0.05 , -0.25 , -0.35 , -0.6 , and -0.7 , respectively. The black arrow in each domain represents the local wavevector direction of the corresponding plane-wave state, which shows a clockwise or counter-clockwise configuration depending on the sign of γ . For sufficiently large values of $|\gamma|$, condensates distribute around a ring in space forming a giant vortex. The color scales for the density and phase distributions are the same as that in Fig. 4. The black circle with an arrow indicates the the direction of the circulation around the vortex core. The unit of length for the figures is l .

one more vortex with the negative phase winding in the spin up component than in the spin down component, which reflects the “anti-paramagnetic” feature. On the contrary, if $\gamma < 0$, the anti-clockwise patterns of wavevectors is favored. Similarly, the spin-down component also carries one more vortex than the up component.

At small values of $|\gamma|$, two domains are formed as depicted in Fig. 7 (c) and (d). The vortices organize into straight-lines between two domains. A variational wavefunction is constructed as

$$\tilde{\psi}(\vec{r}) \sim \left[f_{-}(x)e^{-i\frac{\theta}{2}}\psi_{-,\vec{k}_0} + f_{+}(x)e^{i\frac{\theta}{2}}\psi_{-,\vec{k}_0} \right] \times \frac{e^{-r^2/(2a^2)}}{\sqrt{\pi}\sigma}, \quad (14)$$

where without loss of generality, we choose the wavevector $\vec{k}_0 = k_0\vec{e}_y$; a is radius of the condensate; θ is the relative phase difference between the two plane wave domains; $|f_{-,+}(x)|^2 = (e^{\pm x/W} + 1)^{-1}$ are smeared step functions with W the width of the domain wall. We assume $\sigma \gg (W, 1/k_0)$. Such variational wavefunction has neglectable contribution to the energy term $\langle H_{rot} \rangle$. This explains why the two domain pattern is absent by increasing the rotational angular velocity ρ only, but appears immediately even at small values of $|\gamma|$. With increasing $|\gamma|$, the condensate breaks into more and more domains as in Fig. 7 (b), (e) and (f).

As further increasing $|\gamma|$, domains connect together as a giant vortex as shown in Fig. 7 (a, g, h). The condensates of both spin up and down components distribute around a ring with the radius of $\alpha|\gamma|l$ and overlap each other. This is a giant vortex configuration with a texture of spin aligned along the radial direction. The phase winding numbers of the spin-up and down components differ by one due to the SO coupling.

V. CONCLUSION

To summarize, we have considered the vortex structures of SO coupled BECs in a rotating trap combined

with an external spatially dependent Zeeman field. In the case of strong confining potentials and weak interactions, the condensate exhibit vortex-lattice structures. As varying the magnitude of the external Zeeman field, the configuration evolves from a half-quantum vortex-lattice to a normal one. In the opposite limit, the condensate develops multi-domain patterns with the external Zeeman field. Each domain represents a local plane-wave state, whose wavevector exhibit a clockwise or counter-clockwise configuration. Domain boundaries play the role of like vortices.

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Note added Near the completion of this manuscript, we notice a recent paper studying the rotating Rashba SO coupled BEC, which considered a special case in the presence of the extra term of Eq. 5 with $\gamma = -\rho$ [40]. Our work has studied the general cases, including the pure rotation without the external fields which corresponds to $\gamma = 0$. We have also noticed a recent work [41] by J. Radić *et al.*, where the effective Hamiltonians under different kinds of experimental situations have been discussed. In that framework, the effective magnetic field \vec{B}_R appears when we rotate the entire experimental setup along \hat{z} -axis.

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