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Universality versus material dependence of fluctuation forces between metallic wires

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We calculate the Casimir interaction between two parallel wires and between a wire and a metall plate. The dielectric properties of the objects are described by the plasma, Drude and perfect metal models. We find that at asymptotically large separation interactions involving plasma wires and/or plates are independent of the material properties, but depend on the dc conductivity σ for Drude wires. Counterintuitively, at intermediate separations the interaction involving Drude wires can become independent of σ. At smaller separations, we compute the interaction numerically and observe an approach to the proximity approximation.

Effective interactions between nanowires and nanotubes have attracted lots of attention due to their growing applications in micro- and nanomechanical systems [1–3]. The knowledge of the interactions between single walled carbon nanotubes (SWCNT) with different chirality and hence electromagnetic response is important to separate a polydisperse solution of SWCNT in fractions of equal chirality [4]. Under many circumstances, van der Waals or Casimir forces are the dominant interaction and hence a precise understanding of them is needed. Furthermore, cylindrical shapes are important for precision Casimir force measurements, in comparison to spheres, because of the larger effective area of interaction [5, 6]. Approximations of the Casimir force between cylinders and plates [6] have shown that the temperature dependence varies based on the description of the material properties. Thus there is a need for exact calculations of the Casimir force for cylindrical shapes taking into account realistic material response.

It has been demonstrated that Casimir interactions strongly depend on the combined effects of shape and material properties, see, e.g., [7, 8]. The interplay is particularly strong for quasi one-dimensional conducting materials due to strongly anisotropic collective charge fluctuations. Indeed, for two parallel perfectly conducting wires of distance d the retarded interaction energy per length is $\mathcal{E}/L \sim \hbar c/d^2$, apart from a logarithmic factor [9]. It decays only slowly compared to the retarded interaction $\mathcal{E}/L \sim \hbar c R^4/d^6$ between insulating cylinders that do not support collective fluctuations. Most studies of interactions between one-dimensional systems over a wide range of separations concentrate on these two situations. However, low dimensionality in combination with finite conductivity and plasmon excitations should give rise to interesting new effects that might be probed experimentally using, e.g., the coupling to mechanical oscillation modes. The often employed technique for these effects, the proximity force approximation (PFA) cannot capture the correlations of shape and material response since it is based on the interaction between planar surfaces. There have been attempts to compute the van der Waals interaction between cylinders (and plates) for particular frequency dependent permittivities [9–15]. More specifically, a number of studies have been performed for short sepration regime with the main focus on the corrections to the Proximity Force Approximation [**? ? ?**]. However, the interplay between shape and material effects is not transparent in these works as they are limited either to perfect metals or to asymptotic limits.

In this Letter, we employ the scattering approach to study the interaction between two infinitely long, parallel metallic wires and a wire and a plate that are described either by the plasma or the Drude dielectric function. We model the wires as circular cylinders of radius R and obtain analytical results for the interactions at distances much larger than R. We find regimes of different amplitude and power-law, depending on the relation of the radius to the length $\lambda_\sigma = 2\pi c/\sigma$ with conductivity σ and the plasma wave length λ_p.

Most interestingly, we find that the interaction involving Drude wires approaches the universal interaction between perfect metal wires at intermediate separations and becomes *non-universal* (material dependent) at distances $d \gtrsim R^2/\lambda_\sigma$. This behavior is explained in terms of the size of collective charge fluctuations in a Drude metal. For wires that support plasma oscillations, the interaction does show universality at asymptotically large distances. An estimate of the interaction between two gold wires based on the asymptotic expressions that we found with $R = 10$nm, length $L = 100\mu$m, $\lambda_p = 137$nm and $\lambda_\sigma = 5$nm at a distance $d = 200$nm yields a force of ≈ 1pN within the plasma description and ≈ 27pN within the Drude model. These forces are experimentally detectable and allow a much clearer distinction between plasma and Drude model predictions as compared to two plates or a plate and sphere [8]. This is of particular importance in view of recent experimental findings that interactions between metals might not be consistent with the Drude model [16]. In order to confirm the validity of our large distance expansions and to compare with the PFA at short distances we have performed numerical computations of the interaction.

To investigate the Casimir interaction between a cylinder parallel to a plate or another cylinder, we employ the scattering approach, which allows us to calculate the energy over a large range of separations between the objects. Because of the translational symmetry along the cylinder axis, the Casimir interaction can be written as [17]

$$\mathcal{E} = \frac{\hbar c L}{4\pi^2} \int_0^\infty d\kappa \int_{-\infty}^{\infty} dk_z \, \ln\det(\mathbf{1} - \mathbb{N}) \,, \qquad (1)$$

with L the length of the cylinder, κ the Wick rotated frequency, k_z the wave number along the cylinder axis, and $\mathbf{1}$ the identity matrix. The matrix $\mathbb{N}$ factorizes into the object's scattering amplitudes (T-matrices) that encode the material dependence and distance dependent translation matrices that described the coupling between multipoles on distinct objects.

Parallel wires – For two infinitely long parallel cylinders with radii R, aligned along the z-axis, the elements of the matrix $\mathbb{N}$ for polarizations $\alpha, \beta = E, M$ and partial waves m, m' are

$$\mathbb{N}^{\alpha\beta}_{mm'} = \sum_{\gamma=E,M} T^{\alpha\gamma}_m \sum_{n=-\infty}^{\infty} \mathcal{U}^{12}_{mn} \, T^{\gamma\beta}_n \, \mathcal{U}^{21}_{nm'} \,, \qquad (2)$$

with T the T-matrix of the cylinder (see below). The translation matrix, $\mathcal{U}^{12}$, relates the regulare vector waves $\mathbf{M}^{\rm reg}_{k_z n} = (\kappa^2 + k_z^2)^{-1/2} \nabla \times [I_n(p\rho) e^{i(n\theta + k_z z)}]$ and $\mathbf{N}^{\rm reg}_{k_z n} = \kappa^{-1} \nabla \times \mathbf{M}^{\rm reg}_{k_z n}$ with $p = \sqrt{\kappa^2 + k_z^2}$ in the cylindrical coordinates (ρ, θ, z) of one cylinder to the outgoing vector waves of the other cylinder that are given through replacing the Bessel functions I_n by the Bessel functions of second kind, K_n. The elements are given by

$$\mathcal{U}^{12}_{nn'} = (-1)^{n'} K_{n-n'}(p\,d) \qquad \mathcal{U}^{21}_{nn'} = (-1)^{n-n'} \mathcal{U}^{12}_{nn'} \,, \quad (3)$$

where d is the distance between the centers of cylinders. Note that the translation matrix conserves polarization and thus is independent of the polarization index. The T-matrix for a cylinder with dielectric function $\epsilon(ic\kappa)$ and magnetic permeability $\mu(ic\kappa)$ is diagonal in the partial wave number n but couples polarizations. Its elements are [17]

$$T^{EE}_n = -\frac{I_n(pR)}{K_n(pR)} \frac{\Delta_{2,n}\Delta_{3,n} + P_n^2}{\Delta_{1,n}\Delta_{2,n} + P_n^2} \,, \qquad (4)$$

$$T^{EM}_n = -\frac{P_n}{\sqrt{\epsilon\mu}(pR)^2 K_n^2(pR)} \frac{1}{\Delta_{1,n}\Delta_{2,n} + P_n^2} \,, \qquad (5)$$

with $P_n = (nk_z/(\sqrt{\epsilon\mu} R^2 \kappa)) \left(1/p'^2 - 1/p^2\right)$, $p' = \sqrt{\epsilon\mu\kappa^2 + k_z^2}$ and

$$\Delta_{1,n} = \frac{I'_n(p'R)}{p'R I_n(p'R)} - \frac{1}{\epsilon} \frac{K'_n(pR)}{pR K_n(pR)} \,. \qquad (6)$$

$\Delta_{2,n}$ is obtained from Eq. (6) by interchanging ϵ with μ, and $\Delta_{3,n}$ follows from Eq. (6) by replacing K'_n with I'_n and K_n with I_n. The elements T^{MM}_n are given by T^{EE}_n after interchanging ϵ with μ. Finally, antisymmetry in polarization yields $T^{ME}_n = -T^{EM}_n$.

In order to investigate the impact of the material properties on the Casimir interaction, we consider plasma, Drude and perfect metal cylinders with the magnetic permeability $\mu = 1$. The Drude dielectric function is

$$\epsilon(ic\kappa) = 1 + \frac{(2\pi)^2}{(\lambda_{\rm p}\kappa)^2 + \lambda_\sigma \kappa/2} \,, \qquad (7)$$

and reproduces the plasma model for $\lambda_\sigma \to 0$. Using Eq. (7), the asymptotic behavior of the T-matrix element of Eq. (4) for $n = 0$ at small frequencies ($\kappa \ll 1$, k_z/κ fixed) is given by

$$T^{EE}_0 \approx -\frac{p^2 R^2}{C(\kappa) - p^2 R^2 \ln(pR/2)} \,, \qquad (8)$$

where $C(\kappa)$ depends on the dielectric function. For the plasma model one has $C(\kappa) \approx {\lambda_{\rm p}}^2 \kappa^2/(2\pi^2)$ if the plasmon oscillations cannot build up transverse to the wire axis since the diameter is too small, i.e., $R \ll \lambda_p$. For the Drude model, $C(\kappa) = \lambda_\sigma \kappa/(4\pi^2)$ if $\kappa \ll \lambda_\sigma/\lambda_p^2$, $1/\lambda_\sigma$. The first of the two conditions implies that Drude behavior dominates over plasma behavior, i.e., the second term in the denominator of Eq. (7) is larger than the first term. The second condition ensures that the Drude dielectric function is large compared to one, i.e., metallic behavior is pronounced. For a perfect metal one has $C(\kappa) = 0$. At small frequencies κ but fixed k_z/κ, one has for Drude cylinders $T^{EE}_0 \sim \kappa$, while for plasma and perfect metal cylinders one has $T^{EE}_0 \sim 1$. Since $T^{MM}_0 \sim \kappa^2$, $T^{EM}_0 = T^{ME}_0 = 0$ and higher order elements associated with $n \neq 0$ scale as $\kappa^{2|n|}$, it is sufficient for the large distance interaction to consider only the element T^{EE}_0.

The Casimir interaction between metallic wires is in general complicated and no simple analytical expression that applies to all length scales can be obtained. However, using Eqs. (1) and (2) along with T^{EE}_0 given in Eq. (8), the asymptotic interaction at large separations, $d \gg R$, can be calculated in various limiting cases. For metals with diverging response at zero frequency, one usually observes universal behavior for the interaction at large separations. Indeed, that is what we obtain for the interaction between plasma wires (or plasma and perfect metal wires) which then becomes $\mathcal{E} = -\hbar c L/(8\pi d^2 \ln^2(d/R))$ [9, 13]. This universal form is only applicable beyond an exponentially large crossover length $d \sim R \exp(\lambda_{\rm p}^2/R^2)$. Below this scale the interaction becomes material dependent. For two plasma wires or a plasma wire and a perfect metal wire, with $\lambda_{\rm p}/R \gg 1$, the energy scales as $-R/(\lambda_p d^2 \ln^{3/2}(d/R))$. For numerical coefficients, see Fig. 1(a).

For configurations involving at least one Drude cylinder we find a rather distinct behavior that deviates from naive expectations for universality. For large distances $d \gg R$, λ_σ we obtain two different scaling regimes that are separated, up to logarithmic corrections, by the curve $d/R \sim \sqrt{d/\lambda_\sigma}$, see Fig. 1(b). The unexpected feature is that the interaction is universal in the regime where $d \ll R^2/\lambda_\sigma$. If the distance is increased beyond this crossover scale (with all other length scales kept fixed, see arrow (1) in Fig. 1(b)), the interaction becomes material dependent and up to logarithmic corrections, scales as $-R^2/(\lambda_\sigma d^3)$ for a Drude wire interacting with another Drude wire or a plasma or perfect metal wire. For detailed forms of the interactions in this limit, see Fig. 1(b). If, however, the radii of the wires are increased in the same way as their distance (d/R fixed, see arrow (2) in Fig. 1(b)), finite conductivity becomes unimportant at large distances and the interaction assumes the perfect metal form. An intuitive explanation of this non-universal large distance behavior is given below. Note that the decay of interactions between insulating wires for $d \gg R$ scales as $\hbar c L R^4/d^6$ with a material dependent coefficient.

Wire parallel to a plane – Now we consider a wire that is parallel to a plate. We assume that the plate is in the $y-z$ plane and its distance from the cylinder (z) axis is d. Then the matrix $\mathbb{N}$ in Eq. (1) can be written as

$$\mathbb{N}^{\alpha\beta}_{mm'} = \sum_{\gamma,\gamma'=E,M} T^{\alpha\gamma}_{m} \int_{-\infty}^{\infty} dk_y \frac{e^{-2d\sqrt{\mathbf{k}_\perp^2+\kappa^2}}}{2\sqrt{\mathbf{k}_\perp^2+\kappa^2}} \times D_{mk_z\gamma,\mathbf{k}_\perp\gamma'}\, T^{\gamma'}_{\mathbf{k}_\perp}\, D^\dagger_{\mathbf{k}_\perp\gamma',m'k_z\beta}\,(1-2\delta_{\gamma',\beta})\,, \qquad (9)$$

where k_y is y-component of the wave vector, $\mathbf{k}_\perp \equiv (k_y, k_z)$, and the matrix $D_{mk_z\gamma,\mathbf{k}_\perp\gamma'}$ converts between plane and cylindrical waves [17]. Further, $T^\gamma_{\mathbf{k}_\perp}$ are the T-matrix elements for the plate which are given by the usual Fresnel coefficients [17]. For perfect metal plates $T^E_{\mathbf{k}_\perp} = T^M_{\mathbf{k}_\perp} = 1$ and for small κ at fixed $k_\perp/\kappa$ one has for the plasma model $T^E_{\mathbf{k}_\perp} = T^M_{\mathbf{k}_\perp} = 1 + \mathcal{O}(\lambda_p\kappa)$ and for the Drude model $T^E_{\mathbf{k}_\perp} = T^M_{\mathbf{k}_\perp} = 1 + \mathcal{O}(\lambda_\sigma\kappa)$. Due to this identical behavior of the plate's T-matrix at small κ, the interaction at asymptotically large distances is independent of the model that describes the metal plate.

We calculate the Casimir energy between a wire and a plate at large separations ($d/R \gg 1$), using Eqs. (1), (8) and (9). We again find two different scaling regimes that are separated by curves that are given by the same expressions that we found for two wires, see Fig. 1. In the universal regime (perfect metal), the interaction is $\mathcal{E} = -\hbar c L/(16\pi d^2 \ln^2(d/R))$ [9, 13] for a plasma wire at asymptotically large $d \gg R$ and for a Drude wire at intermediate distances with λ_σ, $\lambda_p^2/\lambda_\sigma \ll d \ll R^2/\lambda_\sigma$, see Fig. 1(b). In the other regime the interaction is non-universal with the energies up to logarithmic accuracy scaling as $\mathcal{E} \sim \hbar c L R/(\lambda_p d^2)$ for a plasma wire and $\mathcal{E} \sim \hbar c L R^2/(\lambda_\sigma d^3)$ for a Drude wire. For the precise form see Fig. 1(b). This can be compared to the faster decay that is observed for an insulating wire interacting with a plane which has $\mathcal{E} \sim \hbar c L R^2/d^4$ [17].

Numerical results - It is interesting to compare the asymptotic results with exact numerical calculations to identify the regions in which the asymptotics are correct. We calculate numerically the integrals and determinant in Eq. (1). The infinite matrix $\mathbb{N}$ given by Eq. (2) is truncated at a finite number $n = n_{\max}$ of partial waves. $n_{\max}$ depends on the separation between the objects and is chosen such that the energy varies by less than 0.01% upon increasing $n_{\max}$ by 10. As the separation becomes shorter, $n_{\max}$ increases. While for $d/R = 10$, $n_{\max} = 9$ is sufficient, we need to use $n_{\max} = 91$ for $d/R = 2.1$.

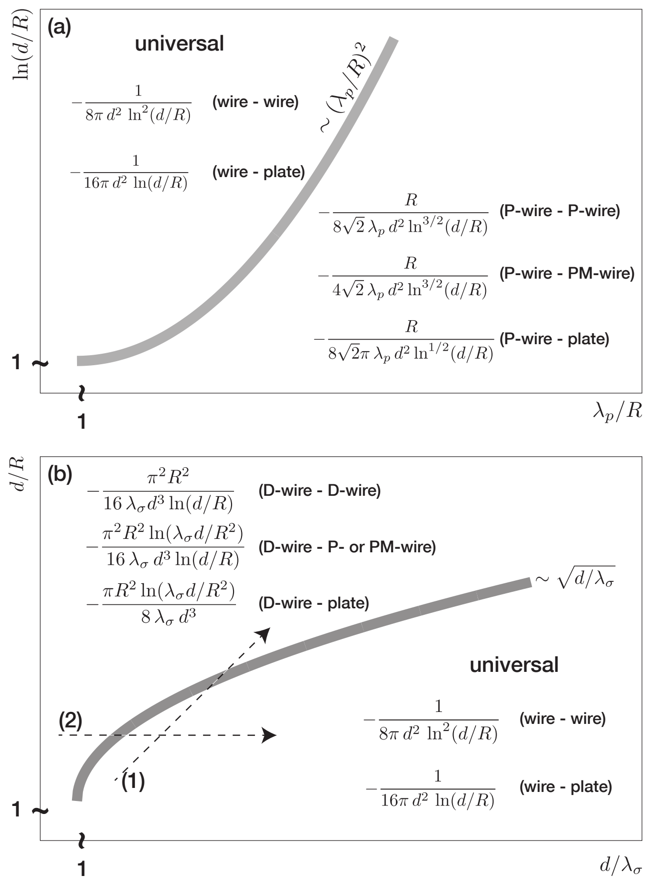

FIG. 1. Summary of the different forms of interaction between two wires and a wire and a plate. Shown are the rescaled interaction energies per cylinder length, $\mathcal{E}/(\hbar c L)$. (a) Interaction involving a plasma (P) wire with another plasma wire, a perfect metal (PM) wire or a plate. The asymptotic results apply sufficiently far away from the separating curve $\ln(d/R) \sim (\lambda_p/R)^2$ and for d/R, $\lambda_p/R \gg 1$. (b) Interaction involving a Drude (D) wire with another Drude wire, a plasma wire, a perfect metal wire or a plate. The separating curve is given, up to logarithmic corrections, by $d/R \sim \sqrt{d/\lambda_\sigma}$. The shown expressions hold for d/R, $d/\lambda_\sigma \gg 1$ and $d \gg \lambda_p^2/\lambda_\sigma$. Depending on the relative size of length scales, different regimes can be reached: Dashed arrow (1) corresponds to an increasing distance d which ultimately leads to a *non-universal* interaction. Dashed arrow (2) indicates an overall increase of the geometry (i.e., d/R fixed) with constant conductivity leading to a *universal* interaction.

Figure 2 shows the ratio of the numerically computed force between two Drude wires and the corresponding asymptotic results (universal and non-universal regimes, see Fig. 1(b)) versus $\ln(2d/R)$. The material parameters are chosen as $\lambda_\text{p}/R = 0.5$ and $\lambda_\sigma = \lambda_\text{p}/27.4$ which correspond to gold with $\lambda_\text{p} = 137$ nm and $\lambda_\sigma \approx 5$ nm. At intermediate separations, the force normalized to the universal result approaches unity whereas at asymptoti-

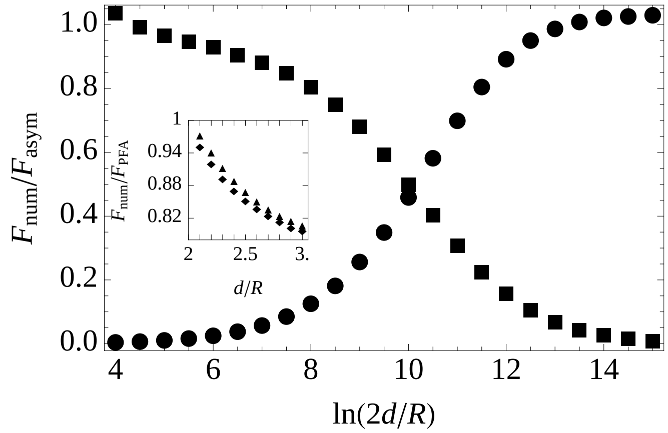

FIG. 2. Ratio of the numerically computed force and the universal (squares) and non-universal (circles) asymptotic results of Fig. 1 for two Drude wires with $\lambda_{\rm p}/R = 0.5$ and $\lambda_\sigma = \lambda_{\rm p}/27.4$. The inset shows the ratio of the numerical force to the proximity force approximation (PFA) for Drude cylinders with $\lambda_{\rm p}/R = 0.5$ (diamonds) and $\lambda_{\rm p}/R = 0.05$ (triangles).

cally large separations the force normalized to the non-universal result tends to unity. This confirms the validity of the crossover shown in Fig. 1(b). We also compare our numerical results for the force at short separations with those obtained from the proximity force approximation (PFA) which is based on the Lifshitz formula for parallel plates made of the same material as the wires. The inset in Fig. (2) shows the ratio of our numerical result for the force and the PFA result versus d/R for Drude wires with $\lambda_{\rm p}/R = 0.5$ and $\lambda_{\rm p}/R = 0.05$. Our data support the validity of the PFA in the limit of vanishing separations.

In summary, we have calculated the Casimir force between two metallic cylinders and a metallic cylinder parallel to a plate. We find that a significant feature of the interaction between a Drude Cylinder with another Drude cylinder or a plate is that upon increasing the separation, the interaction can move from a universal regime to a non-universal one. This behavior can be understood from the wave equation for the electric field inside a Drude cylinder. For imaginary frequencies $\omega = ic\kappa$, the Helmholtz operator $\nabla^2 + \epsilon(\omega)(\omega/c)^2$ for a good Drude conductor becomes $\nabla^2 - 8\pi^2\kappa/\lambda_\sigma$. We are interested in the maximal scale of the field and hence charge fluctuations for a given κ. With the smallest transverse wave vector k_x, $k_y \sim 2\pi/R$ we find the dispersion relation

$$|k_z| \sim R^{-1}\sqrt{\kappa/\kappa_c - 1}\,, \quad \kappa_c = \lambda_\sigma/R^2\,. \tag{10}$$

Hence, collective charge fluctuations on arbitrarily large scales exist only for $\kappa > \kappa_c$ which is a consequence of dimensionality that does not appear in the absence of transverse constraints ($R \to \infty$). For $\kappa < \kappa_c$ charge fluctuations break up into clusters of typical size $\sim R/\sqrt{1-\kappa/\kappa_c}$ due to finite conductivity. The spectral contribution to the interaction between cylinders at distance d is peaked around $\kappa \sim 1/d$. If $d \lesssim 1/\kappa_c$ ($d/R \lesssim \sqrt{d/\lambda_\sigma}$, see Fig. 1(b)), collective charge fluctuations contribute strongly to the interaction and render it universal as for perfect metal cylinders for which $\kappa_c \sim 1/\sigma \to 0$. In the asymptotic regime with $d \gtrsim 1/\kappa_c$ ($d/R \gtrsim \sqrt{d/\lambda_\sigma}$, see Fig. 1(b)), finite conductivity prevents fluctuations on arbitrarily large scales and hence the interaction is proportional to σ, i.e., non-universal. It is important to note that as R goes to zero, κ_c becomes larger, and in consequence the finite conductivity of cylinder becomes more important.

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