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Enhancement of Compton Scattering by an Effective Coupling Constant

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A robust thermodynamic argument shows that a small reduction of the effective coupling constant α of QED greatly enhances the low energy Compton scattering cross section and that the Thomson scattering length is connected to a fundamental scale λ. A discussion provides a possible quantum interpretation of this enormous sensitivity to changes in the effective coupling constant α.

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The process of the energy interchange between radiation and matter provided by Compton scattering is relevant in many areas of physics. For example, in cosmology it keeps the matter at the same temperature as radiation [1]. Compton scattering is also a unique spectroscopy for condensed matter physics, which has acquired greater importance with the advent of modern synchrotron sources [2–4]. For instance, it has been used to extract information about wave functions of valence electrons in a variety of systems ranging from ice [5, 6], water [7], alloys [8] and correlated electron systems [9]. Moreover, Compton scattering can potentially help delineate confinements [10] and spin polarization effects [11] in nanoparticles.

The Compton scattering cross section strength is determined by the classical electron radius, also known as the Thomson scattering length,

$$r_0 = \frac{e^2}{4\pi\epsilon mc^2} \approx 2.82 \times 10^{-13}\text{cm} , \qquad (1)$$

where e is the electron charge, m is the electron mass, c is the speed of light and ϵ is the dielectric constant. Unfortunately, the small size of r_0 makes Compton experiments in condensed matter systems difficult. This is why only few experiments have been done, even with the best synchrotron sources. The classical proton radius is even smaller by a factor $M/m \approx 1863$, where M is the proton mass. Therefore, nuclei are practically invisible in X-ray Compton scattering experiments.

In 1952 Max Born suggested that the electronic radius r_0 is connected to an absolute length scale λ [12]. Thus, if the electromagnetic interaction strength is modified, λ must change as well. Understanding this variation could enable us to enhance the Compton scattering cross sections by *engineering* an effective quantum electro-dynamics (QED) interaction. The effective coupling constant

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar c} , \qquad (2)$$

can be modified through the dielectric response ϵ, for instance, if the incident photon energy is tuned near to the binding energy of a deep core electron level in certain materials.

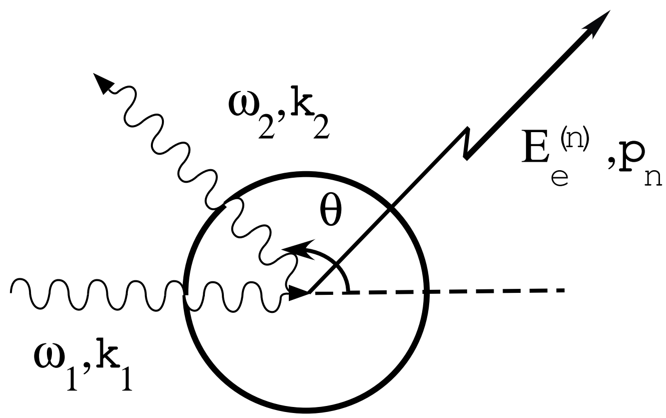

FIG. 1: Schematic diagram of the elementary scattering event involved in the Compton scattering process. The incoming photon scatters from the target to produce an outgoing photon and an electron and leaves the singly ionized target.

This work shows that the Compton cross section can depend strongly on the effective coupling constant α and that a reduction of α as small as 1% may lead to an increase the cross section by a factor 4. Moreover, the present results connect r_0 to a fundamental length λ and thus are consistent old hypothesis by Max Born.

The triple-differential scattering cross section for the process shown in Fig. 1, which is the elementary step underlying Compton scattering, is given by [13, 14]

$$\frac{d^3\sigma^{(n)}}{d\omega_2 d\Omega_2 d\Omega_e} = r_0^2 \frac{\omega_2}{\omega_1}(1+\cos^2\theta) \times |g_n(\mathbf{q})|^2 \delta(\omega_1 - \omega_2 - E_b^{(n)} - \frac{p_n^2}{2m}) , \qquad (3)$$

where θ is the scattering angle, $g_n(\mathbf{q})$ is the Fourier transform of the occupied Dyson orbital $g_n(\mathbf{r})$ with binding energy $E_b^{(n)}$, $\mathbf{q}$ is the momentum transferred to the final system, ω_1 and ω_2 are respectively the energies of the photon before and after the collision. The ejected electron state is usually approximated by a plane wave with momentum $\mathbf{p}_n$ and energy $E_e^{(n)} = p_n^2/(2m)$ if $E_b^{(n)} \ll E_e^{(n)}$. In this regime, Compton scattering is a unique window on the electronic structure of matter because in contrast with most structural analysis techniques which can only deliver information on the total electron densities, this spectroscopy allows direct measurements in momentum space of the electron density associated to a single ionization channel (i.e. a Dyson orbital in a one-electron picture). In the low-energy limit (i.e. $\omega_1 \ll mc^2$), Thirring [15] has shown that the Compton scattering cross section with all radiative corrections reduces in the non-relativistic expression given by Eq. (3). The only effect of the vacuum or the medium is to renormalize the Thomson scattering length r_0. The *Thirring theorem* is a consequence of Lorentz and gauge invariance [16, 17].

We now turn to a general thermodynamic argument in order to derive how the electron volume $V = 4\pi r_0^3/3$ depends on the the effective coupling constant α. Since the classical electron radius r_0 is the length at which QED renormalization effects become important, our argument must be consistent with differential equations of the renormalization group [18]. Thermodynamics is widely considered as a fundamental theory, since it has a universal applicability [19, 20]. Indeed it does not need any modification due to either relativity or quantum theory [21]. The first law of thermodynamics gives the variation of internal energy

$$dE = TdS - PdV + mc^2 d\alpha \ , \tag{4}$$

where T is the temperature, S is the entropy and $P = -E_s/V$ is a pressure imposed by a fictitious piston on the volume V in order to set the units scale for a given α [22]. Thus, the energy scale is characterized by $E_s = \alpha^x \ mc^2$, where x represent a positive integer exponent to be determined. The negative sign of the pressure P is explained by the fact that the electromagnetic vacuum fluctuation (i.e. the Casimir effect) try to pull the piston back into the system. Similar inward pressures are produced by cosmological constants [23]. The third term in Eq. (4) is similar to a chemical potential term since the number of virtual photons is proportional to the effective coupling constant α. Thus, we are assuming that the electron mass m determines the chemical potential of the virtual photons and that it is generated by the Coulomb field of the electron. In adiabatic conditions the term TdS vanishes. Moreover, at equilibrium $dE = 0$, thus the renormalization group β function [18] deduced from Eq. (4) is given by

$$\beta(\alpha) = r\frac{d\alpha}{dr} = -3\alpha^x \ . \tag{5}$$

The solutions for $x = 0, 1, 2$ show that the electron localizes (i.e. r_0 becomes small) when the interaction strength increases. When $x = 0$, the radius scales as

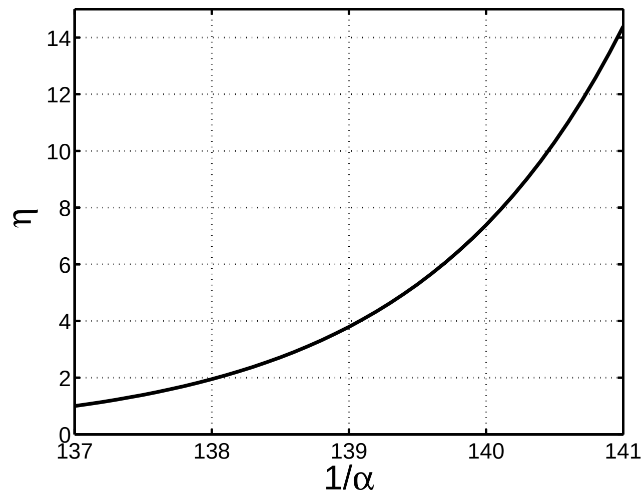

FIG. 2: Cross section enhancement η as a function of the inverse of the effective coupling constant α. Both η and α are pure numbers (without units).

$$r_0 = r_{max} \exp(-\alpha/3), \tag{6}$$

and has a maximal finite size r_{max} corresponding at $\alpha = 0$ while for $x = 1$, the scaling is

$$r_0 = \frac{\lambda_1}{\alpha^{1/3}}, \tag{7}$$

where λ_1 is radius corresponding at $\alpha = 1$. The exponent $x = 2$ is consistent with the QED β function [18]. The Born hypothesis is also verified when $x = 2$, since the corresponding solution admits a minimal length λ different from zero. In this case, the Thomson scattering length depends on $1/\alpha$ by an exponential function

$$r_0 = \lambda \exp\left(\frac{1}{3\alpha}\right) \ , \tag{8}$$

where λ is a certain small length to be determined. Moreover, the corresponding pressure $P = \alpha^2 \ mc^2$ sets the the atomic energy units. In fact, the atomic units are as natural as the fundamental Planck units [24]: their ratios to the fundamental units can be explained within our present argument connecting the Thomson scattering length to the fundamental scale. Interestingly, the volume renormalization factor $Q(\alpha)$ is $\exp(1/\alpha)$ for $x = 2$. This term is similar to the Boltzmann distribution in statistical mechanics (where α plays the role of an effective temperature).

The cross section enhancement defined by

$$\eta = \left[\frac{Q(\alpha)}{Q(1/137)}\right]^{2/3} \tag{9}$$

is shown in Fig. 2 for the case $x = 2$: a reduction of α by few percents induces a huge increase in η. Therefore, cross section enhancements obtained by tuning the incident photon energy near the binding energy of a deep core electron level can be described by the behavior for $x = 2$ while the cases $x = 0, 1$ give negligible cross section enhancements for small variations of α. The trend of η illustrated in Fig. 2 can be produced by a change $\Delta\epsilon$ of the dielectric response near an absorption edge.

Standard inelastic X-ray scattering experiments without the measurement of the kinematics of the outgoing (recoil) electron contain many other processes in addition to the elementary scattering event of Fig. 1. Therefore, coincidence $(\gamma, e\gamma)$ experiments [13] are needed in order to separate the X-ray Compton scattering with nearly free electrons from complicated processes. Some $(\gamma, e\gamma)$ spectrometers are already available for hard X-rays [25]. Unfortunately, standard $(\gamma, e\gamma)$ experiments can be tremendously challenging. Instead, one could use a soft-x-ray fluorescence spectrometer by Carlisle *et al.* [26]. By tuning the incident photon at the K edge of graphite, enhancement effects of the total cross section have been already observed. A coincidence measurement detecting the electrons escaping from the sample can then be used to separate Compton from other types of inelastic scattering. In this much simpler setup multiple scattering of the electrons in the sample are not an impediment for extracting the Compton contribution.

Realistic dielectric data for graphite provided by Draine [27] illustrates how tuning the incident photon energy near the binding energy of the K core level changes the dielectric response and thus the effective coupling constant for the valence electrons. When $x = 2$, a Compton cross section enhancement η of almost a factor four is predicted in graphite by using Draine's dielectric data as shown in Fig. 3. We note that a similar variation of the dielectric function for diamond has been previously reported by Nithianandam and Rife [28]. Besides, a calculation based on the finite difference method for near-edge structure? (FDMNES) [29] agrees with dielectric data of Draine. FDMNES shows that the anomalous scattering factor near the K edge of graphite becomes greater in amplitude than the number of electrons causing the real part of ϵ/ϵ_0 to be greater than unity (ϵ_0 is the vacuum permittivity).

Next, we justify a value for λ. According to Veneziano [30], a consistent quantum gravitational theory should obey the Born principle of reciprocity [31], a symmetry law under the interchange of space-time coordinates and the energy-momentum coordinates, which naturally leads to harmonic oscillators and to the normal modes of vibrating strings. In such theory it is natural to take as the action quantum the square of the Planck length [32]

$$\lambda = \ell_P = \sqrt{\frac{hG}{c^3}} \approx 4.05 \times 10^{-33}\text{cm} \;, \qquad (10)$$

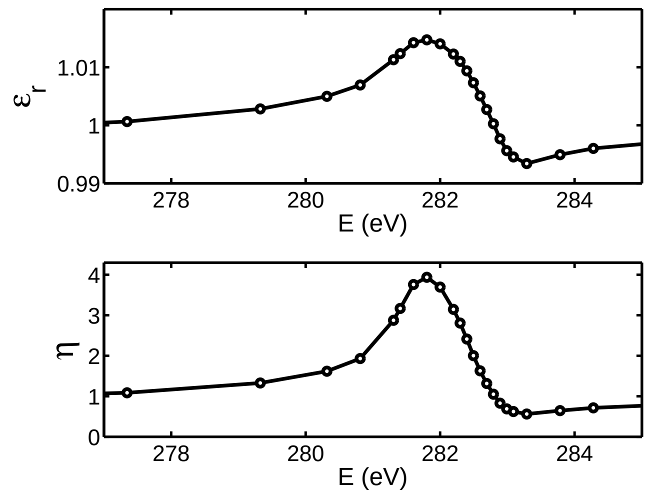

FIG. 3: Top: ϵ_r (ratio of the real part of the dielectric function ϵ and the vacuum permittivity ϵ_0) near the K edge of graphite. E is the incident photon energy. The points are from Draine [27]. Symbol size is representative of error bars. Bottom: corresponding cross section enhancement η.

where h is the Planck constant and G is the gravitational constant. Indeed, by using Eq. (8) with the Planck length λ and $\alpha = 1/137.03604$, the calculated Thomson scattering length is $r_0 = 2.79 \times 10^{-13}$cm, which differs about 1% from its exact value. Minor renormalization effects of the gravitational constant G could improve the agreement [33].

Finally, we could also reverse the logic. In our treatment the length λ is not fixed a priori. Therefore, we can use data from X-rays experiments in graphite to get information about the size of λ. This would strongly vivify a big portion of the existing literature in quantum gravity for which the presence of an effective minimal length is assumed to describe the discretization of a quantum space-time. Presently, in the absence of any experimental signature for quantum gravity such a minimal length is generically set between the electroweak scale $\sim 10^{-16}$ cm and the Planck length. As a result we are opening the door to the possibility of determining an extreme energy effect with sophisticated low energy experiments. In addition since preliminary data seem to support the idea that $\lambda = \ell_P$ up to 1 %, we can get more stringent constraints about the extension of the conjectured additional spatial dimensions with respect to what we currently know from the observed short scale deviations of Newton's law [20, 34].

In conclusion, we suggest that the low energy Compton cross section for the valence (i.e. nearly free) electrons of graphite can be described within the framework of the Thirring theorem implying that the only effect of medium is to renormalize the Thompson scattering length r_0. Besides, a general thermodynamic argument shows that the

Compton scattering cross section grows exponentially if the effective coupling constant α decreases. In particular, a striking enhancement is predicted when the incident photon energy is tuned near the binding energy of the K core level of graphite. The present enhancement effect is also consistent with the QED renormalization group.

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