# Duplex quantum communication through a spin chain

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Data multiplexing within a quantum computer can allow for the simultaneous transfer of multiple streams of information over a shared medium thereby minimizing the number of channels needed for requisite data transmission. Here, we investigate a two-way quantum communication protocol using a spin chain placed in an external magnetic field. In our scheme, Alice and Bob each play the role of a sender and a receiver as two states $\cos(\frac{\theta_1}{2})|0\rangle + \sin(\frac{\theta_1}{2})e^{i\phi_1}|1\rangle$, $\cos(\frac{\theta_2}{2})|0\rangle + \sin(\frac{\theta_2}{2})e^{i\phi_2}|1\rangle$ are transferred through one channel simultaneously. We find that the transmission fidelity at each end of a spin chain can usually be enhanced by the presence of a second party. This is an important result for establishing the viability of duplex quantum communication through spin chain networks.

## I. INTRODUCTION

In classical electronic communications, full-duplex transmission capabilities allow for the simultaneous sending and receiving of data to and from some remote host or process. In instances where real-time information transfer is required between two parties, for example in voice communications and high-performance distributed computing, full-duplex transmission is desired [1]. While two physical twisted-pairs of wires per cable provide the avenue for duplex transmission classically, there has been no counterpart for such communications in quantum machines. It will be shown, however, that spin chains can be used as foundational elements of quantum duplex communications. In principal, full-duplex information transfer can be achieved during quantum computations using the interactions which naturally occur between neighboring sites of a spin chain.

It was Bose who first suggested using an unmodulated spin chain to serve as a mediator for quantum information transfer [2]. The basic idea goes like this: An arbitrary qubit state is encoded at one end of the chain which then evolves naturally under spin dynamics. Later, at some time $\tau$, the state can be received at the other end with some probability. Although his original proposal offers the advantage of simplicity, it does not allow for a perfect state transfer in most situations. In order to improve the fidelity of transmission an extensive investigation has been made regarding state or entanglement transfer through permanently coupled spin chains [2–9]. The transmission fidelity (entanglement) can be significantly enhanced by means of introducing phase shifts, energy currents [5], or by properly encoding the state over more than one site [6, 7]. There are also methods using two parallel spin chains which allow for a perfect state transfer (PST) [8, 9]. In this case PST is achieved using

measurements at the end of the chain. Other methods require a single local on-off switch actuator [10], a single-spin optimal control [11], or via certain classes of random unpolarized spin chains [12]. PST in a strongly coupled antiferromagnetic spin chain has been reported in Ref. [13] which requires weakly coupled external qubits. Furthermore, PST [14] or perfect function transfer [15] can also be realized in a variety of interacting media, including, but not limited to, the spin chain model. Recently, state and entanglement transfer through an unmodulated spin bus with two attached qubits has been investigated. There the authors show that by tuning the couplings between the system and the additional qubits, high-quality communication [16] as well as a two-qubit entangling gate between arbitrary distant qubits [17] can be achieved."

In most scenarios considered in the literature the communication is assumed to occur in one direction, i.e. if Alice sends the information Bob plays the role of the receiver. This type of communication resembles a form of broadcasting where one party sends a signal while the second party simply "listens". Although broadcasting quantum information is certainly an important method of communication, it is by no means the only method needed for quantum computation. Full scale quantum computing will undoubtedly require multiplexing between multiple processes and therefore an analysis of the effects of state transmission in two directions is warranted. Here we study a duplex quantum communication protocol using a spin chain placed in a external magnetic field. In this case Alice and Bob each play the role of a sender and a receiver. Unlike the current trend in spin chain research, our intention is not to improve the quality of state transfer but rather to investigate how the presence of a second party effects the other senders transmission. We focus on the least technically challenging spin chain model and simply require local control over the first and last site for the preparation and reception of the states. We find that in most cases the presence of a second party can significantly enhance the fidelity of state transmission from the other party thereby allowing

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for reliable two-way communication.

The paper will be outlined as follows. In Section II we will describe the physical model we consider and derive expressions for the communication fidelity. The numerical results obtained from these expressions will be presented in Section III. Finally, we will conclude with a summary of our findings in Section IV.

## II. THE MODEL

We depict our scheme in Fig. 1. Alice and Bob are situated at opposite ends of a one-dimensional array of $N$ spin-1/2 systems. We assume the chain has been cooled to the ground state $|\downarrow_1\downarrow_2\ldots\downarrow_N\rangle$ prior to the encoding process, where we have defined the eigenstates of the Pauli operator $\sigma_z$ to be $|\downarrow\rangle \equiv |0\rangle$ and $|\uparrow\rangle \equiv |1\rangle$. Alice and Bob then respectively prepare the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ which they intend to send. To simplify matters, we will assume that both encodings take place simultaneously. After the states of the spin systems at sites 1 and
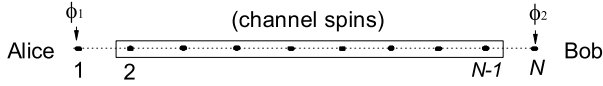


FIG. 1: Schematic illustration of our communication protocol. Alice and Bob respectively encode the states $|\varphi_1\rangle$, $|\varphi_2\rangle$ into the spins located at the first and last sites of the chain. After some time $\tau$, they attempt to receive the state which was sent from the opposite end.

$N$ have been prepared the system as a whole will then be allowed to evolve. This evolution will be generated by nearest-neighbor XY-type interactions and an externally applied magnetic field

$$H = -\frac{J}{2}\sum_{i=1}^{N-1}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y) - h\sum_{i=1}^{N}\sigma_i^z. \quad (1)$$

We assume a ferromagnetic coupling and take the interaction constant to be $J = 1.0$ throughout. The constant $h$ represents the external magnetic field strength of a field applied along the $z$ direction and $\sigma_i^{x,y,z}$ denote the Pauli operators acting on spin $i$. We consider an open ended chain which is perhaps the most natural geometry for a channel. This Hamiltonian can be diagonalized by means of the Jordan-Wigner transformation that maps spins to one-dimensional spinless fermions with creation operators defined by $c_l^\dagger = (\prod_{s=1}^{l-1} -\sigma_l^z)\sigma_l^+$. Here $\sigma_l^+ = \frac{1}{2}(\sigma_l^x + i\sigma_l^y)$ denotes the spin raising operator at site $l$. The action of $c_l^\dagger$ is to flip the spin at site $l$ from down to up. For indices $l$ and $m$, the operators $c_l$ and $c_m^\dagger$ satisfy the anticommutation relations $\{c_l, c_m^\dagger\} = \delta_{lm}$. The
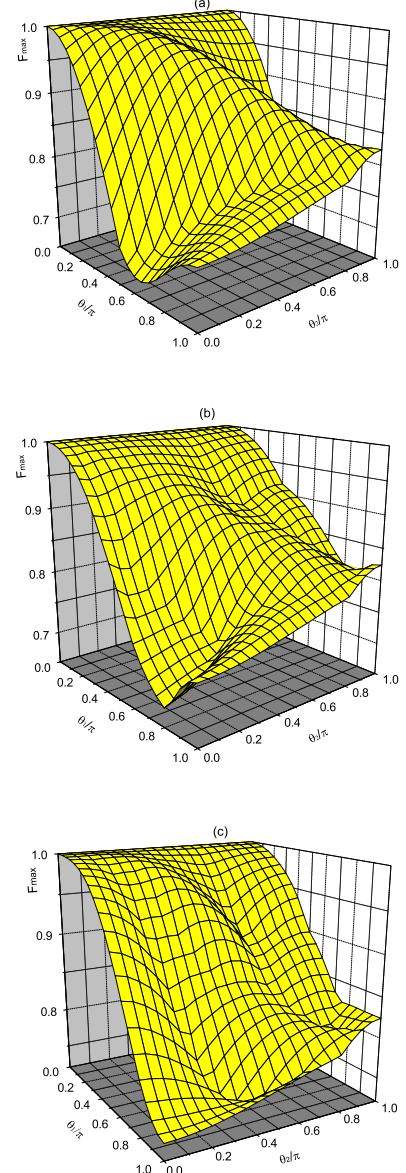


FIG. 2: (Color online) The maximum fidelity $F_{max}$ which can be obtained when transferring a quantum state from Alice to Bob in a time interval $t \in [10, 50]$ as a function of the polar angles $\theta_1$ and $\theta_2$. We have set $N = 10$, $\phi_1 = \phi_2 = 0$ and (a) $h = 0.0$ (b) $h = 0.1$ and (c) $h = 1.0$.

z-component of the total spin is a conserved quantity implying the conservation of the total number of excitations $M = \sum_l c_l^\dagger c_l$ in the system. The evolution of the creation operator $c_j^\dagger$ is given by [18]

$$c_j^\dagger(t) = \sum_l f_{j,l}(t)c_l^\dagger, \quad (2)$$

where

$$f_{j,l} = \frac{2}{N+1} \sum_{m=1}^{N} \sin(q_m j) \sin(q_m l) e^{-iE_m t}, \quad (3)$$

with $E_m = 2h - 2J\cos(q_m)$ and $q_m = \pi m/(N+1)$. When the number of magnon excitations is more than one, the time evolution of the creation operators is [19]

$$\prod_{m=1}^{M} c_{j_m}^\dagger(t) = \sum_{l_1 < \ldots < l_M} \det \begin{vmatrix} f_{j_1,l_1} & f_{j_1,l_2} & \cdots & f_{j_1,l_M} \\ f_{j_2,l_1} & f_{j_2,l_2} & \cdots & f_{j_2,l_M} \\ \cdots & \cdots & \cdots & \cdots \\ f_{j_M,l_1} & f_{j_M,l_2} & \cdots & f_{j_M,l_M} \end{vmatrix} \prod_{m=1}^{M} c_{l_m}^\dagger \quad (4)$$

where $M$ gives the number of excitations. The set $\{j_1, j_2, ..., j_M\}$ denotes the sites where the excitations are created and $\{l_1, l_2, ..., l_M\}$ denotes an ordered combination of $M$ different indices from $\{1, 2, ..., N\}$. In this paper we consider chains which carry no more than two excitations, so we set $M = 2$ in the situations when Eq. (4) is used.

To proceed, let us assume that Alice prepares an arbitrary qubit state $|\varphi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$ at the first site while Bob prepares the state $|\varphi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$ at the $N$th site. The initial state of the chain is then

$$|\Phi(t=0)\rangle = (\alpha_1 |0\rangle + \beta_1 |1\rangle) \otimes |\mathbf{0}\rangle \otimes (\alpha_2 |0\rangle + \beta_2 |1\rangle). \quad (5)$$

The time evolution of $|\Phi(0)\rangle$ will be

$$|\Phi(t)\rangle = [\alpha_1\alpha_2 + \sum_{i=1}^{N} A_i(t)c_i^\dagger + \sum_{i<i'} B_{i,i'}(t)c_i^\dagger c_{i'}^\dagger] |\mathbf{0}\rangle, \quad (6)$$

where $A_i = \alpha_1\beta_2 f_{N,i} + \beta_1\alpha_2 f_{1,i}$, $B_{i,i'} = \beta_1\beta_2(f_{1,i}f_{N,i'} - f_{1,i'}f_{N,i})$. Although we have used $|\mathbf{0}\rangle$ interchangeably in these last two equations, it should be understood that in Eq. (5) the notation $|\mathbf{0}\rangle$ is used to refer to the state $|\downarrow_2\downarrow_3 \ldots \downarrow_{N-1}\rangle$ of the channel spins between the first and last site, while in Eq. (6) it is used to represent the state $|\downarrow_1\downarrow_1 \ldots \downarrow_N\rangle$ of the entire chain. We see from Eq. (6) that the excitations which are created at the ends of the chain begin to spread over time, resulting in a probability distribution of appearing over all sites. Unlike the original one way communication protocol where at most one excitation exists in the chain, here we can find excitations at sites $i$ and $i'$ ($i \neq i'$) with probability $|B_{i,i'}(t)|^2$. Clearly, when $\alpha_2 = 1.0$ and $\beta_2 = 0.0$, our two-way communication scheme reduces to the original scenario [2].

Now consider the dynamics of the system. The fidelity between the received state and initial state $|\varphi_i\rangle$ ($i = 1, 2$) is defined by $F = \sqrt{\langle\varphi_i| \rho(t) |\varphi_i\rangle}$, where $\rho(t)$ is the reduced density matrix associated with the spin state at the receiving position. In what follows we will assume that both parties measure their states simultaneously since the measurement process will necessarily disturb the state of the system. It would be interesting to examine the effect of non-simultaneous measurements on state transmission and will be a subject of later work. For now, with the assumption of simultaneous measurements, the fidelity at Bob's end $F_N$ and Alice's end $F_1$ are calculated to be

$$F_N = \left[ \left||\alpha_1|^2 \alpha_2 + \beta_1^* A_N\right|^2 + \sum_{i=1}^{N-1} |\alpha_1^* A_i + \beta_1^* B_{i,N}|^2 + \sum_{i<(i'\neq N)} |\alpha_1|^2 |B_{i,i'}|^2 \right]^{1/2} \quad (7)$$

$$F_1 = \left[ \left||\alpha_2|^2 \alpha_1 + \beta_2^* A_1\right|^2 + \sum_{i=2}^{N} |\alpha_2^* A_i + \beta_2^* B_{1,i}|^2 + \sum_{(i\neq 1)<i'} |\alpha_2|^2 |B_{i,i'}|^2 \right]^{1/2} \quad (8)$$

We will analyze the behavior of duplex quantum communication in terms of these fidelity measures next. We will show that the transmission fidelity at each end can be significantly enhanced by the presence of the other party.

## III. RESULTS AND DISCUSSION

We assume that Alice and Bob respectively prepare the arbitrary qubit states $\alpha_1 |0\rangle + \beta_1 |1\rangle$ and $\alpha_2 |0\rangle + \beta_2 |1\rangle$ with

each individual state being represented by a point on a Bloch sphere with $\alpha_i = \cos(\frac{\theta_i}{2})$, and $\beta_i = \sin(\frac{\theta_i}{2})e^{i\phi_i}$ ($i = 1, 2$). First we investigate the effect of the polar angles $\theta_i$ on the transmission fidelity and let $\phi_1 = \phi_2 = 0$. In Fig. 2, we plot the maximal fidelity which can be obtained at Bob's side of a $N = 10$ site chain in a time interval $t \in [10, 50]$ as a function of the parameters $\theta_1$ and $\theta_2$. We exclude the time interval $[0, 10]$ since we need to avoid the fact that if Alice and Bob both send a similar state, the fidelity will be very high at $t = 0$ regardless of any actual state transfer. Throughout the paper, the

maximum fidelity $F_{\max}$ is found in the same time interval. When the chain is isolated from an external field ($h = 0$) we find that when $\theta_1 = \theta_2$, i.e. when the two states are identically prepared, $F_{\max}$ is higher than in the other cases (see Fig. 2(a)). Although the fidelity is maximized when both senders encode similar states, any nonzero $\theta_2$ will enhance the fidelity of Alice's transmission when $h = 0$ and $\phi_1 = \phi_2 = 0$. For example, when $\theta_1/\pi = 0.6$, $F_{\max} = 0.67$ for $\theta_2 = 0$, while $F_{\max} = 0.91$ for $\theta_2/\pi = 0.6$. Since the chain has been initialized to the ground state $|\downarrow_1\downarrow_2 \ldots \downarrow_N\rangle$ before Alice and Bob encode their states, a $\theta_2 = 0$ encoding at Bob's end amounts to the same thing as if he were not even present. We can therefore infer that duplex quantum communication has a positive impact on a senders ability to transfer information, at least in the case where $h = 0$ and $\phi_1 = \phi_2 = 0$. In a way, Bob's encoding resembles an "activator" used in chemistry.

Let us now consider the influence of the external magnetic field. Fig. 2(b) and (c) exemplify the weak field ($h = 0.1$) and strong field ($h = 1.0$) regimes. In Fig. 2(b), the behavior of $F_{\max}$ with $\theta_1$ and $\theta_2$ is similar to that given in Fig 2(a), but Fig 2(c) shows that the presence of a strong field can hinder the aforementioned properties as $F_{\max}$ is only slightly enhanced for some range of $\theta_2$. For some values of $\theta_2$, the fidelity of Alice's transmission can actually decrease, though the decrease is small. For example, when $\theta_1/\pi = 0.8$, $F_{\max} = 0.79$ for $\theta_2 = 0$ while $F_{\max} = 0.75$ for $\theta_2/\pi = 0.35$. The fidelity of Bob's transmission can be explicitly calculated from Eq. (8). The results will be similar to those above due to symmetry hence we only consider Alice's state transfer here. We now consider the effect of the phase angles $\phi_i$. A numerical calculation shows that $F_{\max}$ only depends on the difference of the parameters $\phi_1$, and $\phi_2$. In Fig. 3, we plot the maximum fidelity $F_{\max}$ as a function of the difference $\delta\phi = \phi_2 - \phi_1$ for fixed parameters $\theta_1$ and $\theta_2$. Again, we consider the vanishing field ($h = 0.0$), weak field ($h = 0.1$), and strong field ($h = 1.0$) regimes. We find that for an isolated chain ($h = 0.0$) the fidelity of Alice's state transfer will be greatest when the difference $\delta\phi \approx k\pi$ for ($k = -2, -1, 0, 1, 2$). When the difference $\delta\phi/\pi \approx -1.5, -0.5, 0.5, 1.5$ the maximum fidelity which can be obtained in the time interval t$\in [10, 50]$ will minimized with respect to $\delta\phi$. When an external field is applied to the chain it is more difficult to assess the behavior of state transfer as can be seen in Fig. 3(b) and (c). However, in all three cases we find that if Alice and Bob both choose $\theta_1 = \theta_2$ the fidelity will be greater when compared to other values of the parameters $\theta_i$.

In the analysis above, we have only considered a $N = 10$ site chain. We now study the length dependence of the maximal fidelity. In Fig. 4 we compare the success of Alice's state transfer with and without Bob's encoding for various chain lengths. In the figures, the horizontal axes represent the number of sites $N$ and we have selected Alice's state to be $1/2 |0\rangle + \sqrt{3}/2 |1\rangle$ in both figures. For a given $N$, the maximum fidelity $F_{\max}$ is determined nu-
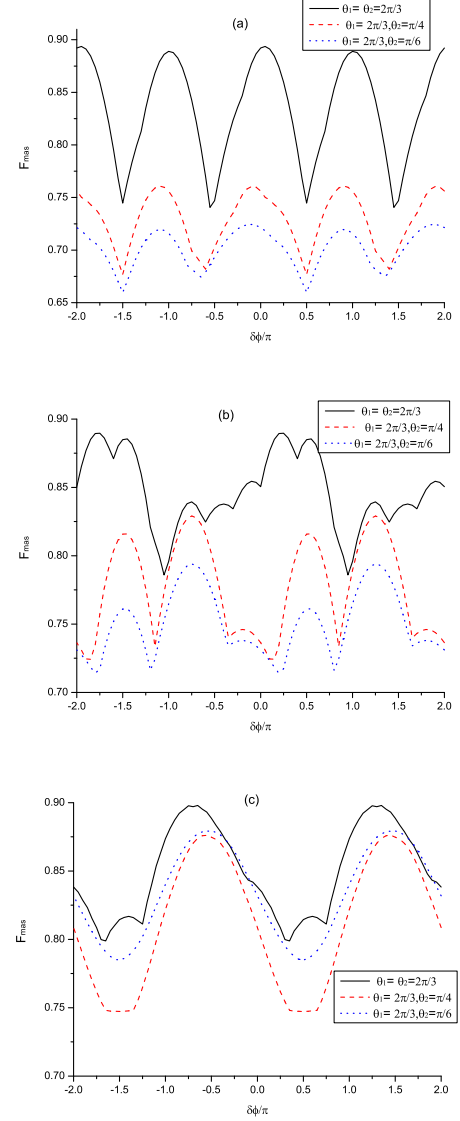


FIG. 3: (Color online) The maximum fidelity $F_{max}$ of Alice's state transfer as a function of $\delta\phi = \phi_2 - \phi_1$. We search for a maximum in the time interval t$\in [10, 50]$ for fixed parameters $\theta_i$. $\theta_1 = \theta_2 = 2\pi/3$(solid line), $\theta_1 = 2\pi/3, \theta_2 = \pi/4$(dashed line), $\theta_1 = 2\pi/3, \theta_2 = \pi/6$(dotted line). Here we have set $N = 10$, (a) $h = 0.0$, (b) $h = 0.1$, and (c) $h = 1.0$.

merically in a range $\theta_2 \in [0, \pi]$ and $\phi_2 \in [0, 2\pi]$. Fig. 4(a) and (b) correspond to the cases $h = 0.0$ and $h = 1.0$, respectively. The plots reveal several interesting features. First of all, we find that the maximum fidelity of Alice's state transfer is generally enhanced when Bob encodes an appropriate state, regardless of the presence or absence of an external field. Our numerical results show that when $N = 5$, a near perfect state transfer ($F_{\max} \approx 1$) can be obtained in many different cases. Secondly, when $h = 0$ (Fig. 4(a)), there are particular chain lengths for which the maximum fidelity is independent of Bob's presence,
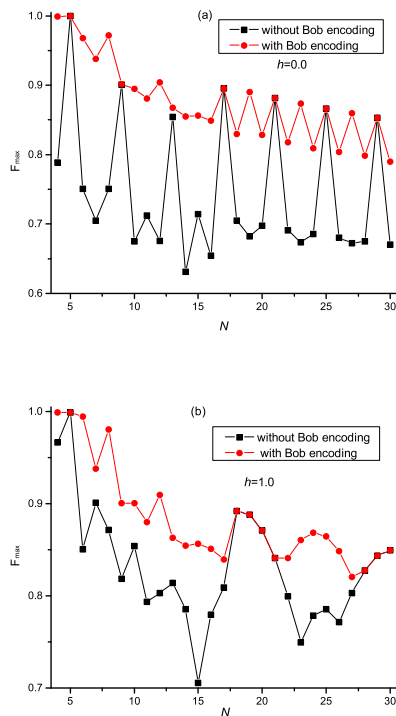
FIG. 4: (Color online) The maximum fidelity $F_{max}$ when transferring the quantum state $1/2\,|0\rangle + \sqrt{3}/2\,|1\rangle$ from Alice's side to Bob's side as a function of $N$. We search for a maximum in the interval t$\in [10, 50]$ and set (a) $h = 0.0$, and (b) $h = 1.0$.

namely chains which have $N = 4n+5$ $(n = 0, 1, 2, ...)$ sites (except for the slight deviation at $N = 13$). This property is lost however when an external field is applied. For instance, when $h = 1.0$ Alice can obtain a higher quality state transfer with Bob's presence for all chain lengths we consider except for $N = 18, 21$ (see Fig. 4(b)). Finally,

for any practical communication protocol it is important to know the time $\tau$ at which the fidelity gains its maximum. As an example, for an isolated ($h = 0.0$) chain consisting of N=10 sites we find that Alice's transmitted state reaches a maximum fidelity at time $\tau = 23.1$ when Bob encodes a similar state while $\tau = 29.2$ when Bob is not present. When h=1.0, $\tau = 23.0$ with Bob's encoding, and $\tau = 29.1$ without Bob's encoding. Thus when Bob encodes an appropriate state Alice can obtain both a higher transmission fidelity as well as an increased transfer speed.

## CONCLUSION

In conclusion, we have investigated the effects of duplex quantum communication through an unmodulated spin chain. A sophisticated quantum computer will undoubtedly rely on data multiplexing so it is an important task to establish potential avenues for two-way communication. We have shown that spin chains are indeed viable candidates for this purpose. Specifically, we have shown that the transmission fidelity at each end of a spin chain can usually be enhanced by the presence of a second party.

Our initiative opens the door for a broad investigation of multi-party communication through spin networks and may find applications in many experimental systems such as quantum dots [20], optical lattices [21], or NMR [22, 23].

## ACKNOWLEDGMENTS

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