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Quantum gates and their coexisting geometric phases

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Geometric phases arise naturally in a variety of quantum systems with observable consequences. They also arise in quantum computations when dressed states are used in gating operations. Here we show how they arise in these gating operations and how one may take advantage of the dressed states producing them. Specifically, we show that that for a given, but arbitrary Hamiltonian, and at an arbitrary time τ, there *always exists* a set of dressed states such that a given gate operation can be performed by the Hamiltonian up to a phase ϕ. The phase is a sum of a dynamical phase and a geometric phase. We illustrate the dressed phase for several systems.

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I. INTRODUCTION

Quantum gates are the building blocks for quantum computers and are necessary for a vast array of quantum mechanical devices. Quantum gates appear in a wide variety of forms and range in complexity from simple single-qubit rotations to highly sophisticated multi-qubit, or more generally, multi-qudit designs. An ideal gating operation will produce a coherent unitary evolution of a quantum state in such a way as to realize the desired transformation of the system of interest with perfect fidelity. Finding a universal set of these operations which can manipulate and entangle collections of qubits is one of the most important tasks in quantum computing. Indeed, unitary-evolution-based quantum computation relies on a processor's ability to manipulate superpositions as well as create and destroy an entanglement using a sequence of these gates. When a particular gate is required, for whatever the reason may be, one typically has a specific Hamiltonian in mind to generate the corresponding evolution. In some cases, the implementation of a quantum gate can be carried out using the interactions which are naturally available to the system, as, for instance, in the case of transferring information between processors using the always-on Heisenberg interactions occurring between neighboring sites in a spin chain [1]. The situation is usually not so simple however, generally one needs to control these gating operations using external fields or other mechanisms, such as measurements. In order to accomplish a specific computational goal one must first identify a physical system which can effectively serve as a collection of qubits, initiate the system to an appropriate state, and then force the system to evolve according to the combined system/external Hamiltonian. We will show here that there always exists a complete set of states which, when acted upon by an arbitrary evolution operator, can produce any given gate, up to an additional phase factor. It is interesting that a part of this phase factor has a geometrical nature similar to the Aharonov-Anandan (A-A) phase [2].

Over the years the geometric phase has been a topic of central interest in quantum mechanics [3]. Unlike the dynamical phase which depends on the dynamical evolution of the system, e.g., the speed at which the parameterized path is followed or the arbitrary choice of gauge, the geometrical phase is completely determined by a closed trajectory of the system in the underlying parameter space and does not depend on the details of the temporal evolution. The geometrical phase is a measurable quantity and can be used to investigate the nontrivial geometric properties of the parameter space. It can be observed in the interference of two identically prepared systems as they develop a relative phase while either system is adiabatically varied. It can also be observed in single quantum systems which are prepared in superpositions of eigenstates of the Hamiltonian. In this case each eigenstate can establish a geometric phase as the Hamiltonian is varied, and the differences between these phases can be observed by the measurable properties of the system.

It was Berry [4] who first noticed the fact that a wave function, originally in a nondegenerate eigenstate of the initial Hamiltonian, will acquire a geometrical phase factor in addition to the familiar dynamical phase if the evolution is induced by a Hamiltonian which is varied adiabatically around a closed path in parameter space. Later, Aharonov and Anandan [2] examined a generalization of the Berry phase without recourse to adiabaticity. Their work showed how any closed path in the projective Hilbert space of state vectors modulo phase factors has a geometric phase associated with it regardless of any adiabatic conditions.

Here, we introduce a family of geometric phases which are associated with quantum gating operations. As we have just mentioned, we will show that it is possible for any Hamiltonian to mimic the action of a specified yet arbitrary gate provided that the system is initialized to an appropriate state. The geometrical contribution of the phase acquired by this quantum evolution can be calculated in a way that is similar to the usual prescription given by Aharonov and Anandan. In this situation the geometrical phase associated with the gating operation can be determined using a dressed Hamiltonian which generates a cyclic evolution.

We will begin our discussion of these results in Sec. II with an example that clearly illustrates the nature of the geometrical phase obtained by the cyclic evolution of a state of a spin chain. The example should help clarify part of the motivation behind this work, as it indicates the origin of a geometrical phase acquired by the non-cyclic evolution of a state through a gating operation. Geometrical effects arising from the transmission of a quantum state through a spin network have been studied and we refer the interested reader to Ref. [5].

We will present our main results in Sec. III where we discuss the geometrical phase which arises in quantum gating operations. We will then provide specific examples in Sec.'s IV, V, and VI. Finally, we describe an experimental proposal for a physical demonstration of these dressed phases in Sec. VII before concluding in Sec. VIII.

II. EXAMPLE OF PERFECT STATE TRANSFER

Let us start with an example of perfect state transfer [6] through a spin chain. Consider an N-site nearest neighbor *XY* model $H = \sum_j J_j(\sigma_j^+\sigma_{j+1}^- + \sigma_{j+1}^+\sigma_j^-)$. When we engineer the tunneling integrals such that $J_j = J\sqrt{j(N-j)}/2$, the Hamiltonian becomes $H = JL_x$, where L_x is the x-component of the quasi-angular momentum operator.

At time τ the evolution operator associated with this Hamiltonian becomes [10]

$$U(\tau) = \exp(-iJ\tau L_x)\,. \tag{1}$$

When $\tau = \pi/2J$, a spin-up state $|\uparrow\rangle_1$ at site 1 is perfectly transferred to $r\,|\uparrow\rangle_N$ at the last site N, where $r = \exp\left(i\pi(N-1)/2\right).$

We first consider the situation when $\tau = \pi/J$, the system evolves cyclically, $|\uparrow\rangle_1 \to r^2\,|\uparrow\rangle_1$, except for a phase factor r^2. If the number of sites N is even, $r^2 = e^{i\pi(N-1)} = -1$, which is apparently a geometric phase factor since it originates from the geometrical length of the chain and is not related to the dynamics. More precisely, it is an Aharonov-Anandan (A-A) phase factor. The A-A phase factor exists for any cyclic evolution of a quantum system, defined by $|\psi(\tau)\rangle = e^{i\phi}\,|\psi(0)\rangle$. It has been shown that cyclic evolutions of this type universally exist in any quantum system, regardless of the specific Hamiltonian which generates the evolution, provided the system begins in an appropriate state [7]. Now, we assume that the wave function is driven by the Schrödinger equation

$$H(t)\,|\psi(t)\rangle = i\frac{d}{dt}\,|\psi(t)\rangle \tag{2}$$

where we let $\hbar = 1$ throughout. Aharonov and Anandan found an expression which removed the the dynamical part from the total phase ϕ

$$\beta = \phi + \int_0^\tau \langle\psi(t)|\,H\,|\psi(t)\rangle\,dt = i\int_0^\tau \left\langle\tilde\psi(t)\right|d\left|\tilde\psi(t)\right\rangle$$

where $\left|\tilde\psi(t)\right\rangle = e^{-if(t)}\,|\psi(t)\rangle$ and $f(\tau) - f(0) = \phi$. β is known as the A-A phase and is uniquely defined up to $2\pi n$ for some integer n. It is a quantity that is independent of both ϕ and the underlying Hamiltonian H. It is easy to check that for the example above we have $e^{i\beta} = r^2$ or $\beta = \pi(N-1)$ mod 2π.

The process of perfect state transfer $|\uparrow\rangle_1 \to r\,|\uparrow\rangle_N$ does not correspond to a cyclic evolution. However, the above analysis implies that the phase factor r, which originates similarly to the A-A phase factor r^2, should possess geometric attributes as well. We will exhibit the occurrence of similar phenomena in various systems after general discussions.

III. GEOMETRIC PHASE ORIGINATING FROM QUANTUM GATES

Consider a quantum unitary gate $\mathcal{G}$, such as a swap gate, or a CNOT gate, and the system evolution operator $U(t)$. The evolution operator is related to the Hamiltonian through the operator Schrödinger equation $H(t) = i\dot U(t)U^\dagger(t)$. The combined operator $W(t) = \mathcal{G}^\dagger U(t)$ is also unitary since $W^\dagger(t)W(t) = U^\dagger(t)\mathcal{G}\mathcal{G}^\dagger U(t) = 1$. As with any unitary operator, the operator $W(\tau)$, at time $t = \tau$, can be diagonalized and has a complete set of orthonormal eigenvectors $\{\Psi_k(0)\}_\tau$ and exponential eigenvalues $\{\exp(i\phi_k)\}_\tau$. A vector $\Psi_k(0)$ in the set obeys the eigenequation:

$$W(\tau)\Psi_k(0) = \exp(i\phi_k)\Psi_k(0). \tag{3}$$

Consequently, if the initial state $|\psi(0)\rangle$ is one of the $\Psi_k(0)$'s, the wave function evolves as $|\psi(\tau)\rangle = \exp(i\phi)\,|\psi(0)\rangle$, driven by an effective Hamiltonian

$$\mathcal{H}(t) = i\dot W(t)W^\dagger(t) = \mathcal{G}^\dagger i\dot U(t)U^\dagger(t)\mathcal{G} = \mathcal{G}^\dagger H(t)\mathcal{G}.$$

In other words, the dynamics driven by the *effective* or *dressed* Hamiltonian $\mathcal{H}(t)$ is cyclic if the initial state $|\psi(0)\rangle$ is one of the $\Psi_k(0)$'s. The corresponding ϕ is a sum of the dynamic phase of the *effective* Hamiltonian $\mathcal{H}(t)$ and the A-A geometric phase β, which we refer to as the *dressed* A-A phase. The essence of the A-A phase β being *geometric* is based on the effective dynamics governed by $\mathcal{H}(t)$. However the effective Hamiltonian may correspond to multiple operators i.e., $\mathcal{H}(t)$ does not change if we replace $W(t) \to W(t)V$ when V is unitary.

On the other hand, the wave function $|\psi(\tau)\rangle$ driven by the *bare* Hamiltonian $H(t)$ satisfies

$$|\psi(\tau)\rangle = U(\tau)\,|\psi(0)\rangle = \exp(i\phi)\mathcal{G}\,|\psi(0)\rangle$$

This indicates that for an arbitrary Hamiltonian and at an arbitrary time τ, there *universally exists* a set of states such that a given gate operation can be performed by this Hamiltonian up to a phase ϕ. In the case where $\mathcal{G}$ is a unit operator, the dressed A-A phase becomes the normal A-A phase [7].

In the example above, a perfect state transfer through a spin chain requires a gate to exchange states at the first and last sites. The operator $\mathcal{G} = r^*\exp(i\pi L_x)$ plays this role, where $\mathcal{G}^2 = 1$. It is easy to check that $\beta = \phi = \pi(N-1)/2 \mod 2\pi$ or $e^{i\beta} = r$. Although the evolution is not cyclic, we are able to extract the geometrical phase associated with the evolution since it can be described by a gating operator which then defines the effective Hamiltonian $\mathcal{H}(t)$. $|\uparrow\rangle_1$ is an eigenstate of the operator $W(\tau)$ for $\tau = \pi/2J$, and thus evolves cyclically under the action of the dressed Hamiltonian $\mathcal{H}(t)$. This allows us to use the expression for the A-A phase, with H replaced with $\mathcal{H}$, in order to determine the *dressed* phase associated with this process.

IV. UNIVERSAL SET OF GATES IN QUANTUM COMPUTATION AND THEIR COEXISTING PHASES

In the case that $\mathcal{G} = \exp(-i\theta_0\sigma_x)$, we will consider a *non-perturbative* time-dependent Hamiltonian,

$$H(t) = \left\{ \begin{array}{c} \varpi\sigma_z, 0 < t < \delta \\ \omega\sigma_x, t > \delta \end{array} \right.$$

where σ_x, σ_y and σ_z are the Pauli matrices. Here, the evolution operator is given by

$$U(t) = \left\{ \begin{array}{c} \exp(-i\varpi t\sigma_z), 0 < t < \delta \\ \exp(-i\omega(t-\delta)\sigma_x)\exp(-i\varpi\delta\sigma_z), t > \delta \end{array} \right. ,$$

We can control the time such that $\omega(\tau - \delta) = \theta_0$ at time $t = \tau$ and $W(\tau) = \exp(-i\varpi\delta\sigma_z)$. The eigenstates and eigenvalues are $\Psi_{\uparrow,\downarrow}(0) = |\uparrow,\downarrow\rangle$ and $\phi_\uparrow = -\varpi\delta$; $\phi_\downarrow = \varpi\delta$. It is easily calculated that $\int_0^\tau \langle\psi(t)|\,\mathcal{H}(t)\,|\psi(t)\rangle\,dt = \pm\varpi\delta\cos 2\theta_0$, where the sign + (-) corresponds to the initial state being $|\uparrow\rangle$ ($|\downarrow\rangle$). The A-A phases here are $\beta_\uparrow = -\varpi\delta(1 - \cos 2\theta_0)$ and $\beta_\downarrow = \varpi\delta(1 - \cos 2\theta_0)$. At time τ, the system evolves as

$$|\psi(\tau)\rangle = e^{-i\varpi\delta}\exp(-i\theta_0\sigma_x)\,|\uparrow\rangle$$

if it is initially in the state $|\uparrow\rangle$. Likewise, $|\psi(\tau)\rangle = e^{i\varpi\delta}\exp(-i\theta_0\sigma_x)\,|\downarrow\rangle$ for the initial state $|\downarrow\rangle$.

V. ADIABATIC CASES

Now consider a slowly changing effective Hamiltonian $\mathcal{H}(t)$, whose instantaneous eigenequation is $\mathcal{H}(t)\,|n(t)\rangle_e = \mathcal{E}_n(t)\,|n(t)\rangle_e$. For non-degenerate systems and a periodic Hamiltonian $\mathcal{H}(\tau) = \mathcal{H}(0)$, the adiabatic theorem shows that

$$|\psi(\tau)\rangle_e = \exp(i\phi_n)\,|n(0)\rangle_e$$

where $\phi_n = -\int_0^\tau \mathcal{E}_n(t)dt + \int_0^\tau {}_e\langle n(t)|\,d\,|n(t)\rangle_e$ for a system that is initially in the state $|n(0)\rangle_e$. For the bare system we have $|\psi(\tau)\rangle = \exp(i\phi)\mathcal{G}\,|n(0)\rangle_e$. In this case, both the dressed $\mathcal{H}(t)$ and bare $H(t)$ Hamiltonians are periodic.

Now let us consider an ion with two ground states $|0\rangle, |1\rangle$ and one excited state $|e\rangle$ [8]. The Hamiltonian for the ion-laser interaction can be approximated by

$$H(t) = |e\rangle\,(\Omega_0\,\langle 0| + \Omega_1\,\langle 1|) + \text{h.c.}$$

in the rotating frame, where Ω_0, Ω_1 are controllable slow-varying Rabi frequencies. If we chose the operator $\mathcal{G} = |1\rangle\langle 0| + |0\rangle\langle 1| + |e\rangle\langle e|$, and if we choose $\Omega_0 = \cos\frac{\theta}{2}$ and $\Omega_0 = -\sin\frac{\theta}{2}e^{i\varphi}$ and $\Omega_1 = \cos\frac{\theta}{2}$, then the effective Hamiltonian $\mathcal{H}(t)$ will have a dark eigenstate (state with the zero-eigenvalue)

$$|D\rangle = \cos\frac{\theta}{2}\,|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}\,|1\rangle\,.$$

The dynamical phase vanishes in this case while the Berry phase is given by $\Phi = \int \sin\theta d\theta d\varphi$. If the parameters (θ, φ) undergo a cyclic evolution, starting and ending at the point $\theta = 0$ in the bare system, then the corresponding evolution will be $|\psi(\tau)\rangle = \exp(i\Phi)\mathcal{G}\,|0\rangle$. In this case the system experiences an evolution from $|0\rangle$ to $|1\rangle$, along with an additional all-geometric phase factor, when the dressed system makes a cyclic evolution.

VI. SUPERPOSITION OF EIGENSTATES

Since the set of eigenvectors $\{\Psi_k(0)\}_\tau$ is complete we can expand any initial wave function $\Psi(0)$ as $\Psi(0) = \sum_k \alpha_k\Psi_k(0)$. Despite the fact that $\Psi(0)$ is generally not an eigenstate of $W(\tau)$, i.e., $W(\tau)\Psi(0) = \sum_k \alpha_k\exp(i\phi_k)\Psi_k(0)$, an arbitrary Hamiltonian can still be used to execute any given gate $\mathcal{G}$ up to a phase ϕ at time τ provided the eigenvalues associated with the states $\{\Psi_k(0)\}_\tau$ obey certain conditions. To establish these conditions notice that if we require

$$U(\tau)\Psi(0) = \exp(i\phi)\mathcal{G}\Psi(0) \tag{4}$$

then we must have

$$\sum_k \alpha_k[\exp(i\phi_k) - \exp(i\phi)]\Psi_k(0) = 0. \tag{5}$$

For indices k such that $\alpha_k \neq 0$ there is set of requirements imposed on the corresponding eigenvalues, namely $\phi = \phi_k + 2\pi m$ for some integer m.

As an example, let us reexamine the case above for $\mathcal{G} = \exp(-i\theta_0\sigma_x)$. If we again choose the time $t = \tau$ such that $\omega(\tau - \delta) = \theta_0$ we obtain the two eigenvalues $\exp(\pm i\varpi\delta)$ of $W(\tau)$. Suppose we expand an arbitrary qubit state $\Psi(0)$ in the basis $\Psi_{\uparrow,\downarrow}(0) = |\uparrow,\downarrow\rangle$ so that $\Psi(0) = \cos(\xi)\,|\uparrow\rangle + \sin(\xi)e^{i\gamma}\,|\downarrow\rangle$. In order to satisfy Eq. (4) the phase ϕ must satisfy both $\phi = -\varpi\delta + 2\pi m$ and $\phi = \varpi\delta + 2\pi m'$. Since this requires that $\varpi\delta = \pi n$ for some integer n, the evolution operator becomes $U(\tau) = \pm\exp(-i\theta_0\sigma_x) = \pm\mathcal{G}$.

For an arbitrary initial state $\Psi(0) \rightarrow \Psi(t)$ we have $\int_0^\tau \langle\Psi(t)|\,\mathcal{H}(t)\,|\Psi(t)\rangle\,dt = \theta_0\sin(2\xi)\cos(\gamma) + \delta\varpi[\cos(2\xi)\cos(2\theta_0) + \sin(2\xi)\sin(2\theta_0)\sin(\gamma)]$. Since $\varpi\delta = \pi n$ and $\phi = \pi n$ for some integer n, the geometric phase angle acquired during the subsequent evolution of $\Psi(0)$ is given by $\beta = \pi n + \theta_0\sin(2\xi)\cos(\gamma) + \pi n[\cos(2\xi)\cos(2\theta_0) + \sin(2\xi)\sin(2\theta_0)\sin(\gamma)]$. Fig. 1 shows the real part of the geometric phase $e^{i\beta}$ as a function of the initial state parameters ξ and γ for $n = \theta_0 = 1$.

Having a knowledge of the geometrical phase acquired by an arbitrary quantum state as it undergoes a unitary evolution can help one to predict the outcome of an interference experiment designed to test the effects associated with the interaction of two or more systems. We will discuss the interference effects which should be obtained in an experiment involving two bosonic spin chains next. There we will consider quantum state transfer through chains described by the Hubbard model in which perfect state transfer has been previously studied [9, 10]. Although we limit our discussion to

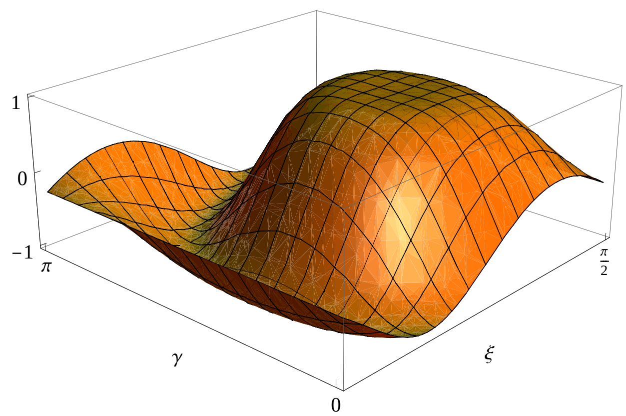

FIG. 1: (Color online) The real part of the dressed phase $e^{i\beta}$ acquired by the initial state $\Psi(0) = \cos(\xi)\,|\uparrow\rangle + \sin(\xi)e^{i\gamma}\,|\downarrow\rangle$ after evolving according to Eq. (4). In this example, we illustrate the behavior of the dressed phase for the gate $\mathcal{G} = \exp(-i\theta_0\sigma_x)$

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only two chains here, the generalization to an arbitrary number of chains should be a straightforward.

VII. EXPERIMENTAL PROPOSAL

It was recently shown that interference can arise in a two-dimensional bosonic lattice when a quantum state is transferred perfectly from one site to another [10]. An experimental setup based on this result can be used to demonstrate the geometric phase according to the field intensity at an appropriate site. To exemplify this effect imagine a ring of bosonic

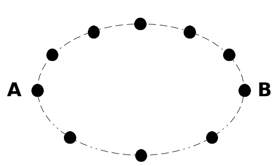

FIG. 2: (Color online) Schematic of a boson ring which can be used to demonstrate the geometric phase according to the intensity at site B. Quantum interference can arise at site B since the optical path lengths of the upper and lower paths are equal but contain different numbers of atoms.

atoms as depicted in Fig. 2. The ring can be thought of as two separate chains which share first and last sites, denoted respectively by A and B in the figure. Both the upper and lower chains are assumed to have equal length but the number of sites contained in either chain can be different. In our example here we have $N_U = 7$ and $N_L = 5$ sites for the upper and lower paths. We will consider the dynamics of bosons governed by the Bose-Hubbard Hamiltonian

$$H = -\sum_{<i,j>} J_{i,j}(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i) + U\sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \epsilon_i \hat{n}_i,$$

where $< i,j >$ indicates that the sum is restricted to nearest neighbors in the lattice and $\hat{a}_i^\dagger(\hat{a}_i)$ denotes the creation (annihilation) operator of a boson at site i. Also, $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ gives the total number of bosonic atoms at site i. This Hamiltonian can allow for a perfect state transfer through both the upper and lower paths of the ring (see Ref. [10]). When this occurs at time t_0 we have

$$U^\dagger(t_0)\hat{a}_i^\dagger U(t_0) = r\hat{a}_{N-i+1}^\dagger,$$

where $r = \exp(-i\pi(N_U - 1)/2)$ for the upper path and $r = \exp(-i\pi(N_L - 1)/2)$ for the lower path. The expression above is given for a linear chain containing N sites. It should be understood that in the situation we are considering, where the chain is not open-ended but instead forms a closed loop, two indices i and i' should be used above, one for the upper chain and one for the lower. Now, let us expand the field operators at $t = 0$ in the Wannier basis

$$\psi(\mathbf{x}) = \sum_i [\hat{a}_i^U w(\mathbf{x} - \mathbf{x}_i^U) + \hat{a}_i^L w(\mathbf{x} - \mathbf{x}_i^L)],$$

where $\hat{a}_i^U$ and $\hat{a}_i^L$ act on the upper and lower chain, respectively. We see that the operators will evolve to

$$\psi(\mathbf{x}, t_0) = \sum_i [r\hat{a}_i^U w(\mathbf{x} - \mathbf{x}_{N_U-i+1}^U) + \hat{a}_i^L w(\mathbf{x} - \mathbf{x}_{N_L-i+1}^L)]$$

at time t_0. Here we have set $r = 1$ for the lower path. Although $r = -1$ for the upper path in Fig. 2, we are keeping the expression in a more general form in order to examine the interference effects when the number of sites is varied. The average field intensity at $\mathbf{x}$ is given by $I(\mathbf{x}, t_0) =< \psi(\mathbf{x}, t_0)^\dagger \psi(\mathbf{x}, t_0) >$, where $< ... >$ denotes the expectation value for the initial state. This can be calculated to be

$$I(\mathbf{x}, t_0) =< \hat{a}_1^\dagger \hat{a}_1 > |w(\mathbf{x} - \mathbf{x}_N)|^2(2 + r + r^*),$$

where $\hat{a}_1^U = \hat{a}_1^L \equiv \hat{a}_1$ and $\mathbf{x}_{N_U}^U = \mathbf{x}_{N_L}^L \equiv \mathbf{x}_N$.

The intensity varies with different values of the signature r, in the situation depicted in Fig. 2 we have $I(\mathbf{x}, t_0) = 0$. Constructive, destructive, and in-between interference effects can be obtained by varying the number of sites in either path.

We note that recent experimental developments support the feasibility of this proposal [11, 12].

VIII. CONCLUSION

We have shown that *any* Hamiltonian can be used to administer *any* given gate up to a phase during an arbitrary time interval provided the system is initialized to an appropriate state. The "dressed" phase accompanying this evolution is found to contain both geometrical and dynamical contributions. The evolution of a quantum system determined by the chosen Hamiltonian will not necessarily be cyclic, nevertheless, we are able to determine the geometrical part of this phase using the dressed Hamiltonian associated with the gating operation. We have also provided an experimental proposal for the demonstration of this dressed phase. The proposal is similar to that which was given in Ref. [10] as a means to verify interference effects arising from the Z_4 group cyclic nature of the signature factor associated with state transfer through bosonic chains. However, the purpose here is to verify interference effects associated with the dressed geometric phase.

It should be mentioned that we have not considered the effects of decoherence in our analysis. It would be interesting to explore the effects a realistic environmental interaction would have on the geometric phase acquired by a mixed state during a particular gating operation.

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