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Topological $p_x + ip_y$ Superfluid Phase of Fermionic Polar Molecules

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We discuss the topological $p_x + ip_y$ superfluid phase in a 2D gas of single-component fermionic polar molecules dressed by a circularly polarized microwave field. This phase emerges because the molecules may interact with each other via a potential $V_0(r)$ that has an attractive dipole-dipole $1/r^3$ tail, which provides p-wave superfluid pairing at fairly high temperatures. We calculate the amplitude of elastic p-wave scattering in the potential $V_0(r)$ taking into account both the anomalous scattering due to the dipole-dipole tail and the short-range contribution. This amplitude is then used for the analytical and numerical solution of the renormalized BCS gap equation which includes the second order Gor'kov-Melik-Barkhudarov corrections and the correction related to the effective mass of the quasiparticles. We find that the critical temperature T_c can be varied within a few orders of magnitude by modifying the short-range part of the potential $V_0(r)$. The decay of the system via collisional relaxation of molecules to dressed states with lower energies is rather slow due to the necessity of a large momentum transfer. The presence of a constant transverse electric field reduces the inelastic rate, and the lifetime of the system can be of the order of seconds even at 2D densities $\sim 10^9$ cm^{-2}. This leads to T_c of up to a few tens of nanokelvins and makes it realistic to obtain the topological $p_x + ip_y$ phase in experiments with ultracold polar molecules.

I. INTRODUCTION

The recent breakthrough in creating ultracold diatomic polar molecules in the ground ro-vibrational state [1, 2] and cooling them towards quantum degeneracy [1] has opened fascinating prospects for the observation of novel quantum phases [3–9]. A serious problem in this direction is related to ultracold chemical reactions, such as KRb+KRb⇒K_2+Rb_2 observed in the JILA experiments with KRb molecules [10], which places severe limitations on the achievable density in three-dimensional samples. In order to suppress chemical reactions and perform evaporative cooling it has been proposed to induce a strong dipole-dipole repulsion between the molecules by confining them to a (quasi)two-dimensional (2D) geometry and orienting their dipole moments (by a strong electric field) perpendicularly to the plane of the 2D translational motion [11]. Nevertheless, in order to prevent the approach of colliding molecules at short separations, where the chemical reactions occur, and to proceed with evaporative cooling, one has to have a fairly strong confinement to the 2D regime [12–14]. The suppression of chemical reactions by nearly two orders of magnitude in the quasi2D geometry has been demonstrated in the recent JILA experiment [15]. It should be noted, however, that these reactions are not expected to occur for all polar molecules of alkali atoms [16], on which experimental efforts are presently focused. While present for KRb molecules and for the molecules containing a Li atom, they are energetically unfavorable, for example, in the case of NaK molecules. We thus expect that future investigations of many-body physics will deal either with molecules which do not undergo ultracold chemical reactions, or otherwise are strongly confined to the 2D regime.

One of the challenging goals in the studies of many-body physics is the creation of the topological $p_x + ip_y$ superfluid phase for identical fermions in two dimensions (2D) (see [17] for a review). This phase, first discussed in relation to superfluid ^{3}He and the fractional quantum Hall effect [18, 19], has exotic topological properties at positive chemical potential $\mu > 0$, i.e. in the BCS regime. The topological nature of this superfluid phase supports a gapless Majorana mode at the boundary to a vacuum, and the quantized vortices in the superfluid carry local zero energy Majorana modes on their cores[19, 20]. These Majorana modes are predicted to cause the vortices to obey non-abelian exchange statistics, which has potential applications in topologically protected quantum information processing [21]. There is significant interest in finding physical realizations of this topological superfluid phase in which the exotic physics of these Majorana modes can be detected.

The $p_x + ip_y$ topological phase can be the ground state of ultracold identical fermionic atoms interacting via a short-range potential [17]. However, away from a p-wave Feshbach resonance the superfluid transition temperature T_c is vanishingly low. On approach to the resonance it increases but the system becomes unstable due to the formation of long-lived diatomic quasibound states and their collisional relaxation into deep molecular states [22, 23]. A stable $p_x + ip_y$ state has been recently proposed for fermionic polar molecules with a large dipole moment [8]. When these molecules are confined to a 2D geometry and dressed by a microwave field (MW) which is nearly resonant with the transition between the lowest and the first excited rotational levels, they acquire an attractive $1/r^3$ dipole-dipole interaction. This leads to superfluid pairing of $p_x + ip_y$ symmetry, and due to the anomalous contribution of the $1/r^3$ tail to the scattering amplitude, already in the BCS limit the superfluid tran-

sition temperature can be made a sizeable fraction of the Fermi energy. At the same time, collisional decay processes remain sufficiently slow to allow the experimental realization of this phase [8]. Several other ways in which a stable p-wave coupled superfluid phase may be obtained have been proposed in recent years [24–30].

In this paper we present a complete analysis of the BCS limit of the p_x+ip_y phase of MW-dressed fermionic polar molecules. In particular, using the many-body perturbative approach up to second order, we derive a relation for the transition temperature T_c. It is then shown how T_c and the collisional stability may be manipulated by tuning the short-range part of a MW-induced effective molecule-molecule interaction potential or by applying a constant electric field perpendicular to the plane of the translational motion. Our analysis is confirmed by numerical calculations, which are also extended to the regime of moderately strong interactions.

The paper is organized as follows. In Section II we discuss a single polar molecule in the presence of a circularly polarized nearly resonant microwave field and a constant electric field as shown in Fig. 1. We then show that two microwave-dressed polar molecules undergoing a 2D translational motion may interact with each other via a potential $V_0(r)$ which has a repulsive core, a potential well, and an attractive $1/r^3$ tail. In Section III we find the amplitude of elastic p-wave scattering of particles in the potential $V_0(r)$, which takes into account both the anomalous scattering due to the $1/r^3$ tail and the short-range contribution. Section IV is dedicated to the analysis of the decay of the gas due to collisional relaxation of the molecules to lower dressed states. The presence of a constant electric field is found to reduce the inelastic rate, and the lifetime of the system can be of the order of seconds even at 2D densities $\sim 10^9$ cm^{-2}. In Section V we present the renormalized BCS gap equation which takes into account the second order Gor'kov-Melik-Barkhudarov processes and the effective mass of the quasiparticles. In Section VI we obtain analytical and numerical solutions of the gap equation, show that the ground state has p_x+ip_y symmetry, and reveal the influence of the short-range part of the scattering potential $V_0(r)$ on the critical temperature. In Section VII we conclude, emphasizing that it is realistic to obtain the topological p_x+ip_y phase for microwave-dressed polar molecules in the 2D geometry, with a critical temperature of up to a few tens of nanokelvin.

II. INTERACTION POTENTIAL FOR MW-DRESSED POLAR MOLECULES

The microwave dressing of polar molecules has been proposed to tune the molecule-molecule interaction potential [31] and to form a repulsive shield for suppressing inelastic losses [32]. The rotational part of the Hamiltonian of a single polar molecule in a circularly polarized

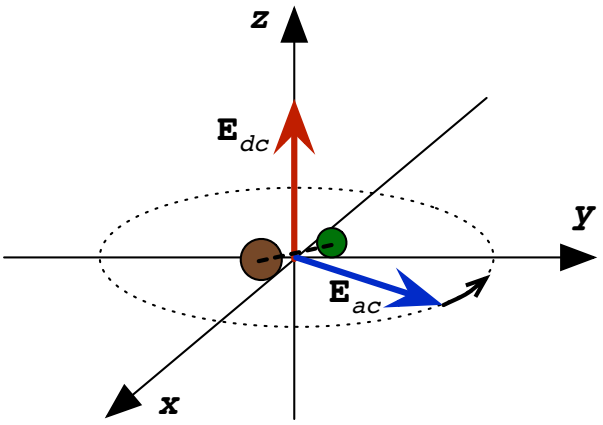

FIG. 1: (color online). Polar molecule (brown and green circles) in the considered configuration of fields. The ac MW field rotates with frequency ω in the plane orthogonal to the dc field

MW-field $\mathbf{E}_{ac}$ and a constant electric field $\mathbf{E}_{dc}$ reads:

$$\hat{H}_0 = B\hat{\mathbf{J}}^2 - \hat{\mathbf{d}}(\mathbf{E}_{dc}+\mathbf{E}_{ac}), \tag{1}$$

where $\hat{\mathbf{J}}$ is the operator of the rotational moment, B is the rotational constant of the molecule, and $\hat{\mathbf{d}}$ is the operator of the dipole moment. In the following we assume that $\mathbf{E}_{dc}$ is parallel to the z-axis, and the vector $\mathbf{E}_{ac}$ rotates in the $\{x,y\}$ plane (see Fig. 1).

In the absence of electric fields, good quantum numbers are the rotational moment J and its projection on the z-axis, M_J, and the diagonal matrix element $\langle J,M_J|\hat{\mathbf{d}}|J,M_J\rangle$ is equal to zero. The dc-field preserves M_J but couples states with different J. We will consider a rather weak dc-field so that the parameter

$$\beta = \frac{dE_{dc}}{B} \tag{2}$$

is small, where d is the molecular dipole moment. Within the manifold that includes the original states with $J=0$ and $J=1$, the new basic states will be called $|\Phi_{00}\rangle$, $|\Phi_{1-1}\rangle$, $|\Phi_{10}\rangle$, and $|\Phi_{11}\rangle$. The transition matrix elements of the operator of the dipole moment, which are needed for further discussions, are given by [31]:

$$d_c = |\langle\Phi_{00}|\hat{d}_{\mp}|\Phi_{1\pm1}\rangle| \approx \frac{d}{\sqrt{3}}\left(1-\frac{49\beta^2}{1440}\right), \tag{3}$$

$$d_g = |\langle\Phi_{00}|\hat{d}_z|\Phi_{00}\rangle| \approx \frac{d\beta}{3}, \tag{4}$$

$$d_e = |\langle\Phi_{1\pm1}|\hat{d}_z|\Phi_{1\pm1}\rangle| \approx \frac{d\beta}{10}, \tag{5}$$

where we have omitted terms that are higher order in β.

Consider the application of a circularly polarized MW-field with frequency ω close to the frequency ω_0 of the transition between the states $|\Phi_{00}\rangle$ and $|\Phi_{11}\rangle$. For the Rabi frequency $\Omega_R = d_cE_{ac}/\hbar$ and the frequency detuning $\delta=\omega-\omega_0$ satisfying the inequality $|\delta|,\Omega_R \ll \omega_0$, the

rotating wave approximation is valid and the MW-field couples only the states $|\Phi_{00}\rangle$ and $|\Phi_{11}\rangle$. The resulting eigenstates are:

$$|+\rangle = a|\Phi_{00}\rangle + b|\Phi_{11}\rangle, \quad (6)$$

$$|-\rangle = b|\Phi_{00}\rangle - a|\Phi_{11}\rangle, \quad (7)$$

with $a = -A/\sqrt{A^2+\Omega_R^2}$, $b = \Omega_R/\sqrt{\Omega_R^2+A^2}$, and $A = (\delta+\sqrt{\delta^2+4\Omega_R^2})/2$. We will consider $\delta \gtrsim \Omega_R$, and choose $\delta > 0$ such that the energy of the state $|+\rangle$ lies *above* the energies of the states $|-\rangle$ and $|\Phi_{1-1}\rangle$. Ramping the MW field on adiabatically then ensures that the ground state $|0,0\rangle$ evolves into the state $|+\rangle$. Thus all molecules can be prepared in this state.

We remark that it was demonstrated in Ref. [33] that polar molecules may be prepared in any hyperfine substate of the ground ro-vibrational state. In particular, we shall assume that the molecules are prepared in the state with maximum projection of the magnetic moment such that the circularly polarized MW field does not cause mixing of hyperfine levels.

We now consider two MW-dressed polar molecules undergoing a 2D motion in the $\{x,y\}$ plane and separated by a distance r which greatly exceeds the radius of the ordinary van der Waals interaction potential. The presence of the *dc* and MW-fields introduces a dipole-dipole interaction between the molecules:

$$\hat{H}_d = \frac{\hat{\mathbf{d}}_1\cdot\hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1\cdot\hat{\mathbf{r}})(\hat{\mathbf{d}}_2\cdot\hat{\mathbf{r}})}{r^3}. \quad (8)$$

Assuming that the molecules are at a fixed separation $\mathbf{r}$ we employ the Born-Oppenheimer approximation and determine an effective intermolecular potential induced by the *dc* and MW-fields. For this purpose we have to diagonalize the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_d$. The dipole-dipole interaction couples the molecular states $|g\rangle \equiv |\Phi_{00}\rangle$, $|e\rangle \equiv |\Phi_{11}\rangle$, and $|\bar{e}\rangle \equiv |\Phi_{1-1}\rangle$, whereas the state $|\Phi_{10}\rangle$ remains decoupled from them. The wavefunction of a two-molecule dressed state can be either symmetric (even parity) or antisymmetric (odd parity) with respect to permutation of the molecules. For studying topological superfluids we will be interested in the states of even parity. We then have a basis of five two-particle states:

$$|g,g\rangle,\ \frac{(|e,g\rangle+|g,e\rangle)}{\sqrt{2}},\ \frac{(|\bar{e},g\rangle+|g,\bar{e}\rangle)}{\sqrt{2}},\ |e,e\rangle,\ \frac{(|\bar{e},e\rangle+|e,\bar{e}\rangle)}{\sqrt{2}},$$

(the state $|\bar{e},\bar{e}\rangle$ is decoupled from these states). The Hamiltonian acting on the spinor of these five states is represented by a matrix [32]:

$$\hat{H} = \hbar\delta \begin{pmatrix} \beta^2/3x^3 & \sqrt{2}\Omega_R/\delta & 0 & 0 & 0 \\ \sqrt{2}\Omega_R/\delta & \left(\beta^2/10-1/2\right)/x^3-1 & 3/2x^3 & \sqrt{2}\Omega_R/\delta & 0 \\ 0 & 3/2x^3 & \left(\beta^2/10-1/2\right)/x^3-1 & 0 & \Omega_R/\delta \\ 0 & \sqrt{2}\Omega_R/\delta & 0 & 3\beta^2/100x^3-2 & 0 \\ 0 & 0 & \Omega_R/\delta & 0 & 3\beta^2/100x^3-2 \end{pmatrix}, \quad (9)$$

where $x \equiv r/r_\delta$, $r_\delta = (d_c^2/\hbar\delta)^{1/3}$, and we omit terms that are higher order in β. Diagonalizing the matrix (9) we obtain a number of effective intermolecular potentials (potential curves) $V_{\text{eff}}(r)$ for MW-dressed polar molecules, depending on the states of the molecules at an infinite separation (see Fig. 2).

For two molecules which at $r \to \infty$ are in the state $|+\rangle$, the long-range tail of such a potential denoted here as $V_0(r)$ is

$$V_0(r\to\infty) = -\frac{\hbar^2}{m}\frac{r_*}{r^3}, \quad (10)$$

with the length scale r_* given by

$$r_* = \frac{md^2}{3\hbar^2}\frac{(\Omega_R/\delta)^2}{1+4(\Omega_R/\delta)^2}\Big\{1-3\beta^2\times \left[49/2160+\left(\frac{7+13\sqrt{1+4(\Omega_R/\delta)^2}}{60\Omega_R/\delta}\right)^2\right]\Big\} \quad (11)$$

and decreasing with an increase in β. For small Ω_R/δ and sufficiently large β the parameter r_* becomes negative, which leads to a repulsive tail of $V_0(r)$. In this case the *dc*-field dominates over the effect of the MW-dressing, so that on average the molecules are perpendicular to the $\{x,y\}$ plane and exhibit a repulsive dipole-dipole interaction at large r. In the following we consider only the case where r_* is positive and the long-range tail of $V_0(r)$ is attractive. Then the quantity r_* is a measure of the radius of the centrifugal barrier experienced by the (fermionic) molecules.

At smaller separations, the dipolar interactions between the molecules cause them to depart from the state $|+\rangle$. This occurs when the characteristic interaction energy d_c^2/r^3 becomes larger than the detuning $\hbar|\delta|$, setting the lengthscale $r_\delta = (d_c^2/\hbar\delta)^{1/3}$ entering the matrix (9). We assume that r_δ is larger than the confinement length in the z-direction, l_z, so that the interaction is 2D. The potential curves of even parity are shown in Fig. 2. The potential $V_0(r)$, being attractive at $r \gtrsim r_\delta$ with a long-

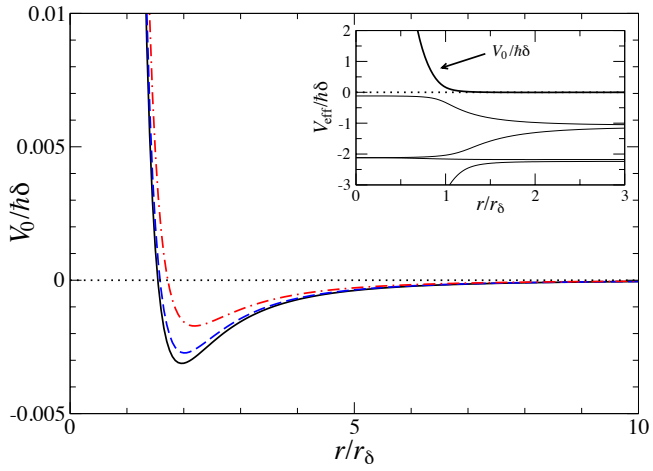

FIG. 2: (color online). Potential energy curve $V_0(r)$ for two $|+\rangle$ state molecules, computed for $\Omega_R = 0.25\delta$. The black (solid) curve corresponds to the value of the perpendicular dc field $\beta = 0$, the blue (dashed) curve to $\beta = 0.1$, and the red (dash-dotted) curve to $\beta = 0.2$. Anticrossings of $V_0(r)$ with other field-dressed levels of even parity occur at distances $r \sim r_\delta$, as shown in the inset for $\Omega_R = 0.25\delta$ and $\beta = 0$.

range $1/r^3$ tail, has a repulsive core for $r \lesssim r_\delta$. The repulsive core prevents low-energy molecules from approaching each other at distances $r \lesssim r_\delta$ and suppresses inelastic collisions, including ultracold chemical reactions observed at JILA in experiments with KRb molecules [10]. Note that in this respect the MW-dressing of polar molecules in the 2D geometry can be used for their evaporative cooling.

Actually, the potential curves of even parity, in particular $V_0(r)$, may cross with potentials of odd parity. However, inelastic transitions between the states of different parity are not possible in two-body collisions, and three-body collisions accompanied by such transitions will be suppressed at the low densities that we consider.

III. ELASTIC SCATTERING

We now consider elastic scattering of fermionic molecules (each in the state $|+\rangle$) undergoing 2D translational motion and interacting with each other via the potential $V_0(r)$, with the attractive dipole-dipole tail (10). At ultralow energies the leading scattering is in the p-wave scattering channel. For the investigation of superfluid pairing we need to know the off-shell scattering amplitude defined as

$$f(\mathbf{k}', \mathbf{k}) = \int \exp(-i\mathbf{k}'\mathbf{r}) V_0(r) \tilde{\psi}_{\mathbf{k}}(\mathbf{r}) d^2 r, \qquad (12)$$

where $\tilde{\psi}_{\mathbf{k}}(\mathbf{r})$ is the true wavefunction of the relative motion with momentum $\mathbf{k}$. The p-wave part of $f(\mathbf{k}', \mathbf{k})$ is $f(k', k) \exp i\phi$, where ϕ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$, and the partial p-wave amplitude is given by

$$f(k', k) = \int_0^\infty J_1(k'r) V_0(r) \tilde{\psi}(k, r) 2\pi r dr, \qquad (13)$$

with J_1 being the Bessel function. The wavefunction of the relative p-wave motion, $\tilde{\psi}(k, r)$, is governed by the Schrödinger equation

$$-\frac{\hbar^2}{m}\left(\frac{d^2\tilde{\psi}}{dr^2} + \frac{1}{r}\frac{d\tilde{\psi}}{dr} - \frac{\tilde{\psi}}{r^2}\right) + V_0\tilde{\psi} = \frac{\hbar^2 k^2}{m}\tilde{\psi}. \qquad (14)$$

The full on-shell scattering amplitude is obtained from equation (12) at $|\mathbf{k}'| = |\mathbf{k}|$, and its p-wave part follows from Eq. (13) at $k' = k$, $f(k) \equiv f(k, k)$. It is related to the p-wave scattering phase shift $\delta(k)$ by [34]

$$f(k) = -\frac{2\hbar^2}{mi}[\exp(2i\delta(k)) - 1] = -\frac{4\hbar^2}{m}\frac{\tan\delta(k)}{1 - i\tan\delta(k)}. \qquad (15)$$

In Eq. (13) the wavefunction $\tilde{\psi}(k, r)$ is normalized such that for $r \to \infty$ we have

$$\tilde{\psi}(k, r) = J_1(kr) - \frac{if(k)}{4} H_1(kr),$$

where $H_1 = J_1 + iN_1$ is the Hankel function, and N_1 is the Neumann function. It is, however, more convenient to normalize the radial wavefunction in such a way that it is real and for $r \to \infty$ one has

$$\psi(k,r) = [J_1(kr) - \tan\delta(k) N_1(kr)] \propto \cos(kr - 3\pi/4 + \delta). \qquad (16)$$

One easily checks that $\tilde{\psi}(k, r) = \psi(k, r)/(1 - i\tan\delta(k))$. Using this relation in Eq. (13) we see that the off-shell scattering amplitude can be written in the form:

$$f(k', k) = \frac{\bar{f}(k', k)}{1 - i\tan\delta(k)}, \qquad (17)$$

where $\bar{f}(k', k)$ is real and is given by Eq. (13) with $\tilde{\psi}(k, r)$ replaced by $\psi(k, r)$. For $k' = k$ one has

$$\bar{f}(k, k) \equiv \bar{f}(k) = -(4\hbar^2/m)\tan\delta(k). \qquad (18)$$

To find the p-wave scattering amplitude in the limit $kr_* \ll 1$ we divide the range of distances into two parts: $r < r_0$ and $r > r_0$, where r_0 lies in the interval $r_* \ll r_0 \ll k^{-1}$ (see Fig. 3). As we will see, this separation reflects the presence of two contributions to the scattering amplitude, the short-range contribution ($r \lesssim r_*$) and the anomalous contribution ($r \sim k^{-1}$).

In region I where $r < r_0$, the p-wave relative motion of two fermionic molecules is governed by the Schrödinger equation with zero kinetic energy:

$$-\frac{\hbar^2}{m}\left(\frac{d^2\psi_I}{dr^2} + \frac{1}{r}\frac{d\psi_I}{dr} - \frac{\psi_I}{r^2}\right) + V_0\psi_I = 0. \qquad (19)$$

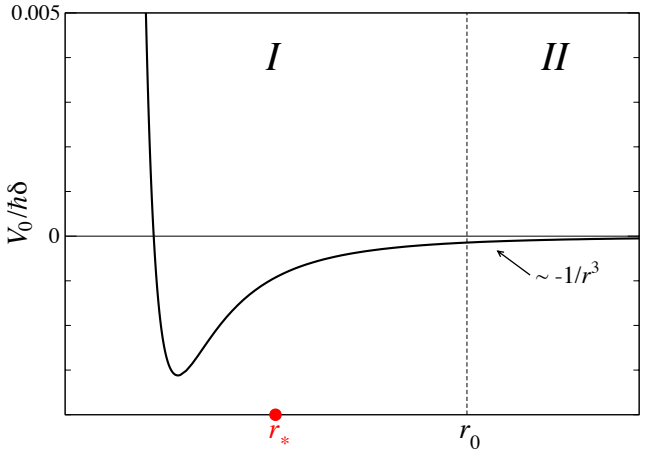

FIG. 3: (color online). The division of ranges into regions I ($r < r_0$) and II ($r > r_0$). The length scales r_* and $r_0 \gg r_*$ are indicated by a circle and a dashed line respectively.

At distances where the interaction potential is behaving as $V_0(r) = -\hbar^2 r_*/mr^3$, the solution of Eq. (19) reads:

$$\psi_I(r) \propto \left[AJ_2\left(2\sqrt{\frac{r_*}{r}}\right) + N_2\left(2\sqrt{\frac{r_*}{r}}\right)\right], \qquad (20)$$

where the constant A is determined by the behavior of $V_0(r)$ at shorter distances.

In region II, at $r > r_0$, the relative motion is practically free and the potential $V_0(r)$ can be considered a perturbation. To zero order we then have for the relative wavefunction:

$$\psi_{II}^{(0)}(r) = J_1(kr) - \tan\delta_I(k) N_1(kr), \qquad (21)$$

where the scattering phase shift δ_I is due to the interaction between particles in region I. Matching the logarithmic derivatives of ψ_I and $\psi_{II}^{(0)}$ at $r = r_0$ we obtain

$$\tan\delta_I = \frac{\pi k^2 r_0 r_*}{8}\left[1 - \frac{r_*}{r_0}\left(2C - \frac{1}{2} + \pi A - \ln\frac{r_0}{r_*}\right)\right], \qquad (22)$$

where $C = 0.5772$ is the Euler constant, and we took into account that $r \gg r_*$ and $kr_0 \ll 1$.

We now include perturbatively the contribution to the p-wave scattering phase shift from distances $r > r_0$ (region II). In this region, to first order in $V_0(r)$ the relative wavefunction is given by

$$\psi_{II}^{(1)}(r) = \psi_{II}^{(0)}(r) - \int_{r_0}^{\infty} G(r,r')V_0(r')\psi_{II}^{(0)}(r')2\pi r' dr', \qquad (23)$$

where the Green function for the free p-wave motion obeys the radial equation:

$$-\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + k^2\right) G(r,r') = \frac{\delta(r-r')}{2\pi r}. \qquad (24)$$

For the asymptotic representation of the relative wavefunction, chosen in Eq. (16), we have

$$G(r,r') = -\frac{m}{4\hbar^2}\begin{cases} \psi_{II}^{(0)}(r')N_1(kr), & r > r' \\ \psi_{II}^{(0)}(r)N_1(kr'), & r < r' \end{cases}. \qquad (25)$$

Then, substituting the Green function (25) into equation (23) and taking the limit $r \to \infty$ we obtain:

$$\tan\delta^{(1)}(k) = \tan\delta_I(k) - \frac{m}{4\hbar^2}\int_{r_0}^{\infty}\left[\psi_{II}^{(0)}(r)\right]^2 V_0(r) 2\pi r dr. \qquad (26)$$

Using Eq. (22) for $\tan\delta_I$ and calculating the integral in Eq. (26) we find:

$$\tan\delta^{(1)}(k) = \frac{2}{3}kr_* - \frac{\pi(kr_*)^2}{8}\left[\ln\frac{r_*}{r_0} + 2C - \frac{3}{2} + \pi A\right], \qquad (27)$$

and we omitted terms that contain higher powers of k.

Adding the second order contribution we have for the relative wavefunction:

$$\psi_{II}^{(2)}(r) = \psi_{II}^{(1)}(r) + \int_{r_0}^{\infty} G(r,r')V_0(r')2\pi r' dr' \times \int_{r_0}^{\infty} G(r',r'')V_0(r'')\psi_{II}^{(0)}(r'')2\pi r'' dr''. \qquad (28)$$

Then, using Eq. (25) and taking the limit $r \to \infty$ we see that including the second order contribution, the scattering phase shift becomes

$$\tan\delta(k) = \tan\delta^{(1)}(k) - \frac{m^2}{8\hbar^4}\int_{r_0}^{\infty}\left[\psi_{II}^{(0)}(r)\right]^2 V_0(r) 2\pi r dr \times \int_{r}^{\infty} N_1(kr')V_0(r')\psi_{II}^{(0)}(r')2\pi r' dr'. \qquad (29)$$

As we are not interested in terms that are proportional to k^3 or higher powers of k and in terms that behave as $(kr_*)^2 r_*/r_0$, we may omit the term $\tan\delta_I(k)N_1(kr)$ in the expression for $\psi_{II}^{(0)}(r)$. Then equation (29) reduces to

$$\tan\delta(k) = \tan\delta^{(1)}(k) - \frac{(\pi kr_*)^2}{2}\int_{kr_0}^{\infty}\frac{J_1^2(x)}{x^2}dx \times \left[\frac{2}{3}x(N_0(x)J_2(x) - N_1(x)J_1(x)) - \frac{1}{2}N_0(x)J_1(x) + \frac{1}{6}N_1(x)J_2(x) - \frac{1}{\pi x}\right]. \qquad (30)$$

For the first four terms in square brackets we may put the lower limit of integration equal to zero. Then, using

the relations:

$$\int_0^\infty J_1^3(x)N_1(x)\frac{dx}{x} = -\frac{1}{4\pi},$$
$$\int_0^\infty J_1^2(x)J_2(x)N_0(x)\frac{dx}{x} = \frac{1}{8\pi},$$
$$\int_0^\infty J_1^3(x)N_0(x)\frac{dx}{x^2} = \frac{1}{16\pi},$$
$$\int_0^\infty J_1^2(x)J_2(x)N_1(x)\frac{dx}{x^2} = -\frac{1}{16\pi},$$
$$\int_{kr_0}^\infty J_1^2(x)\frac{dx}{x^3} \approx \frac{1}{16} - \frac{C}{4} + \frac{\ln 2}{4} - \frac{1}{4}\ln kr_0,$$

we obtain

$$\tan\delta(k) = \tan\delta^{(1)}(k) - \frac{\pi(kr_*)^2}{8}\left\{\frac{7}{12} + C - \ln 2 + \ln kr_0\right\}.$$

Substituting $\tan\delta^{(1)}(k)$ from Eq. (27) we eventually arrive at the scattering phase shift

$$\tan\delta(k) = \frac{2}{3}kr_* - \frac{\pi(kr_*)^2}{8}\ln\rho kr_*, \tag{31}$$

where $\rho = \exp\{3C - \ln 2 - 11/12 + \pi A\} \simeq 1.13\exp(\pi A)$.

Using equations (15) and (18) we then immediately obtain the on-shell scattering amplitude $f(k)$ and the amplitude $\bar{f}(k)$. Note that $\bar{f}(k)$ is conveniently represented as a sum of two terms: $\bar{f}(k) = \bar{f}_1(k) + \bar{f}_2(k)$, where

$$\bar{f}_1(k) = -\frac{8}{3}\frac{\hbar^2}{m}kr_*, \tag{32}$$
$$\bar{f}_2(k) = \frac{\pi}{2}\frac{\hbar^2}{m}(kr_*)^2\ln\rho kr_*, \tag{33}$$

so that in the low-momentum limit the term $\bar{f}_1(k)$ is dominant. The related tangent of the scattering phase shift is contained in the first order contribution from distances $r > r_0$, given by the second term on the right-hand side of Eq. (26) in which one keeps only $J_1(kr)$ in the expression for $\psi_{II}(r)$. This says that $\bar{f}_1(k) = -(4\hbar^2/m)\tan\delta(k) = \int_{r_0}^\infty J_1^2(kr)V_0(r)2\pi r dr$ and for V_0 given by equation (10) the amplitude $\bar{f}_1(k)$ is reduced exactly to the result of Eq. (32).

The off-shell scattering amplitude $\bar{f}(k',k)$ defined by Eq. (17) can also be written as $\bar{f}_1(k',k) + \bar{f}_2(k',k)$, and the leading low-momentum contribution is given by

$$\begin{aligned}\bar{f}_1(k',k) &= \int_{r_0}^\infty J_1(k'r)J_1(kr)V_0(r)2\pi r dr \\ &= -\frac{\pi\hbar^2}{m}kr_*F\left(-\frac{1}{2},\frac{1}{2},2,\frac{k^2}{k'^2}\right),\end{aligned} \tag{34}$$

where we took into account that for $r > r_0$ the interaction potential surely has the form $V_0(r) = -\hbar^2 r_*/mr^3$. The quantity F in Eq. (34) is the hypergeometric function, and the result is written for $k < k'$. For $k > k'$ one should interchange k and k'.

IV. INELASTIC COLLISIONAL PROCESSES

For the considered case of $\delta > 0$ the dressed molecular state $|+\rangle$ is higher in energy than the states $|-\rangle$ and $|\Phi_{1-1}\rangle$. Therefore, molecules in the dressed state $|+\rangle$ may undergo pair inelastic collisions in which one or both are transferred to the state $|-\rangle$ or $|\Phi_{1-1}\rangle$. For $\Omega_R \lesssim \delta$, the released kinetic energy is $\sim \hbar\delta$ and it can cause both molecules to escape from the sample. The kinetic energy release requires a momentum transfer of $\sim \hbar/\lambda_\delta$ with $\lambda_\delta \equiv \sqrt{\hbar/m\delta}$. For $\lambda_\delta/r_\delta \ll 1$ the particles cannot approach each other sufficiently closely to allow the required momentum exchange, and one anticipates a reduction in the loss rate. The same condition can be derived semiclassically as the condition of adiabatic motion in the potential $V_0(r)$.

In order to go beyond this limit and determine the loss rate for λ_δ approaching r_δ, we have solved the full two-body scattering problem, involving states of even parity which at an infinite separation are: $(|+\rangle,|+\rangle)$, $(|+\rangle,|-\rangle)$, $(|+\rangle,|\Phi_{1-1}\rangle)$, $(|-\rangle,|-\rangle)$, and $(|-\rangle,|\Phi_{1-1}\rangle)$ [the state $(|\Phi_{1-1}\rangle,|\Phi_{1-1}\rangle)$ is decoupled]. We calculate numerically the probabilities P_l that two $|+\rangle$-state molecules with relative angular momentum l are scattered into *any* outgoing channel in which at least one of them is in the state $|-\rangle$ or $|1,-1\rangle$. This corresponds to non-adiabatic transitions from the potential $V_0(r)$ to the other potentials shown in the inset of Fig. 2. The Hamiltonian term which causes these non-adiabatic transitions is the kinetic energy (Laplacian) term. Defining the spinor $\chi_i(r) = \sqrt{r}\psi_i(r)$, where the index i labels the two-particles states $(|+\rangle,|+\rangle)$, $(|+\rangle,|-\rangle)$, $(|+\rangle,|\Phi_{1-1}\rangle)$, $(|-\rangle,|-\rangle)$, and $(|-\rangle,|\Phi_{1-1}\rangle)$, we obtain that the Laplacian term acts on $\chi_i(r)$ as

$$\hat{L}\chi(r) = L_{ij}(r)\chi_j(r).$$

The matrix $L_{ij}(r)$ is diagonal, $L_{ij}(r) = \bar{L}_i\delta_{ij}$, with

$$\bar{L}_i(r) = \frac{\hbar^2}{m}\left(-\frac{d^2}{dr^2} + \frac{l^2-1/4}{r^2}\right);\ i = 1,2,4, \tag{35}$$
$$\bar{L}_i(r) = \frac{\hbar^2}{m}\left(-\frac{d^2}{dr^2} + \frac{(l-2)^2-1/4}{r^2}\right);\ i = 3,5. \tag{36}$$

We note that coupling between these scattering channels will affect also the *elastic* scattering of two molecules incident in the $i = 1$ channel, in such a way that the scattering amplitude will differ for molecules in the $l = +1$ and $l = -1$ channels. This difference is a consequence of the fact that time reversal symmetry is broken by the circularly polarized MW field. The change in the scattering amplitude is small for $r_\delta \ll r^*$, which is valid in the situations studied below, so it will be a small effect. Nevertheless, we note that it will cause a slight difference in the energetics (and the transition temperatures) of the $p_x + ip_y$ and $p_x - ip_y$ phases, providing a small symmetry breaking perturbation that will favour one of

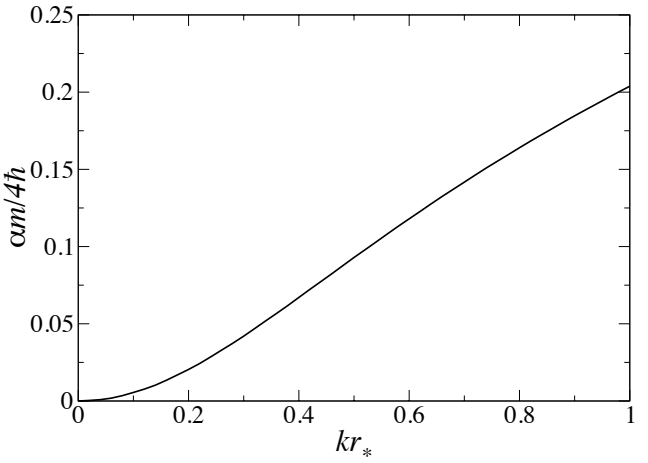

FIG. 4: Inelastic rate constant α as a function of kr_* for $\Omega_R = 0.25\delta$, $r_\delta/\lambda_\delta = 8$, and $\beta = 0$.

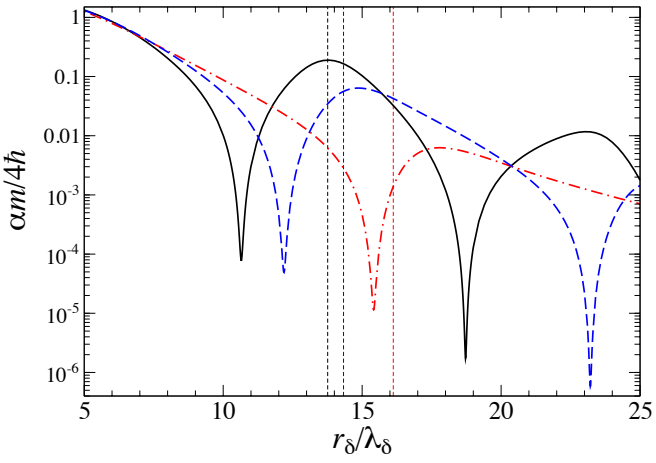

FIG. 5: (color online). Inelastic rate constant α as a function of r_δ/λ_δ for $\Omega_R = 0.25\delta$, $kr_* = 1$, and $\beta = 0$ (black, solid), $\beta = 0.1$ (blue, dashed), and $\beta = 0.2$ (red, dash-dotted). The critical value of r_δ/λ_δ above which a bound state of two molecules exists is indicated with corresponding vertical lines.

these two phases. Which one is favoured depends on the handedness of the circularly polarized MW field.

Taking into account that two molecules are lost in each inelastic collision, and writing the molecule loss rate as

$$\dot{n} = -\alpha n^2, \tag{37}$$

for the 2D inelastic rate constant we obtain:

$$\alpha = 4\hbar/m \sum_l P_l. \tag{38}$$

This rate constant at zero static electric field E_{dc} has been calculated in Ref. [8]. As in [8], we treat particles of the outgoing channel as 2D. This is surely valid for $\lambda_\delta \gg l_z$, where l_z is the size of the molecule wavefunction in the tightly confined z-direction (amplitude of zero point oscillations). In this case the energy release is insufficient to allow transitions to excited states in the z-direction. On the other hand, the 2D treatment of outgoing particles is relevant in the opposite limiting case of $\lambda_\delta \ll l_z$ where they have a very small angle $\sim \lambda_\delta/l_z$ out of the 2D plane. Thus, this approach should give a good result also for the intermediate case $\lambda_\delta \sim l_z$.

The dependence of α on r_δ/λ_δ at a constant kr_*, obtained in Ref. [8] for $\beta = 0$, shows the general trend of a reduction of inelastic losses with increasing r_δ/λ_δ, which is consistent with the semiclassical expectations. In addition, there is a dramatic modulation of the inelastic scattering rate, arising from an interference of incoming and outgoing waves in the scattering potential. We recover these results and investigate the dependence of α on kr_*. Fig. 4 shows α as a function of kr_* for $\Omega_R/\delta = 0.25$, $r_\delta/\lambda_\delta = 8$, and $\beta = 0$. The dependence $\alpha \propto (kr_*)^2$ expected in the low-momentum limit is observed for $kr_* \lesssim 0.1$.

We also consider the influence of a static electric field on the inelastic losses. In Fig. 5 we present the inelastic rate constant α versus the ratio r_δ/λ_δ for various values of β at $kr_* = 1$. One sees that an increase in β shifts the interference minimum of α towards larger values of r_δ/λ_δ and leads to lower values of α at these points. For example, for $\beta = 0.2$ the (first) interference minimum is located at $r_\delta/\lambda_\delta = 15.3$ with $\alpha \simeq 4 \times 10^{-5}\hbar/m$, whereas for $\beta = 0$ the minimum is at $r_\delta/\lambda_\delta = 10.7$ and the corresponding inelastic rate constant is $\alpha \simeq 3 \times 10^{-4}\hbar/m$.

The characteristic lifetime of the sample is $\tau \sim (\alpha n)^{-1}$, and for a deeply degenerate molecular Fermi gas one should take the rate constant α at k equal to the Fermi momentum k_F. We then see that even for $k_F r_* = 1$ the lifetime τ of, for example, $Na^{40}K$ molecules ranges from 20 to 2 s when increasing the density from 10^8 to 10^9 cm^{-2} and considering the first interference minimum at $\beta = 0.2$ (see Fig. 5).

We should avoid the presence of bound states of two molecules in the potential $V_0(r)$. Otherwise three-body recombination will lead to a rapid decay of the gas [8]. A dimensional estimate for the three-body recombination rate constant gives $\alpha_{\rm rec} \sim (\hbar r^{*2}/M)(k_F r^*)^4$. The corresponding decay time is $\tau_{\rm rec} \sim (\alpha_{\rm rec} n^2)^{-1}$, and it can be as short as milliseconds or even lower at densities $n \sim 10^8 - 10^9$ cm^{-2} for not very small values of $k_F r_*$. As we will see below, an efficient superfluid pairing and sufficiently high superfluid transition temperature require $k_F r_*$ approaching unity.

Bound molecule-molecule states in the potential $V_0(r)$ appear under an increase in r_δ/λ_δ. Critical values of r_δ/λ_δ below which bound states are absent for a given β, are shown by dashed vertical lines in Fig. 5. Thus, to the left of these lines the three-body recombination is absent. This is in particular the case at the first interference minimum for which the lifetime of the gas due to two-body inelastic collisions was estimated above.

V. SUPERFLUID PAIRING AND THE GAP EQUATION

We now discuss superfluid pairing in the 2D gas of MW-dressed fermionic polar molecules in the internal state $|+\rangle$, interacting with each other via the potential $V_0(r)$. The Hamiltonian of the system reads:

$$\hat{\mathcal{H}} = \int d^2r\, \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu\right) \hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d^2r d^2r' \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V_0(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r}), \quad (39)$$

where $\hat{\psi}(\mathbf{r})$ is the field operator of the dressed molecules, and μ is the chemical potential. The pairing is due to an effective attractive interaction between the fermions, which is reflected by a negative sign of the scattering amplitude $\bar{f}(k)$ at k close to the Fermi momentum k_F as discussed below using Eqs. (32) and (33). We thus employ a BCS approach [35, 36] and reduce $\hat{\mathcal{H}}$ to a bilinear form:

$$\hat{\mathcal{H}}_{BCS} = \int d^2\hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu\right) \hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d^2r d^2r' [\Delta^*(\mathbf{r},\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r}) + h.c.]. \quad (40)$$

The order parameter (gap) obeys the gap equation

$$\Delta(\mathbf{r},\mathbf{r}') = V_0(\mathbf{r}-\mathbf{r}')\langle\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})\rangle, \quad (41)$$

with the symbol $\langle\,\rangle$ denoting the expectation value. Strictly speaking, in two dimensions this approach is accurate at $T = 0$. At finite temperatures the long-range order is destroyed by long-wave thermal fluctuations of the phase and one has only an algebraic order. However, in the weakly interacting regime the characteristic phase coherence length is exponentially large, and on a distance scale smaller than this length one may still use the order parameter (41) and the BCS approach. In the uniform case this order parameter depends on $\mathbf{r}$ and $\mathbf{r}'$ only through the difference $(\mathbf{r}-\mathbf{r}')$.

The Hamiltonian $\hat{\mathcal{H}}_{BCS}$ is reduced to a diagonal form $\hat{\mathcal{H}}_{BCS} = \sum_{\mathbf{k}} \epsilon_k \hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \text{const}$, by using the Bogoliubov transformation

$$\hat{\psi}(\mathbf{r}) = \frac{1}{\sqrt{S}}\sum_{\mathbf{k}}[u_{\mathbf{k}}\exp(i\mathbf{kr})\hat{b}_{\mathbf{k}} + v^*_{\mathbf{k}}\exp(-i\mathbf{kr})\hat{b}^\dagger_{\mathbf{k}}].$$

Here S is the surface area of the system, while $\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}}$ and $\epsilon_k = \sqrt{\xi_k^2 + |\Delta_{\mathbf{k}}|^2}$ are annihilation/creation operators and energies of single-particle excitations. The functions $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ satisfy the well-known Bogoliubov-de Gennes equations. They are given by

$$u_{\mathbf{k}} = \frac{\xi_k + \epsilon_k}{\sqrt{2\epsilon_k(\xi_k+\epsilon_k)}}; \quad v_{\mathbf{k}} = \frac{\Delta_{\mathbf{k}}}{\sqrt{2\epsilon_k(\xi_k+\epsilon_k)}},$$

where $\Delta_{\mathbf{k}} = \int d^3r \Delta(\mathbf{r}-\mathbf{r}')\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')]$ is the momentum-space order parameter, $\xi_k = \hbar^2k^2/2m - \mu$, and the chemical potential in the weakly interacting regime is close to the Fermi energy $E_F = \hbar^2k_F^2/2m$. In the momentum space, the gap equation (41) takes the form:

$$\Delta_{\mathbf{k}} = -\int \frac{d^2q}{(2\pi)^2} V_0(\mathbf{q}-\mathbf{k})\Delta_{\mathbf{q}}\mathcal{K}(q), \quad (42)$$

where $V_0(\mathbf{q})$ is the Fourier transform of the potential $V_0(r)$, and $\mathcal{K}(q) = \tanh(\epsilon_q/2T)/2\epsilon_q$.

We now renormalize the gap equation (42) by expressing the Fourier transform of $V_0(r)$ through the off-shell scattering amplitude. The relation between these two quantities reads [34]:

$$f(\mathbf{k}',\mathbf{k}) = V_0(\mathbf{k}'-\mathbf{k}) + \int \frac{d^2q}{(2\pi)^2}\,\frac{V_0(\mathbf{k}'-\mathbf{q})f(\mathbf{q},\mathbf{k})}{2(E_k - E_q - i0)}. \quad (43)$$

Multiplying Eq. (43) by $\mathcal{K}(k')\Delta_{\mathbf{k}'}$ and integrating over d^2k', with the help of Eq. (42) we then obtain:

$$\Delta_{\mathbf{k}} = -\int f(\mathbf{k}',\mathbf{k})\Delta_{\mathbf{k}'}\left[\mathcal{K}(k') - \frac{1}{2(E_{k'} - E_k - i0)}\right]\frac{d^2k'}{(2\pi)^2}. \quad (44)$$

Note that in contrast to the commonly used renormalization procedure expressing the Fourier transform of the interaction potential through the vertex function $\Gamma(\mathbf{k},\mathbf{k}',E)$ [37] at an arbitrarily chosen energy E, which coincides with $f(\mathbf{k}',\mathbf{k})$ for $E = E_{k'}$, we use the off-shell scattering amplitude from the very beginning of the renormalization.

At $T = 0$ we put $\tanh(\epsilon_k/2T) = 1$ and, hence, $\mathcal{K}(k) = 1/2\epsilon_k$. We then perform an analysis assuming that in the weak coupling limit the main contribution to the integral in Eq. (44) comes from momenta k' close to k_F. It shows that the dominant pairing instability is in the channel with orbital angular momentum $l = 1$, since for higher angular momenta the interaction (scattering) amplitude is much smaller. The most stable low temperature phase has $p_x \pm ip_y$ symmetry, following from the fact that this phase fully gaps the Fermi surface, in contrast to competing phases [38]. A full numerical solution of the regularized gap equation confirms this analysis.

In fact, equation (42) and, hence, equation (44) are not sufficient for obtaining a correct result for the order parameter. One should calculate the quantity $\delta V(\mathbf{q},\mathbf{k})$ originating from many-body effects and add it to $V_0(\mathbf{q}-\mathbf{k})$ in Eq. (42). The quantity $\delta V(\mathbf{q},\mathbf{k})$ is a correction to the bare interparticle interaction V_0, and the leading terms of $\delta V(\mathbf{q},\mathbf{k})$ are second order in V_0 [39]. The corresponding diagrams are shown in Fig. 6 and they are the same as in the case of superfluid pairing between identical dipolar fermions in three dimensions [40]. They describe processes in which one of the two colliding particles polarizes the medium by creating a particle-hole pair. In Fig. 6(a) the particle-hole pair then annihilates due to the interaction with the other colliding particle. In 6(b),

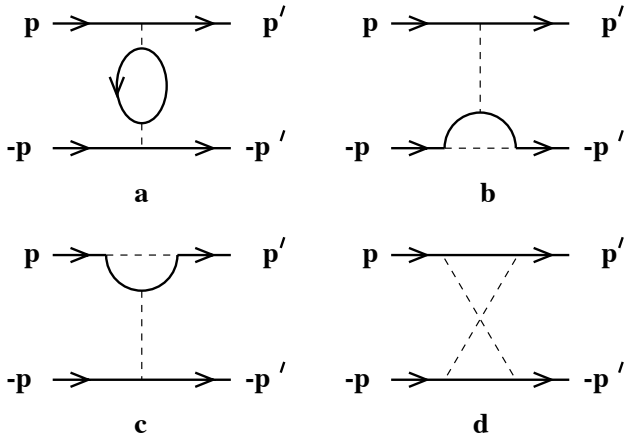

FIG. 6: The lowest order many-body corrections to the effective interparticle interaction.

6(c), and 6d the hole annihilates together with one of the colliding particles. In 6(b) and 6(c) the particle-hole pair is created due to the interaction of the medium with one of the colliding particles, and the hole annihilates with the other colliding partner. In 6(d) these creation and annihilation processes involve one and the same colliding particle. Including these many-body effects, which were introduced by Gor'kov and Melik-Barkhudarov [39], the gap equation becomes:

$$\Delta_{\mathbf{k}} = -\int f(\mathbf{k}',\mathbf{k})\Delta_{\mathbf{k}'}\left[\mathcal{K}(k') - \frac{1}{2(E_{k'}-E_k-i0)}\right]\frac{d^2k'}{(2\pi)^2} - \int \delta V(\mathbf{k}',\mathbf{k})\mathcal{K}(k')\Delta_{\mathbf{k}'}\frac{d^2k'}{(2\pi)^2}. \quad (45)$$

It turns out that we should also take into account the difference between particles and quasiparticles (single-particle excitations). The latter are characterized by the effective mass m_*, and the density of states near the Fermi surface is

$$\nu_F = \frac{m_*}{2\pi\hbar^2}. \quad (46)$$

Thus, dealing with this quantity in the gap equation we should replace m with m_*. Note that for short-range potentials the effective mass correction can be neglected. However, for fairly long-range interactions like $1/r^3$ in 2D, this correction becomes important as was recently demonstrated for the case of s-wave pairing in bilayer dipolar systems [41].

According to the Landau Fermi-liquid theory, the derivative of the quasiparticle energy $\epsilon(k)$ with respect to momentum is given by [35]:

$$\frac{\partial\epsilon(k)}{\partial\mathbf{k}} = \frac{\hbar^2\mathbf{k}}{m} + \int F(\mathbf{k},\mathbf{k}')\frac{\partial N(k')}{\partial\mathbf{k}'}\frac{d^2k'}{(2\pi)^2}, \quad (47)$$

where $N(k')$ is the distribution function of quasiparticles for which we take a step function $N(k') = \theta(k_F - k')$, so that $\partial N(k')/\partial\mathbf{k}' = -(\mathbf{k}'/k')\delta(k'-k_F)$. The leading contribution to the interaction function of quasiparticles, $F(\mathbf{k},\mathbf{k}')$, comes from the scattering by the $1/r^3$ tail of the interaction potential $V_0(r)$. This contribution can be calculated in the first Born approximation and, including all (odd) partial waves, for small momenta it is given by

$$F(\mathbf{k},\mathbf{k}') = [V_0(0) - V_0(\mathbf{k}-\mathbf{k}')] = -\frac{2\pi\hbar^2}{m}|\mathbf{k}-\mathbf{k}'|r_*. \quad (48)$$

This simply follows from the fact that in our weakly interacting system of identical fermionic particles the interaction energy par unit area is

$$E_{\rm int} = \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} V_0(\mathbf{k}_3-\mathbf{k}_2)\langle\hat{a}^\dagger_{\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3}\hat{a}^\dagger_{\mathbf{k}_3}\hat{a}_{\mathbf{k}_2}\hat{a}_{\mathbf{k}_1}\rangle$$
$$= \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2}[V_0(0) - V_0(\mathbf{k}_1-\mathbf{k}_2)]N(k_1)N(k_2),$$

with $\hat{a}^\dagger$ and $\hat{a}$ being the (quasi)particle operators.

For k near k_F equation (47) then yields:

$$v_F = \frac{\hbar k_F}{m} - \frac{1}{\hbar}\int_0^{2\pi} F(2k_F|\sin\phi/2|)\cos\phi\frac{k_F d\phi}{4\pi^2}, \quad (49)$$

where ϕ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$, and $v_F = \partial\epsilon/\hbar\partial k|_{k=k_F}$ is the Fermi velocity. The effective mass is defined as $m_* = \hbar k_F/v_F$. Then, using Eq. (48) we have:

$$\frac{m_*}{m} = 1 + \frac{4}{3\pi}k_F r_*. \quad (50)$$

The Gor'kov-Melik-Barkhudarov corrections and the replacement of the bare mass m by m_* do not change our conclusion that the ground state has $p_x \pm ip_y$ symmetry.

VI. ORDER PARAMETER AND TRANSITION TEMPERATURE

In the 2D geometry that we consider, the transition temperature T_c of a Fermi gas from the normal to superfluid regime is set by the Kosterlitz-Thouless transition. However, in the weak coupling limit the Kosterlitz-Thouless temperature is very close to T_c calculated in the BCS approach [42]. The latter follows from Eq. (45) as the highest temperature at which this equation has a non-trivial solution for the order parameter. Using the $p_x + ip_y$ symmetry we write $\Delta_{\mathbf{k}} = \Delta(k)\exp(i\phi_{\mathbf{k}})$. Then, multiplying both sides of Eq. (45) by $\exp(-i\phi_{\mathbf{k}})$ and integrating over $d\phi_{\mathbf{k}}$ and $d\phi_{\mathbf{k}'}$ we obtain the same equation (45) in which $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{k}'}$ are replaced by $\Delta(k)$ and $\Delta(k')$, the off-shell scattering amplitude $f(\mathbf{k}',\mathbf{k})$ is replaced by its p-wave part $f(k',k)$ defined in Eq. (13), and $\delta V(\mathbf{k}',\mathbf{k})$ by its p-wave part $\delta V(k',k)$. Calculating the contribution of the pole in the second term in the square brackets and expressing the off-shell scattering amplitude

through $\bar{f}(k',k)$ by using Eq. (17), we obtain:

$$\Delta(k)=-\mathrm{P}\int\bar{f}(k',k)\Delta(k')\left[\mathcal{K}(k')-\frac{1}{2(E_{k'}-E_k)}\right]\frac{d^2k'}{(2\pi)^2} - \int\delta V(k',k)\mathcal{K}(k')\Delta(k')\frac{d^2k'}{(2\pi)^2}. \quad (51)$$

The symbol P denotes the principal value. The amplitude $\bar{f}(k',k)$ is real, and Eq. (51) is convenient for analytical and numerical calculations of the order parameter and T_c.

The leading contribution to the integral on the right hand side of Eq. (51) is related to the first term in square brackets and comes from a narrow vicinity of the Fermi surface. Omitting other contributions we then establish a relation between $\Delta(k)$ and the order parameter on the Fermi surface:

$$\Delta(k)=\Delta(k_F)\frac{\bar{f}(k_F,k)}{\bar{f}(k_F)}. \quad (52)$$

For small momenta it is sufficient to use Eq. (34) for $\bar{f}(k_F,k)$ and Eq. (32) for $\bar{f}(k_F)$. This yields:

$$\Delta(k)=\frac{3\pi\Delta(k_F)}{8}\begin{cases}\frac{k}{k_F}F\left(-\frac{1}{2},\frac{1}{2},2,\frac{k^2}{k_F^2}\right), & k<k_F\\ F\left(-\frac{1}{2},\frac{1}{2},2,\frac{k_F^2}{k^2}\right), & k>k_F\end{cases} \quad (53)$$

so that $\Delta(k)\propto k$ for $k<k_F$, and it becomes k-independent for $k>k_F$. In Fig. 7 we compare the result of Eq. (53) with the result of full scale numerics at $T=0$ for the interaction potential $V_0(r)$ at $\Omega_R=0.25\delta$ and $\beta=0$.

Equation (52) can be used to calculate the contribution to the integral in Eq. (51) from momenta k' away from the Fermi surface. This immediately leads to a relation between the zero-temperature order parameter on the Fermi surface, $\Delta_0(k_F)$, and the critical temperature T_c. Putting $k=k_F$ in Eq. (51) and taking into account that in the weak coupling regime one has $|\Delta(k)|,T_c\ll E_F$, we divide the region of integration over k' into two parts: $|E_{k'}-E_F|<\omega$, and $|E_{k'}-E_F|>\omega$, where $|\Delta(k)|,T_c\ll\omega\ll E_F$. In the second region we may put $\epsilon_{k'}=|\xi_{k'}|$ and $\tanh(\epsilon_{k'}/2T)=1$, so that $\mathcal{K}(k')=1/2|\xi_{k'}|$. Then, making use of Eq. (52) for $\Delta(k')$ we see that the integration in this region gives $\Delta(k_F)\mathcal{A}$, where the quantity $\mathcal{A}$ is temperature independent. Dividing both sides of the gap equation by $\Delta(k_F)$ we obtain:

$$-\frac{m}{2\pi\hbar^2}\int_0^\omega[\bar{f}(k_F)+\delta V(k_F,k_F)]\tanh\left(\frac{\sqrt{\xi_{k'}^2+|\Delta(k_F)|^2}}{2T}\right) \times\frac{d\xi_{k'}}{\sqrt{\xi_{k'}^2+|\Delta(k_F)|^2}}+\mathcal{A}=1. \quad (54)$$

As $T\to T_c$ the order parameter tends to zero and we may put $\Delta(k_F)=0$ in Eq. (54). Then, subtracting this equation at $T\to T_c$ from the equation at $T=0$ we have:

$$\int_0^\omega\left\{\frac{\tanh(\xi_{k'}/2T_c)}{\xi_{k'}}-\frac{1}{\sqrt{\xi_{k'}^2+|\Delta_0(k_F)|^2}}\right\}d\xi_{k'}=0.$$

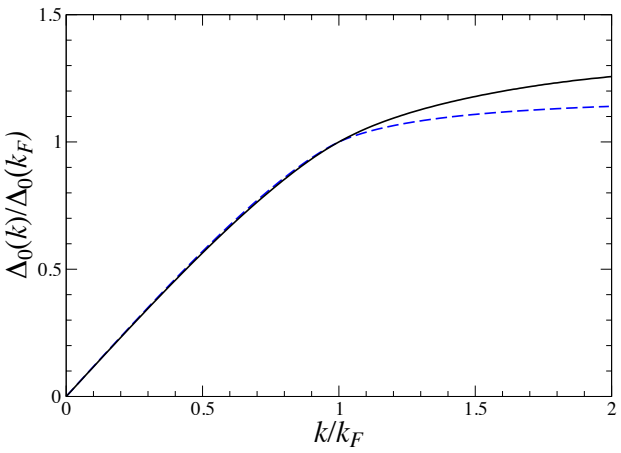

FIG. 7: (color online). The zero-temperature gap $\Delta_0(k)$ in units of $\Delta_0(k_F)$ as a function of k/k_F for $\Omega=0.25$, $\beta=0$, and $k_Fr_*=0.4$ near the minimum of inelastic losses ($r_\delta/\lambda_\delta=10.5$). The solid curve is the result of full scale numerics on the basis of Eq. (51), and the dashed curve is the result of Eq. (53).

The integral converges at $\xi_{k'}$ of the order of T_c or $|\Delta_0(k_F)|$, and we may extend the upper limit of integration to infinity. This gives:

$$T_c=\left(\frac{e^C}{\pi}\right)|\Delta_0(k_F)|, \quad (55)$$

which is the same relation as in the case of the s-wave pairing in a two-component weakly interacting 3D Fermi gas [35]. Note that equation (54) allows one to establish a relation between T_c or $|\Delta_0(k_F)|$ and $|\Delta(k_F)|$ at any temperature. Having in mind that the momentum dependence of the order parameter follows from Eq. (52) we see that the calculation of T_c provides us with the order parameter at any k and temperature.

We now calculate the critical temperature T_c on the basis of the gap equation (51) at $k=k_F$. In the first line of this equation we again divide the region of integration into two parts: $|E_{k'}-E_F|<\omega$, and $|E_{k'}-E_F|>\omega$. In the first region we put $\Delta(k')=\Delta(k_F)$ and $\bar{f}(k',k_F)=\bar{f}(k_F)$. The contribution of the second term in square brackets is then equal to zero. In the first term we set $\epsilon_{k'}=|\xi_{k'}|$ and, hence, $\mathcal{K}(k')=(2|\xi_{k'}|)^{-1}\tanh(|\xi_{k'}|/2T_c)$. Denoting the result of the integration of this term as $\Delta_1(k_F)$ we have:

$$\Delta_1(k_F)=\Delta(k_F)\frac{4k_Fr_*}{3\pi}\left(1-\frac{3\pi}{16}k_Fr_*\ln(\rho k_Fr_*)\right) \times\ln\left(\frac{2e^C\omega}{\pi T_c}\right), \quad (56)$$

where we used $\bar{f}(k_F)=\bar{f}_1(k_F)+\bar{f}_2(k_F)$, with $\bar{f}_1$, $\bar{f}_2$

given by equations (32), (33) and the numerical factor ρ introduced after Eq. (31).

In the second region, $|E_{k'} - E_F| > \omega$, we put $\mathcal{K}(k') = 1/2|\xi_{k'}|$. The contribution of this region is small as $\sim k_F r_*$ compared to the result of Eq. (56). Therefore, it is sufficient to retain only the leading low-momentum contribution to the off-shell amplitude $\bar{f}(k', k_F)$. Thus, using Eq. (34) for $\bar{f}(k', k_F)$ and Eq. (53) for $\Delta(k')$ we write this contribution as

$$\Delta_2(k_F) = \Delta(k_F)\frac{3\pi r_*}{8k_F}\int_0^{k_\omega}\frac{k'^3 dk'}{(k_F^2 - k'^2)}F^2\left(-\frac{1}{2},\frac{1}{2},2,\frac{k'^2}{k_F^2}\right),$$

where $k_\omega = \sqrt{2m(E_F-\omega)/\hbar^2}$. Taking into account that $\omega \ll E_F$ we then obtain:

$$\Delta_2(k_F) = \Delta(k_F)\frac{4k_F r_*}{3\pi}\left[\ln\left(\frac{E_F}{\omega}\right) - \eta\right], \qquad (57)$$

where

$$\eta = 1 - \frac{9\pi^2}{64}\int_0^1\left[F^2\left(-\frac{1}{2},\frac{1}{2},2,x\right) - F^2\left(-\frac{1}{2},\frac{1}{2},2,1\right)\right] \times\frac{xdx}{1-x} \simeq 0.78.$$

The main contribution to the term in the second line of Eq. (51), describing Gor'kov-Melik-Barkhudarov corrections, comes from the vicinity of the Fermi surface, i.e. from the region where $|E_{k'} - E_F| < \omega$. The result of the integration reads:

$$\Delta_3(k_F) = -\Delta(k_F)\frac{\delta V(k_F,k_F)}{2\pi}\ln\left(\frac{2e^C\omega}{\pi T_c}\right). \qquad (58)$$

In the Appendix we show that in the low-momentum limit one has

$$\delta V(k_F,k_F) = \alpha\frac{\hbar^2}{m}(k_F r_*)^2, \qquad (59)$$

where $\alpha \simeq 2.3$.

Then, making a summation of the contributions (56), (57), and (58), and dividing the gap equation by $\Delta(k_F)$ we obtain:

$$\frac{4k_F r_*}{3\pi}\left[\ln\left(\frac{2e^C E_F}{\pi T_c}\right) - \eta\right] - (k_F r_*)^2\ln\left(\frac{E_F}{T_c}\right) \times\left\{\frac{1}{4}\ln(\rho k_F r_*) + \frac{\alpha}{2\pi}\right\} = 1. \qquad (60)$$

Note that we put $\omega \sim E_F$ in the argument of the logarithm in the Gor'kov-Melik-Barkhudarov term and in the term proportional to $(k_F r_*)^2\ln(\rho k_F r_*)$. This is justified because ω can be chosen as a small numerical fraction of E_F, and these terms are small as $\sim k_F r_*$ compared to the leading term.

We now recall that the bare mass m should be replaced by the effective mass m_* following from Eq. (50). The relative difference between m_* and m is proportional to $k_F r_*$ and is small. Therefore it is sufficient to replace m with m_* only in the multiple $r_* \propto m$ in the first term of Eq. (60). Using Eq. (50) this leads to the appearance of a new term $(16/9\pi^2)(k_F r_*)^2\ln(E_F/T_c)$ in equation (60), which is equivalent to replacing α by $\tilde{\alpha} = \alpha - 32/9\pi \simeq 1.17$. Relying on the inequality $k_F r_* \ll 1$ we then immediately find the transition temperature [43]:

$$\frac{T_c}{E_F} = \frac{\kappa}{(k_F r_*)^{9\pi^2/64}}\exp\left(-\frac{3\pi}{4k_F r_*}\right), \qquad (61)$$

where

$$\kappa = \exp\left\{-\frac{9\pi^2}{64}\ln\rho - \frac{9\pi}{32}\tilde{\alpha} + C - \eta + \ln\left(\frac{2}{\pi}\right)\right\}.$$

Substituting the values of ρ, η, and $\tilde{\alpha}$ specified above we obtain:

$$\kappa \simeq 0.16\exp\left(-\frac{9\pi^3 A}{64}\right). \qquad (62)$$

Let us make several important statements regarding equation (61). First of all, the factor in the exponent is $\sim -(k_F r_*)^{-1}$, and the ratio T_c/E_F can be made larger than 10^{-2} even for relatively small $k_F r_*$. This is a consequence of the anomalous scattering due to the attractive $1/r^3$ tail of the potential $V_0(r)$, which gives a scattering amplitude proportional to k. Moreover, an accurate calculation of the scattering amplitude reveals a $(kr_*)^2\ln(kr_*)$ term, which leads to the appearance of a power law factor $(k_F r_*)^{-9\pi^2/64}$ in front of the exponent and provides an additional increase of the critical temperature. This behavior is in contrast to the p-wave pairing of short-range interacting atoms, where the factor in the exponent is inversely proportional to $-k_F^2$ and the BCS critical temperature is vanishingly low.

The dependence of T_c on the short-range behavior of the potential $V_0(r)$ is contained in the factor κ through the coefficient A. In Fig. 8 we present A and κ as functions of r_δ/λ_δ for $\Omega_R = 0.25\delta$ and β ranging from 0 to 0.2. These results show that κ can be varied by two orders of magnitude by changing r_δ/λ_δ [*i.e.* by changing the depth of the potential $V_0(r)$] while remaining in the regime without bound states in this potential, such that a rapid three-body decay is absent.

In Fig. 9 we display the ratio T_c/E_F as a function of $k_F r_*$ for $\Omega_R = 0.25\delta$ and r_δ/λ_δ close to the minimum of the loss rate at a given β. We compare the full scale numerical solution of the gap equation (51) with the analytic expression, Eq. (60). The discrepancy between analytic and numerical results as $k_F r_*$ approaches 1 is to be expected as the analytic method is perturbative in this parameter. From the results of Fig. 9 we conclude that a critical temperature of the order of 5% of the Fermi energy is realistic. For typical 2D densities $n \sim 10^8 - 10^9$ cm^{-2} the Fermi energy of alkali atom molecules is of the

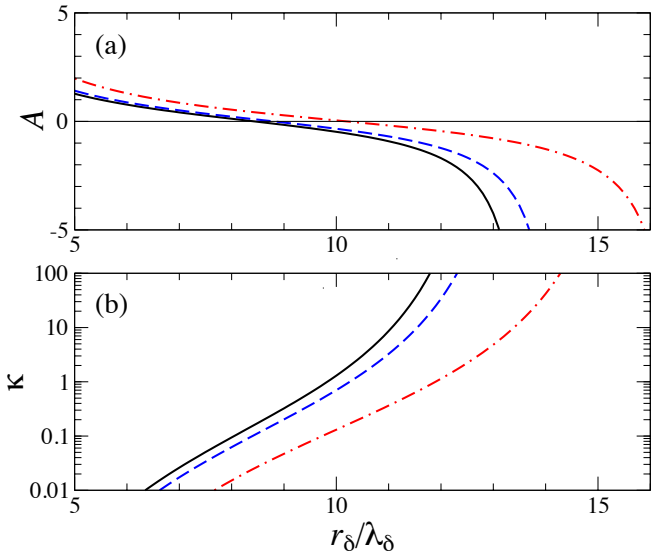

FIG. 8: (color online). The coefficients A and κ versus r_δ/λ_δ for $\Omega_R/\delta = 0.25$ and $\beta = 0$ (black, solid), 0.1 (blue, dashed), and 0.2 (red, dash-dotted). The coefficients are presented for the values of r_δ/λ_δ at which bound states in the potential $V_0(r)$ do not exist.

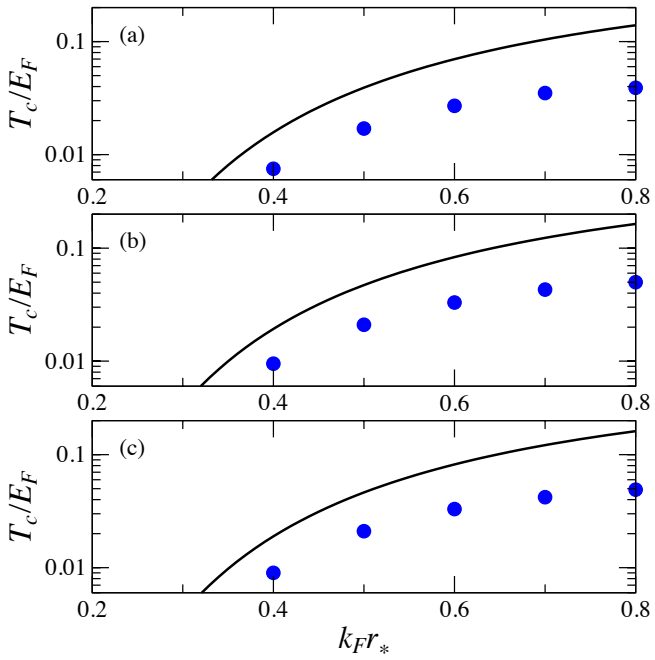

FIG. 9: (color online). Critical temperature T_c in units of the Fermi energy as a function of $k_F r_*$ for $\Omega = 0.25$ near the minimum of inelastic losses. In (a) $\beta = 0$, $r_\delta/\lambda_\delta = 10.7$, in (b) $\beta = 0.1$, $r_\delta/\lambda_\delta = 11.4$, and in (c) $\beta = 0.2$, $r_\delta/\lambda_\delta = 13.2$. The data points are obtained from full scale numerics on the basis of Eq. (51), and the curves from Eq. (60) [43].

order of hundreds of nanokelvins. Then, as we see from Fig. 9, the superfluid transition temperature can be as high as 10 or 20 nK.

VII. CONCLUDING REMARKS

The results described in this paper indicate that it is realistic to create the superfluid topological p_x+ip_y phase with alkali atom polar molecules. The key point is the anomalous scattering by the attractive $1/r^3$ tail of the interaction potential $V_0(r)$, which leads to a relatively large (and negative) p-wave scattering amplitude and thus to an achievable transition temperature T_c even in the BCS regime. What we have shown in the present paper is the possibility to manipulate T_c by modifying the short-range part of the potential $V_0(r)$. As is seen from Fig. 8, the pre-exponential coefficient κ can be varied within 2 or 3 orders of magnitude, and we may still remain in the BCS regime with fairly small inelastic losses.

Another result of this paper is that the addition of a *dc* electric field reduces the inelastic losses, and one can work at higher densities. Consider, for example, ^{6}LiCs molecules which have a permanent dipole moment of 5.5 D [44, 45]. For $\Omega_R = 0.12\delta$ and $\beta = 0.165$ the dipole-dipole distance is $r_* = 100$ nm and $k_F r_* = 1$ at 2D densities of 10^9 cm^{-2} corresponding to a Fermi energy of 200 nK. Then, our calculations give the rate constant 10^{-9} cm^2/s for $r_\delta/\lambda_\delta = 39$, which for the selected Ω_R/δ and β corresponds to the absence of bound states in the potential $V_0(r)$ and is not far from the minimum of inelastic losses. Thus, the lifetime of the gas is about 1 s at these densities. With $k_F r_* = 1$ we are beyond the BCS regime. However, the results of Fig. 9 indicate that in this case we are likely to have a critical temperature of at least a twentieth part of E_F, which is $T_c \simeq 10$ nK. Note that the ratio $r_\delta/\lambda_\delta = 39$ corresponds to $r_\delta \simeq 15$ nm and using the rotational constant $B \simeq 0.27$ K [45] we find that $\beta = 0.165$ corresponds to $E_{dc} \simeq 350$ V/cm.

In order to provide the 2D regime of scattering and superfluid pairing one should have the confinement length in the z-direction, $l_z \lesssim r_\delta$. For the above example of ^{6}LiCs molecules with $r_\delta \simeq 15$ nm this requires a confinement frequency $\omega_z = \hbar/ml_z^2$ of the order of 300 kHz. Such very high frequencies are not unrealistic and are achievable by a tight optical confinement. In particular, frequencies close to 100 kHz have been used for sideband cooling of cesium atoms [46–48].

Similar estimates are obtained for LiK and NaK molecules which have dipole moments of 3.5 and 2.7 D, respectively [44]. Here one can have the Fermi energy above 300 nK ($n \gtrsim 10^9$ cm^{-2}) and T_c on the level of tens of nanokelvins, with a lifetime of the order of a second. However, the necessary confinement frequencies are about 500 kHz.

The requirement for the confinement frequency can be relaxed by introducing a shallow optical lattice and thus increasing the effective mass of the molecules. For example, increasing the effective mass by a factor of 5 allows one to increase r_δ to 25 nm for ^{6}LiCs molecules, which

corresponds to the confinement frequency of 100 kHz. Then, however, we have to have larger r_* and even for $k_F r_* \sim 1$ the Fermi energy is about 20 nK, so that T_c will be on the level of nanokelvins. Similar estimates are obtained for LiK and NaK molecules where under the same increase of the effective mass the confinement frequency can be decreased to 200 kHz, with T_c of the order of nanokelvins. Note that increasing the effective mass by a factor of 10 makes the situation promising even for ^{40}KRb molecules which have a much smaller dipole moment $d = 0.6$ D [1]. Then, taking $r_\delta \simeq 20$ nm, which corresponds to the confinement frequency of 150 kHz, and $r_* = 100$ nm we find T_c on the level of nanokelvins ($E_F \simeq 30$ nK) at densities of 10^9 cm^{-2} and with a lifetime of the order of seconds.

The formation of the $p_x + ip_y$ superfluid phase should be apparent in numerous observables. Superfluidity itself can be detected by the means that have been used to detect s-wave superfluids [49, 50]. The most interesting new properties arise in the presence of quantized vortices, which are predicted to carry zero energy Majorana modes on their cores [19]. Vortices can be induced in superfluid Fermi gases by causing the gas to rotate [49, 51]. The appearance of zero energy Majorana modes on the vortex cores will lead to signatures in the RF absorption spectrum: the Majorana modes are predicted to lead to a series of sharp lines below the superfluid gap. These lines arise from processes in which a fermion is excited out of the Majorana modes [52]. Ultimately one would hope to probe non-abelian exchange statistics of these vortices [53].

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Appendix

In this Appendix, we demonstrate how the Gor'kov-Melik-Barkhudarov corrections to the bare interparticle interaction may be computed. The corrections consist of the four diagrams depicted in Fig. 6 [40]. In the low-momentum limit they are dominated by the long-range $1/r^3$ tail of the potential $V_0(r)$ and, hence, we may approximate this potential as

$$V_0(r) \approx \begin{cases} 0, & r < r_0 \\ -(\hbar^2/m) r_* / r^3, & r > r_0 \end{cases}, \tag{63}$$

where r_0 is much smaller than the de Broglie wavelength of particles. The Fourier transform of the potential takes the form:

$$V_0(\mathbf{q}) \approx -\frac{2\pi\hbar^2}{m}\frac{r_*}{r_0} + \frac{2\pi\hbar^2}{m}|\mathbf{q}| r_*. \tag{64}$$

The expressions for the four diagrams of Fig. 6 are

$$\delta V_a(\mathbf{p}', \mathbf{p}) = \int \frac{d^2q}{(2\pi)^2} \frac{N(\mathbf{q}+\mathbf{p}_-/2) - N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2} - \xi_{\mathbf{q}-\mathbf{p}_-/2}} \times V_0^2(\mathbf{p}_-), \tag{65}$$

$$\delta V_b(\mathbf{p}', \mathbf{p}) = -\int \frac{d^2q}{(2\pi)^2} \frac{N(\mathbf{q}+\mathbf{p}_-/2) - N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2} - \xi_{\mathbf{q}-\mathbf{p}_-/2}} \times V_0(\mathbf{p}_-) V_0(\mathbf{q}-\mathbf{p}_+/2), \tag{66}$$

$$\delta V_c(\mathbf{p}', \mathbf{p}) = -\int \frac{d^2q}{(2\pi)^2} \frac{N(\mathbf{q}+\mathbf{p}_-/2) - N(\mathbf{q}-\mathbf{p}_-/2)}{\xi_{\mathbf{q}+\mathbf{p}_-/2} - \xi_{\mathbf{q}-\mathbf{p}_-/2}} \times V_0(\mathbf{p}_-) V_0(\mathbf{q}+\mathbf{p}_+/2), \tag{67}$$

$$\delta V_d(\mathbf{p}', \mathbf{p}) = -\int \frac{d^2q}{(2\pi)^2} \frac{N(\mathbf{q}+\mathbf{p}_+/2) - N(\mathbf{q}-\mathbf{p}_+/2)}{\xi_{\mathbf{q}+\mathbf{p}_+/2} - \xi_{\mathbf{q}-\mathbf{p}_+/2}} \times V_0(\mathbf{q}-\mathbf{p}_-/2) V_0(\mathbf{q}+\mathbf{p}_-/2), \tag{68}$$

with $\mathbf{p}_\pm = \mathbf{p}' \pm \mathbf{p}$, and $\xi_\mathbf{q} = \hbar^2\mathbf{q}^2/2m - \mu$. In the weakly interacting regime we may put the chemical potential μ equal to the Fermi energy, and at very low temperatures $T < T_c$ we may take the step function $N(\mathbf{q}) = \Theta(k_F - |\mathbf{q}|)$ for the Fermi-Dirac distribution.

The p-wave part of the diagrams (65)-(68) is

$$\delta V(p', p) = \sum_j \delta V_j(p', p) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \sum_{j\in\{a,b,c,d\}} \delta V_j(\mathbf{p}', \mathbf{p}), \tag{69}$$

where ϕ is the angle between $\mathbf{p}'$ and $\mathbf{p}$. One can easily check that the momentum-independent term of $V_0(\mathbf{q})$ does not contribute to the sum of the four diagrams.

For the analytical calculation of T_c we only need the contributions $\delta V_j(p', p)$ on the Fermi surface, i.e. for $p' = p = k_F$. In this case each contribution can be represented in the form $\delta V_j(k_F, k_F) = \alpha_j(\hbar^2/m)(k_F r_*)^2$. The calculation of the diagram 6(a) is straightforward and it gives $\alpha_a = 2\pi$. The other diagrams we calculated numerically. The values of the coefficients are

$$\alpha_a = 2\pi, \quad \alpha_b = \alpha_c \simeq -1.5, \quad \alpha_d \simeq -1.0, \tag{70}$$

and thus we find the result of Eq. (59):

$$\delta V(k_F, k_F) \equiv \alpha \frac{\hbar^2}{m} (k_F r_*)^2, \tag{71}$$

with $\alpha \simeq 2.3$.

For the numerical solution of the gap equation we used a simplified dependence of δV on p' and p, namely

the dependence following from the diagram (a) with $V_0(|\mathbf{p}_-|) = (2\pi\hbar^2/m)|\mathbf{p}_-|r_*$. This dependence is given by:

$$\delta V(p',p) = \delta V(k_F,k_F)\Big\{\frac{pp'}{k_F^2} + \int_0^{2\pi}\frac{d\phi}{2\pi}\Theta(p_- - 2k_F)\cos\phi\frac{p_-\sqrt{p_-^2-4k_F^2}}{k_F^2}\Big\}. \quad (72)$$

For $(p+p') < 2k_F$ we have $\delta V(p',p) = \delta V(k_F,k_F)pp'/k_F^2$, and this quantity rapidly decays at larger momenta.

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