This is the accepted manuscript made available via CHORUS. The article has been published as:

Molecular-beam optical Stark and Zeeman study of the A $\wedge\{2\} \Pi--$ ^ $^{\wedge}\{2\} \Sigma \wedge\{+\}(0,0)$ band system of BaF
Timothy C. Steimle, Sarah Frey, Anh Le, David DeMille, David. A. Rahmlow, and Colan Linton
Phys. Rev. A 84, 012508 - Published 19 July 2011
DOI: 10.1103/PhysRevA.84.012508

# A molecular-beam optical Stark and Zeeman study of the $A^{2} \Pi-X^{2} \Sigma^{+}$ $(0,0)$ band system of BaF 

Timothy C. Steimle*, Sarah Frey and Anh Le<br>Department of Chemistry and Biochemistry<br>Arizona State University<br>Tempe, AZ 85287-1604

David DeMille and David. A. Rahmlow ${ }^{\dagger}$<br>Department of Physics<br>Yale University<br>PO Box 208120<br>New Haven, CT 06520

Colan Linton<br>Centre for Laser Atomic and Molecular Sciences<br>Physics Department<br>University of New Brunswick<br>P.O. Box 4400

Fredericton, New Brunswick, E3B 5A3

Last Modified: May 21, 2011
Send Proofs to: Prof. T. C. Steimle, Department of Chemistry and Biochemistry, Arizona State University, Tempe, Arizona 85287-1604
${ }^{\dagger}$ Current Address:
Department of Physics, University of Connecticut, 2152 Hillside Road, U-3046 Storrs, CT 06269-3046

- e-Mail: Tsteimle@ASU.edu


#### Abstract

The $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system of barium monofluoride ( BaF ) has been recorded using high-resolution laser-induced fluorescence spectroscopy both field-free and in the presence of a static magnetic and electric fields. The field-free spectra for the ${ }^{135} \mathrm{BaF}$, ${ }^{137} \mathrm{BaF}$ and ${ }^{138} \mathrm{BaF}$ isotopologues were modeled to generate an improved set of spectroscopic constants for the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states. The observed optical Stark shifts for the ${ }^{138} \mathrm{BaF}$ isotopologue were analyzed to produce the permanent electric dipole moments of $1.50(2) \mathrm{D}$ and $1.31(2) \mathrm{D}$ for the $A^{2} \Pi_{1 / 2}(v=0)$ and $A^{2} \Pi_{3 / 2}(v=0)$ states, respectively. The observed optical Zeeman shifts for the ${ }^{138} \mathrm{BaF}$ isotopologue were analyzed to produce a set of magnetic g-factors for the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states.


## I. INTRODUCTION

There is renewed interest in measuring and predicting the properties of barium monofluoride, BaF , because of its use as a sensitive experimental venue for quantification of $P$ - and $P, T$-odd effects [1-9]. The determination of the electric dipole moment (EDM) of the electron, $d_{e}$, provides insight into the violation of parity inversion symmetry $(P)$ and time-reversal invariance $(T)$. Such effects are nuclear spin independent and studies of both the even and odd BaF isotopologues are relevant. The interaction of the nuclear anapole moment, a $P$-odd magnetic moment induced by weak interactions within the nucleus, with the spin of the penetrating nucleus is a nuclear spin dependent parity violation effect. This latter interaction can be probed via studies of only the odd BaF isotopologues. The effective Hamiltonian operator for the $X^{2} \Sigma^{+}$electronic state of BaF, including the PNC relevant terms, is $[1,9]$ :

$$
\begin{align*}
& \mathbf{H}^{e f f}\left({ }^{2} \Sigma\right)=B \mathbf{N}^{2}+\gamma \mathbf{N} \cdot \mathbf{S}+b_{F} \mathbf{I} \cdot \mathbf{S}+c\left(I_{z} S_{z}-\frac{1}{3} \mathbf{I} \cdot \mathbf{S}\right) \\
& +e q_{0} Q \frac{3 I_{z}-\mathbf{I}^{2}}{4 I(2 I-1)}+W_{A} k_{A} \mathbf{n} \times \mathbf{I} \cdot \mathbf{S}+\left(W_{S} k_{S}+W_{d} d_{e}\right) \mathbf{S} \cdot \mathbf{n} \tag{1}
\end{align*}
$$

where $\mathbf{N}$ is the angular momentum operator excluding electronic spin, $\mathbf{S}$, and nuclear spin, $\mathbf{I}$, and $\mathbf{n}$ is a unit vector along the bond. The first five terms in Eq. 1 are commonly used to model the fine (via the $B$ and $\gamma$ terms), Fermi contact and dipolar magnetic hyperfine (via the $b_{F}$ and $c$ terms), and the electric quadrupole hyperfine (via the $e q_{0} Q$ term) interactions. The sixth term describes the interaction of the anapole moment of the nucleus, $k_{A}$, with the electron spin, the seventh a scalar electron-nucleus interaction, and the last the interaction of $d_{e}$ with the effective electric field, $W_{d}$. Determination of the
${ }^{135} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ hyperfine parameters is particularly relevant to parity violation studies, because these parameters are the most sensitive probe of the electronic wavefunction in the region of the Ba nucleus. In parity non-conservation studies the constants $W_{A}, W_{S}$ and $W_{d}$ of Eq. 1 are calculated as expectation values of appropriate symmetry-violating operators over the electronic wavefunction in the region of the Ba nucleus. Thus, a comparison of experimentally measured hyperfine constants, which also depend upon the electronic wavefunction in the region of the Ba nucleus, is the most rigorous gauge of the reliability of the predicted $W_{A}, W_{S}$ and $W_{d}$ values.

Experimental schemes for measuring $P$ - and $P, T$-odd effects in BaF exploit laser induced fluorescence detection using the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system. In addition, the study of $P$-odd effects relies on Zeeman tuning of opposite-parity levels to near degeneracy in a large magnetic field, and on mixing of these levels due to an applied electric field. Consequently, a spectroscopic understanding of the energy levels of both the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states, as well as Stark and Zeeman effects in these states, is required. Here we report on the analysis of the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system recorded at near the natural linewidth limit (full width at half maximum $($ FWHM $) \approx 30$ MHz ). Spectra were recorded field-free and in the presence of static electric and magnetic fields. An improved set of fine and hyperfine parameters for the $A^{2} \Pi(v=0)$ state is determined for the ${ }^{135} \mathrm{BaF},{ }^{137} \mathrm{BaF}$ and ${ }^{138} \mathrm{BaF}$ isotopologues. The permanent electric dipole moment, $\mu_{\mathrm{e}}$, for the $A^{2} \Pi_{1 / 2}(v=0)$ and $A^{2} \Pi_{3 / 2}(v=0)$ states and the magnetic $g$-factors for the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states of ${ }^{138} \mathrm{BaF}$ were determined from an analysis of the observed Stark and Zeeman shifts, respectively.

The field-free energies of the $X^{2} \Sigma^{+}(v=0)$ state of the ${ }^{138} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ isotopologues are well characterized. The pure rotational spectra of ${ }^{138} \mathrm{BaF}$ [10] and ${ }^{137} \mathrm{BaF}$ [11] have been recorded and analyzed to produce a set of fine and hyperfine constants for the $X^{2} \Sigma^{+}(v=0)$ state. An improved set of ${ }^{19} \mathrm{~F}$ magnetic hyperfine constant for the ${ }^{138} \mathrm{BaF}$ isotopologue was obtained from the analysis of the molecular beam lasermicrowave double resonance spectrum [12]. In the same study, the electric field induced shifts of the rotational levels of the $X^{2} \Sigma^{+}(v=0)$ state were measured from which a $\mu_{\mathrm{e}}$ of 3.170(3) D was determined. There have been no reported experimental measurement of $\mu_{\mathrm{e}}$ for the $A^{2} \Pi$ state. The $X^{2} \Sigma^{+}(v=0)$ state has also been studied by high-resolution Fourier transform infrared emission spectroscopy [13]. In that study, numerous vibration-rotation bands were analyzed for the three most abundant isotopologues from which Dunham coefficients were derived.

The $A^{2} \Pi$ state is less well characterized. The $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system of the main isotopologue, ${ }^{138} \mathrm{BaF}$, has been previously characterized from analysis of the near Doppler limited resolution Fourier transform spectra of the thermal emission and the $\mathrm{Ar}^{+}$and $\mathrm{Kr}^{+}$ion laser induced fluorescence [14-16]. It was observed that the $A^{2} \Pi$ state was perturbed by the nearby $A^{\prime 2} \Delta$ and $B^{2} \Sigma^{+}$states. Effective molecular constants were obtained by treating the trio of states as a $d$-complex. The radiative lifetime of the $A^{2} \Pi_{1 / 2}(v=0)[17]$ and $A^{2} \Pi_{3 / 2}(v=0)[18]$ states were measured to be 46.1 ns and 56.0 ns , respectively.

There are no high level ab initio predictions for the $X^{2} \Sigma^{+}$and $A^{2} \Pi$ states. A semiempirical electrostatic polarization model developed by Törring et al [19] predicts that $\mu_{\mathrm{e}}$
$=3.47 \mathrm{D}$ and 4.95D for the $X^{2} \Sigma^{+}$and $A^{2} \Pi$ states, respectively. A similar model developed by Mestdagh and Visticot [20] predicts nearly identical values. A ligand field theory predictions [21] predicts $\mu_{\mathrm{e}}=3.89 \mathrm{D}$ and 3.45D for the $X^{2} \Sigma^{+}$and $A^{2} \Pi$ states, respectively.

## II. EXPERIMENT

The supersonic molecular beam production and laser induced fluorescent detection schemes are similar to those used in the previous measurements of $\operatorname{SrF}$ [22] and YbF [23,24]. A continuously rotating barium metal rod was ablated in a supersonic expansion of approximately $5 \%$ sulfur hexafluoride $\left(\mathrm{SF}_{6}\right)$ seeded in argon carrier gas with a backing pressure of approximately 3 MPa . The pulsed free-jet expansion was skimmed to form a well-collimated molecular beam which was crossed with a single longitudinal mode, continuous wave, Ti:Sapphire laser approximately 50 cm downstream from the source. The laser power was attenuated to approximately 20 mW and lightly focused (f.l. $=1 \mathrm{~m}$ ) to avoid power broadening. Spectral line widths of less than 40 MHz FWHM were observed.

The absolute wave numbers were determined to an accuracy of $\pm 0.003 \mathrm{~cm}^{-1}$ by simultaneously recording the Doppler limited $\mathrm{I}_{2}$ absorption spectrum [25,26]. Interpolation between $\mathrm{I}_{2}$ absorption features was achieved by simultaneously recording the transmission of two confocal étalons. One étalon was actively stabilized and calibrated to have a free spectral range of 749.14 MHz . A second, unstablized étalon with a free spectral range of 75.7 MHz was used to interpolate between transmission peaks of the stabilized étalon.

The optical Zeeman spectrometer [27] and optical Stark spectrometer [28] have been described previously. A $0-1.0 \mathrm{kG}$, homogeneous, magnetic field was generated using a homemade electromagnet in the interaction region of molecular beam and laser. The magnetic field was measure using a commercial Gauss meter. Static electric field strengths of up to $2000 \mathrm{~V} / \mathrm{cm}$ were generated by application of a voltage across a pair of conducting plates straddling the region of molecular fluorescence. The electric field was measure by combination of a commercial volt meter and mechanical measurement of the plate separation. A polarization rotator and polarizing filter were used to orient the static electric or magnetic field vector of the linearly polarized laser radiation either parallel "||" or perpendicular " " to that of the applied field resulting in $\Delta M_{\mathrm{F}}=0$ or $\Delta M_{\mathrm{F}}= \pm 1$ selection rules. The combined systematic error associated with the measurement of the magnetic and electric field induced frequency shifts and the field strength is estimated to be less than $2 \%$.

## III. OBSERVATION

## A. Field-free Spectra

Portions of the field-free excitation spectrum in the region of the $A^{2} \Pi_{1 / 2}-X^{2} \Sigma^{+}(0,0)$ and $A^{2} \Pi_{3 / 2}-X^{2} \Sigma^{+}(0,0)$ sub-bands are presented in Figures 1 and 2, respectively. Enlarged portions of the field-free LIF spectra in the regions of the ${ }^{R} R_{11}(4)\left(v=11634.25 \mathrm{~cm}^{-1}\right)$ and ${ }^{Q} Q_{2}(6)\left(\mathrm{v}=12260.60 \mathrm{~cm}^{-1}\right)$ lines are presented in Figures 3 and 4 , respectively. The relative positions of the lines associated with the various isotopologues are evident in Figures 3 and 4. The spectra are congested in part because there are seven naturally occurring isotopes of barium: ${ }^{130} \mathrm{Ba}(0.11 \%),{ }^{132} \mathrm{Ba}(0.10 \%),{ }^{134} \mathrm{Ba}(2.42 \%),{ }^{135} \mathrm{Ba}(6.59 \%)$,
${ }^{136} \mathrm{Ba}(7.85 \%),{ }^{137} \mathrm{Ba}(11.23 \%)$ and ${ }^{138} \mathrm{Ba}(71.70 \%)$. Transitions associated with the ${ }^{135} \mathrm{BaF}$, ${ }^{136} \mathrm{BaF},{ }^{137} \mathrm{BaF}$ and ${ }^{138} \mathrm{BaF}$ isotopologues are readily identified in Figures 1-3. Spectral lines of the odd isotopologues exhibit a large splitting due to a Ba-hyperfine interaction in the $X^{2} \Sigma^{+}(v=0)$ state. The nuclear magnetic g-factors for ${ }^{135} \mathrm{Ba}(\mathbf{I}=3 / 2)$ and ${ }^{137} \mathrm{Ba}(\mathbf{I}=3 / 2)$ are +0.5586 and +0.6249 , respectively, and the quadrupole moments are $+18 \mathrm{fm}^{2}$ and $+28 \mathrm{fm}^{2}$, respectively. The ${ }^{19} \mathrm{~F}(\mathrm{I}=1 / 2)$ magnetic hyperfine splitting is much smaller because the unpaired electron in the $A^{2} \Pi$ and $X^{2} \Sigma^{+}$states is centered on the metal nucleus. The predicted energy levels associated with lines of the ${ }^{137} \mathrm{BaF}$ isotopologue in the region of the ${ }^{R} R_{11}(4)$ and ${ }^{Q} Q_{2}(6)$ lines of the ${ }^{138} \mathrm{BaF}$ isotopologue, obtained using the finalized optimized parameters, are presented in Figures 5 and 6, respectively, and illustrates the relative hyperfine splitting.

The energy level patterns for the low-rotational levels of the $X^{2} \Sigma^{+}$state of the even barium isotopologues are those of a molecule near the Hund's case $\left(b_{\beta J}\right)$ limit with the approximately good intermediate quantum number $J$ resulting from coupling the rotational angular momentum, $\mathbf{N}$, with the electron spin angular momentum, S. Each rotational level of a given $J$ of the even isotopologues splits into two levels designated by the total angular momentum $F$ due to the ${ }^{19} \mathrm{~F}(\mathrm{I}=1 / 2)$ magnetic hyperfine splitting. The resulting Hund's case $\left(b_{\beta J}\right)$ vector coupling appropriate for even barium isotopologues can be written as:

$$
\begin{equation*}
\mathbf{S}+\mathbf{N}=\mathbf{J} ; \mathbf{I}_{1}\left({ }^{19} \mathrm{~F}\right)+\mathbf{J}=\mathbf{F}, \tag{2}
\end{equation*}
$$

which corresponds to the basis set $|\eta \Lambda\rangle\left|(S N) J\left(J I_{I}\right) F\right\rangle$.

The large ${ }^{137,135} \mathrm{Ba}$ magnetic hyperfine interaction in the $X^{2} \Sigma^{+}$state causes the energy level pattern for the low-rotational levels of the odd isotopologues to be those of a molecule near the Hund's case $\left(b_{\beta} s\right)$ limit. The approximately good intermediate quantum number $G$ ( $=1$ and 2 ) results from coupling the ${ }^{137,135} \mathrm{Ba}$ nuclear spin angular momentum, $\mathbf{I}_{\mathbf{1}}$, with the total electron spin angular momentum, $\mathbf{S}$. The ${ }^{19} \mathrm{~F}$ nuclear spin is weakly coupled to $\mathbf{G}$ to produce the total angular momentum, $\mathbf{F}$. The Hund's case $\left(b_{\beta}\right.$ ) vector coupling for low rotational levels can be written as:

$$
\begin{equation*}
\mathbf{S}+\mathbf{I}_{1}\left({ }^{137,135} \mathrm{Ba}\right)=\mathbf{G}\left({ }^{137,135} \mathrm{Ba}\right) ; \mathbf{N}+\mathbf{G}\left({ }^{137,135} \mathrm{Ba}\right)=\mathbf{F}_{1} ; \quad \mathbf{F}_{1}+\mathbf{I}_{2}\left({ }^{19} \mathrm{~F}\right)=\mathbf{F}, \tag{3}
\end{equation*}
$$

which corresponds to the basis function $\left.|\eta \Lambda\rangle\left(S I_{1}\right) G(G N) F_{1}\left(F_{1} I_{2}\right) F\right\rangle$. The electron spin decouples from the nuclear spin with increasing rotation and the energy level pattern of the $X^{2} \Sigma^{+}$state of odd isotopologues gradually transforms to that of a molecule near the Hund's case $\left(b_{\beta J}\right)$ limit. As an illustration, the energy level pattern of the $X^{2} \Sigma^{+}(v=0)$ state of ${ }^{137} \mathrm{BaF}$ as a function of rotation, predicted using the previously determined fine and hyperfine parameters [11], is illustrated in Figure 7. The contribution due to rotation $(\approx B \times N(N+1))$ has been subtracted to emphasize the spin-rotation and hyperfine contributions.

The Ba hyperfine interactions are much smaller in the $A^{2} \Pi$ state and the energy level patterns for the low-rotational levels of both the even and odd barium isotopologues are those of a molecule near the Hund's case $\left(a_{\beta}\right)$ limit with the electron spin quantized
in the molecular frame and having a projection, $\Sigma$. A Hund's case $\left(a_{\beta J}\right)$ coupling scheme, with the nuclear spin sequentially coupled, is appropriate for all the isotopologues;

$$
\begin{equation*}
\mathbf{J}+\mathbf{I}_{1}\left({ }^{135,137} \mathrm{Ba}\right)=\mathbf{F}_{1} ; \quad \mathbf{F}_{1}+\mathbf{I}_{2}\left({ }^{19} \mathrm{~F}\right)=\mathbf{F} . \tag{4}
\end{equation*}
$$

The corresponding basis function is $\left|\eta \Lambda>|\mathrm{S} \Sigma>| J \Omega\left(J I_{1}\right) F_{1}\left(F_{1} I_{2}\right) F\right\rangle$. The inclusion of $\mathbf{I}_{2}$ $\left({ }^{19} \mathrm{~F}\right)$, even though the ${ }^{19} \mathrm{~F}(I=1 / 2)$ interaction is not fully resolved, facilitates spectral simulation.

A conventional ${ }^{2} \Pi$ (case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}$(case $b_{\beta J}$ ) labeling scheme of ${ }^{\Delta N} \Delta J_{F_{i} F_{i}}\left(N^{\prime \prime}\right)$ [29], where $F_{i}^{\prime \prime}$ and $F_{i}^{\prime}$ subscripts designate the spin component of the $X^{2} \Sigma^{+}$and $A^{2} \Pi$ states, respectively, is appropriate for the even isotopologues. In the cases where $\Delta N=\Delta J$ and $F_{i}^{\prime \prime}=F_{i} \quad$ it is customary to drop the superscript and the second subscript. The twelve branches of the ${ }^{2} \Pi$ (case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}\left(\right.$case $\left.b_{\beta J}\right)$ labeling scheme $\left(P_{1}, Q_{1}, R_{1},{ }^{P} Q_{12}\right.$, ${ }^{O} P_{12},{ }^{Q} R_{12}, P_{2}, Q_{2}, R_{2},{ }^{R} Q_{21},{ }^{Q} P_{21}$, and ${ }^{S} R_{21}$ ) are readily assigned (see Figures 1-4). The intermediate quantum number $J$ is not appropriate for the low-rotational levels of the $X^{2} \Sigma^{+}$state of the odd isotopologues and hence neither is the " $F_{i}$ " subscript in the conventional ${ }^{2} \Pi\left(\right.$ case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}\left(\right.$case $\left.b_{\beta J}\right)$ branch designation. It is customary to replace " $F_{i}^{\prime \prime \prime}$ " with the intermediate approximately good quantum number $G$ " of the Hund's case $\left(b_{\beta \mathrm{S}}\right)$ coupling scheme. The twelve branches of the ${ }^{2} \Pi\left(\right.$ case $\left.a_{\beta \mathrm{J}}\right)-{ }^{2} \Sigma^{+}$(case $\left.b_{\beta \mathrm{J}}\right)$ labeling scheme regroup into sixteen branch features of the ${ }^{2} \Pi\left(\right.$ case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}\left(\right.$case $\left.b_{\beta}\right)$ scheme. The branches are designated as ${ }^{O} P_{I G},{ }^{P} P_{l G}+{ }^{P} Q_{I G},{ }^{Q} Q_{I G}+{ }^{Q} R_{I G}$, and ${ }^{R} R_{l G}$ for the ${ }^{2} \Pi_{1 / 2}$
(case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}\left(\right.$case $\left.b_{\beta S}\right)$ sub-band and ${ }^{P} P_{2 G},{ }^{Q} P_{2 G}+{ }^{Q} Q_{2 G},{ }^{R} Q_{2 G}+{ }^{R} R_{2 G}$, and ${ }^{S} R_{2 G}$ for the ${ }^{2} \Pi_{3 / 2}\left(\right.$ case $\left.a_{\beta J}\right)-{ }^{2} \Sigma^{+}\left(\right.$case $\left.b_{\beta} \mathrm{S}\right)$ sub-band with $G=1$ and 2 . The abbreviation ${ }^{P} P_{I G},{ }^{Q} Q_{I G}$, ${ }^{Q} P_{2 G}$ and ${ }^{R} Q_{2 G}$ for the ${ }^{P} P_{I G}+{ }^{P} Q_{I G},{ }^{Q} Q_{I G}+{ }^{Q} R_{I G},{ }^{Q} P_{2 G}+{ }^{Q} Q_{2 G}$ and ${ }^{R} Q_{2 G}+{ }^{R} R_{2 G}$ branches, respectively, is used in Figures 3-6 and below.

A total of 439 transitions in the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band were measured and assigned: 73, 99 and 267 for the ${ }^{135} \mathrm{BaF},{ }^{137} \mathrm{BaF}$ and ${ }^{138} \mathrm{BaF}$ isotopologues, respectively. The observed and calculated transition wave number, based upon the final optimized parameters, and associated assignments for the ${ }^{138} \mathrm{BaF},{ }^{137} \mathrm{BaF}$, and ${ }^{135} \mathrm{BaF}$ isotopologues are listed in Tables I, II and III, respectively, of the Supplemental Tables [30].

## B. Stark Spectra

The ${ }^{S} R_{21}(0)\left(v=12262.88 \mathrm{~cm}^{-1}\right)$ and ${ }^{O} P_{12}(2)\left(v=11628.96 \mathrm{~cm}^{-1}\right)$ lines of the ${ }^{138} \mathrm{BaF}$ isotopologue were selected for optical Stark studies. The observed and predicted spectra for the ${ }^{S} R_{21}(0)$ line recorded field-free and in the presence of a $899.4 \mathrm{~V} / \mathrm{cm}$ static field with perpendicular orientation are presented in Figure 8. The associated energy levels as a function of applied field are also presented. The field free line rapidly splits into four components due to the near degeneracy of $\Lambda$-doublets associated with the $J=3 / 2$ level of the $A^{2} \Pi_{3 / 2}$ state. The pattern of four features slightly shifts to higher wave number due to the second order Stark tuning of the $N=0$ rotational level of the $X^{2} \Sigma^{+}$state. The observed and predicted spectra for the ${ }^{O} P_{12}(2)$ line recorded field-free and in the presence of a 2940 V/cm static field with parallel orientation are presented in Figure 9. The associated energy levels as a function of applied field are also presented. The field free spectral feature weakly tunes to lower wave number due primarily to the second order Stark
tuning in the $e$-parity component of the $J=1 / 2$ level of the $A^{2} \Pi_{1 / 2}$ state. The $\Lambda$-doubling of the $J=1 / 2$ level (i.e. the separation between the $e$ - and $f$-parity components) is $0.2589 \mathrm{~cm}^{-1}$ $(=p+2 q)$. A total of 17 Stark induced shifts for the ${ }^{S} R_{21}(0)$ line and 14 Stark induced shifts for the ${ }^{O} P_{12}(2)$ line were assigned and precisely measured. Measured shifts and the difference between the measured and predicted shifts are listed in Table I.

## C. Zeeman Spectra

The $R_{1}(0)\left(v=11631.28 \mathrm{~cm}^{-1}\right)$ and ${ }^{S} R_{21}(0)\left(v=12262.88 \mathrm{~cm}^{-1}\right)$ lines of the ${ }^{138} \mathrm{BaF}$ isotopologue were selected for optical Zeeman studies. The $R_{1}(0)$ line recorded field free and in the presence of 714 G parallel and 724 G perpendicular magnetic fields are presented in Figure 10. The associated energy levels as a function of applied magnetic field, and the assignments, are also presented. The Zeeman tuning of the upper energy terminus of the $R_{1}(0)$ line (i.e. the "-" parity component of the $J=3 / 2$ rotational level in the $A^{2} \Pi_{1 / 2}$ spin-orbit sub-state) is small and would be nearly zero in the Hund's case (a) limit. The ${ }^{S} R_{21}(0)$ line recorded field-free and in the presence of a 192 G perpendicular magnetic field is presented in Figure 11. The associated energy levels as a function of applied magnetic field, and the assignments, are also presented. The Zeeman tuning of the upper energy terminus of the ${ }^{S} R_{21}(0)$ line (i.e. "-" parity component of the $J=3 / 2$ rotational level in the $A^{2} \Pi_{3 / 2}$ spin-orbit component) is significant because the energy level pattern is near Hund's case (a) limit. The measured thirty four magnetic induced shifts of the ${ }^{S} R_{21}(0)$ line and forty two shifts of the $R_{1}(0)$ line, along with the quantum number assignments, are presented in Table II. The difference between the observed shifts and those predicted using the optimized magnetic g-factors (see below) is also given.

## IV. ANALYSIS

## A. Field Free

The observed transition wave numbers of the molecular beam optical spectrum were directly fit to an effective Hamiltonian for the $X^{2} \Sigma^{+}(v=0)$ and $A^{2} \Pi(v=0)$ state. The effective Hamiltonian for the $X^{2} \Sigma^{+}(v=0)$ state for the ${ }^{138} \mathrm{BaF}$ isotopologue was taken as:

$$
\begin{equation*}
\mathbf{H}^{\text {eff }}\left({ }^{2} \Sigma^{+}\right)=B \mathbf{N}^{2}-D \mathbf{N}^{4}+\gamma \mathbf{N} \cdot \mathbf{S}+b_{F}\left({ }^{19} \mathrm{~F}\right) \mathbf{I} \cdot \mathbf{S}+c\left({ }^{19} \mathrm{~F}\right)\left(I_{z} S_{z}-\frac{1}{3} \mathbf{I} \cdot \mathbf{S}\right) \tag{5}
\end{equation*}
$$

where $\mathbf{N}$ is the angular momentum operator excluding electronic and nuclear spin, $\mathbf{S}$ and I, respectively. The effective Hamiltonian for the $X^{2} \Sigma^{+}(v=0)$ state for the ${ }^{135} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ isotopologues were taken as that of Eq. 5 with the addition of Fermi contact, dipolar and electric quadrupole terms for ${ }^{135} \mathrm{Ba}$ and ${ }^{137} \mathrm{Ba}$ :

$$
\begin{equation*}
b_{F}\left({ }^{135,137} \mathrm{Ba}\right) \mathbf{I} \cdot \mathbf{S}+c\left({ }^{135,137} \mathrm{Ba}\right)\left(I_{z} S_{z}-\frac{1}{3} \mathbf{I} \cdot \mathbf{S}\right)+e q_{0} Q\left({ }^{(135,137} \mathrm{Ba}\right) \frac{3 I_{z}-\mathbf{I}^{2}}{4 I(2 I-1)} . \tag{6}
\end{equation*}
$$

The field-free eigenvalues and eigenvectors for the $X^{2} \Sigma^{+}(v=0)$ state were obtained by numerical diagonalization of the representation constructed in a sequentially coupled Hund's case $\left(a_{\beta J}\right)$ basis set. The representation for ${ }^{138} \mathrm{BaF}$ was of dimension 4 (= $\left.(2 S+1) \times\left(2 I\left({ }^{19} \mathrm{~F}\right)+1\right)\right)$ whereas the representation for the odd isotopologues was $16(=$ $(2 S+1) \times\left(2 I\left({ }^{19} \mathrm{~F}\right)+1\right) \times\left(2 I\left({ }^{135,137} \mathrm{Ba}\right)+1\right)$. The expressions for the matrix elements were taken from Ref. 31.

Combination difference revealed that the $A^{2} \Pi_{3 / 2}(v=0)$ sub-state levels of the ${ }^{135} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ isotopologues exhibited no magnetic hyperfine splitting. Furthermore, no $\Lambda$-doubling in the $A^{2} \Pi_{3 / 2}(v=0)$ sub-state was observed. The $\Lambda$-doubling in Hund's case (a) limit ${ }^{2} \Pi_{3 / 2}$ and ${ }^{2} \Pi_{1 / 2}$ sub-state goes approximately as $q J^{2}$ and $(p+2 q) J$ [31], respectively. Therefore, the energies for the $A^{2} \Pi(v=0)$ state were modeled by including the origin $\left(T_{0,0}\right)$, spin-orbit interaction $(A)$ and the associated centrifugal distortion correction $\left(A_{\mathrm{D}}\right)$, rotation $(B)$ and associated centrifugal distortion correction $(D)$, the $\Lambda$ doubling $(p+2 q)$ (which affects the $A^{2} \Pi_{1 / 2}(v=0)$ sub-band), and the $\Lambda$-type magnetic hyperfine $d\left({ }^{135,137} \mathrm{Ba}\right)$ interaction terms :

$$
\begin{align*}
\mathbf{H}^{\mathrm{eff}}\left({ }^{2} \Pi\right)= & T_{0,0}+A L_{z} S_{\mathrm{z}}+1 / 2 A_{D}\left[L_{z} S_{z}, \mathbf{R}\right]^{+}+B \mathbf{R}^{2}-D\left(\mathbf{R}^{2}\right)^{2} \\
& +1 / 2(p+2 q)\left(e^{-2 i \phi} J_{+} S_{+}+e^{+2 i \phi} J_{-} S_{-}\right)+1 / 2 d\left(e^{-2 i \phi} I_{+} S_{+}+e^{+2 i \phi} I_{-} S_{-}\right) . \tag{7}
\end{align*}
$$

In Eq. $7 J_{ \pm}, S_{ \pm}$and $I_{ \pm}$are the shift operators of the total angular momentum in the absence of nuclear spin, $\mathbf{J}$, the total electron spin, $\mathbf{S}$, and the ${ }^{135,137} \mathrm{Ba}$ nuclear spin angular momentum, $\mathbf{I}_{\mathbf{1}}$, and $\phi$ is the azimuthal coordinate of the electrons. $\left[L_{z} S_{\mathrm{z}}, \mathbf{R}\right]^{+}$is the anticommutator. The field-free eigenvalues and eigenvectors for the $A^{2} \Pi(v=0)$ state were obtained by numerical diagonalization of the representation constructed in a sequentially coupled Hund's case $\left(a_{\beta J}\right)$ basis set. The representation for ${ }^{138} \mathrm{BaF}$ was of dimension 8 (= $\left.2 \times(2 S+1) \times\left(2 I\left({ }^{19} \mathrm{~F}\right)+1\right)\right)$ whereas the representation for the odd isotopologues was $32\left(=2 \times(2 S+1) \times\left(2 I\left({ }^{19} \mathrm{~F}\right)+1\right) \times\left(2 I\left({ }^{135,137} \mathrm{Ba}\right)+1\right)\right.$. The expressions for the matrix elements were taken from Ref. 31.

Various fits of the measured field-free spectrum were performed. The 267 measured field-free transitions [30] for the ${ }^{138} \mathrm{BaF}$ isotopologue were modeled by varying
$T_{0,0}, A, A_{D}, B$ and $(p+2 q)$. The centrifugal distortion correction to the rotational parameters, $D$, for the $A^{2} \Pi(v=0)$ state was held fixed to the value determined from the analysis of the high temperature optical spectrum [15]. For the $X^{2} \Sigma^{+}(v=0)$ state, the $B$ and $\gamma$ parameters were constrained to the microwave values [10], the magnetic hyperfine parameters $b_{F}\left({ }^{19} \mathrm{~F}\right)$ and $c\left({ }^{19} \mathrm{~F}\right)$ were constrained to MODR values [12] and $D$ to the high temperature optical spectrum [15]. The optimized parameters and associated errors are given in Table III. The standard deviation of the fit $\left(=0.0013 \mathrm{~cm}^{-1}\right)$ is commensurate with the estimated measurement error.

Least squares fitting of the 99 measured lines for the ${ }^{137} \mathrm{BaF}$ isotopologue [30] proceeded in a similar fashion. The fine and magnetic hyperfine parameters of the $X^{2} \Sigma^{+}$ $(v=0)$ state were constrained to values extracted from the analysis of the pure rotational spectrum [11] with the exception of $D$ which was constrained to the value obtained from the analysis of ${ }^{138} \mathrm{BaF}$ optical spectrum [15] scaled by the expected mass ratio (i.e. scaled by $\left[\mu\left({ }^{138} \mathrm{BaF}\right)\right]^{2} /\left[\mu\left({ }^{137} \mathrm{BaF}\right)\right]^{2}$ where $\mu=$ reduced mass $)$. Five parameters for the $A^{2} \Pi(v=0)$ state were varied: $A, B,(p+2 q), d\left({ }^{137} \mathrm{Ba}\right)$ and $T_{00} . A_{D}$ was constrained to the value extracted from the analysis of the ${ }^{138} \mathrm{BaF}$ measurements [11] and $D$ to the value obtained from isotopically scaling the ${ }^{138} \mathrm{BaF}$ value. The optimized parameters and associated errors are given in Table III. The standard deviation of the fit $\left(=0.0014 \mathrm{~cm}^{-1}\right)$ is commensurate with the estimated measurement error.

There are no previously determined parameters for the ${ }^{135} \mathrm{BaF}$ isotopologue. Fitting of the 73 measured and assigned transitions [30] was performed by varying a subset of both the $X^{2} \Sigma^{+}(v=0)$ and $A^{2} \Pi(v=0)$ state parameters. In the $X^{2} \Sigma^{+}(v=0)$ state only $B$ and $b_{F}(\mathrm{Ba})$ were varied. The parameter $D$ was constrained to the ${ }^{138} \mathrm{BaF}$ scaled by
the known mass dependence and the $c\left({ }^{135} \mathrm{Ba}\right)$ and $e Q q_{0}\left({ }^{135} \mathrm{Ba}\right)$ parameters to those for ${ }^{137} \mathrm{Ba}$ scaled by the nuclear g-factors and quadrupole moment, respectively. The fluorine magnetic hyperfine parameters for ${ }^{137} \mathrm{BaF}, b_{F}\left({ }^{19} \mathrm{~F}\right)$ and $c\left({ }^{19} \mathrm{~F}\right)$, were constrained to the ${ }^{138} \mathrm{BaF}$ values [10]. The origin, $T_{00}$, rotational constant, $B$, and spin-orbit parameter, $A$, for the $A^{2} \Pi(v=0)$ state of the ${ }^{137} \mathrm{BaF}$ isotopologue were varied. The parameters $A_{D}$ and $(p+2 q)$ were constrained to the ${ }^{138} \mathrm{BaF}$ values, $D$ to the mass scaled ${ }^{138} \mathrm{BaF}$ value and $d(\mathrm{Ba})$ to the ${ }^{137} \mathrm{BaF}$ value scaled by the nuclear g-factors. The standard deviation of the fit $\left(=0.0015 \mathrm{~cm}^{-1}\right)$ is commensurate with the estimated measurement error.

## B. Stark Effect and Intensities

The interaction between the static electric field, $\mathbf{E}$, and the molecular electric dipole moment operator, $\mu$, is represented by the Stark Hamiltonian,

$$
\begin{equation*}
\mathbf{H}^{\text {Stark }}=-\boldsymbol{\mu} \cdot \mathbf{E} . \tag{8}
\end{equation*}
$$

The matrix representation of $\mathbf{H}^{\text {Stark }}$ is block diagonal in $M_{F}$ but of infinite dimension. The predicted Stark shifts in the $X^{2} \Sigma^{+}(v=0)$ state of the ${ }^{138} \mathrm{BaF}$ isotopologue were obtained by numerical diagonalization of a $16 \times 16$ truncated matrix representation constructed using the Hund's case $\left(a_{\beta J}\right)$ basis functions for $F=0-4$. The observed Stark shifts of Table I were used as input into a non-linear least-squares fitting program in which $\mu_{\mathrm{e}}$ for the $X^{2} \Sigma^{+}(v=0)$ was held fixed to the previously determined value $(=3.170(3) \mathrm{D})[12]$ and $\mu_{\mathrm{e}}$ for the $A^{2} \Pi(v=0)$ state was optimized. Initial attempts to simultaneously fit the shifts of the ${ }^{S} R_{21}(0)$ and ${ }^{O} P_{12}(2)$ lines revealed systematic deviations indicating that the $\mu_{\mathrm{e}}$ values for the $A^{2} \Pi_{1 / 2}$ and $A^{2} \Pi_{3 / 2}$ spin-orbit states are
significantly different. Independent fits of the Stark shifts of the ${ }^{S} R_{21}(0)$ and ${ }^{O} P_{12}(2)$ lines determined $\mu_{\mathrm{e}}$ of $1.50(2) \mathrm{D}$ and $1.31(2) \mathrm{D}$ for the $A^{2} \Pi_{1 / 2}(v=0)$ and $A^{2} \Pi_{3 / 2}(v=0)$ states, respectively. The error estimates represent a $90 \%$ confidence limit. The standard deviation of the fits for the ${ }^{S} R_{21}(0)$ and ${ }^{\circ} P_{12}(2)$ lines were 8 MHz and 5 MHz , respectively, which are commensurate with the measurement uncertainty.

Modeling the spectra greatly assisted in the assignment of quantum numbers, particularly in the case of the overlapped spectra for the ${ }^{135} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ isotopologues. Spectra for the ${ }^{138} \mathrm{BaF}$ isotopologue were modeled by constructing a $4 \times 8$ Hund's case $\left(a_{\beta J}\right)$ electric dipole transition moment matrix and cross multiplying the transition matrix by the $X^{2} \Sigma^{+}$and $A^{2} \Pi(v=0)$ eigenvectors. The transition moment was squared, multiplied by a Boltzmann factor and a Lorentzian line shape was superimposed on each spectral feature and co-added. Spectra for the ${ }^{135} \mathrm{BaF}$ and ${ }^{137} \mathrm{BaF}$ isotopologues were modlled in a similar fashion, but with $16 \times 32$ Hund's case $\left(a_{\beta J}\right)$ electric dipole transition moment matrix.

## C. Zeeman Effect

The effective Zeeman Hamiltonian was taken as [31, 32]:

$$
\begin{align*}
& \hat{\mathbf{H}}^{Z e e}(e f f)=g_{S} \mu_{B} \hat{\mathbf{S}} \cdot \hat{\mathbf{B}}+g_{L}^{\prime} \mu_{B} \hat{\mathbf{L}} \cdot \hat{\mathbf{B}}+  \tag{9}\\
& g_{1} \mu_{B}\left(\hat{S}_{x} \hat{B}_{x}+\hat{S}_{y} \hat{B}_{y}\right)+g_{1}^{\prime} \mu_{B}\left(e^{-2 i \phi} \hat{S}_{+} \hat{B}_{+}+e^{+2 i \phi} \hat{S}_{-} \hat{B}_{-}\right) .
\end{align*}
$$

The rotational g-factors, $g_{r}$ and $g_{r}{ }^{\prime}$, were not included because their effects are expected to be negligibly small for the low-rotational levels investigated. In the effective Hamiltonian model, both $g_{S}$ and $g_{L}^{\prime}$ are treated as adjustable parameters to account for electronic state mixing [31].

The matrix representation of $\hat{\mathbf{H}}^{\text {Zee }}(e f \AA)$ is of infinite order. The very small ${ }^{19} \mathrm{~F}$ magnetic hyperfine splitting was not resolved in the Zeeman spectra (see Figures 10 and 11) due to inhomogeneous and residual fields and the energy levels can be characterized by the projection quantum number $M_{J}$. Eigenvalues and eigenvectors for the lowest rotational levels of the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states investigated here were obtained by numerical diagonalization of an $8 \times 8$ and $16 \times 16$ matrix representations, respectively, constructed from the $J=0.5$ to $J=3.5$ Hund's case (a) basis functions. The expressions for the matrix elements were taken from Ref. 31. The 76 magnetically-induced Zeeman shifts of Table II, and initial estimates of the $g$-factors for the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states, were input for a nonlinear least square fitting process. Initial attempts to simultaneously optimize $g_{S}, g_{L}^{\prime}, g_{l}$ and $g_{l}^{\prime}$ for the $A^{2} \Pi$ state and $g_{l}$ and $g_{\mathrm{s}}$ for $X^{2} \Sigma^{+}$ produced a highly correlated set of parameters. In the end, the $g_{S}$ parameter for the $A^{2} \Pi$ $(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states was held fixed at 2.002 because the non-adiabatic contributions to this parameter are expected to be small [31]. In addition, $g_{l}\left(A^{2} \Pi\right)$, which is predicted to be small (vide infra) was set to zero. The optimized values for $g_{l}$ $\left(X^{2} \Sigma^{+}(v=0)\right), g_{l}^{\prime}\left(A^{2} \Pi(v=0)\right)$ and $g_{L}^{\prime}\left(A^{2} \Pi(v=0)\right)$ along with their associated errors and correlation matrix, are listed in Table V. The standard deviation of the fit was 34 MHz , which is commensurate with the estimated measurement accuracy.

## V. DISCUSSION

A primary objective of the current study was to determine a set of spectroscopic parameters that can be used to predict the eigenvalues and eigenvectors for the $A^{2} \Pi(v=0)$
and $X^{2} \Sigma^{+}(v=0)$ states of the three major isotopologues of barium monofluorides, both field-free and in the presence of static magnetic and electric fields. As can be seen from the comparison of the predicted and observed spectra given in Figures 3-6, 8 and 9 the determined spectroscopic constants accurately reproduce the energies and eigenvectors at fields appropriate for future PNC measurements. The observed ${ }^{135} \mathrm{Ba}$ and ${ }^{137} \mathrm{Ba}$ hyperfine structure in the low-rotational levels of the $A^{2} \Pi_{1 / 2}(v=0)$ sub-state can be modeled using solely the $d(\mathrm{Ba})$ magnetic hyperfine parameter. The $A^{2} \Pi_{3 / 2}(v=0)$ sub-state exhibited no hyperfine structure.

Surprisingly, the semi-empirical electrostatic model predictions by Törring et al [19] and Mestdagh and Visticot [20] which predict $\mu_{\mathrm{e}}$ values of approximately 5D for the $A^{2} \Pi$ which are in very poor agreement with observation. The LFT prediction [21] of 3.45D for the $A^{2} \Pi$ states is only slightly better. The determined $\mu_{\mathrm{e}}$ values for BaF are compared with values for isovalent $\operatorname{CaF}[33,34]$ and $\operatorname{SrF}[35,36]$ in Table V. Only the $\mu_{\mathrm{e}}$ values for the $A^{2} \Pi_{3 / 2}(v=0)$ spin-orbit state were measured for CaF and SrF . For all three molecules there is an observed large decrease in $\mu_{\mathrm{e}}$ upon excitation from the $X^{2} \Sigma^{+}$state to the $A^{2} \Pi$ state. This is qualitatively explained as the result of promoting the sole unpaired electron from a metal ion-centered $\mathrm{n} s$ orbital $(\mathrm{n}=4,5$, and 6 for $\mathrm{Ca}, \mathrm{Sr}$ and Ba$)$ to a more highly polarizable $\mathrm{n} p_{ \pm l} /(\mathrm{n}-1) d_{ \pm l}$ hybrid orbital. The effect becomes more pronounced down the series with the ratios $\mu_{\mathrm{e}}\left(A^{2} \Pi_{3 / 2}\right) / \mu_{\mathrm{e}}\left(X^{2} \Sigma^{+}\right)$being $0.79,0.59$ and 0.41 for CaF , SrF and BaF , respectively. This mimics the trend in static average electric dipole polarizabilities for the ground states of $\mathrm{Ca}\left(=23 \AA^{3}\right), \mathrm{Sr}\left(=28 \AA^{3}\right)$ and $\mathrm{Ba}\left(=56 \AA^{3}\right)$.

The observation that $\mu_{\mathrm{e}}\left(A^{2} \Pi_{1 / 2}\right)(=1.50(2) \mathrm{D})$ is larger than $\mu_{\mathrm{e}}\left(A^{2} \Pi_{3 / 2}\right)(=1.31(2)$ D) must result from a complicated spin-orbit mixing of the $A^{2} \Pi$ state with the nearby $A^{\prime 2} \Delta\left(T_{\mathrm{e}} \approx 10940 \mathrm{~cm}^{-1}\right), B^{2} \Sigma^{+}\left(T_{\mathrm{e}} \approx 14040 \mathrm{~cm}^{-1}\right)$ and $C^{2} \Pi\left(T_{\mathrm{e}} \approx 20091 \mathrm{~cm}^{-1}\right)$ states. LFT [20], which does not account for spin-orbit mixing, predicts that $\mu_{\mathrm{e}}\left(A^{\prime 2} \Delta\right)$ and $\mu_{\mathrm{e}}\left(C^{2} \Pi\right)$ are greater than $\mu_{\mathrm{e}}\left(A^{2} \Pi\right)$ which in turn is comparable to $\mu_{\mathrm{e}}\left(B^{2} \Sigma^{+}\right)$. According to LFT, $\mu_{\mathrm{e}}$ $\left(A^{\prime 2} \Delta\right)$ is large because, to a first approximation, the sole unpaired electron is in a compact $\mathrm{Ba}^{+}$-centered $5 d$ orbital and $\mu_{\mathrm{e}}\left(C^{2} \Pi\right)$ is large because the sole unpaired electron is in a $\mathrm{Ba}^{+}$-centered $5 d-6 p$ hybrid orbital that is pointed towards the F-nucleus. Furthermore, according to LFT $\mu_{\mathrm{e}}\left(A^{2} \Pi\right) \approx \mu_{\mathrm{e}}\left(B^{2} \Sigma^{+}\right)$because the LFT predicted $5 d$ and $6 p$ composition for the two states is approximately identical. Evidently the $A^{2} \Pi_{1 / 2}$ state is more strongly mixed with the $C^{2} \Pi$ and $A^{\prime 2} \Delta$ states than is the $A^{2} \Pi_{3 / 2}$ state. A similar effect has been observed in CaOH and SrOH [36].

The determined Zeeman parameters can also be qualitatively understood. An estimate for $g_{l}\left(X^{2} \Sigma^{+}\right)$is obtainable from the Curl relationship [32]:

$$
\begin{equation*}
g_{l} \approx-\gamma / 2 B, \tag{8}
\end{equation*}
$$

where $\gamma$ and $B$, are the spin-rotation and rotational parameters. Using the parameters from Table III, Eq. 8 gives $g_{l}\left(X^{2} \Sigma^{+}\right)=-0.00625$ which is the correct sign but about a factor of four too small. The parity-dependent anisotropic $g$-factor for the $A^{2} \Pi$ state can be approximated using an equation analogous to the Curl relationship [31]:

$$
\begin{equation*}
g_{l}^{\prime} \approx p / 2 B \tag{9}
\end{equation*}
$$

where $p$ is the $\Lambda$-doubling parameter. It is reasonable to assume that $q=0$, and thus that $p=-0.2575 \mathrm{~cm}^{-1}$, since no $\Lambda$-doubling in the $A^{2} \Pi_{3 / 2}$ levels was observed. Eq. 9 predicts that $g_{l}^{\prime} \approx-0.2575 /(2 \times 0.21954)=-0.5866$, which is in very good agreement with the experimental value $(=-0.536(23))$. In fitting the Zeeman shifts, $g_{l}$ for the $A^{2} \Pi$ was constrained to zero. It is difficult to estimate $g_{l}$ for the $A^{2} \Pi$ from the Curl relationship (Eq. 8) because the required spin-rotation parameter, $\gamma$, for the $A^{2} \Pi$ state is not determined. Brown and Watson [36] have shown that for a ${ }^{2} \Pi$ state $\gamma$ and $A_{D}$, the centrifugal distortion correction to the spin-orbit coupling, are totally correlated. When $\gamma$ is constrained to zero, as was done here, then the fitting parameter $A_{D}$ is a linear combination of the true values for $A_{D}$ and $\gamma$ [37]:

$$
\begin{equation*}
A_{D}^{\text {Fiting }} \approx A_{D}^{\text {True }}-\gamma \frac{2 B}{(A-2 B)} . \tag{10}
\end{equation*}
$$

An upper limit for $\gamma\left(A^{2} \Pi\right)$ of $-0.046 \mathrm{~cm}^{-1}$ is obtained from Eq. 10 by assuming that $A_{D}^{\text {True }}$ $=0$. The Curl relationship then predicts that an approximate value for $g_{l}\left(A^{2} \Pi\right)$ of +0.11 .

## VI. CONCLUSION

A cold molecular beam sample of BaF has been generated by laser vaporization techniques and the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system recorded at near the natural linewidth limit both field-free and in the presence of static magnetic and electric fields. A set of effective spectroscopic parameters has been generated that can reproduce the energies at
field strengths appropriate for proposed PNC measurements. The determined hyperfine parameters, magnetic g-factors and permanent electric dipole moments are benchmarks for evaluation of future predictions of the excited $A^{2} \Pi$ state.

## Acknowledgements.

This research has been supported by the National Science Foundation -Experimental Physical Chemistry (Grant 943360412) and Physics (Grant PHY-0758045). The authors would like to thank Dr. Jinhai Chen for assistance in the measurements.

## Figure Captions

Figure 1. The field-free spectrum of $A^{2} \Pi_{1 / 2}-X^{2} \Sigma^{+}(0,0)$ sub-band system of BaF.

Figure 2. The field-free spectrum of $A^{2} \Pi_{3 / 2}-X^{2} \Sigma^{+}(0,0)$ sub-band system of BaF .

Figure 3. The observed and predicted spectra of the $A^{2} \Pi_{1 / 2}-X^{2} \Sigma^{+}(0,0)$ sub-band system of BaF in the region of the $R_{1}(4)$ line $\left(v=11634.25 \mathrm{~cm}^{-1}\right)$ of the ${ }^{138} \mathrm{BaF}$ isotopologue. The predicted spectra were obtained using the optimized parameters of Table III and a linewidth of 20 MHz .

Figure 4. The observed and predicted spectra of the $A^{2} \Pi_{3 / 2}-X^{2} \Sigma^{+}(0,0)$ sub-band system of BaF in the region of the $Q_{2}(6)$ line $\left(v=12260.60 \mathrm{~cm}^{-1}\right)$ and ${ }^{Q} P_{21}(6)$ line $\left(v=12260.58 \mathrm{~cm}^{-1}\right)$ of the ${ }^{138} \mathrm{BaF}$ isotopologue. The predicted spectra were obtained using the optimized parameters of Table III and a linewidth of 20 MHz .

Figure 5. The predicted energy level pattern associated with the ${ }^{R} R_{12}(4)$ line $\left(v=11634.19 \mathrm{~cm}^{-1}\right)$ and the ${ }^{R} R_{11}(4)$ line $\left(v=11634.35 \mathrm{~cm}^{-1}\right)$ line of the ${ }^{137} \mathrm{BaF}$ isotopologue. The assignment of the spectral features of Figure 3 are indicated. The energies were calculated using the optimized parameters of Table III.

Figure 6. The predicted energy level pattern associated with the ${ }^{Q_{P 22}}(6)$ line $\left(v=12260.52 \mathrm{~cm}^{-1}\right)$ and the ${ }^{Q_{P_{21}}(6)}$ line $\left(v=12260.68 \mathrm{~cm}^{-1}\right)$ line of the ${ }^{137} \mathrm{BaF}$
isotopologue. The assignment of the spectral features of Figure 4 are indicated. The energies were calculated using the optimized parameters of Table III.

Figure 7. The ${ }^{137} \mathrm{BaF}$ isotopologue spin-rotation and hyperfine energy level pattern for the $X^{2} \Sigma^{+}(v=0)$ state as a function of rotational quantum number, $N$. The low-rotational levels are those of a Hund's case $\left(b_{\beta S}\right)$ molecule with $G$ being the appropriate intermediate quantum number and those of the high rotational levels a Hund's case $\left(b_{\beta}\right)$ molecule where $J$ is the appropriate intermediate quantum number.

Figure 8. The observed and predicted ${ }^{S} R_{21}(0)$ line of the ${ }^{138} \mathrm{BaF}$ isotopologue recorded field-free and in the presence of a $899.4 \mathrm{~V} / \mathrm{cm}$ static field with perpendicular orientation and the associated energy levels as a function of applied field. The near degeneracy of the $\Lambda$-doublets in the $J=3 / 2$ level of the $A^{2} \Pi_{3 / 2}$ state results in a linear Stark tuning of the energy levels.

Figure 9. The observed and predicted ${ }^{O} P_{12}(2)$ line of the ${ }^{138} \mathrm{BaF}$ isotopologue recorded field-free and in the presence of a $2940.9 \mathrm{~V} / \mathrm{cm}$ static field with parallel orientation and the associated energy levels as a function of applied field. The large $\Lambda$-doubling $(=0.257$ $\mathrm{cm}^{-1}$ ) of the $J=1 / 2$ levels of the $A^{2} \Pi_{1 / 2}$ state results in a second order Stark effect.

Figure 10. The $R_{1}(0)$ line of the ${ }^{138} \mathrm{BaF}$ isotopologue recorded field free (bottom) and in the presence of 713 G parallel (middle) and a 724 G perpendicular (top) magnetic fields.

The associated energy levels as a function of applied magnetic field, and the assigned transitions, are also presented.

Figure 11. The ${ }^{S} R_{21}(0)$ line of the ${ }^{138} \mathrm{BaF}$ isotopologue recorded field-free (bottom) and in the presence of a 192 G perpendicular (top) magnetic field. The associated energy levels as a function of applied magnetic field, and the assigned transitions, are also presented.

## Reference

1. M. G.Kozlov, A. V.Titov, N. S.Mosyagin, and P. V. Souchko, Phys. Rev. A: Atomic, Molecular, and Optical Physics 56, R3326 (1997).
2. A. V.Titov, N. S.Mosyagin, A. N. Petrov, and T. A. Isaev, Int. J. of Quantum Chemistry 104, 223 (2005).
3. M. K. Nayak, and R. K. Chaudhuri, J. Phys. B: Atomic, Molecular and Optical Physics 39, 1231 (2006).
4. A. V.Titov, N. S.Mosyagin, A. N. Petrov, T. A. Isaev, and D. P. DeMille, Progress in Theoretical Chemistry and Physics, 15 (Recent Advances in the Theory of Chemical and Physical Systems), 253 (2006).
5. M. K.Nayak, R.K.Chaudhuri, and B. P. Das, Phys. Rev. A: Atomic, Molecular, and Optical Physics 75, 022510 (2007).
6. M. K. Nayak and R. K.Chaudhuri, Phys. Rev. A: Atomic, Molecular, and Optical Physics 78, 012506 (2008).
7. M. K.Nayak and B. P. Das, Phys. Rev. A: Atomic, Molecular, and Optical Physics 79, 060502 (2009).
8. D. DeMille, S. B. Cahn, D. Murphree, D. A. Rahmlow, and M. G. Kozlov, Phys. Rev. Lett. 100, 023003(2008)
9. M. G. Kozlov and L. N. Labzowsky, J. Physics B: Atomic, Molecular and Optical Physics 28, 1933 (1995).
10. Ch. Rayzlewicz and T. Törring, Chem. Phys. 51, 329 (1980).
11. Ch. Rayzlewicz, H.-U. Schütze-Pahlmann, J. Hoeft and T. Törring, Chem. Phys. 71, 389 (1982).
12. W.E. Ernst, J. Kändler, and T. Törring, J. Chem. Phys. 84, 4769 (1986).
13. B. Guo, K. Q. Zhang, and P.F. Bernath, J. Mol. Spectrosc. 170, 59, (1995).
14. C. Effantin, A. Bernard, J. D'Incan, G. Wannous, J. Verges, and R. F. Barrow, Mol. Physics 70, 735 (1990).
15. A. Bernard, C. Effantin, J. D'Incan, J. Verges, and R. F. Barrow, Mol. Physics 70, 747 (1990).
16. A. Bernard, C. Effantin, E. Andrianavalona, J. Verges, and R. F. Barrow, J. Mol. Spectrosc. 152, 174 (1992).
17. L.-E. Berg, N. Gador, D. Husain, H. Ludwigs, and P. Royen, Chem. Phys. Lett. 287, 89 (1998).
18. L.-E.Berg, T. Olsson, J. C. Chanteloup, A. Hishikawa, and P. Royen, Mol. Phys. 79, 721 (1993).
19. T. Törring, W.E. Ernst, and J. Kändler, J. Chem. Phys. 90, 4927 (1989).
20. J.M. Mestdagh and J.P. Visticot, Chem. Phys. 155, 79 (1991).
21. A. R. Allouche, G. Wannous and M. Aubert-Frécon, Chem. Phys. 170, 11 (1993).
22. A.T. Le, H. Wang, and T.C. Steimle, Phys. Rev. 80, 062513 (2009).
23. T.C. Steimle, T. Ma and C. Linton, J. Chem. Phys. 127, 234316 (2007).
24. T. Ma, C. Butler, J.M. Brown, C. Linton and T.C. Steimle, J. Phys. Chem. A. 113, 8038 (2009).
25. S. Gerstenkorn and P. Luc, Atlas du spectre d'absorption de la mol $\odot c u l e ~ d e ~$ l'iode(Editions du C.N.R.S.) (1978); Rev. Phys. Appl. 14, 791 (1979).
26. S. Gerstenkorn, P. Luc and R. Vetter, Rev. Phys. Appl. 16529 (1981).
27. T. C. Steimle, J. Gengler, and J. Chen, Can. J. Chem. 82, 779 (2004).
28. T. C. Steimle, Int. Rev. Phy. Chem. 19, 455 (2000).
29. G. Herzberg, Molecular Spectra and Molecular Structure (Van Nostrand Reinhold Company, New York, 1950).
30. See EPAPS Document No. [number will be inserted by publisher] for supplemental tables. This document can be reached via a direct link in the online article's HTML reference section or via the EPAPS homepage http://0www.aip.org.library.lib.asu.edu/pubservs/epaps.html.
31. J. M. Brown and A. Carrington, Rotational Spectroscopy of Diatomic Molecules (Cambridge University Press, Cambridge, 2003).
32. W.Weltner, Jr., 1983, Magnetic Atoms and Molecules, (New York, Dover Publications Inc.)
33. W. J. Childs, L. S. Goodman, U. Nielsen, and V. J. Pfeufer, Chem. Phys. 80, 2283 (1984).
34. W. E. Ernst and J. Kaendler, Phys. Rev. A: Atomic, Molecular, and Optical Physics 39, 1575 (1989).
35. W. E. Ernst, J. Kaendler, S.Kindt, and T. Toerring, Chem. Phys. Lett. 113, 351 (1985).
36. J. Kaendler, T. Martell, and W. E. Ernst, Chem. Phys. Lett. 155, 470 (1989).
37. T.C. Steimle, D.A. Fletcher, K.Y. Jung and C.T. Scurlock, J. Chem. Phys. 96, 2556 (1992).
38. J. M. Brown and J .K. G. Watson J. Mol. Spectrosc. 65, 65 (1977).

Supplemental Table I. The observed and calculated transition wave numbers $\left(\mathrm{cm}^{-1}\right)$ for the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system of ${ }^{138} \mathrm{BaF}$.

| Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. | Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P_{1}$ | 1 | 11630.0816 | -0.0010 | $Q_{1}$ | 12 | 11630.5669 | 0.0012 |
|  | 2 | 11629.9810 | -0.0002 |  | 13 | 11630.5366 | 0.0013 |
|  | 3 | 11629.8720 | 0.0008 |  | 14 | 11630.4971 | 0.0009 |
|  | 4 | 11629.7541 | 0.0015 |  | 15 | 11630.4504 | 0.0020 |
|  | 5 | 11629.6262 | 0.0007 |  | 16 | 11630.3924 | 0.0004 |
|  | 6 | 11629.4892 | -0.0005 |  | 17 | 11630.3279 | 0.0010 |
|  | 7 | 11629.3446 | -0.0008 |  | 18 | 11630.2548 | 0.0016 |
|  | 8 | 11629.1931 | 0.0005 |  | 19 | 11630.1700 | -0.0007 |
|  | 9 | 11629.0316 | 0.0005 |  | 20 | 11630.0816 | 0.0020 |
|  | 10 | 11628.8612 | 0.0002 |  | 21 | 11629.9810 | 0.0013 |
|  | 11 | 11628.6827 | 0.0003 |  | 22 | 11629.8720 | 0.0009 |
|  | 12 | 11628.4951 | -0.0001 |  | 23 | 11629.7541 | 0.0003 |
|  | 13 | 11628.2977 | -0.0017 |  | 24 | 11629.6284 | 0.0007 |
|  | 14 | 11628.0966 | 0.0016 |  | 25 | 11629.4933 | 0.0004 |
|  | 15 | 11627.8832 | 0.0012 |  | 26 | 11629.3497 | 0.0003 |
| ${ }^{0} P_{12}$ | 2 | 11628.9683 | 0.0007 |  | 27 | 11629.1980 | 0.0010 |
|  | 3 | 11628.1797 | 0.0002 |  | 28 | 11629.0366 | 0.0007 |
|  | 4 | 11627.3818 | -0.0010 |  | 29 | 11628.8665 | 0.0006 |
|  | 10 | 11622.4214 | -0.0015 |  | 30 | 11628.6872 | 0.0000 |
| $P_{2}$ | 3 | 12260.2951 | -0.0008 |  | 31 | 11628.4990 | -0.0006 |
|  | 4 | 12259.6279 | -0.0010 |  | 32 | 11628.3024 | -0.0008 |
|  | 5 | 12258.9527 | -0.0009 |  | 33 | 11628.0966 | -0.0014 |
|  | 6 | 12258.2681 | -0.0020 |  | 34 | 11627.8832 | -0.0006 |
| ${ }^{Q} P_{21}$ | 2 | 12261.5823 | -0.0002 | ${ }^{P} Q_{12}$ | 1 | 11630.0869 | -0.0007 |
|  | 3 | 12261.3437 | -0.0009 |  | 2 | 11629.9883 | -0.0006 |
|  | 4 | 12261.0967 | -0.0018 |  | 3 | 11629.8808 | -0.0009 |
|  | 5 | 12260.8446 | 0.0005 |  | 4 | 11629.7648 | -0.0010 |
|  | 6 | 12260.5807 | -0.0008 |  | 5 | 11629.6406 | -0.0008 |
|  | 7 | 12260.3100 | -0.0007 |  | 6 | 11629.5071 | -0.0012 |
|  | 8 | 12260.0327 | 0.0010 |  | 7 | 11629.3657 | -0.0010 |
|  | 9 | 12259.7444 | 0.0000 |  | 8 | 11629.2161 | -0.0005 |
|  | 10 | 12259.4489 | 0.0000 |  | 9 | 11629.0577 | -0.0001 |
|  | 11 | 12259.1439 | -0.0012 |  | 10 | 11628.8898 | -0.0006 |
|  | 12 | 12258.8337 | 0.0006 |  | 11 | 11628.7136 | -0.0009 |
|  | 13 | 12258.5126 | -0.0003 |  | 12 | 11628.5298 | -0.0002 |
|  | 14 | 12258.1842 | -0.0002 |  | 13 | 11628.3389 | 0.0020 |
|  | 15 | 12257.8477 | 0.0000 |  | 14 | 11628.1358 | 0.0006 |
| $Q_{1}$ | 0 | 11630.2581 | -0.0001 |  | 15 | 11627.9250 | 0.0001 |
|  | 0 | 11630.2601 | -0.0003 | $Q_{2}$ | 2 | 12261.5890 | -0.0013 |
|  | 1 | 11630.3321 | 0.0009 |  | 3 | 12261.3531 | -0.0020 |
|  | 2 | 11630.3956 | 0.0001 |  | 4 | 12261.1093 | -0.0024 |
|  | 3 | 11630.4518 | 0.0005 |  | 5 | 12260.8592 | -0.0008 |
|  | 4 | 11630.4971 | -0.0014 |  | 6 | 12260.5986 | -0.0015 |
|  | 5 | 11630.5366 | -0.0004 |  | 7 | 12260.3299 | -0.0021 |
|  | 6 | 11630.5669 | -0.0001 |  | 8 | 12260.0550 | -0.0007 |
|  | 7 | 11630.5880 | -0.0003 |  | 9 | 12259.7702 | -0.0009 |
|  | 8 | 11630.6004 | -0.0006 |  | 10 | 12259.4770 | -0.0013 |


|  | 9 | 11630.6055 | 0.0004 |  | 11 | 12259.1754 | -0.0018 |
| ---: | ---: | ---: | ---: | :--- | ---: | ---: | ---: |
|  | 10 | 11630.6023 | 0.0017 |  | 12 | 12258.8675 | -0.0004 |
|  | 11 | 11630.5880 | 0.0005 |  | 13 | 12258.5492 | -0.0012 |

Supplemental Table 1 (Continued)

| Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. | Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q_{2}$ | 14 | 12258.2236 | -0.0010 | ${ }^{R} Q_{21}$ | 52 | 12261.6583 | 0.0004 |
|  | 15 | 12257.8892 | -0.0014 |  | 53 | 12261.4192 | 0.0003 |
| ${ }^{R} Q_{21}$ | 1 | 12262.4466 | -0.0010 |  | 54 | 12261.1694 | -0.0016 |
|  | 2 | 12262.6408 | -0.0008 |  | 55 | 12260.9135 | -0.0007 |
|  | 3 | 12262.8273 | -0.0001 |  | 56 | 12260.6476 | -0.0009 |
|  | 4 | 12263.0046 | -0.0003 |  | 57 | 12260.3719 | -0.0018 |
|  | 5 | 12263.1729 | -0.0012 |  | 58 | 12260.0868 | -0.0032 |
|  | 6 | 12263.3338 | -0.0013 |  | 59 | 12259.7952 | -0.0021 |
|  | 7 | 12263.4874 | -0.0004 | $R_{1}$ | 0 | 11631.2805 | 0.0010 |
|  | 8 | 12263.6320 | -0.0003 |  | 0 | 11631.2825 | 0.0008 |
|  | 9 | 12263.7687 | 0.0002 |  | 1 | 11632.0350 | 0.0016 |
|  | 10 | 12263.8973 | 0.0009 |  | 2 | 11632.7792 | 0.0006 |
|  | 11 | 12264.0152 | -0.0008 |  | 4 | 11634.2447 | 0.0016 |
|  | 12 | 12264.1239 | -0.0034 | ${ }^{Q} R_{12}$ | 1 | 11630.3339 | -0.0023 |
|  | 13 | 12264.2297 | -0.0006 |  | 2 | 11630.4028 | -0.0005 |
|  | 14 | 12264.3270 | 0.0020 |  | 3 | 11630.4609 | -0.0009 |
|  | 15 | 12264.4119 | 0.0005 |  | 4 | 11630.5118 | 0.0001 |
|  | 16 | 12264.4894 | -0.0001 |  | 5 | 11630.5525 | -0.0004 |
|  | 17 | 12264.5597 | 0.0005 |  | 6 | 11630.5851 | -0.0005 |
|  | 18 | 12264.6216 | 0.0010 |  | 7 | 11630.6055 | -0.0041 |
|  | 19 | 12264.6732 | -0.0004 |  | 8 | 11630.6244 | -0.0006 |
|  | 20 | 12264.7190 | 0.0007 |  | 9 | 11630.6315 | -0.0003 |
|  | 21 | 12264.7560 | 0.0013 |  | 10 | 11630.6299 | -0.0001 |
|  | 22 | 12264.7845 | 0.0019 |  | 11 | 11630.6196 | 0.0000 |
|  | 23 | 12264.8041 | 0.0020 |  | 12 | 11630.5976 | -0.0029 |
|  | 24 | 12264.8140 | 0.0007 |  | 13 | 11630.5724 | -0.0004 |
|  | 25 | 12264.8166 | 0.0006 |  | 14 | 11630.5366 | 0.0002 |
|  | 26 | 12264.8118 | 0.0015 |  | 15 | 11630.4908 | -0.0005 |
|  | 27 | 12264.7982 | 0.0020 |  | 16 | 11630.4369 | -0.0007 |
|  | 28 | 12264.7751 | 0.0015 |  | 17 | 11630.3755 | 0.0003 |
|  | 29 | 12264.7438 | 0.0012 |  | 18 | 11630.3039 | -0.0003 |
|  | 30 | 12264.7046 | 0.0015 |  | 19 | 11630.2249 | 0.0005 |
|  | 31 | 12264.6575 | 0.0024 |  | 20 | 11630.1381 | 0.0021 |
|  | 32 | 12264.6007 | 0.0021 |  | 21 | 11630.0397 | 0.0009 |
|  | 33 | 12264.5350 | 0.0015 |  | 22 | 11629.9335 | 0.0006 |
|  | 34 | 12264.4613 | 0.0013 |  | 23 | 11629.8188 | 0.0005 |
|  | 35 | 12264.3797 | 0.0018 |  | 24 | 11629.6948 | -0.0001 |
|  | 36 | 12264.2896 | 0.0024 |  | 25 | 11629.5618 | -0.0010 |
|  | 37 | 12264.1893 | 0.0013 |  | 26 | 11629.4209 | -0.0010 |
|  | 38 | 12264.0830 | 0.0029 |  | 27 | 11629.2723 | 0.0000 |
|  | 39 | 12263.9647 | 0.0010 |  | 28 | 11629.1141 | 0.0002 |
|  | 40 | 12263.8421 | 0.0035 |  | 29 | 11628.9463 | -0.0003 |
|  | 41 | 12263.7071 | 0.0022 |  | 30 | 11628.7705 | -0.0001 |
|  | 42 | 12263.5648 | 0.0023 |  | 31 | 11628.5847 | -0.0010 |
|  | 43 | 12263.4126 | 0.0012 |  | 32 | 11628.3909 | -0.0011 |


|  | 44 | 12263.2522 | 0.0006 |  | 33 | 11628.1894 | 0.0000 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 45 | 12263.0831 | 0.0000 |  | 34 | 11627.9776 | -0.0004 |
|  | 46 | 12262.9079 | 0.0020 | $R_{2}$ | 1 | 12262.4509 | -0.0018 |
|  | 47 | 12262.7209 | 0.0010 |  | 2 | 12262.6478 | -0.0006 |
|  | 48 | 12262.5260 | 0.0008 |  | 3 | 12262.8382 | 0.0003 |
|  | 49 | 12262.3216 | 0.0000 |  | 4 | 12263.0167 | -0.0014 |
|  | 50 | 12262.1085 | -0.0007 |  | 5 | 12263.1882 | -0.0018 |
|  | 51 | 12261.8895 | 0.0015 |  |  |  |  |

Supplemental Table 1 (Continued)

| Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. | Branch | $N^{\prime \prime}$ | Observed | Obs.-Cal. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R_{2}$ | 6 | 12263.3517 | -0.0020 | $R_{2}$ | 38 | 12264.1857 | 0.0006 |
|  | 7 | 12263.5073 | -0.0018 |  | 39 | 12264.0719 | 0.0005 |
|  | 8 | 12263.6552 | -0.0011 |  | 40 | 12263.9481 | -0.0009 |
|  | 9 | 12263.7944 | -0.0008 |  | 41 | 12263.8198 | 0.0018 |
|  | 10 | 12263.9256 | -0.0002 |  | 42 | 12263.6796 | 0.0013 |
|  | 11 | 12264.0460 | -0.0021 |  | 43 | 12263.5305 | 0.0006 |
|  | 12 | 12264.1597 | -0.0024 |  | 44 | 12263.3733 | 0.0005 |
|  | 13 | 12264.2679 | 0.0001 |  | 45 | 12263.2071 | 0.0001 |
|  | 14 | 12264.3641 | -0.0011 |  | 46 | 12263.0326 | 0.0001 |
|  | 15 | 12264.4532 | -0.0011 |  | 47 | 12262.8508 | 0.0016 |
|  | 16 | 12264.5348 | -0.0003 |  | 48 | 12262.6570 | -0.0001 |
|  | 17 | 12264.6063 | -0.0012 |  | 49 | 12262.4574 | 0.0011 |
|  | 18 | 12264.6714 | -0.0002 |  | 50 | 12262.2460 | -0.0006 |
|  | 19 | 12264.7267 | -0.0006 |  | 51 | 12262.0269 | -0.0011 |
|  | 20 | 12264.7751 | 0.0004 |  | 52 | 12261.8004 | -0.0003 |
|  | 21 | 12264.8140 | 0.0003 |  | 53 | 12261.5635 | -0.0009 |
|  | 22 | 12264.8448 | 0.0004 |  | 54 | 12261.3181 | -0.0011 |
|  | 23 | 12264.8673 | 0.0007 |  | 55 | 12261.0628 | -0.0023 |
|  | 24 | 12264.8821 | 0.0016 |  | 56 | 12260.8012 | -0.0008 |
|  | 25 | 12264.8865 | 0.0006 |  | 57 | 12260.5276 | -0.0024 |
|  | 26 | 12264.8845 | 0.0016 |  | 58 | 12260.2463 | -0.0027 |
|  | 27 | 12264.8723 | 0.0008 |  | 59 | 12259.9580 | -0.0009 |
|  | 28 | 12264.8531 | 0.0015 |  | 60 | 12259.6573 | -0.0025 |
|  | 29 | 12264.8238 | 0.0005 | ${ }^{S} R_{21}$ | 0 | 12262.8812 | 0.0003 |
|  | 30 | 12264.7873 | 0.0008 |  | 0 | 12262.8832 | 0.0001 |
|  | 31 | 12264.7438 | 0.0026 |  | 1 | 12263.5070 | 0.0002 |
|  | 32 | 12264.6878 | 0.0004 |  | 2 | 12264.1240 | -0.0004 |
|  | 33 | 12264.6260 | 0.0010 |  | 3 | 12264.7341 | 0.0003 |
|  | 34 | 12264.5546 | 0.0004 |  | 4 | 12265.3376 | 0.0027 |
|  | 35 | 12264.4752 | 0.0004 |  | 5 | 12265.9301 | 0.0024 |
|  | 36 | 12264.3877 | 0.0009 |  | 6 | 12266.5162 | 0.0039 |
|  | 37 | 12264.2896 | -0.0007 |  | 7 | 12267.0913 | 0.0028 |

R.M.S $=0.0013 \mathrm{~cm}^{-1}$

SUPPLEMENTAL TABLE II: Observed and calculated line positions of $A^{2} \Pi$ $X^{2} \Sigma^{+}(0,0)$ band system of ${ }^{135} \mathrm{BaF}$

| Branch | $F^{\prime \prime}{ }_{1}, F^{\prime \prime}-F^{\prime}{ }_{1}, F^{\prime}$ | Observed - $10000$ | Obs-cal |
| :---: | :---: | :---: | :---: |
| ${ }^{R} R_{12}(0)$ | 3.0, 2.5-4.0, 3.5 | 1631.2145 | 0.0011 |
|  | 3.0, 2.5-3.0, 2.5 | 1631.2192 | 0.0005 |
|  | 3.0, 2.5-2.0, 1.5 | 1631.2217 | -0.0006 |
| ${ }^{R} R_{11}(0)$ | 2.0, 1.5-3.0, 2.5 | 1631.3618 | 0.0020 |
|  | 2.0, 1.5-2.0, 1.5 | 1631.3646 | 0.0013 |
|  | 2.0, 1.5-1.0, 0.5 | 1631.3666 | 0.0015 |
| ${ }^{R} R_{12}(1)$ | 3.0, 2.5-4.0, 3.5 | 1632.1150 | -0.0015 |
|  | 2.0, 1.5-3.0, 2.5 | 1632.1179 | -0.0020 |
|  | 2.0, 1.5-2.0, 1.5 | 1632.1200 | -0.0022 |
| ${ }^{R} R_{11}(1)$ | 4.0, 3.5-5.0, 4.5 | 1632.8635 | -0.0017 |
|  | 2.0, 1.5-3.0, 2.5 | 1632.8660 | -0.0016 |
|  | 3.0, 2.5-3.0, 2.5 | 1632.8690 | -0.0018 |
| ${ }^{R} R_{12}(4)$ | 7.0, 6.5-8.0, 7.5 | 1634.1865 | 0.0022 |
|  | 6.0, 5.5-7.0, 6.5 | 1634.1931 | 0.0003 |
|  | 5.0, 4.5-6.0, 5.5 | 1634.2012 | 0.0016 |
|  | 4.0, 3.5-5.0, 4.5 | 1634.2078 | 0.0027 |
| ${ }^{R} R_{11}(4)$ | 4.0, 3.5-5.0, 4.5 | 1634.3368 | 0.0002 |
|  | 5.0, 4.5-6.0, 5.5 | 1634.3375 | -0.0010 |
| ${ }^{R} R_{12}(7)$ | 9.0, 8.5-10.0, 9.5 | 1636.3244 | 0.0018 |
|  | $9.0,8.5-10.0,9.5$ | 1636.3306 | -0.0009 |
|  | 8.0, 7.5-9.0, 8.5 | 1636.3381 | -0.0021 |
|  | $7.0,6.5-8.0,7.5$ | 1636.3455 | -0.0025 |
| ${ }^{R} R_{11}(7)$ | 7.0, 6.5-8.0, 7.5 | 1636.4785 | 0.0022 |
|  | 8.0, 7.5-9.0, 8.5 | 1636.4808 | 0.0007 |
| ${ }^{o}{ }_{P_{12}(7)}$ | 9.0, 8.5-8.0, 7.5 | 1624.8484 | -0.0001 |
|  | 8.0, 7.5-7.0, 6.5 | 1624.8498 | -0.0002 |
|  | 7.0, 6.5-6.0, 5.5 | 1624.8516 | 0.0000 |
|  | 6.0, 5.5-5.0, 4.5 | 1624.8535 | 0.0001 |
| ${ }^{R} Q_{22}(1)$ | 3.0, 2.5-4.0, 3.5 | 2262.3838 | 0.0002 |
| ${ }^{R} Q_{21}(1)$ | 2.0, 1.5-2.0,1.5 | 2262.5233 | 0.0003 |
|  | 3.0, 2.5-3.0, 2.5 | 2262.5245 | 0.0007 |
| ${ }^{R} Q_{21}(2)$ | 2.0, 1.5-2.0, 1.5 | 2262.7144 | -0.0014 |
|  | 4.0, 3.5-4.0, 3.5 | 2262.7184 | -0.0020 |
| ${ }^{s} R_{22}(4)$ | 7.0, 6.5-8.0, 7.5 | 2265.2782 | 0.0004 |
|  | 6.0, 5.5-7.0, 6.5 | 2265.2815 | -0.0009 |
|  | 5.0, 4.5-6.0, 5.5 | 2265.2865 | 0.0007 |
|  | 4.0, 3.5-5.0, 4.5 | 2265.2910 | 0.0024 |
| ${ }^{s} R_{21}(4)$ | 4.0, 3.5-5.0, 4.5 | 2265.4193 | -0.0007 |
|  | 5.0, 4.5-6.0, 5.5 | 2265.4233 | -0.0015 |
|  | 6.0, 5.5-7.0, 6.5 | 2265.4263 | -0.0006 |
| ${ }^{s} R_{22}(7)$ | 10.0, 9.5-11.0, 10.5 | 2267.0383 | 0.0016 |
|  | 9.0, 8.5-10.0, 9.5 | 2267.0423 | -0.0007 |
|  | 8.0, 7.5-9.0, 8.5 | 2267.0469 | -0.0015 |
|  | 7.0, 6.5-8.0, 7.5 | 2267.0521 | -0.0012 |
| ${ }^{s} R_{21}(7)$ | $7.0,6.5-8.0,7.5$ | 2267.1840 | 0.0024 |


|  | 8.0, 7.5-9.0, 8.5 | 2267.1880 | -0.0004 |
| :---: | :---: | :---: | :---: |
|  | 9.0, 8.5-10.0, 9.5 | 2267.1920 | -0.0002 |
| ${ }^{Q} P_{22}+{ }^{Q} Q_{22}(2)$ | 4.0, 3.5-4.0, 3.5 | 2261.5170 | 0.0011 |
| $Q^{P_{21}+}{ }^{Q} Q_{21}$ (2) | 4.0, 3.5-4.0, 3.5 | 2261.6578 | 0.0003 |
| $Q_{P_{22}+}{ }^{Q} Q_{22}$ (3) | 3.0, 2.5-2.0, 1.5 | 2261.4146 | -0.0005 |
|  | 4.0, 3.5-3.0, 2.5 | 2261.4197 | 0.0005 |
|  | 5.0, 4.5-5.0, 4.5 | 2261.4228 | 0.0021 |
| $Q_{P_{22}+}{ }^{Q} Q_{22}(4)$ | 7.0, 6.5-6.0, 5.5 | 2261.0280 | 0.0015 |
|  | 6.0, 5.5-6.0, 5.5 | 2261.0292 | -0.0019 |
| ${ }^{Q} P_{21}+{ }^{Q} Q_{21}(4)$ | 4.0, 3.5-3.0, 2.5 | 2261.1689 | 0.0001 |
|  | 5.0, 4.5-4.0, 3.5 | 2261.1723 | -0.0012 |
|  | 6.0, 5.5-6.0, 5.5 | 2261.1763 | 0.0007 |
| ${ }^{Q} P_{22}+{ }^{Q} Q_{22}(7)$ | 10.0, 9.5-9.0, 8.5 | 2260.2388 | 0.0037 |
|  | 9.0, 8.5-9.0, 8.5 | 2260.2408 | -0.0007 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.2454 | -0.0014 |
|  | 6.0, 5.5-6.0, 5.5 | 2260.2491 | -0.0022 |
|  | 7.0, 6.5-7.0, 6.5 | 2260.2536 | 0.0019 |
| ${ }^{Q} P_{21}+{ }^{Q} Q_{21}(7)$ | 7.0, 6.5-6.0, 5.5 | 2260.3799 | -0.0001 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.3861 | -0.0007 |
|  | 9.0, 8.5-9.0, 8.5 | 2260.3911 | 0.0005 |
| ${ }^{Q} P_{22}+{ }^{Q} Q_{22}(6)$ | 8.0, 7.5-7.0, 6.5 | 2260.5060 | -0.0023 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.5110 | -0.0020 |
|  | 7.0, 6.5-7.0, 6.5 | 2260.5160 | -0.0017 |
|  | 6.0, 5.5-6.0, 5.5 | 2260.5210 | -0.0009 |
|  | 5.0, 4.5-5.0, 4.5 | 2260.5257 | 0.0000 |
| ${ }^{Q} P_{21}+{ }^{Q} Q_{21}(6)$ | 6.0, 5.5-5.0, 4.5 | 2260.6525 | 0.0013 |
|  | 7.0, 6.5-7.0, 6.5 | 2260.6590 | 0.0017 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.6630 | 0.0024 |
|  | Sdt. Dev. $=0.00$ | $\mathrm{cm}^{-1}$ |  |

All units are in wave number $\mathrm{cm}^{-1}$.

SUPPLEMENTAL TABLE III. Observed and calculated line positions of $A^{2} \Pi$ $\underline{X^{2} \Sigma^{+}(0,0) \text { band system of }{ }^{137} \mathrm{BaF}}$

| Branch | $F{ }^{\prime \prime}{ }_{1}, F^{\prime \prime}-F^{\prime}{ }_{1}, F^{\prime}$ | Observed 10000 | Obs-cal |
| :---: | :---: | :---: | :---: |
| ${ }^{R} R_{12}(4)$ | 7.0, 6.5-8.0, 7.5 | 1634.1865 | 0.0030 |
|  | 6.0, 5.5-7.0, 6.5 | 1634.1931 | 0.0004 |
|  | 5.0, 4.5-6.0, 5.5 | 1634.2012 | 0.0012 |
|  | 4.0, 3.5-5.0, 4.5 | 1634.2078 | 0.0019 |
| ${ }^{R} R_{11}(4)$ | 4.0, 3.5-5.0, 4.5 | 1634.3518 | -0.0005 |
|  | 5.0, 4.5-6.0, 5.5 | 1634.3525 | -0.0015 |
| ${ }^{o} P_{12}(7)$ | 6.0, 5.5-5.0, 4.5 | 1624.8673 | -0.0015 |
|  | 6.0, 5.5-5.0, 4.5 | 1624.8691 | -0.0014 |
| ${ }^{o} P_{11}(7)$ | 8.0, 7.5-7.0, 6.5 | 1625.0221 | -0.0010 |
|  | 9.0, 8.5-8.0, 7.5 | 1625.0286 | -0.0027 |
| ${ }^{o} P_{12}(10)$ | 12.0, 11.5-11.0, 10.5 | 1622.3420 | 0.0018 |
|  | 11.0, 10.5-10.0, 9.5 | 1622.3430 | -0.0002 |
|  | 10.0, 9.5-9.0, 8.5 | 1622.3453 | -0.0009 |
|  | 8.0, 7.5-7.0, 6.5 | 1622.3482 | -0.0007 |
|  | 9.0, 8.5-8.0, 7.5 | 1622.3496 | -0.0004 |
|  | 10.0, 9.5-9.0, 8.5 | 1622.4894 | 0.0029 |
| ${ }^{o} P_{11}(10)$ | 11.0, 10.5-10.0, 9.5 | 1622.5008 | 0.0015 |
|  | 12.0, 11.5-11.0, 10.5 | 1622.5089 | -0.0001 |
| ${ }^{R} R_{12}(1)$ | 4.0, 3.5-5.0,4.5 | 1631.9721 | 0.0005 |
|  | 3.0, 2.5-4.0,3.5 | 1631.9736 | 0.0008 |
|  | 3.0, 2.5-4.0, 3.5 | 1631.9784 | -0.0013 |
|  | 2.0, 1.5-3.0, 2.5 | 1631.9803 | -0.0006 |
| ${ }^{R} R_{11}(1)$ | 3.0, 2.5-4.0, 3.5 | 1632.1338 | -0.0010 |
|  | 2.0, 1.5-3.0,2.5 | 1632.1371 | -0.0018 |
|  | 2.0, 1.5-2.0, 1.5 | 1632.1400 | -0.0016 |
| ${ }^{P_{22}+}{ }^{Q} Q_{22}$ (7) | 6.0, 5.5-6.0, 5.5 | 2260.2697 | 0.0001 |
|  | 7.0, 6.5-7.0, 6.5 | 2260.2646 | -0.0005 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.2600 | -0.0003 |
|  | 9.0, 8.5-9.0, 8.5 | 2260.2548 | -0.0001 |
|  | 10.0, 9.5-9.0, 8.5 | 2260.2502 | 0.0017 |
| ${ }^{Q_{21}+}{ }^{Q} Q_{21}(7)$ | 9.0, 8.5-9.0, 8.5 | 2260.4196 | 0.0007 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.4149 | -0.0003 |
|  | 7.0, 6.5-6.0, 5.5 | 2260.4088 | 0.0006 |
| ${ }^{Q_{P 22}+}{ }^{Q} Q_{22}$ (8) | 5.0, 4.5-5.0, 4.5 | 2260.5373 | -0.0006 |
|  | 6.0, 5.5-6.0, 5.5 | 2260.5333 | -0.0008 |
|  | $7.0,6.5-7.0,6.5$ | 2260.5293 | -0.0007 |
|  | 8.0, 7.5-8.0, 7.5 | 2260.5257 | 0.0004 |
|  | 8.0, 7.5-7.0, 6.5 | 2260.5210 | 0.0005 |
| ${ }^{P_{21}+}{ }^{Q} Q_{21}$ (8) | 8.0, 7.5-8.0, 7.5 | 2260.6888 | 0.0011 |
|  | 7.0, 6.5-7.0, 6.5 | 2260.6848 | 0.0002 |
|  | 6.0, 5.5-5.0, 4.5 | 2260.6783 | 0.0000 |
| ${ }^{R} Q_{22}+{ }^{R} R_{22}(1)$ | 3.0, 2.5-4.0, 3.5 | 2262.3893 | -0.0004 |
| ${ }^{R} Q_{21}+{ }^{R} R_{21}(1)$ | 3.0, 2.5-3.0, 2.5 | 2262.5425 | -0.0024 |
| $Q_{P_{22}+}{ }^{Q} Q_{22}$ (3) | 6.0, 5.5-5.0, 4.5 | 2261.2820 | -0.0009 |
|  | 5.0, 4.5-5.0, 4.5 | 2261.2851 | -0.0019 |


| ${ }^{Q} P_{22}+{ }^{Q} Q_{22}$ (4) | 4.0, 3.5-4.0, 3.5 | 2261.2882 | -0.0016 |
| :---: | :---: | :---: | :---: |
|  | 3.0, 2.5-3.0, 2.5 | 2261.2906 | -0.0011 |
|  | 7.0, 6.5-6.0, 5.5 | 2261.0375 | 0.0009 |
|  | 6.0, 5.5-6.0, 5.5 | 2261.0422 | 0.0009 |
|  | 5.0, 4.5-5.0, 4.5 | 2261.0443 | -0.0005 |
|  | 4.0, 3.5-4.0, 3.5 | 2261.0455 | -0.0020 |
|  | 3.0, 2.5-3.0, 2.5 | 2261.0467 | -0.0029 |
| ${ }^{o} P_{12}(4)$ | 2.0, 1.5-1.0, 0.5 | 1627.3119 | -0.0008 |
|  | 4.0, 3.5-3.0, 2.5 | 1627.3143 | 0.0000 |
|  | 5.0, 4.5-4.0, 3.5 | 1627.3162 | 0.0007 |
|  | 6.0, 5.5-5.0, 4.5 | 1627.3191 | 0.0018 |
| ${ }^{R} R_{12}(7)$ | 10.0-9.5-11.0, 10.5 | 1636.3194 | 0.0013 |
|  | 9.0, 8.5-10.0, 9.5 | 1636.3281 | -0.0007 |
|  | 8.0, 7.5-9.0, 8.5 | 1636.3372 | -0.0009 |
|  | $7.0,6.5-8.0,7.5$ | 1636.3463 | 0.0000 |
| ${ }^{R} R_{11}(7)$ | $7.0,6.5-8.0,7.5$ | 1636.4902 | 0.0008 |
|  | 8.0, 7.5-9.0, 8.5 | 1636.4921 | -0.0008 |
| ${ }^{s} R_{22}(4)$ | $7.0,6.5-8.0,7.5$ | 2265.2782 | 0.0013 |
|  | 6.0, 5.5-7.0, 6.5 | 2265.2815 | -0.0001 |
|  | 5.0, 4.5-6.0, 5.5 | 2265.2865 | 0.0014 |
|  | 4.0, 3.5-5.0, 4.5 | 2265.2910 | 0.0032 |
| ${ }^{s} R_{21}(4)$ | 4.0, 3.5-5.0, 4.5 | 2265.4368 | 0.0027 |
|  | 5.0, 4.5-6.0, 5.5 | 2265.4404 | 0.0013 |
|  | 6.0, 5.5-7.0, 6.5 | 2265.4428 | 0.0018 |
| ${ }^{s} R_{22}(7)$ | 10.0, 9.5-11.0, 10.5 | 2267.0346 | 0.0022 |
|  | 9.0, 8.5-10.0, 9.5 | 2267.0396 | 0.0007 |
|  | 8.0, 7.5-9.0, 8.5 | 2267.0446 | 0.0003 |
|  | $7.0,6.5-8.0,7.5$ | 2267.0493 | 0.0002 |
| ${ }^{s} R_{21}(7)$ | $7.0,6.5-8.0,7.5$ | 2267.1947 | 0.0025 |
|  | 8.0, 7.5-9.0, 8.5 | 2267.1997 | 0.0005 |
|  | 9.0, 8.5-10.0, 9.5 | 2267.2047 | 0.0019 |
| ${ }^{R_{22}(10)}$ | 13.0, 12.5-14.0, 13.5 | 2268.7139 | 0.0006 |
|  | 12.0, 11.5-13.0, 12.5 | 2268.7190 | -0.0023 |
|  | $11.0,10.5-12.0,11.5$ | 2268.7202 | -0.0018 |
|  | 11.0, 10.5-12.0, 11.5 | 2268.7265 | -0.0020 |
|  | 10.0, 9.5-11.0, 10.5 | 2268.7277 | -0.0011 |
|  | 10.0, 9.5-11.0, 10.5 | 2268.7340 | -0.0014 |
| ${ }^{R} R_{12}(0)$ | 3.0, 2.5-4.0, 3.5 | 1631.2192 | 0.0018 |
|  | 3.0, 2.5-3.0,2.5 | 1631.2242 | 0.0006 |
|  | 3.0, 2.5-2.0,1.5 | 1631.2272 | -0.0005 |
| ${ }^{R} R_{11}(0)$ | 2.0, 1.5-3.0,2.5 | 1631.3816 | 0.0020 |
|  | 2.0,1.5-2.0,1.5 | 1631.3863 | 0.0026 |
|  | 2.0,1.5-1.0, 0.5 | 1631.3875 | 0.0018 |
| ${ }^{R} R_{12}(2)$ | 5.0, 4.5-6.0, 5.5 | 1632.7155 | -0.0019 |
|  | 4.0, 3.5-5.0, 4.5 | 1632.7258 | 0.0000 |
|  | 3.0, 2.5-4.0, 3.5 | 1632.7296 | -0.0021 |
| ${ }^{R} R_{11}(2)$ | 4.0, 3.5-5.0, 4.5 | 1632.8811 | -0.0012 |
|  | 2.0, 1.5-3.0, 2.5 | 1632.8851 | -0.0002 |
|  | 3.0, 2.5-3.0, 2.5 | 1632.8878 | -0.0010 |
| ${ }^{R} Q_{22}+{ }^{R} R_{22}(2)$ | 4.0, 3.5-5.0, 4.5 | 2262.5856 | 0.0010 |


| ${ }^{R} Q_{21}+{ }^{R} R_{21}(2)$ | $2.0,1.5-2.0,1.5$ | 2262.7347 | -0.0018 |
| :---: | :---: | :---: | :---: |
|  | $4.0,3.5-4.0,3.5$ | 2262.7387 | -0.0024 |
| $Q_{P_{22}}+{ }^{Q} Q_{22}(2)$ | $4.0,3.5-4.0,3.5$ | 2261.5249 | 0.0004 |
| ${ }^{Q} P_{21}+{ }^{Q} Q_{21}(2)$ | $4.0,3.5-3.0,2.5$ | 2261.6815 | 0.0006 |
|  | Std. dev $=0.00144 \mathrm{~cm}^{-1}$ |  |  |

All units are in wave number $\mathrm{cm}^{-1}$.

Table I. The observed and calculated Stark shifts for the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system of ${ }^{138} \mathrm{BaF}$.

| Branch, Pol | Field (V/cm) | Assign $^{\text {a }}$ | Shift (MHz) | Obs-Calc ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $P_{12}(2), \\|$ | 806.3 | A | -25 | -9 |
|  | 1611.9 | A | -64 | -1 |
|  | 1741.5 | A | -79 | -7 |
|  |  | 2014.0 | B | -78 |
|  | A | -103 | -8 |  |
|  |  | B | -89 | 3 |
|  | 2341.5 | A | -127 | 2 |
|  | 2416.0 | B | -128 | -2 |
|  |  | A | -136 | 1 |
|  | 2940.9 | B | -125 | 8 |
|  |  | A | -207 | -4 |
|  | 3540.9 | B | -194 | 2 |
|  |  | A | -292 | 4 |
| ${ }^{S_{21}(0)}$ ), $\perp$ | 600.9 | B | -281 | 2 |
|  | 600.9 | D | 259 | -1 |
|  | 600.9 | B | 93 | -8 |
|  | 600.9 | A | -66 | -10 |
|  | 899.4 | D | -223 | -9 |
|  | 899.4 | C | 172.8 | 5.7 |
|  | 899.4 | B | -69.7 | 3.4 |
|  | 899.4 | A | -307.9 | -2.6 |
|  | 1201.5 | D | 549.3 | -4.8 |
|  | 1201.5 | C | 239.2 | -17.3 |
|  | 1201.5 | B | -376.2 | 6.0 |
|  | 1501.2 | B | -67.1 | -11.3 |
|  | 1501.2 | A | -439.2 | 9.9 |
|  | 1742.1 | D | 887.0 | 7.2 |
|  | 1742.1 | C | 415.7 | -3.7 |
|  | 1742.1 | B | -47.2 | -8.3 |
|  | 1742.1 | A | -500.2 | -5.2 |
|  | Std. dev. $=8.4 ~ M H z$ |  |  |  |

a) Refers to transitions indicated in Figures 6 and 7.
b) Calculated shifts obtained using optimized $\mu_{\mathrm{e}}$ of 1.50(2) D and $1.31(2) \mathrm{D}$ for the $A^{2} \Pi_{1 / 2}(v=0)$ and $A^{2} \Pi_{3 / 2}(v=0)$ states and previously determined $\mu_{\mathrm{e}}$ of $3.170(3) \mathrm{D}$ for the $X^{2} \Sigma^{+}(v=0)$ state [12].

Table II. The observed and calculated Zeeman shifts for the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$ band system of ${ }^{138} \mathrm{BaF}$.

| Branch | Field (Gauss) | $M_{J^{\prime}}$ | $M_{J^{\prime \prime}}$ | $\begin{gathered} \hline \mathrm{Obs} \\ (\mathrm{MHz}) \end{gathered}$ | $\begin{gathered} \hline \hline \text { Dif } \\ (\mathrm{MHz}) \end{gathered}$ | Branch | $\begin{gathered} \text { Field } \\ \text { (Gauss) } \end{gathered}$ | $M_{J^{\prime}}$ | $M_{J \prime \prime}$ | $\begin{gathered} \mathrm{Obs} \\ (\mathrm{MHz}) \end{gathered}$ | $\begin{gathered} \hline \hline \text { Dif } \\ (\mathrm{MHz}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} R_{1}(0) \\ \\| \end{gathered}$ | 948 | 1/2 | 1/2 | -1387 | 26 | $\begin{gathered} R_{1}(0) \\ \perp \end{gathered}$ | 250 | 3/2 | 1/2 | -402 | 21 |
|  | 948 | -1/2 | -1/2 | 1408 | -3 |  | 250 | -1/2 | 1/2 | -297 | 25 |
|  | 931 | 1/2 | 1/2 | -1442 | -54 |  | 250 | 1/2 | -1/2 | 331 | 10 |
|  | 931 | -1/2 | -1/2 | 1419 | 33 |  | 250 | -3/2 | -1/2 | 423 | 0 |
|  | 833 | 1/2 | 1/2 | -1227 | 14 | ${ }^{s_{R_{21}}(0)}$ | 137 | 1/2 | 1/2 | -111 | 4 |
|  | 833 | -1/2 | -1/2 | 1243 | 3 |  | 137 | -1/2 | -1/2 | 128 | 15 |
|  | 713 | 1/2 | 1/2 | -1127 | -65 |  | 141 | 1/2 | 1/2 | -106 | 12 |
|  | 713 | -1/2 | -1/2 | 1094 | 33 |  | 141 | -1/2 | -1/2 | 117 | 0 |
|  | 482 | 1/2 | 1/2 | -766 | -48 |  | 192 | 1/2 | 1/2 | -166 | -4 |
|  | 482 | -1/2 | -1/2 | 742 | 24 |  | 192 | -1/2 | -1/2 | 172 | 13 |
|  | 248 | 1/2 | 1/2 | -412 | -43 |  | 228 | 1/2 | 1/2 | -183 | 9 |
|  | 248 | -1/2 | -1/2 | 387 | 18 |  | 228 | -1/2 | -1/2 | 192 | 4 |
|  | 136 | 1/2 | 1/2 | -249 | -46 |  | 230 | 1/2 | 1/2 | -206 | -12 |
|  | 136 | -1/2 | -1/2 | 211 | 8 |  | 230 | -1/2 | -1/2 | 175 | -14 |
| $\begin{gathered} R_{1}(0) \\ \perp \end{gathered}$ | 948 | 3/2 | 1/2 | -1578 | 28 |  | 253 | 1/2 | 1/2 | -207 | 7 |
|  | 948 | -1/2 | 1/2 | -1176 | 44 |  | 253 | -1/2 | -1/2 | 230 | 21 |
|  | 948 | 1/2 | -1/2 | 1211 | -7 |  | 254 | 1/2 | 1/2 | -226 | -12 |
|  | 948 | -3/2 | -1/2 | 1612 | 9 |  | 254 | -1/2 | -1/2 | 193 | -16 |
|  | 833 | 3/2 | 1/2 | -1372 | 39 |  | 307 | 1/2 | 1/2 | -244 | 16 |
|  | 833 | -1/2 | 1/2 | -1035 | 37 |  | 307 | -1/2 | -1/2 | 271 | 19 |
|  | 833 | 1/2 | -1/2 | 1060 | -11 |  | 310 | 1/2 | 1/2 | -279 | -17 |
|  | 833 | -3/2 | -1/2 | 1417 | 8 |  | 310 | -1/2 | -1/2 | 242 | -13 |
|  | 724 | 3/2 | 1/2 | -1214 | -18 | ${ }^{s_{R_{21}}(0)}$ | 141 | -1/2 | 1/2 | -265 | 10 |
|  | 724 | -1/2 | 1/2 | -984 | -52 |  | 141 | 1/2 | -1/2 | 267 | -6 |
|  | 724 | 1/2 | -1/2 | 912 | 19 |  | 192 | -1/2 | 1/2 | -374 | 1 |
|  | 724 | -3/2 | -1/2 | 1207 | -18 |  | 192 | 3/2 | 1/2 | 51 | -1 |
|  | 604 | 3/2 | 1/2 | -1017 | 6 |  | 192 | -3/2 | -1/2 | -49 | 5 |
|  | 604 | -1/2 | 1/2 | -759 | 18 |  | 192 | 1/2 | -1/2 | 383 | 11 |
|  | 604 | 1/2 | -1/2 | 774 | -2 |  | 228 | -1/2 | 1/2 | -459 | -13 |
|  | 604 | -3/2 | -1/2 | 1009 | -13 |  | 228 | 1/2 | -1/2 | 445 | 4 |
|  | 489 | 3/2 | 1/2 | -816 | 12 |  | 230 | -1/2 | 1/2 | -459 | -11 |
|  | 489 | -1/2 | 1/2 | -612 | 17 |  | 230 | 3/2 | 1/2 | 48 | -14 |
|  | 489 | 1/2 | -1/2 | 612 | -17 |  | 230 | -3/2 | -1/2 | -78 | -14 |
|  | 489 | -3/2 | -1/2 | 820 | -7 |  | 254 | -1/2 | 1/2 | -517 | -21 |
|  | 364 | 3/2 | 1/2 | -611 | 5 |  | 254 | 3/2 | 1/2 | 58 | -10 |
|  | 364 | -1/2 | 1/2 | -458 | 10 |  | 254 | -3/2 | -1/2 | -85 | -14 |
|  | 364 | 1/2 | -1/2 | 470 | 2 |  | 310 | 3/2 | 1/2 | 63 | -20 |
|  | 364 | -3/2 | -1/2 | 625 | 9 |  | 310 | -3/2 | -1/2 | -107 | -19 |
|  |  |  |  |  |  | Standard deviation of fit $=21.5 \mathrm{MHz}$. |  |  |  |  |  |

Table III. The field-free spectroscopic parameters in wave numbers $\left(\mathrm{cm}^{-1}\right)$ for the $A^{2} \Pi$ $X^{2} \Sigma^{+}(0,0)$ band system of BaF

| Parameter | ${ }^{138} B a F$ | ${ }^{137} \mathrm{BaF}$ | ${ }^{135} \mathrm{BaF}$ |
| :---: | :---: | :---: | :---: |
| $X^{2} \Sigma^{+}(v=0)$ |  |  |  |
| $B$ | $0.21594802^{\mathrm{a}}$ | $0.21613878^{\mathrm{d}}$ | $0.21675(5)$ |
| $10^{7} \times D$ | $1.85^{\mathrm{b}}$ | $1.85^{\mathrm{b}}$ | $1.85^{\mathrm{e}}$ |
| $\gamma$ | $0.00269930^{\mathrm{a}}$ | $0.002702703^{\mathrm{d}}$ | $0.00270270^{\mathrm{e}}$ |
| $b_{F}(\mathrm{Ba})$ | NA | $0.077587^{\mathrm{d}}$ | $0.0702(5)$ |
| $c(\mathrm{Ba})$ | NA | $0.0250173^{\mathrm{d}}$ | $0.002237381^{\mathrm{f}}$ |
| $e q_{0} Q(\mathrm{Ba})$ | NA | $-.00390270^{\mathrm{d}}$ | $-0.003490348^{\mathrm{f}}$ |
| $b_{F}(\mathrm{~F})$ | $0.002209862^{\mathrm{c}}$ | $0.002209873^{\mathrm{d}}$ | $0.002209873^{\mathrm{d}}$ |
| $c(\mathrm{~F})$ | $0.000274323^{\mathrm{c}}$ | $0.000274323^{\mathrm{d}}$ | $0.000274323^{\mathrm{d}}$ |
| $A^{2} \Pi(\mathrm{v}=0)$ |  |  |  |
| $A$ | $632.28175(6)$ | $632.2802(8)$ | $632.2803(10)$ |
| $10^{5} \times A_{D}$ | $3.1(2)$ | $3.1^{\mathrm{e}}$ | $3.1^{\mathrm{e}}$ |
| $B$ | $0.2117414(10)$ | $2.11937(12)$ | $0.2125(4)$ |
| $10^{7} \times D$ | $2.00^{\mathrm{b}}$ | $2.00^{\mathrm{b}}$ | $2.00^{\mathrm{b}}$ |
| $(p+2 q)$ | $-0.25755(10)$ | $-0.2581(2)$ | $-0.25755^{\mathrm{e}}$ |
| $d(\mathrm{Ba})$ | NA | $0.0076(10)$ | $0.00685^{\mathrm{f}}$ |
| $T_{00}$ | $11946.3168(2)$ | $11945.3152(6)$ | $11946.3034(10)$ |
| $S t d . D e v$. | 0.0013 | 0.0014 | 0.0015 |

[a] Ref. 10 .
[b] Ref. 15.
[c] Ref. 12.
[d] Ref. 11.
[e] Constrained to value scaled from the fit of the ${ }^{138} \mathrm{BaF}$ transitions.
[f] Constrained to value obtained from scaling the value for ${ }^{137} \mathrm{BaF}$.

Table IV. The optimized magnetic g-factors $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states of ${ }^{138} \mathrm{BaF}$.

| Parameter $^{\mathrm{a}}$ | Value | Correlation <br> Matrix |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{g}_{\mathrm{l}}{ }^{\prime}\left(A^{2} \Pi\right)$ | $-0.536(23)$ | 1 |  |  |
| $\mathrm{~g}_{\mathrm{L}}{ }^{\prime}\left(A^{2} \Pi\right)$ | $0.980(35)$ | -0.6497 | 1 |  |
| $\mathrm{~g}_{\mathrm{l}}\left(X^{2} \Sigma^{+}\right)$ | $-0.028(13)$ | 0.2233 | 0.2314 | 1 |

a) $\mathrm{g}_{\mathrm{s}}$ for the $A^{2} \Pi(v=0)$ and $X^{2} \Sigma^{+}(v=0)$ states constrained to $2.002, g_{l}\left(A^{2} \Pi\right)$ was set to zero.

Table V. The permanent electric dipole moments (Debye, D) for the alkaline earth monofluorides

| State | CaF |  | SrF |  | BaF |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Value | Ref. | Value | Ref. | Value | Ref. |
| $X^{2} \Sigma^{+}$ | $3.07(7)$ | 33 | $3.4963(6)$ | 35 | $3.170(3)$ | 12 |
| $A^{2} \Pi_{1 / 2}$ | - | - | - |  | $1.50(2)$ | present |
| $A^{2} \Pi_{3 / 2}$ | $2.45(6)$ | 34 | $2.06(5)$ | 36 | $1.31(2)$ | present |

FIGURE 1


## FIGURE 2



Figure 2

## FIGURE 3



Figure $3 \quad$ AD10803 27MAY2011

Figure 4


Figure $4 \quad$ AD10803 27MAY2011

Figure 5
$A^{2} \Pi_{1 / 2}(\nu=0)$


Figure $5 \quad$ AD10803 27MAY2011

Figure 6

$$
\begin{gathered}
A^{2} \prod_{3,2}(v=0) \\
J=5.5
\end{gathered}
$$

Energy $=12269.65 \mathrm{~cm}^{-1}$


Figure $6 \quad$ AD10803 27MAY2011

Figure 7


Figure 8


Laser wave number -12262 $\left(\mathrm{cm}^{-1}\right)$


Figure 8

Figure 9


Figure 9

FIGURE 10


Figure 10

Figure 11


