Quantum pumping of interacting bosons

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Quantum Pumping of Interacting Bosons

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Quantum pumping in an interacting one-dimensional Bose-Einstein condensate is analyzed. It is shown that a spatially periodic potential, oscillating adiabatically in time with frequency ω_0 acts as a quantum pump inducing an atom current from broken spatiotemporal symmetries of the driven potential. The current generated by the pump is strongly affected by the interactions. It has a power law dependence on the frequency with the exponent determined by the interaction, while the coupling to the pump affects the amplitudes. It depends on the phase difference between two umklapp terms of the drive, providing evidence for the full quantum character of the boson transport. The results suggest the realization of a quantum pump with laser-cooled atoms.

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I. INTRODUCTION

Quantum transport phenomena are fundamental problems in physics, chemistry and biology. An intriguing example is the quantum pumping, where in the absence of a biased force a directed current of particles is produced along a periodic structure[1, 2]. An adiabatic quantum pump is a device that generates a dc current by a periodic slow variation of some system characteristic, the variation being slow enough so that the system remains close to its ground state throughout the pumping cycle. From a fundamental physics standpoint, this mechanism represents a new macroscopic quantum phenomenon reminiscent of the quantum Hall effect and of superconductivity where current flows without dissipation. Possible applications of charge pumping are metrological applications, moreover the ability of the pumps to control the position of single electron or atom could be used in various quantum information processing schemes[3, 4].

Experimentally, quantum pumping has been studied in solid state systems where some quantum effects were observed, as in quantum dots[5–7]. More recently other type of quantum transport effect, based on a ratchet mechanism has been observed in atomic physics systems[8–12].

The system of cold atoms in optical lattices allows for studies of quantum transport in the effective absence of dissipation, when optical fields detuned far from resonance are used. In remarkable experiments till now, tunnelling oscillations and quantum resonances have been observed with moving lattice potentials[13–15]. In recent ratchet experiments with laser-cooled atoms, the relationship between the appearance of transport and symmetry breaking has been investigated, and the scaling of transport versus dissipation has been studied[8–11]. From the theoretical point of view quantum transport in cooled atoms systems has been proposed in the form of ratchet transport[16, 17] or of quantum stirring[19]. The interest in such proposed devices is that they would provide a blueprint for new quantum machinery[18] or a probe for superfluidity[19].

Here are reported the results on atom quantum pumping in an interacting one-dimensional Bose Einstein Condensate (BEC). The proposed pump consists of a Bose-Einstein condensate exposed to a spatially periodic optical lattice potential oscillating wave like with frequency ω_0 and momentum q_0. When the driving lattice potential breaks spatial and temporal symmetries, directed transport is obtained with characteristics different from a ratchet. The low-energy properties of the interacting one-dimensional BEC are described by a Luttinger liquid, and the pumping is carried out at small ω_0, staying this way in the neighborhood of the Luttinger liquid fixed point. An anomalous response is observed and a nonuniversal behavior different from that of interacting one-dimensional fermions[20] emerges.

II. THE LOW-ENERGY MODEL IN 1D AND PUMPING CURRENT

A system of interacting one-dimensional bosons in an optical lattice is considered and since the study of the *adiabatic* pumping involves the low energy physics of such a model, it suffices to consider the system close to its fixed point - a Luttinger liquid - to which it flows under the action of the renormalization group (RG) [21–23] with Hamiltonian

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[21, 22]:

$$H=\frac{\hbar}{2\pi}\int dx[\frac{v_s}{K}(\nabla\phi(x))^2+\frac{v_sK}{\hbar^2}(\pi\Pi(x))^2]. \quad (1)$$

($\hbar=1$ hereafter). This Hamiltonian is a standard sound wave one in which the fluctuations of the phase $\phi(x)$ represent the phonon modes of the density wave

$$\rho(x)=[\rho_0-\frac{1}{\pi}\nabla\phi(x)]\sum_{p=-\infty}^{\infty}e^{i2p(\pi\rho_0x-\phi(x))}, \quad (2)$$

ρ_0 being the average particle density; $\pi\Pi(x)=\hbar\nabla\theta(x)$ is the conjugate momentum of $\phi(x)$, $[\phi(x),\nabla\theta(x')]=i\pi\delta(x-x')$. In the case of contact interaction between bosons $U\delta(x-x')$, the Luttinger parameters v_s and K used in (1) were determined in[23]. When the interaction goes to zero, K goes to infinity, while $K=1$ for infinitely strong hard-core interactions (Tonks-Girardeau limit)[24]. The situation $K<1$ can be instead realized in Bose-gases with long-range interactions, e.g. in dipolar gases.

An external potential $V(x)$ which is periodic (with period l and period wavevector $Q=\frac{2\pi}{l}$) acting on a length L of the system is considered. Such potential couples to the density (2) and leading terms of such coupling are of the form: $H_{n,m}\sim g_{n,m}\int dxe^{ik_{n,m}x}e^{i2n\phi(x)}$, the so called umklapp terms, where $k_{n,m}=n2\pi\rho_0-mQ$ represents the momentum transfer. A commensurability between the boson density and the imposed periodicity implies $k_{n,m}=0$ for some n,m. At commensurate filling some of these operators may become relevant for a critical value of the Luttinger parameter $K_c=2/n^2$ and drive the system towards a Mott-insulator phase. One then allows the external periodic potential to oscillate with frequency ω_0 and propagate with characteristic momenta $\{q\}$ centered around a value q_0, $V(x)\to V(x,t)=\sum_q A_q\cos(\omega_0t-qx)V(x)$, i.e. it acts as a pump $H^{ext}(t)=H^{pump}$. Close to the Luttinger fixed point, the umklapp terms will have time and phase dependent coupling constant of the form $g_{n,m}(t)=g_{n,m}e^{i\omega_0t-\varphi_{n,m}}$. The momenta $\{q\}$ in the driving potential break the reflection symmetry and are reflected in the effective low-energy Hamiltonian by the umklapp phases $\varphi_{n,m}$ that for weak periodic potential are given by $\varphi_{n,m}\sim nq_0/\omega_0$. When reflection symmetry is present $\varphi_{n,m}=0$ and no current will flow in the system.

The bosonized version of the pump Hamiltonian reads:

$$H^{pump}=\sum_{n,m}H_{n,m}(t)=\sum_{n,m}\frac{g_{n,m}}{(2\pi\rho_0^{-1})}\int dx\{e^{i(\omega_0t-\varphi_{n,m})}e^{ik_{n,m}x}e^{i(2n\phi(x))}+h.c.\}, \quad (3)$$

where the sum runs over n,m integers (m is the order of commensurability, e.g. $m=1$ corresponds to one boson per site). Beyond the pump, also boundary terms may be generated under the renormalization group (RG) flow, hence backscattering terms at the edges of the pump of size L may be present, but on a large distance the bulk contribution will dominate.

We wish to study the effect of the oscillating terms on the current operator that in bosonized version is given by $j=\frac{1}{\pi}\partial_t\phi=\Pi(x)$. In the following, we shall consider the oscillating lattice as a perturbation around the Luttinger liquid fixed point and compute the current perturbatively, by the Keldysh technique[25]. Within this technique the boson current is given by:

$$\langle j(x,t)\rangle=\langle T_K\{j(x,t)e^{-i\int dt_1H^{pump}(t_1)}\}\rangle, \quad (4)$$

where T_K is the time ordering operator along the Keldysh contour. Explicit calculations show that to get a dc current out of the pumping potential at least two umklapp operators with a nonzero phase difference are required, in agreement with the general idea of pumping and differently from a ratchet mechanism. Thus the first nonvanishing contribution to the pump comes from the second order perturbation expansion:

$$\langle j(x,t)\rangle^{(2)}=-\frac{1}{2}\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\langle T_K\{j(x,t^\eta)\int dt_1\int dt_2H^{pump}(t_1^{\eta_1})H^{pump}(t_2^{\eta_2})\}\rangle, \quad (5)$$

where T_K is the ordering/anti-ordering operator along the Keldysh contour and $\eta,\eta_{1,2}=\pm$ identify the upper/lower branch. By using the bosonic expression of H^{pump} the d.c. current can be calculated and its expression is:

$$j^{d.c.}(\omega_0) \;=\; Kv_s\sum_{n,m}\sum_{n',m'}(n-n')\,(\rho_0L)^{-(n-n')^2K}\,\mathcal{A}_{n,m}^{n',m'}\,I_{nm}^{n'm'}(\omega_0,[k_+]_{n,m}^{n',m'})\frac{\sin([k_-]_{n,m}^{n',m'}\frac{L}{2})}{[k_-]_{n,m}^{n',m'}} \quad (6)$$

where $[k_\pm]_{n,m}^{n',m'} = (\frac{k_{n,m} \pm k_{n',m'}}{2})$ and,

$$\mathcal{A}_{n,m}^{n',m'} = \frac{g_{n,m}}{(2\pi\rho_0^{-1})} \frac{g_{n',m'}}{(2\pi\rho_0^{-1})} \sin\varphi_{n,m}^{n',m'} \tag{7}$$

is the area enclosed in a pumping cycle by the periodic parameters $g_{n,m}(t)$ and $g_{n',m'}(t)$, $\varphi_{n,m}^{n',m'}$ is the phase difference $(\varphi_{n,m} - \varphi_{n',m'})$.

$$I_{nm}^{n'm'}(\omega_0, [k_+]_{n,m}^{n',m'}) \;=\; \mathrm{Sgn}(\omega_0) \frac{(2v_s\rho_0)^{-2K^{nn'}}}{\Gamma^2(2K^{nn'})} \left| |\omega_0| - |[k_+]_{n,m}^{n',m'}| \right|^{2K^{nn'}-1} \theta(|\omega_0| - |v_s[k_+]_{n,m}^{n',m'}|) \tag{8}$$

where $K^{nn'} = nn'K$, the function $\mathrm{Sgn}(\omega_0)$ is defined as $\mathrm{Sgn}(\omega_0) = 0$ for $\omega_0 = 0$ in addition to the usual definition $\mathrm{Sgn}(\omega_0) = \pm 1$ for ω_0 positive/negative and Γ is the gamma function.

III. DISCUSSION

The nonlinear dependence on the size of the pumping region is suppressed for large $\rho_0 L$ in the terms with large $n - n'$. Therefore, the leading contribution to the current comes from terms with $n - n' = \pm 1$. The particle transport is not quantized but depends linearly on the area $\mathcal{A}_{n,m}^{n',m'}$ enclosed by the interaction (cfr. Eq.(7); this reflects the fact that while the external periodic potential couples to atoms the quasiparticles of the interacting system are Luttinger-like bosons. Moreover at least two umklapp terms are needed to have a nonzero dc particle current, this agrees with the picture that the quantum pump is induced by the out-of-phase variation of any pair of independent parameters making the difference from a ratchet mechanism. The current would vanish if under the RG a single umklapp term is induced, even if associated with several phases. In case of reflection symmetry we have $\varphi_{n,m} = 0$, resulting in a zero dc current. Thus the breaking of reflection symmetry is a necessary condition for the pumping. The condensate prepared initially at rest with respect to the lattice reference frame, is accelerated to positive (negative) momentum values for a phase difference between the two driving umklapp terms $\varphi_{n,m} = \pm\frac{\pi}{2}$. Most importantly, the response of the Bose Luttinger liquid to the external perturbation produces anomalous frequency dependence in the pumped current. When the commensurate case is considered, $k_+ = 0$, the pumped current, Eq.(8), reduces to a power law in frequency dependence with an exponent $2K^{nn'} - 1$. In the hard-core limit, i.e. for the Tonks-Girardeau gas $K = 1$, thus the lowest value for the exponent will correspond to one, giving the expected linear ω_0 behavior at commensurability. With interaction, the frequency dependence of the current is generally nonlinear with an exponent depending on the strength of the Luttinger interaction as shown in Fig.1. For $K^{nn'} > 1$ and at increasing K, the current goes smoothly to zero in the zero frequency limit and for a finite size of the pump (see Fig.2) indicating that, differently from electrons[20], one cannot pump noninteracting bosons due to the superfluid character of the ground state. In the range $K^{nn'} < 1$, the Luttinger fixed point would become unstable at commensurability and a charge density wave (CDW) ground state forms, where the considerations above do not apply. In the incommensurate case, the current vanishes in the frequency window $|\omega_0| < |v_s[k_+]_{n,m}^{n',m'}|$. This reflects the physical requirement that sufficient energy must be supplied from the pumping source in order to make the transition. The non-trivial power law appears again immediately beyond the frequency threshold.

Our proposal can be realized with a rubidium Bose-Einstein condensate exposed to an optical lattice potential that is periodically modulated in time and space. The fundamental spatial frequency of periodicity $l/2$ is generated by a usual standing wave formed with two counterpropagating fields, where l denotes the wavelength of the driving laser. In each of the beams, an acoustooptic modulator generates all required optical frequencies in the corresponding beam. After the interaction with the potential, the BEC freely expand and then an absorption image is recorded. By this time of flight technique, the atomic velocity distribution can be analyzed. The interaction of the BEC with the spatially periodic potential results in a diffraction into several discrete diffraction peaks. The mean atomic momentum, and consequently the mean atom current, along the lattice axis can be deduce from such time of flight images. The generated atomic current can also be determined from a measurement of the momentum distribution of the particle flow, since the average current can be written as $j = (\hbar/m)\int |\psi(k)|^2 k dk$, where ψ is the bosons wavefunction. Bragg spectroscopy is ideally suited for measurements of the momentum distribution, since it can be selectively applied to atoms in the 1-d channel and combined with fluorescence imaging for high signal-to-noise detection. For ^{87}Rb in the F = 2 state, the quasi-one-dimensional character of the gas is governed by the condition $n^{-1} \gg l_r$, where l_r is the radial confinement, which could be achieved for the transverse and axial trap frequencies of the order $\omega_r = 2\pi \times 5.$ kHz and $\omega_a = 2\pi \times 4$ Hz with 10^3 ^{87}Rb atoms. The pump is adiabatic as long as the bosons remain in the transverse ground

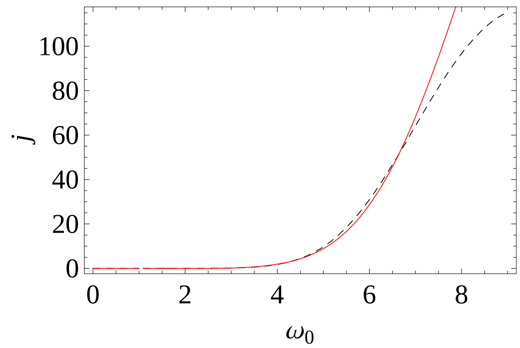

FIG. 1: (Color online) Pumping current having taken into account umklapp terms g_{20}, g_{21} for the Luttinger parameter $K = 1.8$. The dashed line is for system length (in units of $1/k_{20}$) $L = 1.$ and the straight (red) for L=0.8 The current is measured in units of v_s, ω_0 in units of $v_s k_{20}$.

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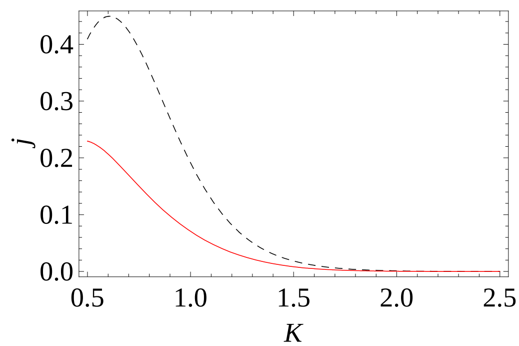

FIG. 2: (Color online) Pumping current having taken into account umklapp terms g_{20}, g_{21} as a function of the Luttinger liquid exponent (K goes to infinity for noninteracting bosons). The dashed line is for system length (in units of $1/k_{20}$) $L = 0.8$ and the straight (red) for L=1.5. The current is measured in units of v_s, $\omega_0 = 0.5$ is in units of $v_s k_{20}$.

state which implies that the pumping frequency must be small compared to trapping frequency and the bosonization is applicable for length scales $\tilde{l}$ such that $\hbar v_s/\tilde{l} \ll \hbar\omega_r$, i.e. when the kinetic energy of longitudinal motion of the particles is much smaller than the trapping energy.

A generalization of the present proposal could be provided by considering a ring-shaped[26, 27] optical lattice.

IV. CONCLUSIONS

To conclude, a parametric quantum pump for an interacting one-dimensional Bose gas has been proposed. It has been demonstrated that the pump, consisting of a periodic potential oscillating in space and in time induces a dc current. The current arises from the interference of two out-of-phase umklapp operators, requiring for a spatiotemporal breaking symmetry. It has been shown that the pumped current is strongly affected by the interaction displaying a nonuniversal behavior that depends on the interaction itself. In the hard-core limit the expected linear frequency behavior, as for noninteracting electrons, is recovered while for noninteracting bosons the pumped current goes smoothly to zero in the limit of small frequency and finite size of the pump. Our results demonstrate that quantum pumping pave the way to the realization of a quantum motor providing directed transport of interacting bosons in the absence of a net directed force using operational principles fully quantum in nature and offering a check of Luttinger liquid physics.

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