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Adiabatic quantum state transfer in non-uniform triple-quantum-dot system

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We introduce an adiabatic quantum state transfer scheme in a non-uniform coupled triple-quantum-dot system. By adiabatically varying the external gate voltage applied on the system, the electron can be transferred between two spatially separated dots. We numerically study the effect of the system parameters on transfer fidelity and find a perfect matching between them. We also find that there is a relatively large tolerance range of difference between two coupling constants to permit high fidelity quantum state transfer.

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The coherent control of transitions between individual discrete quantum states is central to the study of quantum state transfer (QST) [1, 2]. Due to the promise of scalability and long decoherence times, the applications of adiabatic passage for coherent QST have been widely investigated in solid-state systems [3–15]. Such methods are relatively insensitive to gate errors and other external noises and do not require an accurate control of the system parameters, thus they can realize high-fidelity QST. Zhang et al. [5] have describe a scheme for using an all-electrical, adiabatic population transfer between two spatially separated dots in a triple-quantum-dot (TQD) system by adiabatically engineering external gate voltage. Recently, a robust method, termed Coherent Tunneling by Adiabatic Passage (CTAP), has also been introduced to spatial transport of physical particles both in optical microtraps [6] and quantum dots [7] which is a spatial analogue of the well-known Stimulated Raman Adiabatic Passage (STIRAP) technique [16] from quantum optics. In such technique, the basic idea is to use the existence of a spatial dark state which is a coherent superposition state of two "distant" spatial trapping sites. Via adiabatic manipulation of the dark-state wavefunction, it is possible to transport electrons from one trapping site to another.

In this paper we consider a different adiabatic protocol to achieve population transfer between two spatially separated dots. We introduce a non-uniform coupled TQD array which can be manipulated by external gate voltage applied on the two external dots (sender and receiver). Through maintaining the system in the ground state we show that the electron initially in the left dot can be transferred to the right dot occupation with high fidelity. Furthermore, we study in details the dynamic competition between the adiabatic QST and the decoherence. There are two time scales τ_A and τ_D depicting such competition, where τ_A represents the adiabatic time limited by the adiabatic conditions and τ_D represents the decoherence time.

We first introduce the isolated (no coupling to the leads) TQD array $|L,\sigma\rangle$, $|M,\sigma\rangle$, $|R,\sigma\rangle$ ($\sigma =\uparrow,\downarrow$), where $|\kappa,\sigma\rangle$ ($\kappa = L, M, R$) corresponds to an electron in the dot κ with spin σ. The scheme is schematically shown in Fig. 1(a). Specifically, we consider the interactions between nearest-neighbor quantum dots are time-independent and slightly different. We term this model non-uniform TQD system. The system are controlled by external time-varying gates voltage $\mu_\alpha(t)$ ($\alpha = L,\ R$), which control the site energies of two end of the array. We will show that the information encoded in electronic spin can be transported from $\cos\theta\,|L,\uparrow\rangle + e^{i\phi}\sin\theta\,|L,\downarrow\rangle$ to $\cos\theta\,|R,\uparrow\rangle + e^{i\phi}\sin\theta\,|R,\downarrow\rangle$. Notice that the polarization of the spin of an electron is not changed as time evolves. Then the problem about the quantum information transfer (QIT) can be reduced to the issue of QST and a complete QST can achieve perfect QIT. In this sense, we can ignore spin degrees of freedom to illustrate the principles of QST from $|L\rangle$ to $|R\rangle$.

Using $\{|L\rangle, |M\rangle, |R\rangle\}$ as basis of the Hilbert space, the Hamiltonian for non-uniform TQD system in matrix form reads as

$$H(t) = \begin{bmatrix} \mu_L(t) & J_1 & 0 \\ J_1 & 0 & J_2 \\ 0 & J_2 & \mu_R(t) \end{bmatrix}, \tag{1}$$

where J_i ($i = 1, 2$) is the fixed coupling constant between nearest-neighbor dots, assumed to be real (negative) for the sake of simplicity. The on-site energies $\mu_L(t)$ and $\mu_R(t)$ are modulated in Gaussian pulses to realize the adiabatic transfer, according to (shown in Fig. 2)

$$\mu_L(t) = -\mu_L^{\max}\exp\left[-\frac{1}{2}\alpha^2 t^2\right], \tag{2a}$$

$$\mu_R(t) = -\mu_R^{\max}\exp\left[-\frac{1}{2}\alpha^2\,(t-\tau)^2\right], \tag{2b}$$

where τ and α are the total adiabatic evolution time and standard deviation of the control pulse modulating the chemical potential of states $|L\rangle$ and $|R\rangle$. For simplicity we set the peak voltage of each dot to be equal $\mu_L^{\max} = \mu_R^{\max} = \mu_0$ and satisfy $\mu_0 \gg |J_i|$ ($i = 1, 2$). We will see below that this simplicity has no relevance to the problem.

At time $t = t_0$, the Hamiltonian $H(t_0)$ has eigenvectors $|\psi_n(t_0)\rangle$ ($n = 0, 1, 2$) which are superpositions of

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$|L\rangle, |M\rangle, |R\rangle$ and the eigenvalues are denoted by $\varepsilon_n(t_0)$, sorting in ascending order $\varepsilon_0 < \varepsilon_1 < \varepsilon_2$. Under adiabatic evolution, these eigenstates evolve continuously to $|\psi_n(t)\rangle$. The instantaneous Hamiltonian's eigen equation is

$$H(t)\,|\psi_n(t)\rangle = \varepsilon_n(t)\,|\psi_n(t)\rangle\,. \tag{3}$$

We use ground state $|\psi_0(t)\rangle$ of Eq. (3) to induce population transfer from state $|L\rangle$ to $|R\rangle$ (see Fig. 1(b)). One advantage of this proposal is that there can be no heat dissipation during the transfer.

Starting from $t = 0$, the left quantum dot is approximate decoupled from the array in the case $\mu_0 \gg |J_1|$ and the Hamiltonian (1) is reduced to

$$H(t=0) \simeq -\mu_0\,|L\rangle\langle L| + J_2\,(|M\rangle\langle R| + \text{h.c.})\,. \tag{4}$$

The ground state of this Hamiltonian is $|\psi_0\,(t=0)\rangle = |L\rangle$.

In the time limit, $t \to \tau$, the parameter $\mu_L(t)$ goes to zero and $\mu_R(t)$ goes to $-\mu_0$. The Hamiltonian (1) evolves to

$$H(t=\tau) \simeq -\mu_0\,|R\rangle\langle R| + J_1\,(|L\rangle\langle M| + \text{h.c.})\,, \tag{5}$$

and the corresponding ground state is $|\psi_0\,(t=\tau)\rangle = |R\rangle$.

Our aim is to induce population transfer from state $|L\rangle$ to $|R\rangle$ by maintaining the system in ground state. Providing adiabaticity is satisfied [17]

$$|\varepsilon_0 - \varepsilon_{1,2}| \gg |\langle\psi_0|\dot{\psi}_{1,2}\rangle|, \tag{6}$$

the overall system will remain in its instantaneous ground state. Preparing the system in state $|\psi_0\,(t=0)\rangle = |L\rangle$ and adiabatially changing $\mu_L(t)$ and $\mu_R(t)$, we can see that the system will end up in $|R\rangle$.

Providing the length of evolution time τ is sufficiently large, the fidelity of QST is also determined by peak gate voltage μ_0. Notice that the square of the module of fidelity $|F(t)|^2 = |\langle R\,|\psi_0\,(t)\rangle|^2$ denotes the probability of finding $|R\rangle$ in the ground state $|\psi_0\,(t)\rangle$. Now we suppose to get analytical expression of transfer fidelity using first order perturbation theory. We start from Eq. (1) at $t = \tau$ and consider the coupling term $J_2\,(|R\rangle\langle M| + |M\rangle\langle R|)$ as a weak perturbation. The Hamiltonian

$$H(t=\tau) = H_0 + H_I, \tag{7}$$

contains two parts

$$H_0 = J_1\,(|L\rangle\langle M| + |M\rangle\langle L|) - \mu_0\,|R\rangle\langle R|\,, \tag{8a}$$
$$H_I = J_2\,(|R\rangle\langle M| + |M\rangle\langle R|)\,. \tag{8b}$$

Our aim is to find the approximate expression for the ground state $|\psi_0\rangle$ of the perturbed Hamiltonian $H(t=\tau)$. The eigenfunctions of unperturbed Hamiltonian H_0 is

$$|\psi_0^{(0)}\rangle = |R\rangle, |\psi_\pm^{(0)}\rangle = \frac{1}{\sqrt{2}}\,(|L\rangle \pm |M\rangle)\,. \tag{9}$$

The energies of these states are

$$\varepsilon_0^{(0)} = -\mu_0, \varepsilon_\pm^{(0)} = \pm J_1. \tag{10}$$

As the first order perturbation, we have the corrected ground state to be

$$\begin{aligned} |\psi_0\rangle &= |\psi_0^{(0)}\rangle + \sum_{\eta=\pm} \frac{\langle\psi_\eta^{(0)}|H_I|\psi_0^{(0)}\rangle}{\varepsilon_0^{(0)} - \varepsilon_\eta^{(0)}} |\psi_\eta^{(0)}\rangle \\ &= \frac{J_1 J_2}{\mu_0^2 - J_1^2}|L\rangle - \frac{\mu_0 J_2}{\mu_0^2 - J_1^2}|M\rangle + |R\rangle. \end{aligned} \tag{11}$$

So the transfer fidelity of adiabatic QST at $t = \tau$ is

$$|F(\tau)|^{-2} = 1 + \frac{J_2^2\left(\mu_0^2 + J_1^2\right)}{\left(\mu_0^2 - J_1^2\right)^2}, \tag{12}$$

which shows that the peak voltage μ_0 determined the fidelity of QST. As $\mu_0 \gg |J_i|$ is satisfied, the fidelity is near to unity.

The analysis above is based on the assumption that the adiabaticity is satisfied. In order to demonstrate the QST in the system (1) and to show how exact the approximation is, we numerically solve the schrödinger equation and the above central conclusion can be get confirmed.

Initialize an electron in the left dot, i.e., the initial state is $|\Psi\,(0)\rangle = |L\rangle$, the consequent time evolution of the state is given by (assuming $\hbar = 1$)

$$i\frac{d}{dt}\,|\Psi\,(t)\rangle = H(t)\,|\Psi\,(t)\rangle\,. \tag{13}$$

The time evolution creates a coherent superposition:

$$|\Psi\,(t)\rangle = c_1(t)\,|L\rangle + c_2(t)\,|M\rangle + c_3(t)\,|R\rangle\,. \tag{14}$$

The probability of finding the target state $|R\rangle$ is given by $|F(t)|^2 = |c_3(t)|^2$.

Firstly, one must to make sure that no level crossings occur during adiabatic evolution, i.e., $\varepsilon_0(t) - \varepsilon_j(t) < 0$. To calculate the energies is generally only possible numerically. As an example, in Fig. 3(a) we present the results showing the eigenenergy gap $\Delta(t) = \varepsilon_1(t) - \varepsilon_0(t)$ between the first-excited state and ground state of the non-uniform TQD system for $\mu_0 = 20$, $J_1 = -0.8$, $J_2 = -1.0$, $\tau = 375$ and $\alpha = 3/\tau$, $4/\tau$, $5/\tau$, $6/\tau$. It shows that for the given evolution time τ the minimum of the energy gap increase as standard deviation α increasing. The reason is that the overlap of the pulses ($\mu_L(t)$ and $\mu_R(t)$) play the important role during sufficient long time τ. For instance $t = \tau/2$, setting the pulse overlap $\zeta \equiv \mu_{L,R}(\tau/2) = -\mu_0 \exp\left[-\alpha^2\tau^2/8\right]$, we have

$$\Delta\,(\tau/2) = \frac{2\left(J_1^2 + J_2^2\right)}{\sqrt{\left(2J_1\right)^2 + \left(2J_2\right)^2 + \zeta^2} + \zeta}. \tag{15}$$

This energy gap depends both on peak voltage μ_0 and standard deviation α of pulse. For a given μ_0, a bigger

$\alpha\tau$ would give a smaller ζ, and the energy gap $\Delta(\tau/2)$ would become larger. Besides, the asymmetrical shape of energy gap $\Delta(t)$ results from the difference between J_1 and J_2. From Eq. (15) one can see that the adiabaticity also can be improved by reducing μ_0, but it will leads to a low fidelity because the final instantaneous energy eigenstate is not the desired one which is shown in Fig. 5(a).

In Fig. 3 we also show the time evolution of the populations $|c_1(t)|^2$, $|c_2(t)|^2$ and $|c_3(t)|^2$ with $\alpha = 4/\tau$ and $\alpha = 5/\tau$. Note that for $\alpha = 4/\tau$, as illustrated in Fig. 3(b), the population on state $|R\rangle$ is nearly decoupled and stays at 0.92. The fraction of population left in states $|L\rangle$ and $|M\rangle$ is $|c_1(\tau)|^2 + |c_2(\tau)|^2 = 0.08$ and executes Rabi oscillations because the quantum dots L and M are coupled with $J_1 = -0.8$. Whereas for $\alpha = 5/\tau$, as shown in Fig. 3(c), one can see that the fidelity of adiabatic QST has been improved considerably by this slight change. The fidelity of QST achieve 0.995 and only 0.5% of population remains in states $|L\rangle$ and $|M\rangle$. This is consistent with the results shown in Fig. 3(a) because the eigenenergy gap plays opposite role for transition probability.

The above numerical results show that the electron move along a designed trajectory and result in QST from left QD to right as long as the evolution of the Hamiltonian is slow enough to satisfy the perturbation theory. In practice the minimum possible transfer time will be a few times greater than μ_0/J_1^2 which is illustrated in Fig. 4. Note that the transfer fidelity becomes stable when the total evolution time satisfy $\tau \geq 6\mu_0/J_1^2$.

The preceding discussion is based on the assumption that the system parameters are setup with arbitrary precision that is the system is coupled with $J_1 = -0.8$ and $J_2 = -1.0$. However, it is difficult to fabricate such precise Hamiltonian in experiment. Next we will show that the adiabatic passage like us is relatively insensitive to the system parameters. From the analytical results, the fidelity of adiabatic QST depends on the contrast ratio between peak voltage μ_0 and coupling constants J_i. To determine the parameter range needed to achieve high fidelity transfer, we numerically integrate the Eq. (13), with varying the peak voltage μ_0 and systematic error J_1/J_2 in the coupling constants. In Fig. 5(a) we present results showing the square of fidelity $|F(\tau)|^2 = |c_R(\tau)|^2$ as a function of μ_0 with $J_1 = -0.8$, $J_2 = -1.0$, $\tau = 375$ and $\alpha = 5/\tau$. We can see that the population transfer is close to one ($|F(\tau)|^2 \geq 0.99$) and stable when μ_0 is achieved for $|\mu_0/J_2| \geq 14$. The plot in Fig. 5(a) is in agreement with the analytical results Eq. (12) with high accuracy. On the other hand, the difference between J_1 and J_2 has a little effect upon transfer fidelity within certain range. We have illustrated this in Fig. 5(b) where the fidelity $|F(\tau)|^2$ as a function of difference J_1/J_2 has been modeled for peak voltage $\mu_0 = 20$ and $\tau = 375$. Note that the ratio as much as 0.35 still permits $|F(\tau)|^2 \approx 0.994$.

In summary, we have introduced a method of coherent QST through a non-uniform TQD system by adiabatic passage. This scheme is realized by modulation of gate voltage of quantum dots. Different from the CTAP scheme, our method is to induce population transfer by maintaining the system in its ground state which is more stable than dark state. By numerically solving the schrödinger equation under different system parameters, the results show that it is a high fidelity process for a proper choose of system parameters and also robust against experimental parameter variations.

In a real system, quantum decoherence is the main obstacle to the experimental implementation of quantum information. For coupled QDs, experiments [18] show that the coupling strength J is about 0.25 meV while $\mu_0 \sim 20J$. We can estimate a time of ~ 50 ps required for adiabatic operation. On the other hand, the typical decoherence time T_2 for electron-spin has been indicated experimentally [19] to be longer than 80 ± 9 μs at 2.5 K which is much longer than adiabatic operation time. So our scheme has applicability in practice.

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Figures

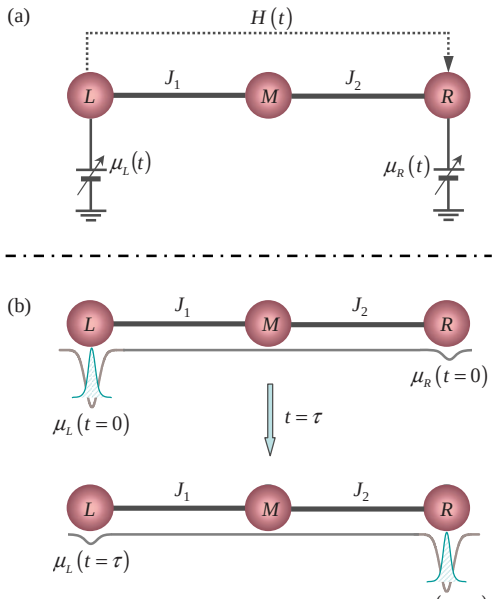

FIG. 1: (Color online) Schematic illustrations of quantum state transfer in non-uniform triple-dot system: (a) the system is controlled by gates voltage $\mu_\alpha(t)$ ($\alpha = L,\ R$); (b) by adiabatically change the $\mu_\alpha(t)$ ($\alpha = L,\ R$) one can achieve QST from $|L\rangle$ to $|R\rangle$.

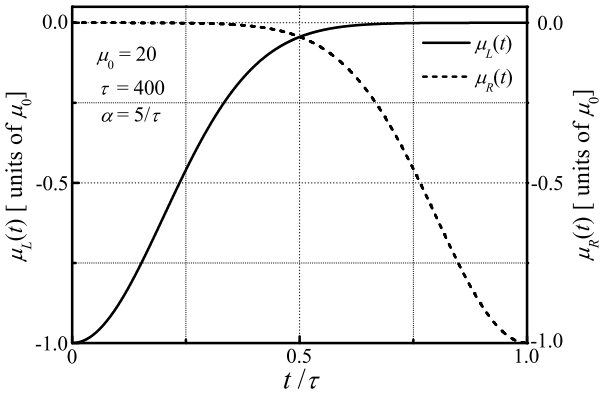

FIG. 2: Gate voltages as a function of time (in units of τ) , $\mu_L(t)$ is the solid line and $\mu_R(t)$ is the dash line.

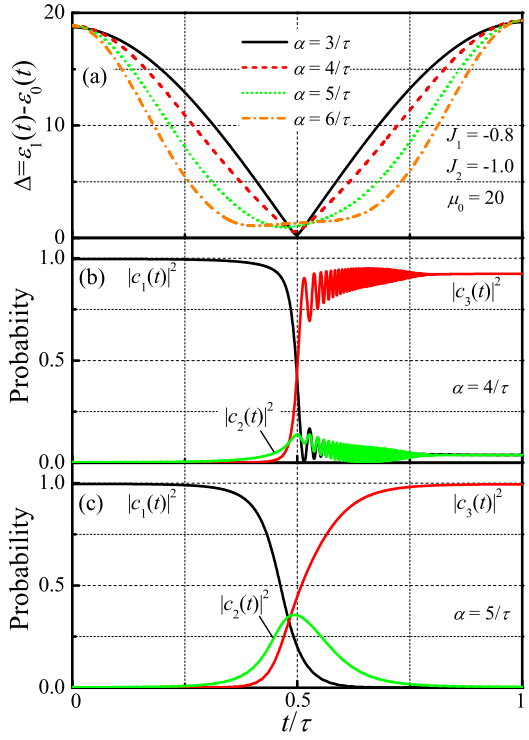

FIG. 3: (Color online) (a) The energy gap $\Delta(t) = \varepsilon_1(t) - \varepsilon_0(t)$ between the first-excited state and ground state of the non-uniform TQD system for $\mu_0 = 20$, $J_1 = -0.8$, $J_2 = -1.0$, $\tau = 375$ and $\alpha = 3/\tau$ (solid line), $4/\tau$ (dash line), $5/\tau$ (dot line), $6/\tau$ (dash-dot line). The time evolution of the probabilities induced by the pulses in Fig. 2 for (b) $\alpha = 4/\tau$ and (c) $\alpha = 5/\tau$. Initially the population is on left QD (black line) and finally mainly on right QD (red line). The population on the intermediate QD is shown as a green line.

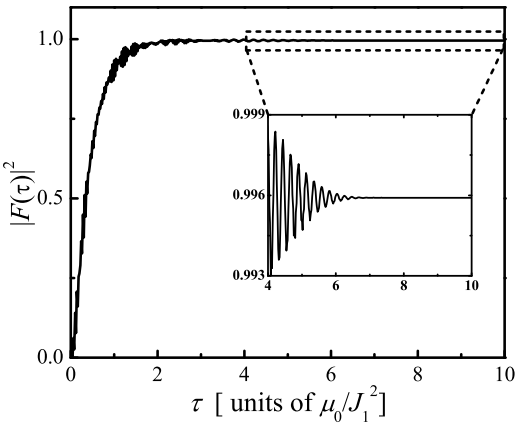

FIG. 4: Fidelity as a function of total adiabatic evolution time τ (in units of μ_0/J_1^2) and the standard deviation of the pulse is $\alpha = 5/\tau$. The insert shows an enlarged portion of the main figure with same scale. When $\tau \geq 6\mu_0/J_1^2$, the fidelity of QST becomes stable.

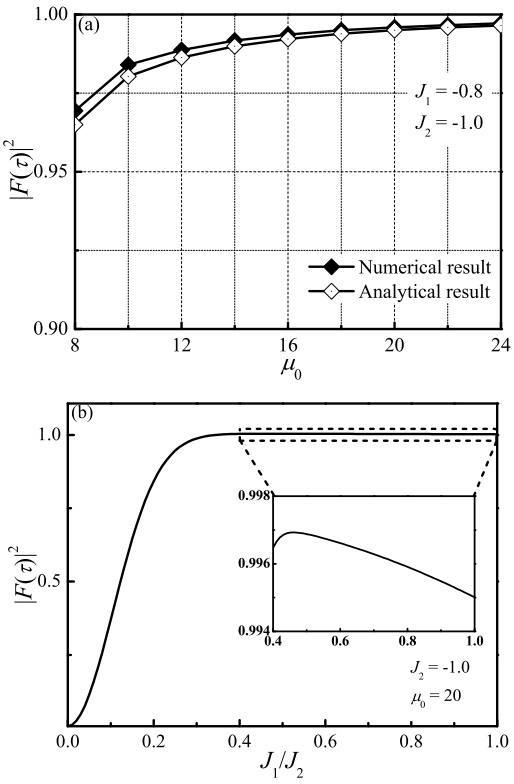

FIG. 5: The plot of the square of fidelity $|F(\tau)|^2$ as a function of system parameters: (a) the peak voltage μ_0 and (b) the ratio J_1/J_2. If the condition is satisfied when $|\mu_0/J_{max}| \geq 14$ and $J_1/J_2 \geq 0.4$, the transfer fidelity is near to one.