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Hybrid Entanglement Swapping of Photons: Creating the Orbital Angular Momentum Bell States and Greenberger-Horne-Zeilinger States

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Twisted photons offer a high-dimensional Hilbert space with the degree of freedom of orbital angular momentum (OAM). Entanglement swapping allows entangling photons that never interact. We report in this paper the hybrid entanglement swapping from multiphoton spin entangled states to multiphoton OAM entangled states with the aid of N-pair hybrid spin-OAM entangled photons. Our scheme provides a feasible scheme for creating the two-photon OAM Bell states ($N = 2$) or multihoton multidimensional OAM Greenberger-Horne-Zeilinger states ($N \geq 3$). We highlight the advantage of multiparticle multidimensional entangled states in some applications of quantum information protocols.

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I. I. INTRODUCTION

Quantum Entanglement, describing correlations between quantum systems, such as the Bell states [1] or Greenberger-Horne-Zeilinger (GHZ) states [2], is at the heart of Einstein-Podolsky-Rosen (EPR) paradox and of many quantum information protocols [3]. The entanglement between two photons is normally produced by the same source via the process of spontaneous parametric down-conversion (SPDC) [4]. There is another method of nonlinear scissors proposed to generate the finite dimensional and maximally entangled states [5, 6]. Besides, entanglement swapping also represents an intriguing and useful technique, which can entangle photons that never interacted in the past [7]. Entanglement swapping has been experimentally demonstrated with both the pulsed source [8] and continuous-wave source [9]. The quantum feature of entanglement swapping was also confirmed by observing a violation of Bell's inequality [10]. Through a prior distribution of entangled singlets, Bose *et al.* [11] has proposed to establish multiparticle entanglement between distant users by the generalized entanglement swapping and Pan *et al.* [12] has demonstrated experimentally the multiparticle entanglement swapping. Today, entanglement swapping has been an arguably essential ingredient for both quantum repeater and quantum relay, which are usually implemented in terms of photonic qubits encoded into the polarization state [13, 14]. The polarization is in essence related to the spin angular momentum of light in a two-dimensional Hilbert space. In contrast, with the degree of freedom of orbital angular momentum (OAM), twisted photons can offer a higher-dimensional Hilbert space and realize encoding qudits (quantum state in d dimensions) [15]. It has been shown that for some applications it is more advantageous

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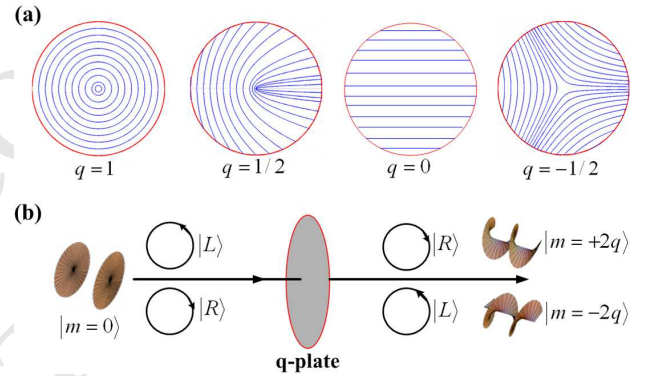†Electronic address: `shewl@mail.sysu.edu.cn`

FIG. 1: (Color online). (a), (b), (c) and (d) are illustrations of some typical q-plates with $q = 1$, $q = 1/2$, $q = 0$, and $q = -1/2$, respectively, where the orientation of optical axis is locally tangent to the line. (e) visualizes the spin-controlled OAM generation of q-plates.

to use high-dimensional entanglement rather than multiple qubits [16]. In this paper we combine entanglement swapping with OAM and propose hybrid entanglement swapping to transfer multiphoton spin entangled qubits to multiphoton OAM entangled qudits. Our scheme provides a feasible method to create the multiparticle multidimensional entanglements, such as the OAM Bell states or OAM Greenberger-Horne-Zeilinger (GHZ) states.

II. II. THEORY AND APPLICATIONS

It was Allen *et al.* who recognized that a light beam with a helical phase structure of $\exp(i\ell\phi)$ carries an OAM of $\ell\hbar$ per photon, where ϕ denotes the azimuthal angle [17]. The strong EPR correlations in angle-orbital angular momentum variables were very recently demonstrated, which creates a new opportunity for increasing bandwidth in quantum cryptography [18]. As an ideal candidate to represent higher-dimensional quantum al-

phabet, OAM has been employed to test local realism with the violation of a two-photon, three-dimensional Bell inequality [19] and OAM-based quantum cryptographic scheme for key distribution has been reported [20]. Besides, trigger qutrits for point-to-point communication protocol and the complete characterization of entangled qutrits for quantum bit commitment are both demonstrated [21, 22]. Recently, much attention was also focused on the interaction between spin and orbital angular momentum. An interesting example is the spin-controlled OAM generation by the Pancharatnam-Berry phase optical elements, q-plates [23]. A general q-plate is a planar slab of a uniaxial birefringent medium, with an inhomogeneous orientation of optical axis in x-y plane and a homogeneous phase retardation of π along z-axis. The orientation of optical axis in a polar coordinate can be described by $\alpha(\phi) = q\phi + \alpha_0$, where q and α are two constants. Some typical q-plates with $\alpha_0 = 0$ are illustrated in Fig. 1(a) and such q-plates can be made by photoaligned liquid crystal polymers [24]. Based on q-plates, increase of Shannon dimensionality [25], entanglement transfer from spin to OAM [26], and quantum cloning of OAM qubits [27] or spin-orbit ququarts [28] were recently reported. In the single photon space, the function of a q-plate is visualized by Fig. 1(b) and can be described by a quantum operator [29]

$$\hat{Q}(q) = |R, m + 2q\rangle\langle L, m| + |L, m - 2q\rangle\langle R, m|, \quad (1)$$

where $|L\rangle$ and $|R\rangle$ are spin eigenstates, namely, left- and right-handed circular polarizations, respectively, while $|m\rangle$ are the OAM eigenstates. Here, the q-plates will be used for hybrid entanglement swapping to transfer multiphoton spin entangled qubits to multiphoton OAM entangled qudits.

In an original entanglement swapping protocol [8], there are two independent pairs of entangled photons, 1-1' and 2-2'. If photons 1 and 2 from each pair are subjected to a joint Bell-state measurement, then the remainder photons, 1' and 2', will fall into an entangled state, albeit they never interacted with each other in the past. Obviously, the entangled degree of freedom before and after swapping remains the same, i.e., both in polarization states (spin qubits). In our hybrid entanglement swapping, however, the entanglements before and after swapping are different, namely in spin (qubits) and OAM states (qudits), respectively. To achieve this, it is necessary to use hybrid spin-OAM entanglement instead of the traditional polarization entanglement. It should be noted that hybrid entanglement that two photons are entangled in different degrees of freedom is actually not new [30]. Based on spin-OAM hybrid entanglement we demonstrated how to teleport a controllable OAM generator [31]. Besides, experimental generation of hybrid spin-OAM entanglement with the aid of a single q-plate having $q = 1$ has been recently realized [32, 33]. In contrast, here we employ $N(N \geq 2)$ pieces of different q-plates and take the advantage of the wide SPDC spiral bandwidth to create N-pair different hybrid spin-OAM
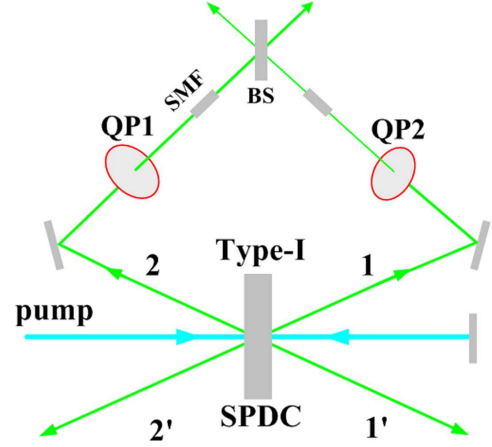

FIG. 2: (Color online). The proposed experimental configuration for generating arbitrary OAM Bell states.

entangled photons. Based on the multi-pair hybrid entangled photons, we propose the hybrid entanglement swapping with optical angular momentum and demonstrate the schemes for creating two-photon OAM Bell states and multiphoton OAM GHZ states.

A. A. Creation of arbitrary OAM Bell states (N = 2)

Bell's theorem, formulated in 1964, proved that Einstein's point of view of local realism leads to algebraic predictions that are contradicted by quantum mechanics [1]. Polarization formed the basis of the early work for testing the violation of the Bell inequalities [34]. In analogy with polarization (spin), it is possible to project orthogonal OAM states onto two-dimensional subspaces that can be represented by a Bloch sphere, equivalent to the Poincare sphere for polarization [35]. Recently, the violation of a suitable Bell inequality was demonstrated in a number of two-dimensional subspaces of the higher dimensional OAM Hilbert space, where the OAM Bell states are prepared in a symmetric form: $|\psi\rangle_\ell = \frac{1}{\sqrt{2}} (|\ell\rangle_s| - \ell\rangle_i + | - \ell\rangle_s|\ell\rangle_i)$, with s and i denoting the signal and idler photons, respectively [36]. Based on hybrid entanglement swapping, our aim here is to create the OAM Bell states with a more general format that the signal OAM ($| \pm k\rangle_s$) and idler OAM ($| \pm l\rangle_i$) are not necessarily equal, namely k and l are in principle arbitrary.

The proposed experimental scheme is sketched in Fig. 2. The pump UV pulse is slightly focused and incident on a quadratic nonlinear crystal. During the first passage type-I SPDC is done and a photon pair 1-1' with a high-dimensional OAM entanglement is created[37],

$$|\Psi\rangle_{1,1'}^{(0)} = \sum_\ell C_{\ell,-\ell}|\ell\rangle_1| - \ell\rangle_{1'}|H\rangle_1|H\rangle_{1'}, \quad (2)$$

where $C_{\ell,-\ell}$ are the probability amplitudes of finding a signal photon with an OAM of $\ell\hbar$ and an idler photon with $-\ell\hbar$. While the direct product $|H\rangle_1|H\rangle_{1'}$ indicates that there does not exist polarization entanglement between two photons, due to the type-I phase matching condition. Again, after reflected by a mirror, type-I SPDC is done once more and another entangled photon pair 2-2' is created during the second passage,

$$|\Psi\rangle_{2,2'}^{(0)} = \sum_\ell C_{\ell,-\ell}|\ell\rangle_2|-\ell\rangle_{2'}|H\rangle_2|H\rangle_{2'}. \qquad (3)$$

However, these two prior high-dimensional OAM entangled photon pairs cannot be directly used for our purpose. Similarly to [33], we direct photons 1 and 2 onto two q-plates, QP_1 and QP_2, respectively. Interactions with q-plates described by Eq. (1) therefore modify both Eqs. (2) and (3) as follows:

$$|\Psi\rangle_{1,1'}^{(1)} = \hat{Q}(q_1)|\Psi\rangle_{1,1'}^{(0)}$$
$$= \sum_\ell C_{\ell,-\ell}\left(|R,\ell+k\rangle_1 + |L,\ell-k\rangle_1\right)|-\ell,H\rangle_{1'}, \quad (4)$$

$$|\Psi\rangle_{2,2'}^{(1)} = \hat{Q}(q_2)|\Psi\rangle_{2,2'}^{(0)}$$
$$= \sum_\ell C_{\ell,-\ell}\left(|R,\ell+l\rangle_2 + |L,\ell-l\rangle_2\right)|-\ell,H\rangle_{2'}, \quad (5)$$

where $k = 2q_1$ and $l = 2q_2$ are OAM increase imparted by QP_1 and QP_2, respectively. The subsequent single-mode fibers (SMF) exclusively select the fundamental Gaussian mode with zero OAM of photons 1 and 2 and this postselection simultaneously makes Eqs. (4) and (5) fall into the following hybrid patterns:

$$|\Psi\rangle_{1,1'}^{(2)} = \frac{1}{\sqrt{2}}\left(|R\rangle_1|k\rangle_{1'} + |L\rangle_1|-k\rangle_{1'}\right), \qquad (6)$$

$$|\Psi\rangle_{2,2'}^{(2)} = \frac{1}{\sqrt{2}}\left(|R\rangle_2|l\rangle_{2'} + |L\rangle_2|-l\rangle_{2'}\right), \qquad (7)$$

In the derivation of Eqs. (6) and (7), we have utilized the relation $C_{-\ell,\ell} = C_{\ell,-\ell}$, owing to the symmetry of the SPDC process [38]. In contrast to the case with only a single pair of hybrid entangled photons, here we have two pairs and thus we could write the combined state as

$$|\Xi\rangle_{11'22'} = \frac{1}{2}\left(|R\rangle_1|k\rangle_{1'} + |L\rangle_1|-k\rangle_{1'}\right)$$
$$\otimes \left(|R\rangle_2|l\rangle_{2'} + |L\rangle_2|-l\rangle_{2'}\right), \qquad (8)$$

Key to our procedure is that quantum mechanics allows rewriting Eq. (8) in terms of the spin Bell states of photons 1-2 and OAM Bell states of 1'-2', which reads

$$|\Xi\rangle_{11'22'} = \frac{1}{2}\left(|\Phi^+\rangle_{12}|\Gamma^+\rangle_{1'2'} + |\Phi^-\rangle_{12}|\Omega^+\rangle_{1'2'}\right.$$
$$\left. + i|\Psi^+\rangle_{12}|\Omega^-\rangle_{1'2'} - i|\Psi^-\rangle_{12}|\Gamma^-\rangle_{1'2'}\right), \qquad (9)$$

where $|\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2)$ and $|\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2)$ are the four spin Bell states

while $|\Omega^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|k\rangle_{1'}|l\rangle_{2'} \pm |-k\rangle_{1'}|-l\rangle_{2'})$ and $|\Gamma^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|k\rangle_{1'}|-l\rangle_{2'} \pm |-k\rangle_{1'}|l\rangle_{2'})$ are the four OAM Bell states. For Eq. (9) the relations between horizontal and circular polarizations, $\langle H|L\rangle = \frac{1}{\sqrt{2}}$, $\langle H|R\rangle = \frac{1}{\sqrt{2}}$, $\langle V|L\rangle = \frac{i}{\sqrt{2}}$, and $\langle V|R\rangle = -\frac{i}{\sqrt{2}}$ have been used. Equation (9) tells us the principle of hybrid entanglement swapping: a spin Bell-state measurement of photons 1 and 2 - each from two pairs of entangled photons - results in the entanglement of photons 1' and 2' in OAM degree of freedom, even though they never interacted with each other in the past. Alternatively, one can also interpret this entanglement swapping as the teleportation of quantum information from a spin Bell state to an OAM Bell state. The practical difficulty lies in unambiguously identifying the four Bell states $|\Phi^{(\pm)}\rangle$ and $|\Psi^{(\pm)}\rangle$. Without losing generality, it is however sufficient to project photons 1 and 2 onto $|\Psi^-\rangle_{12}$ by superposing them at a non-polarizing beam splitter (BS) and registering coincidence counts between the outputs of BS [39]. In this case, the remaining photons 1' and 2' will be correspondingly projected to

$$|\Gamma^-\rangle_{1'2'} = \frac{1}{\sqrt{2}}\left(|k\rangle_{1'}|-l\rangle_{2'} - |-k\rangle_{1'}|l\rangle_{2'}\right), \qquad (10)$$

which is just an arbitrary OAM Bell state we are looking for.

In order to verify the quantum nature of the hybrid entanglement swapping, we could quote the violation of Clauser-Horne-Shimony-Holt (CHSH)-Bell inequality as an indicator [40]. To do this, we investigate the rotational effect of q-plates on the fourth-order coincidence counts. Label β_1 and β_1 as the rotational angles of QP_1 and QP_2, respectively, and thus we know the quantum operator of Eq. (1) becomes

$$\hat{Q}(q) = e^{-i2(q-1)\beta}|R,m+2q\rangle\langle L,m| +$$
$$e^{i2(q-1)\beta}|L,m-2q\rangle\langle R,m|. \qquad (11)$$

Subsequently, the coincidence count will vary as a function of both β_1 and β_1,

$$P(\beta_1,\beta_2) \propto \cos^2(R_1\beta_1 - R_2\beta_2), \qquad (12)$$

where $R_1 = 2(q_1-1)$ and $R_2 = 2(q_2-1)$ denotes the folds of rotational symmetry of QP_1 and QP_2, respectively [41], see Fig. 1(a). The sinusoidal manner of Eq. (12) therefore undoubtedly predicts the violation of a suitable CHSH-Bell inequality with $S = 2\sqrt{2}$, following directly from the quantum mechanics.

B. B. Creation of multiphoton OAM GHZ states $(N \geq 3)$

Since the seminal work of Greenberger, Horne, and Zeilinger, the multiparticle maximally entangled states, known as GHZ states, has been receiving more and more attention [2]. They played a crucial role in fundamental
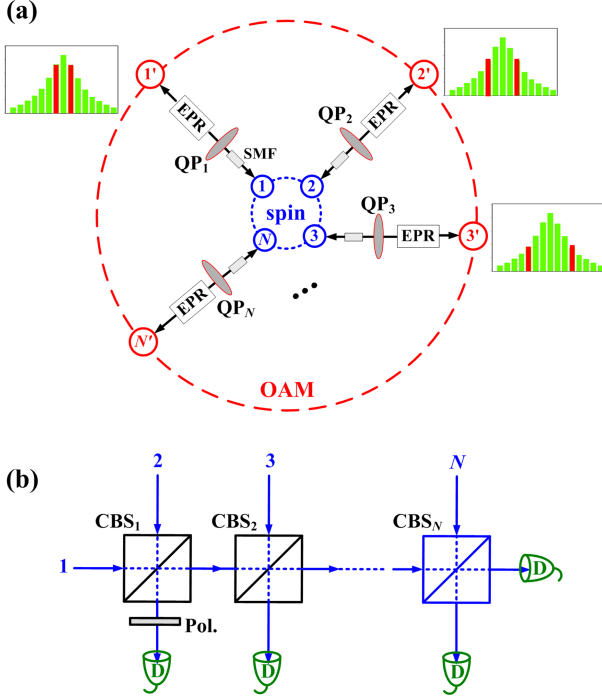
FIG. 3: (Color online). (a) Configuration of multiparticle hybrid entanglement swapping: Inner circle represents the spin GHZ projection and outer circle denotes the creation of an OAM GHZ state. Insets show the OAM occupation in the spiral bandwidth, which is determined by the use of each specific q-plate. (b) an N-particle spin GHZ state analyzer, where CBS are circular polarization beam splitters.

tests of quantum mechanics versus local realism [42–44] and in new quantum information protocols [45]. As inspired by Bose *et al* [11], we generalize our hybrid entanglement swapping to three or more photon pairs, which provides a possible method to produce the OAM version of GHZ states. The general scheme proposed is depicted in Fig. 3. There are N angular EPR entangled photons pairs serving as the prior quantum resources, each of which takes a similar form,

$$|\Psi\rangle_{i,i'}^{(0)} = \sum_\ell C_{\ell,-\ell} |\ell\rangle_i |-\ell\rangle_{i'} |H\rangle_i |H\rangle_{i'}. \qquad (13)$$

One member (i) of each pair is sent for the exchange center (the small inner circle) while the other (i') is sent to a distant user in the OAM-based quantum network to be constructed (the great outer circle). Before reaching the exchange center, each photon i passes through a q-plate (QP_i). Similarly to the case of $N = 2$, after interacting with the q-plates and post-selections by SMF, the combined state of the resultant N hybrid entangled photon pairs can be written as

$$
\begin{aligned}
|\Xi\rangle_N &= (\frac{1}{\sqrt{2}})^N \left(|R\rangle_1| + m_1\rangle_{1'} + |L\rangle_1| - m_1\rangle_{1'}\right) \\
&\otimes \left(|R\rangle_2| + m_2\rangle_{2'} + |L\rangle_2| - m_2\rangle_{2'}\right) \otimes \cdots \\
&\otimes \left(|R\rangle_N| + m_N\rangle_{N'} + |L\rangle_N| - m_N\rangle_{N'}\right), \quad (14)
\end{aligned}
$$

where $m_i = 2q_i$. Key to our procedure is that we could rewrite Eq. (14) in terms of the generalized spin Bell states of photons $1, 2, \cdots, N$ and the generalized OAM Bell states of photons $1', 2', \cdots, N'$, which reads

$$
\begin{aligned}
|\Xi\rangle_N &= \sum \left(\prod_{i=1}^N |\sigma(i)\rangle_i + \prod_{i=1}^N |\sigma^\perp(i)\rangle_i\right) \\
&\otimes \left(\prod_{i'=1}^N |m(i')\rangle_{i'} + \prod_{i=1}^N |-m(i')\rangle_{i'}\right). \quad (15)
\end{aligned}
$$

where $\sigma(i)$ stand for binary spin variables $\in L, R$ and $\sigma^\perp(i)$ denotes its complement, namely, *if $\sigma(i) = R$ (or L), then $\sigma^\perp(i) = L$ (or R)*. Besides, the correspondence between $|\sigma(i)\rangle_i$ and $|m(i')\rangle_{i'}$ are mapped as follows: for an arbitrary i, *if $\sigma(i) = R$ (or L), then $m(i') = m_i$ (or $-m_i$)*. There are 2^N different sets in the sum of Eq. (15) and $2^N = \sum_{j=1}^N \binom{N}{j}$, where $\binom{N}{j}$ is the number of different sets of cat states with j particles of state $|L\rangle$ and other $N - j$ particles of state $|R\rangle$. Of particular importance is that Eq. (15) also reveals that if we bring photons $1, 2, \cdots, N$ together and perform a joint generalized Bell-state measurement upon them, we know photons $1', 2', \cdots, N'$ will end up with an generalized OAM Bell state. Owing to the orthogonality of these generalized Bell states, we are able to formulate from Eq. (15) this procedure as

$$
\begin{aligned}
&\left(\prod_{i=1}^N \langle\sigma(i)|_i + \prod_{i=1}^N \langle\sigma^\perp(i)|_i\right) |\Xi\rangle_N \\
&= \left(\prod_{i'=1}^N \langle m(i')|_{i'} + \prod_{i=1}^N \langle-m(i')|_{i'}\right). \quad (16)
\end{aligned}
$$

Using polarizing beam splitters (PBS) and half-wave plates, Pan and Zeilinger [46] has presented an analyzer to conveniently identify two of the generalized N-particle GHZ states, $|\Phi^\pm\rangle_{123} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3 \pm |V\rangle_1|V\rangle_2|V\rangle_3)$. Inspired by their idea, if we replace PBS with circular polarization beam splinters (CBS), we are able to identify $|\Phi^\pm\rangle_{123} = \frac{1}{\sqrt{2}}(|L\rangle_1|L\rangle_2|L\rangle_3 + |R\rangle_1|R\rangle_2|R\rangle_3)$. As shown in Fig. 3(b), each CBS is configured such that it transmits L while reflects R polarization photons. If the photons are perfectly overlapped spatially and temporally at each CBS, then a three-fold coincidence count will make $|L\rangle_1|L\rangle_2|L\rangle_3$ and $|R\rangle_1|R\rangle_2|R\rangle_3$ indistinguishable. Along the same line, when $N - 1$ CBS are cascaded, we will be able to project photons $1, 2, \cdots, N$ to $|\Phi^\pm\rangle_{12\cdots N} = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^N |L\rangle_i \pm \prod_{i=1}^N |R\rangle_i\right)$. Correspondingly, we conclude from Eq. (16) that the

remainder of $1', 2', \cdots, N$ will fall into

$$|\Omega^{(+)}\rangle_{1'2'\ldots N'} = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{N}|m_i\rangle_{i'} + \prod_{i=1}^{N}|-m_i\rangle_{i'}\right) \text{ or}$$

$$|\Omega^{(-)}\rangle_{1'2'\ldots N'} = \frac{1}{\sqrt{2}}\left(\prod_{i=1}^{N}|m_i\rangle_{i'} - \prod_{i=1}^{N}|-m_i\rangle_{i'}\right), (17)$$

Obviously, equations (17) have exhibited the multiphoton multidimensional OAM GHZ entanglement. Owing to the discrete and high-dimensional nature of the OAM spiral bandwidth, it can thus be considered as a mutiple-qudit entanglement, in contrast to the spin GZH states only associated with the multiple-qubit entanglement. As illustrated by Fig. 2(a), employing different q-plates (QP$_i$), we could create spin-OAM hybrid entangled states with different OAM numbers of $m_i = 2q_i$, occupying different locations in the bandwidth. For example, if we choose $q_1 = 1/2$, $q_2 = 1$ and $q_3 = 3/2$, then we can obtain a three-photon OAM GHZ state, $|\Omega^{(+)}\rangle_{1'2'3'} = (|1\rangle_{1'}|2\rangle_{2'}|3\rangle_{3'} + |-1\rangle_{1'}|-2\rangle_{2'}|-3\rangle_{3'})/\sqrt{2}$. Therefore, our scheme provides a feasible method to establish a multiphoton multiple-qudit entanglement. We anticipate that such multiphoton multiple-qudit entanglement will be a fascinating quantum system, which holds promise to reveal a much stronger violation of EPR locality and motivate some new quantum information protocols. An interesting example is that our hybrid entanglement swapping suggests a new model of OAM-based quantum repeater to establish the OAM entanglement between two or more distant nodes in a quantum network [13].

III. III. CONCLUSION

In contrast to that with only a single pair of hybrid entangled photons, we have reported in this paper the creation of multiple pairs of spin-OAM hybrid entangled photons with different OAM numbers. We are therefore able to combine these multiple hybrid entangled photon pairs with entanglement swapping to realize the hybrid entanglement swapping from multiphoton spin entangled qubits to multiphoton OAM entangled qudits. It is illustrated that our scheme provides a feasible and promising approach to create the two-photon OAM Bell states or multiphoton OAM GHZ states. We anticipate such multiphoton multiple-qudit entanglement has potential applications in both fundamental and applied quantum information field.

IV. ACKNOWLEDGEMENTS

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