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Long-lived metastable-qubit memory

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Coherent storage of quantum information is crucial to many quantum technologies. Long coherence times have been demonstrated in trapped-ion qubits, typically using the hyperfine levels within the ground state of a single ion. However, recent research suggests qubits encoded in metastable states could provide architectural benefits for quantum information processing, such as the possibility of effective dual-species operation in a single-species system and erasure-error conversion for fault-tolerant quantum computing. Here we demonstrate long-lived encoding of a quantum state in the metastable states of a trapped ion. By sympathetically cooling with another ion of the same species, employing a tuned CPMG dynamical-decoupling sequence, and constantly monitoring for leakage errors, we demonstrate a coherence time of 136(42) seconds with a qubit encoded in the metastable $5D_{5/2}$ state of a single $^{137}\text{Ba}^+$ ion. In agreement with a model based on empirical results from dynamical-decoupling-based noise spectroscopy, we find that dephasing of the metastable levels is the dominant source of error once leakage errors are removed.

Qubits encoded in metastable states of atomic systems show great potential for quantum information processing [1–5]. Such states provide access to additional electronic manifolds with high isolation from the more generally used ground-state manifold—coherent and dissipative operations may be performed almost completely independently in these different subspaces, leading to novel utilization of trapped ions and neutral atoms for quantum logic. For instance, the flexibility that comes with access to separate sets of states suggests new architectures [6, 7] for quantum computing that can circumvent some of the challenges presented by dual-species methods, including those stemming from mass differences between the constituent qubit species and increased control overhead.

One possible limitation of metastable states for quantum information processing when compared to the often-employed ground states is the set of additional channels for scattering and decay that the former possess. Perhaps counterintuitively however, using metastable states could be more advantageous for erasure error conversion [4, 8, 9], due to the much smaller branching ratio for scattering back to the qubit space. It also provides a simpler scheme for erasure error correction when compared to ground-state qubits, since the latter would require coherent transfer of the qubits before leakage detection. These properties strengthen the prospect of quantum fault tolerance, as erasure quantum error correcting codes can have very high thresholds, due to the fact that erasure errors are easier to detect than more general Pauli errors [10]. Thus, it could be advantageous to use metastable states for stimulated-Raman-based quantum logic [6, 8], even with the challenges caused by spontaneous decay. It is therefore of significant interest to investigate the behavior of metastable-states when storing quantum information, especially in the presence of other atomic systems that are being used for ancillary operations relevant to carrying out quantum algorithms and quantum error correction.

Previous pioneering investigations have demonstrated few-to-tens-of-millisecond-scale [11, 12] to second-scale [13] co-

herence in metastable levels in neutral atoms and ions, respectively. However, such times are not commensurate with the longer coherence times shown for ground state qubit encodings, which range from minutes to hours [14, 15], and on the order of a minute [16], with and without dynamical decoupling, respectively. This leaves missing the experimental validation of metastable-state qubit coherence times sufficient for effective use in high-fidelity quantum-state storage and manipulation, despite general awareness that metastable state qubits could be competitive with ground state qubit encodings, given sufficient experimental advances.

Here we demonstrate the storage of a quantum state in the metastable $D_{5/2}$ level of a $^{137}\text{Ba}^+$ ion with a coherence time of 136 (42) seconds. Going beyond previous implementations of a metastable qubit memory, we combine four experimental techniques which have not been employed together before, to enable this result. Specifically, we (a) apply tuned Carr-Purcell-Meiboom-Gill (CPMG) sequences and (b) perform simultaneous leakage detection for (c) a hyperfine qubit controlled with RF signals, while (d) cooling the memory ion sympathetically with another ion of the same species. Our measured coherence time represents the lifetime of a metastable qubit with dynamical decoupling, removing influence from spontaneous decay. This memory time is approximately four times the metastable state’s lifetime, and is comparable to ground-state memory times. The use of mid-algorithm measurement allows us to observe leakage errors explicitly to extrapolate the expected qubit lifetime that an erasure error correction scheme should provide, and hence validates the use of metastable states for high-fidelity quantum logic more generally.

While we use leakage detection to evaluate only experimental trials in which the metastable qubit does not decay to the ground state, we point out that the method demonstrated here is generally useful, well beyond post-selected experiments. Crucially, since the process of monitoring for leakage errors does not affect the memory qubit, the signal from the leakage detection can be used in real-time to verify that no error has

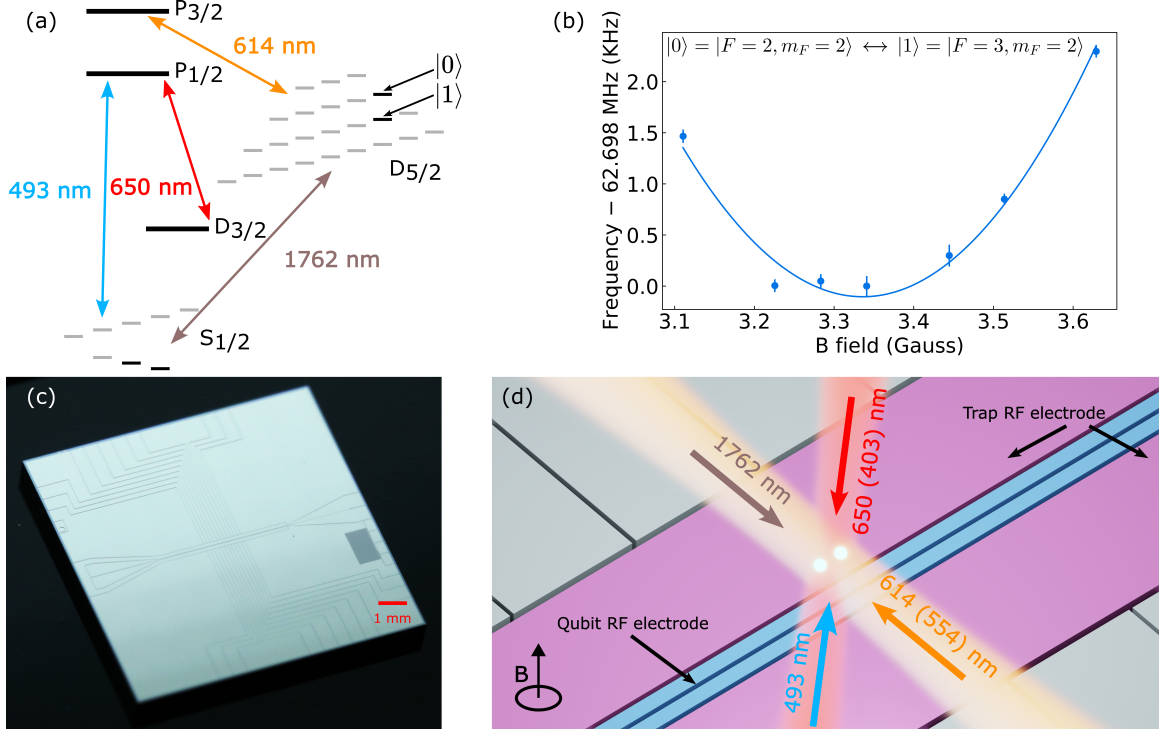


FIG. 1. Experiment setup and level structure. (a) Level diagram of $^{137}\text{Ba}^+$. The highlighted states in the $D_{5/2}$ and $S_{1/2}$ states are the memory qubit and cabinet shelving [17] states respectively. The hyperfine structure of other levels is neglected for clarity. (b) Metastable qubit transition frequency vs. ambient magnetic field. The solid line is a fit to the data with free parameters being the curvature, an overall frequency shift, and a conversion factor between coil current and magnetic field strength. (c) A photograph of the microfabricated trap used in this work. (d) Surface electrode trap and laser configuration. The electrodes for applying RF currents to drive transitions between hyperfine states are marked in blue and the trapping RF electrodes are marked in purple. The DC electrodes are marked in grey. The laser propagation directions are labeled by arrows. Lasers propagating in the same direction are combined and labeled with parentheses. All beams address both ions equally.

occurred [18] and to continue a quantum algorithm; this is to be contrasted with post-selection in a system in which measurement outcomes can be acted upon only after the quantum information is destroyed. A key consequence is that the long coherence time measured here could be realistically utilized for quantum information processing if leveraged as part of an erasure-based error-correcting code.

As we utilize OMG [6]/dual-type [7] encoding in these experiments, individual addressing is required to change the qubit type of specific ions starting in the nominally same state after ion initialization. This is achieved here using a simple micromotion-assisted individual addressing technique and projection. Furthermore, metastable-qubit transition energies are subject to environmental perturbations, chief among these being magnetic field fluctuations. We mitigate dephasing from this source by encoding in a field-insensitive qubit, by surrounding the ion with a superconducting shield, and by applying dynamical decoupling to the memory qubit.

We use $^{137}\text{Ba}^+$ ions for both the memory and the ancilla, with the relevant level structure shown in Fig. 1(a). The memory qubit is encoded in the metastable $D_{5/2}$ level, which has a lifetime of $T_D = 30.14(30)$ s [19]. Hyperfine sublevels $|F=2, m_F=2\rangle$ and $|F=3, m_F=2\rangle$ are the $|0\rangle$ and $|1\rangle$ qubit states, respectively. Based on the measured properties of this

species [20], we expect the qubit transition frequency to be approximately 62.7 MHz and first-order insensitive to magnetic field fluctuation at a magnetic field of 3.34 G. To verify the magnetic field sensitivity, we perform spectroscopy at different ambient fields and the result is shown in Fig. 1(b). The second-order field sensitivity is calculated to be $32.13 \text{ kHz/Gauss}^2$.

As shown in Fig. 1(c,d), we use a microfabricated, surface-electrode, RF Paul trap in a cryogenic system (see more detail in [21]) for confining the ions. The ion-electrode distance is approximately $50 \mu\text{m}$ and the trap RF drive is at 36 MHz. The trap is cooled to 6 K. For driving RF transitions within the $D_{5/2}$ manifold, we apply an RF current of a few milliamperes through the middle electrodes (marked in blue), with the other end of the electrodes shorted to ground. Only one of these electrodes is used in this manner for the results presented here.

Two coils mounted on the intermediate (50 K) stage of the cryocooler in a near-Helmholtz configuration are used for generating a magnetic field perpendicular to the trap surface. There are four rings of niobium (superconducting critical temperature approximately 9.2 K) shielding placed co-axially to the trap-chip normal above and below the chip, with central circular holes of various sizes. These rings significantly suppress magnetic field variation along the vertical direction

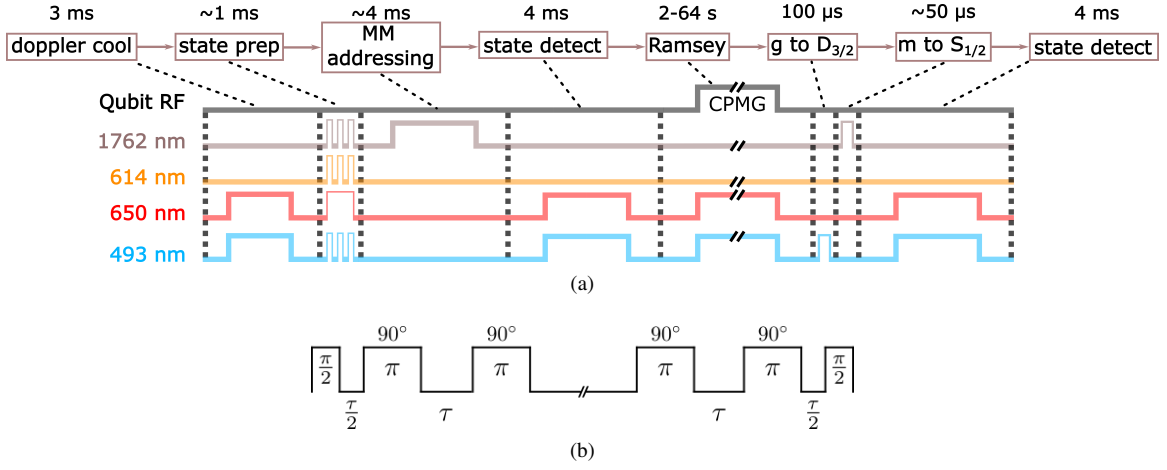


FIG. 2. Experiment sequence and dynamical decoupling scheme. (a) Eight-step operation sequence for sympathetically cooled quantum memory experiment. "MM": micromotion; "g": ground state qubit; "m": metastable qubit. (b) The CPMG sequence. π pulses with 90° phases relative to the initial $\pi/2$ pulse are applied interspersed with a delay time τ which sets the filter function that suppresses (or samples) the noise spectrum.

(chip-normal). More information about, and characterization of, the niobium shielding can be found in the supplementary material [22].

For loading ions, two photons at 554 nm and 403 nm [23] ionize atoms produced by an effusive oven. For Doppler cooling and state detection, the 493 nm laser is frequency-modulated using an electric-optical modulator (EOM). The laser frequency is set so that the carrier drives the $S_{1/2} |F=2\rangle$ to $P_{1/2} |F=2\rangle$ transition, and two microwave tones at approximately 1.47 GHz and 4.0 GHz drive the other three transitions between the hyperfine-split $S_{1/2}$ and $P_{1/2}$ levels. The two tones are switchable using RF switches to allow for state preparation [17] in a particular sublevel. For repumping population from the $D_{3/2}$ state, the 650-nm laser is also modulated by an EOM with two tones so that, along with the carrier, these tones drive the $F=2 \leftrightarrow 1$, $1 \leftrightarrow 1, 0 \leftrightarrow 1$, $3 \leftrightarrow 2$ transitions between the $D_{3/2}$ and $P_{1/2}$ level.

For the 614 nm laser, which is used to quench the $D_{5/2}$ state, two acousto-optic modulators (AOMs) set up in a double-pass configuration are used for frequency modulation and switching. We find that it is sufficient to have a single frequency tone for effective quenching of all the sublevels in the $D_{5/2}$ state. An optical shutter is added to the beam path for higher extinction of the 614 nm laser beam during memory-qubit coherence measurements, to avoid unwanted AC Stark shifts and quenching of the $D_{5/2}$ state. A 1762 nm laser locked to a high finesse cavity (StableLaser Systems) and amplified by a thulium-doped fiber amplifier (Cybel, Perseus) is used for state preparation and pre-readout shelving of the memory qubit.

The pulse sequence shown in Fig. 2a is used for experiments with two ions. The ions are first Doppler-cooled using the 493 nm and the 650 nm laser beams. We then prepare both ions in the $S_{1/2} |F=1, m_F=0\rangle$ state using a similar scheme as in [17], except we do not perform a polarization-based step

with the 493 nm laser. To have a consistent assignment of cooling and memory ion, a micromotion-assisted individual addressing technique [22] is used to excite the memory ion to the $D_{5/2}$ state. State detection is conducted to verify that only one ion is shelved to the $D_{5/2}$ state. The process is repeated until a one-ion-dark event is detected.

To measure the memory-qubit coherence time, an RF current is sent to the through-wire electrode to drive the transition between the $|0\rangle$ and $|1\rangle$ states. The radio-frequency signal is generated via direct digital synthesis (DDS, M-labs, Urukul) synchronized to an ovenized crystal oscillator, and we drive pulses to implement a CPMG dynamical decoupling sequence [24], a modified Ramsey sequence [see Fig. 2b], tuned to the measured noise spectrum. The Doppler cooling beams are kept on during the Ramsey sequence for sympathetic cooling and leakage detection. Without the cooling beams, the memory ion's temperature will increase due to heating from the trap, which lowers the fidelity of the 1762 nm pulses and could lead to ion loss, especially at longer delay times. To detect leakage errors, we measure scattered photons using a photomultiplier tube every 4.05 ms to check if the memory ion decayed from the $D_{5/2}$ state. The sequence is terminated when spontaneous decay is detected via fluorescence from both ions and the trial is discarded. The ratio of discarded trials to total trials increases exponentially as a function of delay time, and is approximately 92% for the measurement at a delay of 64 s.

After the Ramsey sequence, the qubit $|0\rangle$ state is shelved back to the ground state for state detection. To avoid shelving the cooling ion to the $D_{5/2}$ state during the same operation, the 493 nm laser is first switched on for 1 ms to shelve the cooling-ion population into the $D_{3/2}$ state. Two "cabinet shelving" pulses [17] are then used to drive the $|0\rangle$ population to the $S_{1/2} |F=1, m_F=0, 1\rangle$ states to achieve a higher shelving fidelity. The state of the memory qubit is then extracted by fluorescence detection (the cooling ion remaining dark).

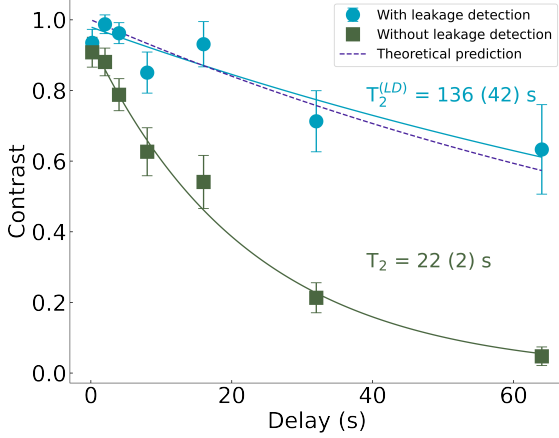


FIG. 3. Experimental measurement of qubit coherence with no leakage detection (green squares), with real-time leakage detection (blue circles), and expected coherence time based on measured noise spectrum (purple dashed line) [22]. A coherence time of 136(42) seconds is measured with leakage detection, while 22(2) seconds is measured without leakage detection, limited by decay from the qubit states [see text]. Error bars correspond to one standard deviation. Solid lines are exponential decay fits to the data points.

	Bare Ramsey	CPMG	CPMG + LD
Exp.	1.7(4) s	22(2) s	136(42) s
Predicted	3 s	24 s	116 s

TABLE I. Coherence times under different conditions. Predictions are based on the noise spectroscopy and analysis described in the text. LD: leakage detection.

The result of the coherence time measurement with leakage detection $T_2^{(LD)}$ is shown in Fig. 3. A coherence time of 136(42) s is extracted from an exponential fit. We also perform the experiment without leakage detection, and the coherence time in that case, T_2 , is measured to be 22(2) s. These two coherence times taken together are consistent with the lifetime of the $D_{5/2}$ state T_D if we assume that the coherence time of the metastable qubit in the presence of decay (T_2) is dependent on the combination of decay from the qubit states and pure dephasing, the latter characterized by a time T_ϕ . In that case, we would assume the relationship $1/T_2 \geq 1/T_1 + 1/T_\phi$ (note that the appropriate decoherence rate due to state decay here is $1/T_1$ rather than $1/(2T_1)$ since both qubit states can decay). For $T_\phi = T_2^{(LD)} = 136(42)$ s and $T_1 = T_D$, we obtain $T_2 \leq 24.6(7.6)$ s for the decoherence time in the presence of leakage, consistent with the measured 22(2) s. A comparison of the coherence times under different experimental conditions can be found in Table I.

The long coherence time we obtain is made possible by a detailed characterization and modeling of our noise spectrum (via a noise spectroscopy analysis [22]), which also in-

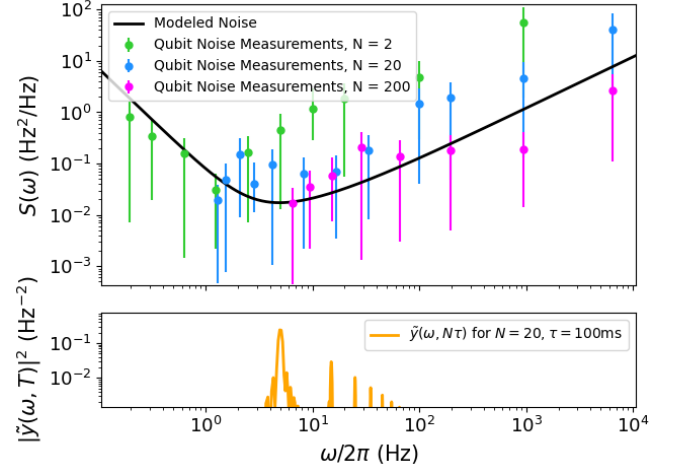


FIG. 4. Qubit-noise spectroscopy and fitted qubit-splitting frequency-noise spectrum. The green, blue, and pink data points signify noise amplitudes based on coherence measurements of the metastable qubit using CPMG sequences of varying length and number of pulses N . The noise spectrum, modeled as described in the text, is a fit to these data. The effective filter function of the CPMG sequence used in the coherence-time measurement shown in Fig. 3 is depicted in the lower panel.

forms our understanding of achievable limits. Fluctuations of the qubit energy splitting are modeled by including a dephasing term $\beta(t)$ in addition to the bare qubit splitting ω_0 in the Hamiltonian $H = \frac{\hbar}{2}(\omega_0 + \beta(t))\sigma_z$, where σ_z is the usual Pauli matrix. This model is then used to analyze the set of CPMG pulse sequences, each consisting of a series of N total π pulses spaced by time τ , which are applied to empirically estimate the frequency noise spectrum $S(\omega)$ associated with the noise process $\beta(t)$ via qubit-noise spectroscopy[14].

The effect of applying a CPMG sequence is similar to applying a filter to the noise spectrum with a center frequency of $\omega_c = 2\pi \times \frac{1}{2\tau}$ and width $1/T$, for total delay time $T = N\tau$, as described by a frequency domain filter function $|\tilde{y}(\omega, T)|$. For the set of noise spectroscopy CPMG sequences, we vary N and measure the coherence decay $\chi(T)$:

$$\chi(T) = \int_0^\infty S(\omega) |\tilde{y}(\omega, T)|^2 d\omega \quad (1)$$

[see [22] for a full derivation]. We fit (1) with experimental coherence decay data points $\{\chi_{\text{exp}}^{(N)}(T)\}$ to extract $S(\omega)$ at measured total delays as shown in Fig. 4. These data provide an empirical power-law spectrum that has a minimum at $\omega \approx 2\pi \times 5$ Hz and increases rapidly at both lower and higher frequencies. Using this spectrum and the applied CPMG sequence parameters for the coherence time measurement ($\tau = 0.1$ s), we predict a coherence decay as shown by the dashed line in Fig. 3, consistent with the measured coherence.

Environmental noise and imperfections in control fields both contribute to the observed noise spectrum. The qubit

drive is performed via direct digital synthesis and this was carefully considered as a high-frequency noise source, but it is filtered and its measured phase noise is too low to account for the full noise amplitude. Magnetic-field fluctuations also contribute and were characterized, since the attenuation provided by our superconducting shield decreases with increasing frequency.[25].

Low-frequency noise is dominated by fluctuations in the amplitude of the applied RF voltage used for ion confinement. The asymmetry of the trap-electrode structure in the vertical (z) direction leads to uncanceled magnetic fields arising from the current in the trap-RF electrodes. Due to the small frequency difference between the trap RF frequency and the qubit frequency, this magnetic field off-resonantly couples to the qubit and causes AC Zeeman shifts. Fluctuations of the RF amplitude thus lead to qubit-frequency fluctuations. This noise could be suppressed by active stabilization [26]. Variation of the trapping field also induces frequency fluctuation due to the quadrupole moment of the $D_{5/2}$ state [27], but we calculate [22], that this contributes insignificantly. Decoherence also arises from spontaneous Raman photon scattering from the Doppler cooling light [28], which is also applied during the memory storage period, but this effect is negligible due to the large detunings from relevant transitions [22].

The coherence time observed here for clock states in metastable levels of a trapped ion is comparable in scale to those achievable in the ground-state manifolds of atoms. This suggests that such a state encoding could be useful for quantum computing, for instance, using multiple $^{137}\text{Ba}^+$ ions as a register. As shown here, select ions could be addressed to encode metastable-state qubits, while other ions left in the ground-state manifold could be used for sympathetic cooling, allowing simultaneous periodic erasure-error detection and qubit readout. Such a mid-circuit measurement capability indicates quantum error correction could also be applied, providing significant functionality in a single-species system. This demonstration of several key OMG / dual-type-architecture elements thus provides concrete steps toward the realization of trapped-ion quantum computing at scale.

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