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# Simulating Heisenberg Interactions in the Ising Model with Strong Drive Fields

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The time-evolution of an Ising model with large driving fields over discrete time intervals is shown to be reproduced by an effective XXZ-Heisenberg model at leading order in the inverse field strength. For specific orientations of the drive field, the dynamics of the XXX-Heisenberg model is reproduced. These approximate equivalences, valid above a critical driving field strength set by dynamical phase transitions in the Ising model, are expected to enable quantum devices that natively evolve qubits according to the Ising model to simulate more complex systems.

## INTRODUCTION

With the recent advances in digital and analog quantum computers and simulators, there is a growing effort toward mapping quantum field theories onto arrays of qubits and, more generally, qudits [1–68]. While classical computers are used successfully in studies of the static properties of quantum systems, time dependent evolution and finite-density systems typically suffer from non-polynomial scaling computational resource requirements [69–71]. This places important quantities of interest in Standard Model physics, and in many other areas, beyond the capabilities of classical computation. In contrast, quantum simulations are expected to be able to address some of these quantities with polynomially scaling computational costs [72, 73], and quantum advantages in scientific applications are being sought. A number of platforms for digital quantum computation are being developed, such as those utilizing superconducting qubits, optical qubits, quantum dots, trapped ions and Rydberg atoms [74–84]. In addition to performing universal digital quantum computations, these platforms can be used for analog simulations of systems that can be mapped onto their native Hamiltonians. This approach is more limited than universal digital quantum computation, but recent work suggests that the error rates on existing hardware may be low enough to enable a useful quantum advantage through analog simulation [85]. While the study of spin systems, and systems comprised of finite-dimensional Hilbert spaces more generally, are interesting by themselves, they are now central to advancing quantum simulations for scientific applications.

Despite the apparent simplicity of a regular lattice of qubits or qudits, it has been widely appreciated since the discovery of quantum mechanics that spin systems exhibit a wide variety of non-trivial properties and dynamics. One of the most studied spin systems is the Heisenberg model which is able to describe critical points and phase transitions in magnetic materials. In addition to condensed matter applications, the Heisenberg model also has an important role in high energy physics as, for example, it can be used to study the lattice  $O(3)$  nonlinear  $\sigma$  model [61–65, 86–88]. This theory is one of the “sandboxes” used to better understand quantum chromodynamics (QCD) as it shares a number of qualitative aspects such as asymptotic freedom and  $\theta$ -vacua [63–65]. In

the electroweak sector, the dynamics of coherent collective flavor oscillations of neutrinos can be mapped to the Heisenberg model [89, 90]. This is of particular importance in extreme astrophysical environments, where neutrino-neutrino interactions can be significant. After extensive studies using classical computers, including of entanglement, for example Refs. [91–99], it is expected that quantum simulations [98, 100–104] of the Heisenberg model will enable the study of dynamics beyond the reach of classical computers and provide new insights into these problems. Due to this wide applicability, a number of methods for quantum simulation of the Heisenberg model have been developed, including digital approaches [105, 106], hybrid digital-analog approaches [107], and analog simulation on trapped ions [108], Rydberg atoms using dipole-dipole interactions [109] and nuclear spins [110, 111]. Dissipative versions of the Heisenberg model have also been studied on digital quantum computers [112, 113].

In this work, the potential for using Ising systems for analog simulations of physical systems that can be mapped to the Heisenberg model is investigated. The Ising model is considered because a number of platforms available for quantum simulation, such as Rydberg atoms, trapped ions and superconducting qubits, can be natively described by this Hamiltonian [114–122]. Unlike previous approaches, this method of analog simulation of the Heisenberg model can be implemented with a time-independent Hamiltonian which could be beneficial for some platforms. There is an extensive literature regarding the time evolution of Ising models in background fields [123–139], including recent work related to transitions to chaotic phases induced by finite-time steps in Trotterized time evolution in digital quantum simulations [138, 139], identified through studying the system at periodic times. Previous work has shown that Ising interactions with a transverse field generate time evolution according to the XY model [125–128, 140]. Studying the time evolution of the Ising model has also shown that the Ising model undergoes confinement analogous to QCD near its critical point with a field in the  $\hat{x}$  and  $\hat{z}$  directions [123, 124, 141]. Known to be universal in a computational sense [142], developing analog simulations of the Heisenberg model is expected to also advance simulations of other physical systems. We show how constant driving fields in the Ising model generate an effective Heisenberg model

Hamiltonian to leading order in the inverse field strength at periodic times. The systematic errors associated with the periodic dynamics are quantified, revealing that the convergence of the expansion in the inverse field strength is limited by the location of dynamical quantum phase transitions (DQPTs) in the Ising model with an external field.

### HEISENBERG FROM ISING WITH STRONG FIELDS

The Ising Hamiltonian with constant global driving fields is given by

$$\hat{H}^{\text{Ising}} = \sum_{i>j} J_{ij} \hat{Z}_i \hat{Z}_j + \frac{1}{2} \sum_i \Omega_x \hat{X}_i + \Omega_y \hat{Y}_i + \Omega_z \hat{Z}_i . \quad (1)$$

To analyze the evolution of this Hamiltonian, it is helpful to transform into the interaction picture, where the driving fields are taken to be the “free” term in the Hamiltonian. The interaction-picture Hamiltonian is

$$\hat{H}_I^{\text{Ising}}(t) = \sum_{i>j} J_{ij} \hat{Z}_{I,i}(t) \hat{Z}_{I,j}(t) , \quad (2)$$

where  $\hat{Z}_{I,i}(t) = \hat{U}_0^\dagger(t) \hat{Z}_i \hat{U}_0(t) = \vec{e}(t) \cdot \vec{S}$ ,  $\hat{U}_0(t) = \prod_j e^{-it(\Omega_x \hat{X}_j + \Omega_y \hat{Y}_j + \Omega_z \hat{Z}_j)}$ ,  $\vec{S}$  is a vector of Pauli matrices and  $\vec{e}(t)$  is a unit vector. From this perspective, the driving fields can be viewed as rotating  $\vec{e}(t)$  from the north pole to other points on the unit sphere. By choosing periodic driving fields that generate closed paths on the sphere, it is possible to engineer evolution according to different Hamiltonians. The use of periodic dynamics to generate different Hamiltonians, known as Floquet engineering, has been used to simulate a range of interactions [108, 109, 143–153], including the Ising Hamiltonian from the Heisenberg interaction in quantum-dot systems [154, 155]. Floquet engineering has also been previously applied to static Hamiltonians in the interaction picture to understand how some systems prethermalize to a Hamiltonian that is not the generator of their evolution [156–159]. In particular, it has been used to show that the dynamics of the XYZ-Heisenberg model with a strong external field are approximated by the XXZ-Heisenberg model for times that are exponential in the driving field [156]. We will show that in the Ising model, evolution according to the XXZ-Heisenberg Hamiltonian can be approximated by taking  $\Omega_x = \Omega \sin \theta$ ,  $\Omega_y = 0$ , and  $\Omega_z = \Omega \cos \theta$ . With these driving fields, the interaction Hamiltonian becomes periodic over time intervals  $\frac{2\pi}{\Omega}$ , and the Schrodinger picture becomes equivalent to the interaction picture at these periods. A representative path generated by such fields on the unit sphere is shown in Fig. 1. The time-evolution of the system after discrete time intervals, and the associated Magnus expansion is given by

$$\begin{aligned} \hat{U}_F &= \mathcal{T} \exp \left\{ -i \int_0^{\frac{2\pi}{\Omega}} dt' \hat{H}_I^{\text{Ising}}(t') \right\} \\ &= \hat{U}_B^\dagger \exp \left\{ -i \frac{2\pi}{\Omega} \left( \hat{H}_1 + O \left( \frac{1}{\Omega} \right) \right) \right\} \hat{U}_B , \quad (3) \end{aligned}$$

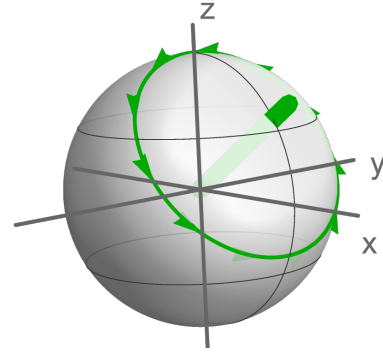


FIG. 1. A representative path on the unit sphere taken by  $\vec{e}(t)$  that generates time evolution according to the XXX-Heisenberg model. The green line corresponds to the direction of the driving field.

where

$$\hat{H}_1 = \sum_{i>j} J_{ij} \left[ \cos^2 \theta \hat{Z}_i \hat{Z}_j + \frac{\sin^2 \theta}{2} (\hat{X}_i \hat{X}_j + \hat{Y}_i \hat{Y}_j) \right] , \quad (4)$$

where  $\hat{U}_B$  is a local change of basis given by  $\hat{U}_B = \prod_j e^{+i\theta \hat{Y}_j/2}$  (that aligns the driving field with the  $z$  axis). Therefore, time evolution of the Ising model with this choice of driving fields approximates that of the XXZ-Heisenberg model between discrete intervals of  $\Delta t = \frac{2\pi}{\Omega}$ . Note that while the formalism of Floquet engineering was used to derive this result, the Hamiltonian is time independent and the periodicity is only manifest in the interaction picture. Also, the Ising model with a fast oscillating drive field could be used to simulate an XXZ-Heisenberg model because the dynamics of a transversely driven Ising model are equivalent to that of a time-independent Ising model with external fields in the  $\hat{z}$  and  $\hat{x}$  directions [129]. The  $O(\frac{1}{\Omega^2})$  higher-order terms in the Magnus expansion of the Floquet operator in Eq. (3) have one-body and three-body operators. The one-body operators can be eliminated by renormalization of the “free” Hamiltonian employed to transform to the interaction picture, but the three-body terms are a genuine deviation from the Heisenberg Hamiltonian. Such higher-order terms in the Magnus expansion can be removed through the use of time-dependent driving fields [160].

This approach to simulating the XXZ-Heisenberg model is similar in spirit to recent proposals for simulating gauge theories by adding terms to the Hamiltonian that generate gauge symmetries [161–165]. In these proposals, an energy penalty for breaking gauge invariance decouples the gauge invariant sector from the rest of Hilbert space analogously to how dynamical decoupling can be used to decouple systems from their environment [164]. In this work, the addition of driving fields to the Ising model can be interpreted as adding an energy penalty for violating the global  $U(1)$  symmetry generated by the driving fields. This causes the different symmetry sectors to decouple, leading to time evolution that can be described by the XXZ-Heisenberg model.

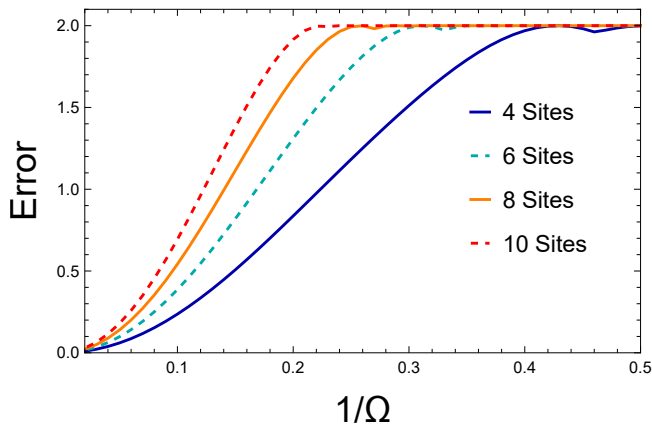


FIG. 2. The spectral norm (magnitude of the largest eigenvalue)  $\|e^{-i\frac{2\pi}{\Omega}\hat{H}_{\text{XXX}}^{\text{Heis}}} - \hat{U}_F\|$  of the difference between the XXX-Heisenberg chain time-evolution operator derived from Eq. (5) and the Floquet engineered approximation in Eqs. (3) and (4) as a function of the driving field strength  $\Omega$ , for a selection of chain lengths.

### BEYOND LEADING ORDER IN THE MAGNUS EXPANSION AND DYNAMICAL PHASE TRANSITIONS

The derivation of the approximate Heisenberg time evolution indicates that systematic errors from the Magnus expansion are suppressed by  $\Omega^{-1}$  compared to leading order. However, the Magnus expansion is known to have a finite radius of convergence, and a priori it is not obvious what the minimum value of  $\Omega$  is for the leading order term to accurately describe dynamics. As mentioned previously, numerical studies of digital quantum simulations have been used to show that Trotterized time evolution transitions into chaotic dynamics for sufficiently large time steps [138, 139]. In this context, the Floquet-period  $\Delta t = \frac{2\pi}{\Omega}$  is analogous to a Trotter time step, and we show that at small  $\Omega$  the breakdown of the Magnus expansion is associated with a dynamical quantum phase transition in the Ising model.

As an example, we focus on the special point  $\theta = \tan^{-1}\sqrt{2}$ , where the leading order Eq. (4) becomes an XXX-Heisenberg Hamiltonian with enhanced non-abelian  $O(3)$ -symmetry

$$\hat{H}_{\text{XXX}}^{\text{Heis}} = \frac{1}{3} \sum_i \hat{X}_i \hat{X}_{i+1} + \hat{Y}_i \hat{Y}_{i+1} + \hat{Z}_i \hat{Z}_{i+1}. \quad (5)$$

The systematic errors in the time evolution (of any state) are bounded by the spectral norm (magnitude of the largest eigenvalue) of the difference between the exact Heisenberg time-evolution operator and the Floquet engineered approximation given in Eqs. (3) and (4),  $\|e^{-i\frac{2\pi}{\Omega}\hat{H}_{\text{XXX}}^{\text{Heis}}} - \hat{U}_F\|$ . This is shown for the one-dimensional XXX-Heisenberg model with  $J = 1/3$  in Fig. 2 as a function of  $\Omega$  for varying chain lengths. At large values of  $\Omega$ , systematic deviations in the spectral norm decrease with increasing  $\Omega$  as predicted by the Magnus expansion. At small values of  $\Omega$ , the spectral norm saturates below a critical value  $\Omega_c$ . Unfortunately, the lattice sizes for which the spectral norm can be efficiently computed are not large enough to determine the scaling of  $\Omega_c$  with chain length. For

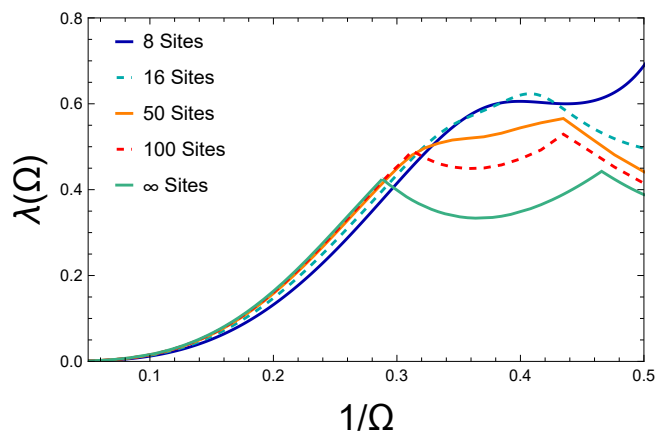


FIG. 3. The rate function, defined in Eq. 8, for the ground state of XXX-Heisenberg chains of different lengths.

longer chain lengths, Loschmidt echoes of the ground state of the XXX-Heisenberg model in Eq. (5),  $|\psi_G\rangle$ , time evolved over  $t = \frac{2\pi}{\Omega}$  with the driven Ising model,

$$\hat{H}^{\text{Ising}} = \sum_i \hat{Z}_i \hat{Z}_{i+1} + \frac{\Omega}{2\sqrt{3}} \left( \hat{Z}_i + \sqrt{2}\hat{X}_i \right), \quad (6)$$

are computed. If the time evolution of the XXX-Heisenberg model were perfectly reproduced by the driven Ising model, the Loschmidt echo, defined as the probability to return to the initial state, i.e.,

$$\mathcal{L}(\Omega) = \left| \langle \psi_G | e^{-i\frac{2\pi}{\Omega}\hat{H}^{\text{Ising}}} | \psi_G \rangle \right|^2, \quad (7)$$

would equal unity, and deviation from unity provide an estimate of contributions beyond leading order in the Magnus expansion. As  $\log \mathcal{L}(\Omega)$  is an extensive quantity, the rate function

$$\lambda(\Omega) = -\log(\mathcal{L}(\Omega)) / L, \quad (8)$$

is computed to compare chains of different lengths  $L$ , as shown in Fig. 3. For chains of  $L \leq 16$ , time evolution was computed using exact diagonalization. The ground states of the  $L = 50$  and  $L = 100$  chains were computed using DMRG and time evolution was performed using TDVP [166–171]. The ground state and time evolution of the infinite Heisenberg chain was computed using iTEDB [172–174]. At large  $\Omega$ ,  $\lambda(\Omega)$  decreases with increasing  $\Omega$ , indicating that the leading order Magnus expansion is correctly describing the dynamics of the model. This asymptotic behavior only occurs beyond a “kink” in  $\lambda(\Omega)$ , indicating that at small values of  $\Omega$  the Magnus expansion is failing to converge. The presence of a kink (non-analytic behavior) in  $\lambda(\Omega)$  is the defining characteristic of a dynamical quantum phase transition [130]. Note that other inequivalent definitions of dynamical phase transitions have been introduced in the literature [175]. DQPTs have previously been studied in spin systems and have been shown to be associated with unstable renormalization group fixed points [130–135, 176]. Our results show that the Ising model with a constant driving

field  $\vec{\Omega} = \Omega \left( \frac{1}{\sqrt{3}} \hat{z} + \sqrt{\frac{2}{3}} \hat{x} \right)$  undergoes a DQPT into a regime with an approximate  $O(3)$  symmetry at discrete time intervals, as seen in Fig. 3.

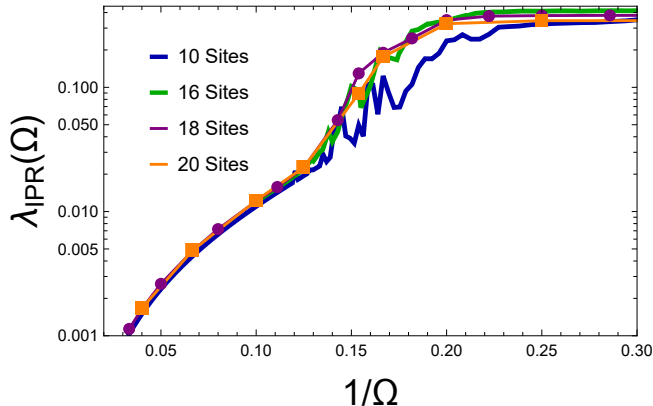


FIG. 4. The log IPR for the ground state of XXX-Heisenberg chains of different lengths. The IPR for the chain of length 10 was computed with exact diagonalization and the IPR for larger chains was computed by averaging the Loschmidt echo over 1000 periods.

These calculations demonstrate that for short time scales Heisenberg evolution is being successfully simulated, provided a sufficiently large driving field strength is used. However, this does not guarantee that the dynamics are reproduced at long times. Generically, periodically driven systems are expected to heat at late times [177–179], however in the context of digital quantum simulation it has been shown that quantum localization prevents this in Trotterized time evolution [138, 139]. The Floquet engineering technique used in this work uses a static Ising Hamiltonian so one would expect that at large field strengths the eigenstates of the Ising model are perturbatively close to those of the Heisenberg Hamiltonian. This would guarantee that long time dynamics are correctly reproduced as in the case of Trotterized time evolution. This perturbative argument can be verified through the calculation of the inverse participation ratio (IPR). For a given state,  $|\psi\rangle$ , and eigenstates,  $|n\rangle$  of some Hamiltonian, the IPR is defined by

$$\text{IPR} = \sum_n |\langle n|\psi\rangle|^4 . \quad (9)$$

The IPR measures how localized  $|\psi\rangle$  is relative to the eigenbasis  $|n\rangle$ . In practice, it can be evaluated by averaging the Loschmidt echo over long periods of time. To compare systems of different sizes, a normalized IPR, defined by  $\lambda_{\text{IPR}}(L) = -\frac{1}{L} \log(\text{IPR})$  for a chain of length  $L$ , was computed for the ground state of the XXX-Heisenberg model in Fig. 4. For the chain of length 10, the IPR was computed by explicitly evaluating Eq. (9), while for the larger chains the IPR was computed by averaging Loschmidt echos. As this figure shows, for large  $\Omega$  the log IPR is small which indicates the perturbative argument holds and the XXX-Heisenberg ground state is localized with respect to the Ising Hamiltonian. This

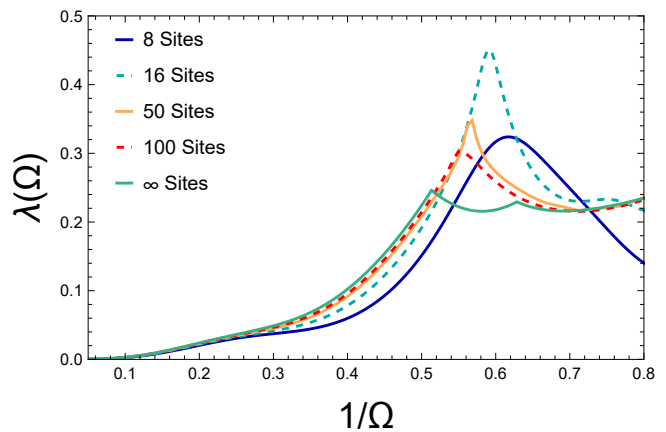


FIG. 5. The rate function for the ground state of XY-Heisenberg chains of different lengths.

indicates that the long time dynamics of the XXX-Heisenberg model is being successfully simulated with this technique.

When the constant driving field is taken to be in another direction, an approximate  $O(2)$  symmetry emerges. The arguments above suggest that there should be a DQPT that occurs in this case as well. As an example, the traditional one-dimensional transverse field Ising model with the driving field purely in the  $\hat{x}$  direction will generate evolution according to the XY-Heisenberg model (up to a change of basis),

$$\hat{H}_{XY}^{\text{Heis.}} = \frac{1}{2} \sum_i \hat{Y}_i \hat{Y}_{i+1} + \hat{Z}_i \hat{Z}_{i+1} . \quad (10)$$

The rate function  $\lambda(\Omega)$  for the ground state of  $\hat{H}_{XY}$  evolved under the transverse field Ising Hamiltonian for one period is shown in Fig. 5. As is the case for the XXX-Heisenberg model, there is a series of kinks indicating a DQPT before the rate function begins to decrease. It is interesting to note that the final kink is at  $\Omega^* \approx 1.948$  which is close to, but not quite at the critical point of the transverse field Ising model at  $\Omega = 2$ . While this work shows that the Ising model can be used to simulate  $\hat{H}_{XY}$  in the strong field limit, it has been shown in previous work that in  $2+1$  dimensions, the weak field limit of the Ising model also reproduces the dynamics of the XY-Heisenberg model [136, 137].

These results explicitly show that in 1D, this technique can be used to simulate Heisenberg model physics for long times with a driving field that is not extensive with the system size. While there are classical computational tools that enable the study of large 1D systems such as tensor networks, simulating real time evolution in 1D systems still has computational costs that grow exponentially with time and analog quantum simulation may be of practical use. While these calculations were only performed for 1D, this technique can also be applied to simulate higher dimensional Heisenberg models and it is likely that the required driving field strength for simulating dynamics accurately is not extensive with the system size as well. Even if the required driving field strength is extensive with system

size in higher dimensions, this technique may still be of practical importance as real time evolution for even modest sized 2D systems is difficult for classical computers. In addition to enabling analog quantum simulation of the Heisenberg model on platforms with Ising interactions such as Rydberg atoms and superconducting qubits, this technique could be combined with the results of Ref. [142] to potentially perform analog quantum simulation of an arbitrary Hamiltonian. This could potentially enable analog simulations of any quantum field theory of physical interest on these platforms.

## DISCUSSION

In this work, a new method for analog simulation of the Heisenberg model has been proposed that can be implemented on platforms whose natural evolution is described by the Ising model with constant external fields. For a specific driving field, the time-evolution operator of the Ising model is approximately that of the Heisenberg model over periodic time intervals. Interestingly, the leading-order effective Heisenberg operator has enhanced symmetry over the intrinsic Hamiltonian.

The systematic errors associated with this method at small external-field strength are limited by non-analytic behavior in the Ising model, associated with dynamical quantum phase transitions, which indicates the Magnus expansion is failing to converge. Beyond a critical value of the driving field, the effective Hamiltonian describing the time-evolution can be determined from a Magnus expansion, with each increasing order in  $1/\Omega$  introducing operators involving an increasing number of spins.

The technique presented in this work could be implementable on a range of quantum devices, including systems of Rydberg atoms or even superconducting qubits, in any dimension. While Rydberg systems are very promising as analog quantum simulators, one of the main challenges is that natively they offer a very narrow class of interactions, which severely limits their applicability. Therefore, being able to engineer new interactions makes an important step forward in expanding the systems that can be studied on these platforms. Furthermore, unlike previous proposals for quantum simulations of Heisenberg models, this method can be implemented using time-independent fields, which is extremely important for experimental platforms with a limited slew rate for the external fields. In the near term, analog quantum simulations will be the only method of probing the long time dynamics of large systems. The dynamics of the Heisenberg model are of particular interest, not only for condensed matter applications, but also for high-energy physics, such as in coherent neutrino oscillations and as a lattice regularization of the  $O(3)$  nonlinear  $\sigma$  model which will be important for developing quantum simulations of QCD. Importantly, this technique easily scales to higher dimensions. By enabling analog simulation of the Heisenberg model on Rydberg atoms and superconducting qubits, systems beyond the reach of classical computers can be simulated and one may be able to achieve a scientifically

useful quantum advantage.

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