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Realizing symmetry-protected topological phases in a spin-1/2 chain with next-nearest neighbor hopping on superconducting qubits

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Quantum simulation on near-term quantum hardware is a topic of intense interest. The preparation of novel quantum states of matter provides a quantitative assessment of the capabilities of near-term digital quantum computers to implement circuits with structure of relevance to quantum simulation. Here, we conduct a benchmark study by realizing symmetry-protected topological (SPT) phases of a spin-1/2 Hamiltonian with next-nearest-neighbor hopping on up to 11 qubits on a programmable superconducting quantum processor using adiabatic state preparation. Using recompilation techniques to reduce the gate count to around 50 two-qubit gates, we observe clear signatures of the two distinct SPT phases, such as excitations localized to specific edges and finite string order parameters. We identify a parasitic phase associated with the two-qubit gate as the dominant imperfection that limits the depth of the circuits, indicating a research topic of interest for future hardware development.

I. INTRODUCTION

Quantum computers have long been of interest for their potential to simulate quantum many-body systems [1–6]. A recent emphasis has been using quantum computers to treat classically challenging chemistry and condensed matter problems [5–9]. Advances in near-term quantum hardware now make prototype versions of these simulations possible, for instance in the computation of the ground state properties of chemical [10–17] and solid-state [12, 18–20] quantum systems as well as simulation of their real-time dynamics for closed [21–31] and open [32–38] systems. Several recent studies have also reported the simulation of finite-temperature physics on near-term devices [39–42].

The realization of topological phases of matter is another area of considerable interest. These phases do not fit within the Landau paradigm of local order parameters associated with symmetry breaking, and the study of their ground-state properties and excitations is an active area of research in condensed matter physics [43–46]. Analog quantum simulators are able to realize some of these phases and associated phenomena such as models with topological band structures [47–51], Thouless charge pumps [52–54], various symmetry-protected topological (SPT) phases [55–60] and quantum spin liquids [61]. On digital quantum computers, the preparation of various topological phases has been reported, including spin-1/2 chain models with three-body interactions [62–65], the toric code [66], and topological Floquet phases [67, 68].

Beyond realizing the quantum phase of interest, a comparison of the hardware data against theory can serve to benchmark quantum devices and identify the hardware imperfections that limit circuit complexity [29, 69]. These imperfections have varying impacts depending on the particular structure of the circuits, implying benchmarking methods which employ dissimilar circuit struc-

tures are inadequate for characterizing devices for quantum simulation applications [70]. For example, many protocols used to assess the performance of quantum computers employ randomized circuits [71, 72] which do not accurately reflect the impact of coherent errors in more structured circuits [73, 74].

Circuits which perform representative quantum simulation tasks are expected to provide more relevant information on the capabilities of near-term hardware. Spin-1/2 models with beyond-nearest-neighbor interactions are an attractive target of study for quantum simulation due to their richer physics [75, 76] and more complex circuits compared to those with only nearest-neighbor (NN) couplings. Of particular interest are topological phases in these models characterized by non-local string order parameters and edge excitations [77]. Hence, the preparation of such phases may serve as a relevant benchmark of the capabilities of present digital quantum computers for quantum simulation.

Here, we report the preparation of SPT phases of a spin-1/2 chain with next-nearest-neighbor interactions on up to 11 qubits of a programmable superconducting quantum processor, Google’s Rainbow processor. Circuits with up to 50 two-qubit gates are found to yield the topological signatures of the SPT phases, including finite or zero values of different string order parameters and edge excitations localized to different sides of the chain depending on the particular SPT phase. Examination of the discrepancies with theory demonstrates that a parasitic phase in the two-qubit gate is the leading limitation on the gate depth. Based on these findings, we estimate the gap between the present achievable gate depth and that required for realization of other novel phases such as chiral spin liquids. Our results provide a benchmark of the current capabilities and limitations of today’s quantum processor in performing quantum simulations based on digitized adiabatic state preparation.

II. THEORY

A. SPT phases of spin-1/2 chain with NNN hopping

We consider a one-dimensional spin-1/2 chain with NN and next-nearest-neighbor (NNN) interactions as shown in Fig. 1(a). Its Hamiltonian is given by

$$H_T = - \sum_k J'_1 (\sigma_{2k}^x \sigma_{2k+1}^x + \sigma_{2k}^y \sigma_{2k+1}^y) \quad (1)$$

$$+ J_1 (\sigma_{2k+1}^x \sigma_{2k+2}^x + \sigma_{2k+1}^y \sigma_{2k+2}^y)$$

$$+ J_2 (\sigma_k^x \sigma_{k+2}^x + \sigma_k^y \sigma_{k+2}^y)$$

where $\sigma^x, \sigma^y, \sigma^z$ are Pauli operators, J'_1 (J_1) denotes the strength of the NN interactions from the even to odd sites (odd to even), and J_2 denotes the strength of the NNN coupling.

The phase diagram of H_T contains two distinct gapped SPT phases known as the even-parity dimer (ED) and singlet-dimer (SD) phases. The model is in the ED (SD) phase when $J'_1 = 2J_1 < 0$; $J_1 > 0$ ($J_1 = -2J_2 > 0$; $J'_1 < 0$). The solution of the edge states in the two phases were derived by Zou et al [77]. Each edge state is two-fold degenerate and protected by time-reversal (TR) symmetry. The two phases are topologically distinct because they cannot be deformed into one another without breaking TR, inversion, and D_2 symmetry of spin rotation by π about the x, y and z axes [78].

These phases can be distinguished by the location of their edge excitations; for a lattice with an odd number of lattice points, the ED (SD) phase has an edge excitation on the right (left) edge of the chain. In addition, the phases can be distinguished by string order parameters, defined as:

$$O_{z_n} = -\lim_{r \rightarrow \infty} \langle (\sigma_n^z + \sigma_{n+1}^z) e^{i\pi \sum_k \sigma_k^z} (\sigma_{2r+n}^z + \sigma_{2r+n+1}^z) \rangle \quad (2)$$

where the sum over k is restricted to $n+2 \leq k \leq 2r+n-1$ and r should be as large as possible. Generally, a non-zero string-order parameter indicates the presence of hidden long-range order and a topologically non-trivial phase. In the present model, the ED (SD) phase exhibits a finite O_{z_n} value for odd (even) n . In this work, n is chosen to be 0 or 1. To select r , we choose the largest value that satisfies the constraint that the number of operators used to construct O_{z_0} and O_{z_1} are the same. This requirement is equivalent to using the maximum integer value of r that satisfies $2r+1+1 \leq M$, where M is the number of sites of the system. For systems with $M = 7, 9, 11$, this constraint corresponds to $r = 2, 3, 4$, respectively.

B. Preparation of SPT phases on a digital quantum processor

We used circuits based on adiabatic state preparation [79, 80] (ASP) to prepare the SPT phases of H_T on the superconducting quantum processor. The system is initialized in the ground state of an initial Hamiltonian H_I and evolved to the ground state of the target Hamiltonian H_T over time duration T using a linear interpolation $H(s) = (1-s)H_I + sH_T$ where $s \equiv t/T$. In this study, H_T is given in Eqn. 1. The initial Hamiltonian is given by

$$H_I = -B_z \sum_k (-1)^k \sigma_k^z \quad (3)$$

with B_z a uniform external field. For $B_z > 0$ and an odd number of sites, the ground state of H_I is given by $|0101\dots 01010\rangle$ which can be prepared by applying X-rotation single-qubit gates on sites labeled by odd indices.

To carry out ASP, we Trotterized the adiabatic evolution to first order and implemented the resulting steps using quantum circuits constructed from single-qubit and two-qubit quantum gates as shown in Fig. 1(b). The two-qubit gate K , known as the fermionic simulation (FSIM) gate, can be constructed using the native gate set available on the Rainbow chip [29]. An example of a circuit that implements a single Trotter step for a system of 7 sites is shown in Fig. 1(c). Despite extensive experimentation, the overall circuit that carries out the full ASP was found to produce qualitative inaccuracies with theory. To assess the maximum gate depth that could be achieved while yielding quantitative agreement, we used a circuit recompilation scheme [42] by fitting the circuits needed to realize the state at each time in the adiabatic evolution to a parameterized circuit. In ref. [42], the parameterized circuits consisted of alternating layers of single-qubit gates and two-qubit gates. We used this ansatz in our benchmark studies by using the native gate \sqrt{iS}^\dagger for the two-qubit gate and the native gate PhasedXZ (ϕ) for the single qubit gate, respectively. A schematic of the final recompiled circuit is shown in Fig. 1(d). The reduction in circuit depth is discussed in Appendix A.

We used the Google Rainbow and Weber quantum processors for this study. The Rainbow and Weber processors consist of a two-dimensional array of 23 and 54 transmon qubits, respectively, with each qubit tunably coupled to its neighbors. The native single-qubit gates are the PhasedXZ gate which consists of a rotation about an axis in the XY plane of the Bloch sphere with an extra phase about the Z axis. The native two-qubit gates are the \sqrt{iS}^\dagger gates. Further information on the device parameters are available in Ref. [81]. Simulated data in the absence of noise are generated using Google's circuit emulator qsim. [82]

We perform 8192 repetitions of each circuit with measurements in the Z-basis for all sites at each Trotter

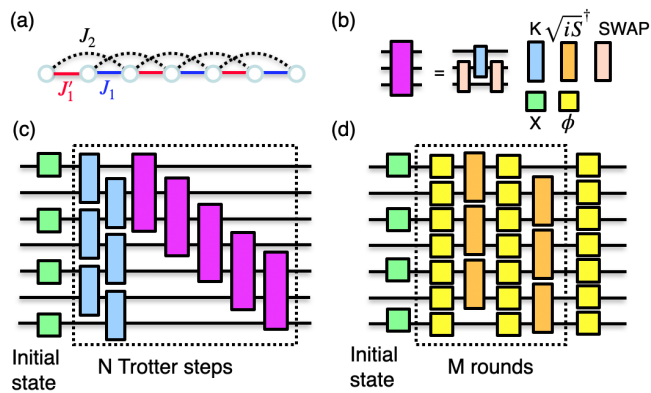


FIG. 1 Schematic of quantum circuits used to prepare the SPT phases. (a) Arrangement of sites in a 1D-lattice of 7 sites. The strength of interactions going from even-labelled to odd-labelled (odd-labelled to even-labelled) sites are given by J_1' (J_1); those for NNN couplings are given by J_2 . For this study, we also consider chains with 9 and 11 sites. (b) Quantum gates used to construct the circuits for the demonstrations. Their matrix representations are provided in ref. [83]. The gates PhasedXZ (ϕ) and \sqrt{iS}^\dagger are the native one-qubit and two-qubit gates on the Google Rainbow processor. (c) Circuit to implement Trotterized ASP for a system with 7 sites. (d) Schematic of the recompiled circuit with M gate rounds.

step. While the quantum circuits implemented conserve the total S_z , the presence of hardware error can lead to S_z non-conservation. We mitigate this error by post-selecting the measurements for $\Delta S_z = 0$. The quantities required to compute string order parameters in Eqn. 2 can be computed from the measurements in the Z-basis. Similarly, the occupancy of the i th site is simply related to the expectation value $\langle \sigma_i^z \rangle$. Both quantities can be directly computed by performing the appropriate sums with the measurement bitstrings. Only those bitstrings with $S_z = 0$ are used in the computation.

III. RESULTS

A. Signatures of SPT phases

We first report calculations of the string order parameters O_{z_1} for the ED phase versus ASP time s for spin chains with 7, 9 and 11 sites. We collected data from 15 configurations of qubits; based on the \sqrt{iS} gate cross-entropy benchmarking (XEB) average error per cycle, we selected the ten best configurations, from which we computed the mean and standard deviation for all observables. To prepare the ED phase, the Hamiltonian parameters were set to $J_1 = 0.2$, $J_1' = -1.5$, $J_2 = -0.1$, $B_z = 2.5$ and $T = 3.0$, and $M = 5$ layers of gate rounds were used for circuit recompilation. Emulated results were obtained by running the Trotterized ASP circuit on

Google's circuit emulator qsim.

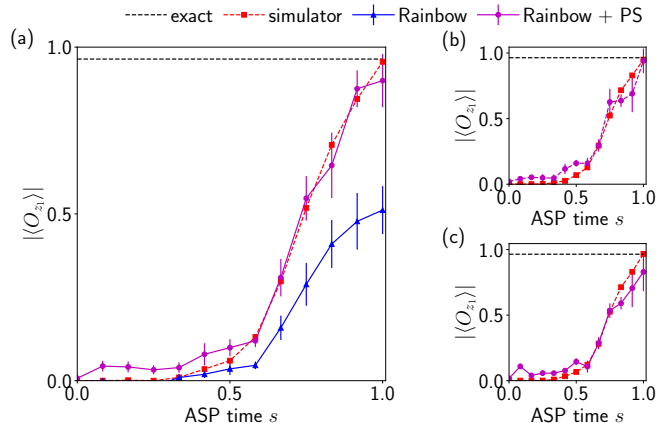


FIG. 2 Preparation of ED phase for increasing system sizes on Rainbow quantum processor.

Absolute value of the string order parameter O_{z_1} versus ASP time s for a system size of (a) 7, (b) 9, and (c) 11 qubits, respectively. Data from Rainbow was collected using 15 different configurations of qubits and only the best 10 configurations were selected based on their \sqrt{iS} gate XEB average error per cycle. The hardware data without any error mitigation (blue triangle) yields qualitative agreement with the emulated ASP trajectory (red square). Quantitative agreement is obtained when post selection is used (purple circle). The ED phase can be prepared reliably for system sizes of up to 11 qubits. The parameters $J_1 = 0.2$, $J_1' = -1.5$, $J_2 = -0.1$, $B_z = 2.5$, and $T = 3.0$ are used to prepare the ED phase. The lines through the symbols are guides to the eye.

We plot $|O_{z_1}|$ versus ASP time s on 7 sites in Fig. 2(a). We observe good agreement between the final value of $|O_{z_1}|$ at $s = 1$ obtained from Trotterized ASP using qsim and the value from exact diagonalization in Fig. 2(a). This result indicates that a Trotter step size of 0.25 is sufficiently small enough to approximate the ASP evolution that yields the ED phase with high fidelity.

We next compare the data obtained by running Trotterized ASP trajectories on qsim with the data obtained by running recompiled circuits on Rainbow without any error mitigation for 7 sites. These circuits required 30 \sqrt{iS}^\dagger gates. Although the trend of O_{z_1} increasing with ASP time is reproduced as seen in Fig. 2(a), a clear discrepancy exists for the final value of O_{z_1} at the end of the adiabatic trajectory. To mitigate this discrepancy, we perform post-selection based on S_z conservation. We observe a marked improvement in the quality of the hardware data, with quantitative agreement obtained between the hardware data and the simulator. With this error mitigation step, the quantum processor is able to reproduce the adiabatic trajectory with sufficient fidelity to arrive at the expected non-zero value of the string order parameter in the ED phase.

We next compute O_{z_1} for system sizes of 9 and 11 qubits. The number of two-qubit gates used in the re-

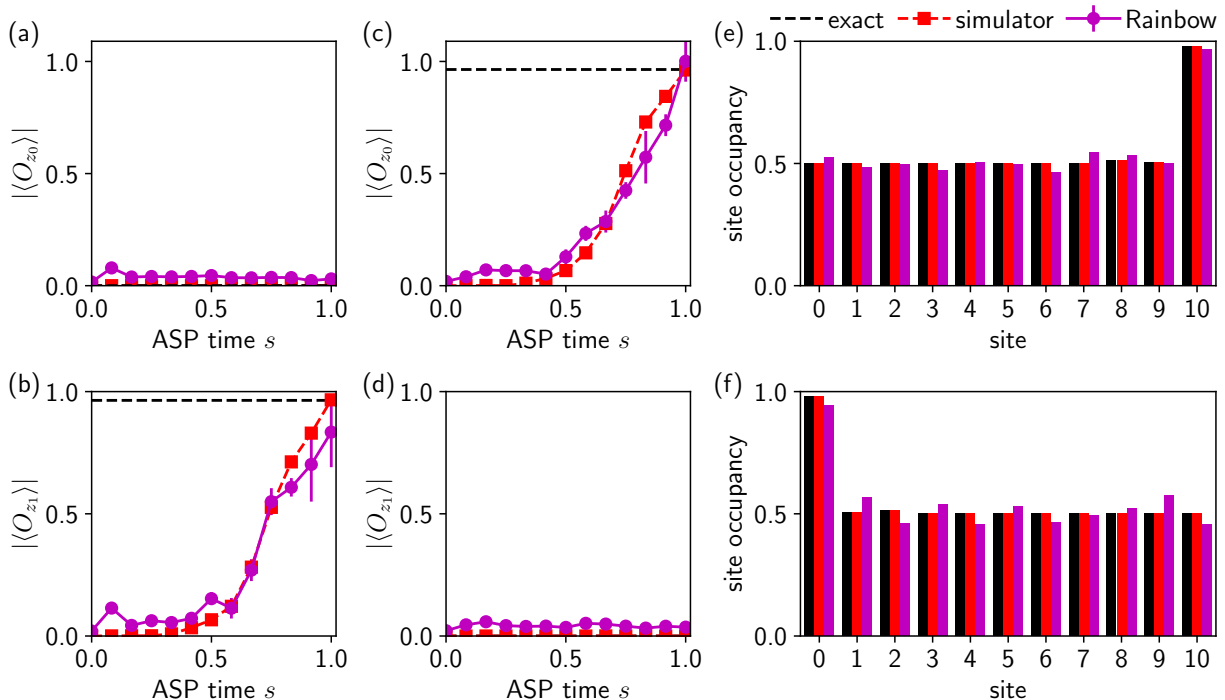


FIG. 3 **Preparation of ED and SD phases using 11 qubits on the Rainbow quantum processor.** Absolute value of the string order parameters (a) O_{z_0} and (b) O_{z_1} versus ASP time (s) in the ED phase. (c, d) Analogous result for the SD phase. Occupancy of each site at the end of the ASP trajectory for the (e) ED and (f) SD phases. The two SPT phases can be prepared and distinguished clearly by finite or zero string-order parameter and location of edge excitation. The parameters $J_1 = 0.2$, $J'_1 = -1.5$, $J_2 = -0.1$, $B_z = 2.5$, $T = 3.0$ were used to prepare the ED phase. The parameters $J_1 = 1.5$, $J'_1 = -0.2$, $J_2 = -0.1$, $B_z = 2.5$, $T = 3.0$ were used to prepare the SD phase.

compiled circuits was 40 and 50, respectively, compared to 30 in the 7 qubit case. Despite the larger number of gates, we observe good agreement in the value of the string order parameter over the adiabatic trajectory in Fig. 2(b) and 2(c), although with a slight degradation that likely arises from the deeper circuits. The data indicates that the SPT phases for a system of 11 sites can be prepared with enough fidelity to observe its topological features on the Rainbow quantum processor.

Next, we verify that we can distinguish the SD and ED phases using the string order parameters. Figure 3(a) and 3(b) shows $|O_{z_0}|$ and $|O_{z_1}|$ versus s on 11 qubits when the model is tuned into the ED phase. We observe good agreement between the hardware data and the simulator over the adiabatic path. At the end of the adiabatic path, we measure 0.029 ± 0.007 and 0.829 ± 0.147 for O_{z_0} and O_{z_1} , respectively, which is in good agreement with the expected values of ~ 0 and 0.964 . Similarly, we tune the model into the SD phase by setting the Hamiltonian parameters to $J_1 = 1.5$, $J'_1 = -0.2$, $J_2 = -0.1$. The string order parameters O_{z_0} and O_{z_1} versus s are given

in Figs. 3(c) and 3(d), respectively. Again, the final values of the string order parameter from the hardware are 0.981 ± 0.085 and 0.034 ± 0.013 , which are in quantitative agreement with the numerically determined exact values of 0.962 and ~ 0 . In both cases, we measured a finite value for the appropriate string order parameters and nearly zero for the other, indicating that the correct SPT phases were successfully prepared.

Finally, we plot the occupancy of each site at the end of the adiabatic evolution for the ED and SD phases in Figs. 3(e) and 3(f), respectively. In the ED (SD) phase, an edge excitation is predicted to exist on the right (left) end of the chain. This feature is indeed observed using the exact solution obtained from exact diagonalization. The results from the hardware clearly indicate a difference in the occupancy on the appropriate edge of the chain for each phase and the rest of the chain, with the value in good agreement with the exact result. This observation provides additional evidence that the SPT states prepared on the hardware exhibit the key features expected of these topological phases.

B. Origin of gate depth limitations

The results obtained above required the use of circuit recompilation techniques to reduce the gate depth for

ASP to a maximum of around 50 two-qubit gates. With-

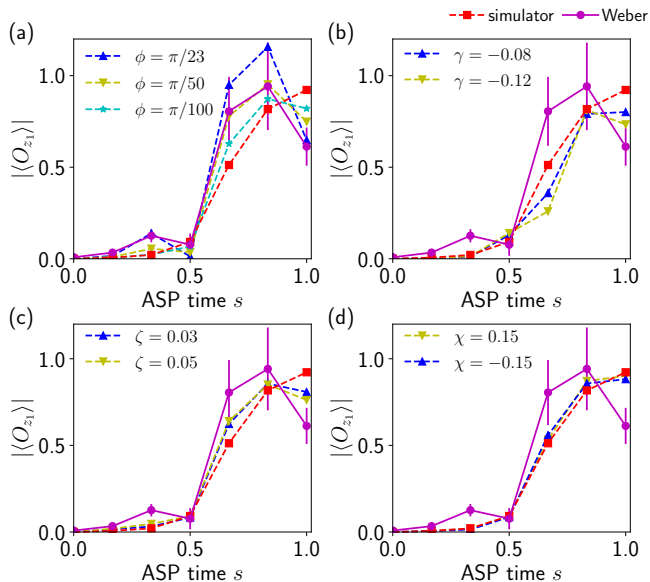


FIG. 4 **Effects of gate imperfections on $|\langle O_{z_1} \rangle|$ in the ED phase.** Absolute value of the string order parameters O_{z_1} versus ASP time (s) in the ED phase when (a) ϕ , (b) γ , (c) ζ , and (d) χ are varied. Error bars for the hardware data (Weber) were obtained from 4 different qubit configurations. The parameter ϕ best explains the observed trend in string order parameter with s . The parameters $J_1 = 0.2$, $J'_1 = -1$, $J_2 = -0.1$, $B_z = 1.5$, $T = 3.0$ were used to prepare the ED phase.

out circuit recompilation, the number of required two-qubit gates was around 170 for 7 sites, and the hardware results were in only qualitative agreement with the expected final string order parameter value. To investigate the origin of this circuit depth limitation, we examined the non-idealities of the two-qubit gates on the Rainbow processor. The most general excitation-number-conserving two-qubit gate, denoted by $U(\theta, \zeta, \chi, \gamma, \phi)$ takes the following form (with the basis states in the order $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$):

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i(\gamma+\zeta)}\cos\theta & -ie^{-i(\gamma-\chi)}\sin\theta & 0 \\ 0 & ie^{-i(\gamma+\chi)}\sin\theta & e^{-i(\gamma-\zeta)}\cos\theta & 0 \\ 0 & 0 & 0 & e^{-i(2\gamma+\phi)} \end{pmatrix} \quad (4)$$

While the ideal native two-qubit gate on Rainbow and Weber is given by $U(\pi/4, 0, 0, 0, 0)^\dagger$, additional interactions lead to non-zero values of ζ , χ , γ and ϕ . We numerically simulated the effects of these non-idealities on the value of the string order parameter along the adiabatic trajectory by plotting $|\langle O_{z_1} \rangle|$ versus ASP time s on 7 sites for different values of ϕ , γ , ζ , and χ .

Figure 4 shows the string order parameter versus ASP time s for various values of ϕ , γ , ζ and χ . We also collected data from 4 different qubits configurations on Weber, a quantum processor with similar specifications as

Rainbow, and plot the mean and standard deviation of the string order parameter. For comparison, we plot the ideal trajectory obtained using qsim [82]. Comparing the ideal trajectory with data from Weber, we observe a non-monotonic trend at the end of the trajectory in the hardware data. Similar behavior was observed in the simulator results for various values of ϕ , γ , ζ and χ . The non-monotonic trend is observed to be most sensitive to ϕ , and the results with $\phi = \pi/50$ yielded the best qualitative agreement with the hardware results. These observations suggest that the parasitic controlled phase ϕ is a dominant factor in limiting the gate depth of the present simulations.

IV. DISCUSSION AND FUTURE OUTLOOK

We now discuss the implications of our findings regarding the role of the parasitic phase in limiting gate depth for quantum simulation. Error mitigation strategies such as randomized compiling [84] may be applicable to mitigate coherent errors. However, in the present simulations, this protocol cannot be implemented because the two-qubit native gate does not commute with the Pauli group. Other strategies to mitigate coherent gate errors may be possible, but their effectiveness is in general problem-dependent. Several strategies were attempted in this work but were ultimately unsuccessful (see Appendix B). Supposing that the parasitic phase can be successfully mitigated, emulations using Cirq indicate that the maximum number of two-qubit gates is around 100 for the present simulations given reported error rates [29] which agrees well with the findings of a recent work which simulated correlated molecules and materials on the same device [85].

Given the capabilities of present quantum processors, we examine the resources required to prepare more complex chiral spin liquids that are thought to exist in a spin-1/2 frustrated honeycomb with similar couplings studied in our work [75, 76, 86]. For a system size of 20 spins in a hexagonal lattice, corresponding to 4 unit cells, the total number of two-qubit gates required to implement a single Trotter step of the ASP trajectory is 210. Assuming six Trotter steps are needed in total, 1260 two-qubit gates would therefore be required. This value exceeds our estimate of the achievable gate depth by around an order of magnitude. Our results provide quantitative metrics regarding the improvements needed for future quantum devices to realize more exotic topological phases using ASP.

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APPENDIX A: Circuit recompilation scheme

This section describes the circuit recompilation technique originally introduced in [42] and which is used to reduce the depth of the circuits implemented on the Rainbow quantum processor. Let the target unitary be U_{targ} and the parameterized circuit be $U_{rec}(\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is a composite vector of all the free variables in the parameterized circuit. Given a reduced density operator ρ on the finite domain acted on by the target unitary, the optimal parameterized circuit is found by performing a gradient descent to maximize the function

$$F(\boldsymbol{\theta}) = |\text{Tr}(U_{rec}(\boldsymbol{\theta})^\dagger U_{targ}\rho)|^2, \quad (\text{A1})$$

which can be interpreted as the fidelity between $U_{rec}(\boldsymbol{\theta})$ and U_{targ} with respect to the reduced density matrix ρ . For the purpose of this work, we set $F(\boldsymbol{\theta}) = 0.999$ as the stopping criterion. We compare the number of two-qubit gates present in the target unitary U_{targ} at the end of 12 Trotter steps and in the parameterized circuit $U_{rec}(\boldsymbol{\theta})$ in Table I. The number of two-qubit gates required decreases by around an order of magnitude with recompilation.

| System size | U_{targ} | $U_{rec}(\boldsymbol{\theta})$ |
|-------------|------------|--------------------------------|
| 7 | 336 | 30 |
| 9 | 468 | 40 |
| 11 | 564 | 50 |

TABLE I Comparison of number of two-qubit gates in target unitary U_{targ} for 12 Trotter steps and in $U_{rec}(\boldsymbol{\theta})$ for different system sizes

APPENDIX B: Parasitic controlled phase in non-recompiled circuits

This section describes attempted strategies to mitigate the parasitic controlled phase. The first approach constructs $\text{CPHASE}(\psi)$ and appends it to the back of the native gate with $\psi = -\phi$ to compensate for the parasitic phase; that is we implement $\sqrt{iS}_{hardware}^\dagger \text{CPHASE}(\psi = -\phi)$. This gate can be constructed exactly by using a series of single-qubit rotations and two $\sqrt{iS}_{hardware}^\dagger$ to compensate for the phase in each \sqrt{iS}^\dagger . By noting that $\sqrt{iS}_{hardware}^\dagger$ is approximately $\sqrt{iS}^\dagger \text{CPHASE}(\phi)$ for some parasitic phase ϕ , a controlled-phase gate between control qubit i and target qubit j , $\text{CPHASE}(\psi = -\phi)_{ij}$ can be constructed exactly as [87]

$$\begin{aligned} \text{CPHASE}(\psi)_{ij} = & (R_{Z_i}(\pi - \psi/2) \otimes R_{Z_j}(-\psi/2)) \\ & (R_{X_i}(-\xi_i) \otimes R_{X_j}(-x_{ij})) \\ & \sqrt{iS}_{hardware,ij}^\dagger \\ & (R_{Z_i}(\pi + \phi/2) \otimes R_{Z_j}(\phi/2)) \\ & (R_{X_i}(-2\alpha) \otimes \mathcal{I}_j) \\ & \sqrt{iS}_{hardware,ij}^\dagger \\ & (R_{Z_i}(\psi/2) \otimes R_{Z_j}(\psi/2)) \\ & (R_{X_i}(\xi_i) \otimes R_{X_j}(x_{ij})) \end{aligned} \quad (\text{B1})$$

where R_Z , R_X are the single-qubit rotations around the z-axis and x-axis, and the decomposition parameters α, ξ_i, ξ_j are given by

$$\sin(\alpha) = \sqrt{\frac{\sin^2(\psi/4) - \sin^2(\phi/2)}{\sin^2(\pi/4) - \sin^2(\phi/2)}} \quad (\text{B2})$$

$$\begin{aligned} \xi_i = & \tan^{-1}\left(\frac{\tan(\alpha)\cos(\pi/4)}{\cos(\phi/2)}\right) \\ & + \frac{\pi}{2}(1 - \text{sgn}(\cos(\phi/2))) \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} \xi_j = & \tan^{-1}\left(\frac{\tan(\alpha)\sin(\pi/4)}{\sin(\phi/2)}\right) \\ & + \frac{\pi}{2}(1 - \text{sgn}(\sin(\phi/2))) \end{aligned} \quad (\text{B4})$$

The cost of this approach is the addition of two native two-qubit gates for each original two-qubit gate, thereby increasing the gate depth by a factor of 3.

We tested this scheme on Weber by performing Floquet characterization to estimate the parasitic phase ϕ present on each qubit [29], then used the average value to construct a compensated $\text{CPHASE}(-\phi_{avg})$ that was appended to the hardware gate $\sqrt{iS}_{hardware}^\dagger$. The results with these compensated circuits are presented in Fig. 5(a). We observe greater deviations from the exact result when the compensated circuits are used. The likely origin of the worse performance is the larger number of two-qubit gates are used in the compensated circuits (510 versus 170 to reach the end of the adiabatic path).

The second approach is based on the observation that the phase present in the $|11\rangle$ can be removed at the expense of adding half the phase to the $|01\rangle$ and $|10\rangle$ using single-qubit Z rotations. Assuming that fidelity is a quadratic function of gate parameters, a higher fidelity can be obtained by splitting the phase into two. We tested this scheme by performing Floquet calibration to estimate the ϕ present on each qubit and used the average to perform single-qubit Z rotations on the qubits. The result is shown in Fig. 5(b). Although some improvement in the final value of the string order parameter is observed, the non-monotonic trend remains largely unchanged, indicating that manipulation of the parasitic phase is inadequate to remove the discrepancy.

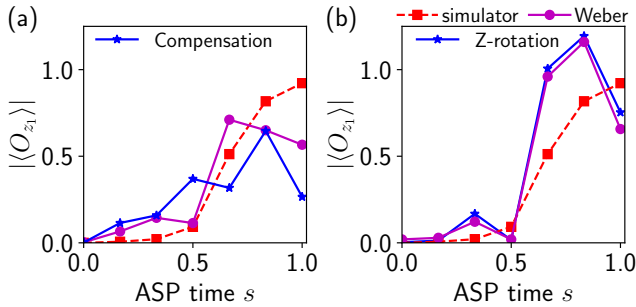


FIG. 5 **Attempts to compensate for parasitic controlled phase.** Absolute value of the string order parameters O_{z_1} versus ASP time (s) in the ED phase for circuits with (a) CPHASE(ϕ) appended to each native two-qubit gate to compensate for the parasitic controlled phase; (b) single-qubit Z rotations added to split the parasitic phase among two basis states. The data obtained from Weber and the noiseless data from the simulator are also shown. The qualitative trend of the string order parameter is qualitatively unchanged by the two strategies. The parameters $J_1 = 0.2$, $J'_1 = -1$, $J_2 = -0.1$, $B_z = 1.5$, $T = 3.0$ were used to prepare the ED phase.

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- [1] Richard P. Feynman, “Simulating physics with computers,” *International Journal of Theoretical Physics* **21**, 467–488 (1982).
- [2] Seth Lloyd, “Universal quantum simulators,” *Science* **273**, 1073–1078 (1996).
- [3] I. M. Georgescu, S. Ashhab, and Franco Nori, “Quantum simulation,” *Rev. Mod. Phys.* **86**, 153–185 (2014).
- [4] Ehud Altman, Kenneth R. Brown, Giuseppe Carleo, Lincoln D. Carr, Eugene Demler, Cheng Chin, Brian DeMarco, Sophia E. Economou, Mark A. Eriksson, Kai-Mei C. Fu, Markus Greiner, Kaden R.A. Hazzard, Randall G. Hulet, Alicia J. Kollár, Benjamin L. Lev, Mikhail D. Lukin, Ruichao Ma, Xiao Mi, Shshank Misra, Christopher Monroe, Kater Murch, Zaira Nazario, Kang-Kuen Ni, Andrew C. Potter, Pedram Roushan, Mark Saffman, Monika Schleier-Smith, Irfan Siddiqi, Raymond Simmonds, Meenakshi Singh, I.B. Spielman, Kristan Temme, David S. Weiss, Jelena Vučković, Vladan Vuletić, Jun Ye, and Martin Zwierlein, “Quantum simulators: Architectures and opportunities,” *PRX Quantum* **2**, 017003 (2021).
- [5] Nan Sheng, Christian Vorwerk, Marco Govoni, and Giulia Galli, “Quantum simulations of material properties on quantum computers,” (2021), [arXiv:2105.04736 \[quant-ph\]](https://arxiv.org/abs/2105.04736).
- [6] Nicolas P. D. Sawaya, Francesco Paesani, and Daniel P. Tabor, “Near- and long-term quantum algorithmic approaches for vibrational spectroscopy,” (2021), [arXiv:2009.05066 \[quant-ph\]](https://arxiv.org/abs/2009.05066).
- [7] Sam McArdle, Suguru Endo, Alán Aspuru-Guzik, Simon C. Benjamin, and Xiao Yuan, “Quantum computational chemistry,” *Rev. Mod. Phys.* **92**, 015003 (2020).
- [8] Bela Bauer, Sergey Bravyi, Mario Motta, and Garnet Kin-Lic Chan, “Quantum algorithms for quantum chemistry and quantum materials science,” *Chem. Rev.* **120**, 12685–12717 (2020).
- [9] M. Cerezo, Andrew Arrasmith, Ryan Babbush, Simon C. Benjamin, Suguru Endo, Keisuke Fujii, Jarrod R. McClean, Kosuke Mitarai, Xiao Yuan, Lukasz Cincio, and Patrick J. Coles, “Variational quantum algorithms,” *Nature Reviews Physics* **3**, 625–644 (2021).
- [10] Alberto Peruzzo, Jarrod McClean, Peter Shadbolt, Man-Hong Yung, Xiao-Qi Zhou, Peter J. Love, Alán Aspuru-Guzik, and Jeremy L. O’Brien, “A variational eigenvalue solver on a photonic quantum processor,” *Nat. Commun.* **5**, 4213 (2014).
- [11] P. J. J. O’Malley, R. Babbush, I. D. Kivlichan, J. Romero, J. R. McClean, R. Barends, J. Kelly, P. Roushan, A. Tranter, N. Ding, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, A. G. Fowler, E. Jeffrey, E. Lucero, A. Megrant, J. Y. Mutus, M. Neeley, C. Neill, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. C. White, P. V. Coveney, P. J. Love, H. Neven, A. Aspuru-Guzik, and J. M. Martinis, “Scalable Quantum Simulation of Molecular Energies,” *Phys. Rev. X* **6**, 031007 (2016).
- [12] Abhinav Kandala, Antonio Mezzacapo, Kristan Temme, Maika Takita, Markus Brink, Jerry M. Chow, and Jay M. Gambetta, “Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets,” *Nature* **549**, 242–246 (2017).
- [13] J. I. Colless, V. V. Ramasesh, D. Dahlen, M. S. Blok, M. E. Kimchi-Schwartz, J. R. McClean, J. Carter, W. A. de Jong, and I. Siddiqi, “Computation of molecular spectra on a quantum processor with an error-resilient algorithm,” *Phys. Rev. X* **8**, 011021 (2018).
- [14] Sam McArdle, Tyson Jones, Suguru Endo, Ying Li, Simon C. Benjamin, and Xiao Yuan, “Variational ansatz

- based quantum simulation of imaginary time evolution,” *npj Quantum Information* **5**, 75 (2019).
- [15] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Sergio Boixo, Michael Broughton, Bob B. Buckley, David A. Buell, Brian Burkett, Nicholas Bushnell, Yu Chen, Zijun Chen, Benjamin Chiaro, Roberto Collins, William Courtney, Sean Demura, Andrew Dunsworth, Daniel Eppens, Edward Farhi, Austin Fowler, Brooks Foxen, Craig Gidney, Marissa Giustina, Rob Graff, Steve Habegger, Matthew P. Harrigan, Alan Ho, Sabrina Hong, Trent Huang, William J. Huggins, Lev Ioffe, Sergei V. Isakov, Evan Jeffrey, Zhang Jiang, Cody Jones, Dvir Kafri, Kostyantyn Kechedzhi, Julian Kelly, Seon Kim, Paul V. Klimov, Alexander Korotkov, Fedor Kostritsa, David Landhuis, Pavel Laptev, Mike Lindmark, Erik Lucero, Orion Martin, John M. Martinis, Jarrod R. McClean, Matt McEwen, Anthony Megrant, Xiao Mi, Masoud Mohseni, Wojciech Mroczkiewicz, Josh Mutus, Ofer Naaman, Matthew Neeley, Charles Neill, Hartmut Neven, Murphy Yuezhen Niu, Thomas E. O’Brien, Eric Ostby, Andre Petukhov, Harald Putterman, Chris Quintana, Pedram Roushan, Nicholas C. Rubin, Daniel Sank, Kevin J. Satzinger, Vadim Smelyanskiy, Doug Strain, Kevin J. Sung, Marco Szalay, Tyler Y. Takeshita, Amit Vainsencher, Theodore White, Nathan Wiebe, Z. Jamie Yao, Ping Yeh, and Adam Zalcman, “Hartree-Fock on a superconducting qubit quantum computer,” *Science* **369**, 1084–1089 (2020).
- [16] William J. Huggins, Bryan A. O’Gorman, Nicholas C. Rubin, David R. Reichman, Ryan Babbush, and Joonho Lee, “Unbiasing fermionic quantum monte carlo with a quantum computer,” (2021), [arXiv:2106.16235 \[quant-ph\]](https://arxiv.org/abs/2106.16235).
- [17] Kushal Seetharam, Debopriyo Biswas, Crystal Noel, Andrew Risinger, Daiwei Zhu, Or Katz, Sambuddha Chattopadhyay, Marko Cetina, Christopher Monroe, Eugene Demler, and Dries Sels, “Digital quantum simulation of nmr experiments,” (2021), [arXiv:2109.13298 \[quant-ph\]](https://arxiv.org/abs/2109.13298).
- [18] Guglielmo Mazzola, Pauline J. Ollitrault, Panagiotis Kl. Barkoutsos, and Ivano Tavernelli, “Nonunitary operations for ground-state calculations in near-term quantum computers,” *Phys. Rev. Lett.* **123**, 130501 (2019).
- [19] He Ma, Marco Govoni, and Giulia Galli, “Quantum simulations of materials on near-term quantum computers,” *npj Computational Materials* **6**, 85 (2020).
- [20] C. Neill, T. McCourt, X. Mi, Z. Jiang, M. Y. Niu, W. Mroczkiewicz, I. Aleiner, F. Arute, K. Arya, J. Atalaya, R. Babbush, J. C. Bardin, R. Barends, A. Bengtsson, A. Bourassa, M. Broughton, B. B. Buckley, D. A. Buell, B. Burkett, N. Bushnell, J. Campero, Z. Chen, B. Chiaro, R. Collins, W. Courtney, S. Demura, A. R. Derk, A. Dunsworth, D. Eppens, C. Erickson, E. Farhi, A. G. Fowler, B. Foxen, C. Gidney, M. Giustina, J. A. Gross, M. P. Harrigan, S. D. Harrington, J. Hilton, A. Ho, S. Hong, T. Huang, W. J. Huggins, S. V. Isakov, M. Jacob-Mitos, E. Jeffrey, C. Jones, D. Kafri, K. Kechedzhi, J. Kelly, S. Kim, P. V. Klimov, A. N. Korotkov, F. Kostritsa, D. Landhuis, P. Laptev, E. Lucero, O. Martin, J. R. McClean, M. McEwen, A. Megrant, K. C. Miao, M. Mohseni, J. Mutus, O. Naaman, M. Neeley, M. Newman, T. E. O’Brien, A. Opremcak, E. Ostby, B. Pató, A. Petukhov, C. Quintana, N. Redd, N. C. Rubin, D. Sank, K. J. Satzinger, V. Shvarts, D. Strain, M. Szalay, M. D. Trevithick, B. Villalonga, T. C. White, Z. Yao, P. Yeh, A. Zalcman, H. Neven, S. Boixo, L. B. Ioffe, P. Roushan, Y. Chen, and V. Smelyanskiy, “Accurately computing the electronic properties of a quantum ring,” *Nature* **594**, 508–512 (2021).
- [21] R. Barends, L. Lamata, J. Kelly, L. García-Álvarez, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Yu Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Hoi, C. Neill, P. J. J. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, E. Solano, and John M. Martinis, “Digital quantum simulation of fermionic models with a superconducting circuit,” *Nature Communications* **6**, 7654 (2015).
- [22] Henry Lamm and Scott Lawrence, “Simulation of nonequilibrium dynamics on a quantum computer,” *Phys. Rev. Lett.* **121**, 170501 (2018).
- [23] A. Chiesa, F. Tacchino, M. Grossi, P. Santini, I. Tavernelli, D. Gerace, and S. Carretta, “Quantum hardware simulating four-dimensional inelastic neutron scattering,” *Nat. Phys.* **15**, 455–459 (2019).
- [24] Adam Smith, M. S. Kim, Frank Pollmann, and Johannes Knolle, “Simulating quantum many-body dynamics on a current digital quantum computer,” *npj Quantum Inf.* **5**, 106 (2019).
- [25] Suguru Endo, Jinzhao Sun, Ying Li, Simon C. Benjamin, and Xiao Yuan, “Variational quantum simulation of general processes,” *Phys. Rev. Lett.* **125**, 010501 (2020).
- [26] Cristina Cîrstoiu, Zoë Holmes, Joseph Iosue, Lukasz Cincio, Patrick J. Coles, and Andrew Sornborger, “Variational fast forwarding for quantum simulation beyond the coherence time,” *npj Quantum Information* **6**, 82 (2020).
- [27] Akhil Francis, J. K. Freericks, and A. F. Kemper, “Quantum computation of magnon spectra,” *Phys. Rev. B* **101**, 014411 (2020).
- [28] Ming-Cheng Chen, Ming Gong, Xiaosi Xu, Xiao Yuan, Jian-Wen Wang, Can Wang, Chong Ying, Jin Lin, Yu Xu, Yulin Wu, Shiyu Wang, Hui Deng, Futian Liang, Cheng-Zhi Peng, Simon C. Benjamin, Xiaobo Zhu, Chao-Yang Lu, and Jian-Wei Pan, “Demonstration of adiabatic variational quantum computing with a superconducting quantum coprocessor,” *Phys. Rev. Lett.* **125**, 180501 (2020).
- [29] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Andreas Bengtsson, Sergio Boixo, Michael Broughton, Bob B. Buckley, David A. Buell, Brian Burkett, Nicholas Bushnell, Yu Chen, Zijun Chen, Yu-An Chen, Ben Chiaro, Roberto Collins, Stephen J. Cotton, William Courtney, Sean Demura, Alan Derk, Andrew Dunsworth, Daniel Eppens, Thomas Eckl, Catherine Erickson, Edward Farhi, Austin Fowler, Brooks Foxen, Craig Gidney, Marissa Giustina, Rob Graff, Jonathan A. Gross, Steve Habegger, Matthew P. Harrigan, Alan Ho, Sabrina Hong, Trent Huang, William Huggins, Lev B. Ioffe, Sergei V. Isakov, Evan Jeffrey, Zhang Jiang, Cody Jones, Dvir Kafri, Kostyantyn Kechedzhi, Julian Kelly, Seon Kim, Paul V. Klimov, Alexander N. Korotkov, Fedor Kostritsa, David Landhuis, Pavel Laptev, Mike Lindmark, Erik Lucero, Michael Marthaler, Orion Martin, John M. Martinis, Anika Marusczyk, Sam McArdle, Jarrod R. McClean, Trevor McCourt, Matt McEwen, Anthony Megrant, Carlos Mejuto-Zaera, Xiao Mi, Masoud Mohseni, Wojciech Mroczkiewicz, Josh Mutus, Ofer Naaman, Matthew Nee-

- ley, Charles Neill, Hartmut Neven, Michael Newman, Murphy Yuezhen Niu, Thomas E. O'Brien, Eric Ostby, Bálint Pató, Andre Petukhov, Harald Putterman, Chris Quintana, Jan-Michael Reiner, Pedram Roushan, Nicholas C. Rubin, Daniel Sank, Kevin J. Satzinger, Vadim Smelyanskiy, Doug Strain, Kevin J. Sung, Peter Schmitteckert, Marco Szalay, Norm M. Tubman, Amit Vainsencher, Theodore White, Nicolas Vogt, Z. Jamie Yao, Ping Yeh, Adam Zalcman, and Sebastian Zanker, "Observation of separated dynamics of charge and spin in the fermi-hubbard model," (2020), [arXiv:2010.07965](https://arxiv.org/abs/2010.07965) [quant-ph].
- [30] Benedikt Fauseweh and Jian-Xin Zhu, "Digital quantum simulation of non-equilibrium quantum many-body systems," *Quantum Information Processing* **20** (2021), [10.1007/s11128-021-03079-z](https://doi.org/10.1007/s11128-021-03079-z).
- [31] Joe Gibbs, Kaitlin Gili, Zoë Holmes, Benjamin Commeau, Andrew Arrasmith, Lukasz Cincio, Patrick J. Coles, and Andrew Sornborger, "Long-time simulations with high fidelity on quantum hardware," (2021), [arXiv:2102.04313](https://arxiv.org/abs/2102.04313) [quant-ph].
- [32] Zixuan Hu, Rongxin Xia, and Sabre Kais, "A quantum algorithm for evolving open quantum dynamics on quantum computing devices," *Scientific Reports* **10**, 3301 (2020).
- [33] Lorenzo Del Re, Brian Rost, A. F. Kemper, and J. K. Freericks, "Driven-dissipative quantum mechanics on a lattice: Simulating a fermionic reservoir on a quantum computer," *Phys. Rev. B* **102**, 125112 (2020).
- [34] Zixuan Hu, Kade Head-Marsden, David A. Mazziotti, Prineha Narang, and Sabre Kais, "A general quantum algorithm for open quantum dynamics demonstrated with the fenna-matthews-olson complex," (2021), [arXiv:2101.05287](https://arxiv.org/abs/2101.05287) [quant-ph].
- [35] Kade Head-Marsden, Stefan Krastanov, David A. Mazziotti, and Prineha Narang, "Capturing non-markovian dynamics on near-term quantum computers," *Phys. Rev. Research* **3**, 013182 (2021).
- [36] Hirsh Kamakari, Shi-Ning Sun, Mario Motta, and Austin J. Minnich, "Digital quantum simulation of open quantum systems using quantum imaginary time evolution," (2021), [arXiv:2104.07823](https://arxiv.org/abs/2104.07823) [quant-ph].
- [37] Anthony W. Schlimgen, Kade Head-Marsden, LeeAnn M. Sager, Prineha Narang, and David A. Mazziotti, "Quantum simulation of open quantum systems using a unitary decomposition of operators," (2021), [arXiv:2106.12588](https://arxiv.org/abs/2106.12588) [quant-ph].
- [38] Brian Rost, Lorenzo Del Re, Nathan Earnest, Alexander F. Kemper, Barbara Jones, and James K. Freericks, "Demonstrating robust simulation of driven-dissipative problems on near-term quantum computers," (2021), [arXiv:2108.01183](https://arxiv.org/abs/2108.01183) [quant-ph].
- [39] Mario Motta, Chong Sun, Adrian T. K. Tan, Matthew J. O'Rourke, Erika Ye, Austin J. Minnich, Fernando G. S. L. Brandão, and Garnet Kin-Lic Chan, "Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution," *Nat. Phys.* (2019), [10.1038/s41567-019-0704-4](https://doi.org/10.1038/s41567-019-0704-4).
- [40] J. Cohn, F. Yang, K. Najafi, B. Jones, and J. K. Freericks, "Minimal effective gibbs ansatz: A simple protocol for extracting an accurate thermal representation for quantum simulation," *Phys. Rev. A* **102**, 022622 (2020).
- [41] D. Zhu, S. Johri, N. M. Linke, K. A. Landsman, C. Huerta Alderete, N. H. Nguyen, A. Y. Mat-suura, T. H. Hsieh, and C. Monroe, "Generation of thermofield double states and critical ground states with a quantum computer," *Proceedings of the National Academy of Sciences* **117**, 25402–25406 (2020), <https://www.pnas.org/content/117/41/25402.full.pdf>.
- [42] Shi-Ning Sun, Mario Motta, Ruslan N. Tazhigulov, Adrian T.K. Tan, Garnet Kin-Lic Chan, and Austin J. Minnich, "Quantum computation of finite-temperature static and dynamical properties of spin systems using quantum imaginary time evolution," *PRX Quantum* **2**, 010317 (2021).
- [43] M. Z. Hasan and C. L. Kane, "Colloquium: Topological insulators," *Rev. Mod. Phys.* **82**, 3045–3067 (2010).
- [44] Xiao-Liang Qi and Shou-Cheng Zhang, "Topological insulators and superconductors," *Rev. Mod. Phys.* **83**, 1057–1110 (2011).
- [45] T. Senthil, "Symmetry-protected topological phases of quantum matter," *Annual Review of Condensed Matter Physics* **6**, 299–324 (2015).
- [46] Xiao-Gang Wen, "Colloquium: Zoo of quantum-topological phases of matter," *Rev. Mod. Phys.* **89**, 041004 (2017).
- [47] Marcos Atala, Monika Aidelsburger, Julio T. Barreiro, Dmitry Abanin, Takuya Kitagawa, Eugene Demler, and Immanuel Bloch, "Direct measurement of the Zak phase in topological Bloch bands," *Nature Physics* **9**, 795–800 (2013).
- [48] Gregor Jotzu, Michael Messer, Rémi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif, and Tilman Esslinger, "Experimental realization of the topological Haldane model with ultracold fermions," *Nature* **515**, 237–240 (2014).
- [49] M. Aidelsburger, M. Lohse, C. Schweizer, M. Atala, J. T. Barreiro, S. Nascimbène, N. R. Cooper, I. Bloch, and N. Goldman, "Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms," *Nature Physics* **11**, 162–166 (2015).
- [50] E. Flurin, V. V. Ramasesh, S. Hacohe-Gourgy, L. S. Martin, N. Y. Yao, and I. Siddiqi, "Observing topological invariants using quantum walks in superconducting circuits," *Phys. Rev. X* **7**, 031023 (2017).
- [51] Xinsheng Tan, Dan-Wei Zhang, Qiang Liu, Guangming Xue, Hai-Feng Yu, Yan-Qing Zhu, Hui Yan, Shi-Liang Zhu, and Yang Yu, "Topological maxwell metal bands in a superconducting qutrit," *Phys. Rev. Lett.* **120**, 130503 (2018).
- [52] Shuta Nakajima, Takafumi Tomita, Shintaro Taie, Tomohiro Ichinose, Hideki Ozawa, Lei Wang, Matthias Troyer, and Yoshiro Takahashi, "Topological thouless pumping of ultracold fermions," *Nature Physics* **12**, 296–300 (2016).
- [53] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, "A thouless quantum pump with ultracold bosonic atoms in an optical superlattice," *Nature Physics* **12**, 350–354 (2016).
- [54] Michael Lohse, Christian Schweizer, Hannah M. Price, Oded Zilberberg, and Immanuel Bloch, "Exploring 4D quantum Hall physics with a 2D topological charge pump," *Nature* **553**, 55–58 (2018).
- [55] Martin Leder, Christopher Grossert, Lukas Sitta, Maximilian Genske, Achim Rosch, and Martin Weitz, "Real-space imaging of a topologically protected edge state with ultracold atoms in an amplitude-chirped optical lattice," *Nature Communications* **7**, 13112 (2016).
- [56] Eric J. Meier, Fangzhao Alex An, and Bryce Gad-

- way, “Observation of the topological soliton state in the su–schrieffer–heeger model,” *Nature Communications* **7**, 13986 (2016).
- [57] Bo Song, Long Zhang, Chengdong He, Ting Fung Jeffrey Poon, Elmur Hajiyev, Shanchao Zhang, Xiong-Jun Liu, and Gyu-Boong Jo, “Observation of symmetry-protected topological band with ultracold fermions,” *Science Advances* **4**, eaao4748 (2018).
- [58] Dizhou Xie, Wei Gou, Teng Xiao, Bryce Gadway, and Bo Yan, “Topological characterizations of an extended su–schrieffer–heeger model,” *npj Quantum Information* **5**, 55 (2019).
- [59] Sylvain de Léséleuc, Vincent Lienhard, Pascal Scholl, Daniel Barredo, Sebastian Weber, Nicolai Lang, Hans Peter Büchler, Thierry Lahaye, and Antoine Browaeys, “Observation of a symmetry-protected topological phase of interacting bosons with rydberg atoms,” *Science* **365**, 775–780 (2019).
- [60] W. Cai, J. Han, Feng Mei, Y. Xu, Y. Ma, X. Li, H. Wang, Y. P. Song, Zheng-Yuan Xue, Zhang-qi Yin, Suotang Jia, and Luyan Sun, “Observation of topological magnon insulator states in a superconducting circuit,” *Phys. Rev. Lett.* **123**, 080501 (2019).
- [61] Giulia Semeghini, Harry Levine, Alexander Keesling, Sepehr Ebadi, Tout T. Wang, Dolev Bluvstein, Ruben Verresen, Hannes Pichler, Marcin Kalinowski, Rhine Samajdar, Ahmed Omran, Subir Sachdev, Ashvin Vishwanath, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, “Probing topological spin liquids on a programmable quantum simulator,” (2021), [arXiv:2104.04119](https://arxiv.org/abs/2104.04119) [quant-ph].
- [62] Kenny Choo, Curt W. von Keyserlingk, Nicolas Regnault, and Titus Neupert, “Measurement of the entanglement spectrum of a symmetry-protected topological state using the ibm quantum computer,” *Phys. Rev. Lett.* **121**, 086808 (2018).
- [63] Daniel Azses, Rafael Haenel, Yehuda Naveh, Robert Raussendorf, Eran Sela, and Emanuele G. Dalla Torre, “Identification of symmetry-protected topological states on noisy quantum computers,” *Phys. Rev. Lett.* **125**, 120502 (2020).
- [64] Adam Smith, Bernhard Jobst, Andrew G. Green, and Frank Pollmann, “Crossing a topological phase transition with a quantum computer,” (2020), [arXiv:1910.05351](https://arxiv.org/abs/1910.05351) [cond-mat.str-el].
- [65] Jin Ming Koh, Tommy Tai, Yong Han Phee, Wei En Ng, and Ching Hua Lee, “Stabilizing multiple topological fermions on a quantum computer,” (2021), [arXiv:2103.12783](https://arxiv.org/abs/2103.12783) [cond-mat.str-el].
- [66] K. J. Satzinger, Y. Liu, A. Smith, C. Knapp, M. Newman, C. Jones, Z. Chen, C. Quintana, X. Mi, A. Dunsworth, C. Gidney, I. Aleiner, F. Arute, K. Arya, J. Atalaya, R. Babbush, J. C. Bardin, R. Barends, J. Basso, A. Bengtsson, A. Bilmes, M. Broughton, B. B. Buckley, D. A. Buell, B. Burkett, N. Bushnell, B. Chiaro, R. Collins, W. Courtney, S. Demura, A. R. Derk, D. Eppens, C. Erickson, E. Farhi, L. Foaro, A. G. Fowler, B. Foxen, M. Giustina, A. Greene, J. A. Gross, M. P. Harrigan, S. D. Harrington, J. Hilton, S. Hong, T. Huang, W. J. Huggins, L. B. Ioffe, S. V. Isakov, E. Jeffrey, Z. Jiang, D. Kafri, K. Kechedzhi, T. Khattar, S. Kim, P. V. Klimov, A. N. Korotkov, F. Kostritsa, D. Landhuis, P. Laptev, A. Locharla, E. Lucero, O. Martin, J. R. McClean, M. McEwen, K. C. Miao, M. Mohseni, S. Mon-
tazeri, W. Mroczkiewicz, J. Mutus, O. Naaman, M. Neeley, C. Neill, M. Y. Niu, T. E. O’Brien, A. Opremcak, B. Pató, A. Petukhov, N. C. Rubin, D. Sank, V. Shvarts, D. Strain, M. Szalay, B. Villalonga, T. C. White, Z. Yao, P. Yeh, J. Yoo, A. Zalcman, H. Neven, S. Boixo, A. Megrant, Y. Chen, J. Kelly, V. Smelyanskiy, A. Kitaev, M. Knap, F. Pollmann, and P. Roushan, “Realizing topologically ordered states on a quantum processor,” (2021), [arXiv:2104.01180](https://arxiv.org/abs/2104.01180) [quant-ph].
- [67] Philipp T. Dumitrescu, Justin Bohnet, John Gaebler, Aaron Hankin, David Hayes, Ajesh Kumar, Brian Neyenhuis, Romain Vasseur, and Andrew C. Potter, “Realizing a dynamical topological phase in a trapped-ion quantum simulator,” (2021).
- [68] Xu Zhang, Wenjie Jiang, Jinfeng Deng, Ke Wang, Jiachen Chen, Pengfei Zhang, Wenhui Ren, Hang Dong, Shibo Xu, Yu Gao, Feitong Jin, Xuhao Zhu, Qiujiang Guo, Hekang Li, Chao Song, Alexey V. Gorshkov, Thomas Iadecola, Fangli Liu, Zhe-Xuan Gong, Zhen Wang, Dong-Ling Deng, and H. Wang, “Digital quantum simulation of floquet symmetry-protected topological phases,” *Nature* **607**, 468–473 (2022).
- [69] C Kokail, C Maier, R van Bijnen, T Brydges, M K Joshi, P Jurcevic, C A Muschik, P Silvi, R Blatt, C F Roos, and P Zoller, “Self-verifying variational quantum simulation of lattice models,” *Nature* **569**, 355–360 (2019).
- [70] Timothy Proctor, Kenneth Rudinger, Kevin Young, Erik Nielsen, and Robin Blume-Kohout, “Measuring the capabilities of quantum computers,” *Nature physics* **18**, 75–79 (2022).
- [71] Sergio Boixo, Sergei V. Isakov, Vadim N. Smelyanskiy, Ryan Babbush, Nan Ding, Zhang Jiang, Michael J. Bremner, John M. Martinis, and Hartmut Neven, “Characterizing quantum supremacy in near-term devices,” *Nature physics* **14**, 595–600 (2018).
- [72] Andrew W. Cross, Lev S. Bishop, Sarah Sheldon, Paul D. Nation, and Jay M. Gambetta, “Validating quantum computers using randomized model circuits,” *Phys. Rev. A* **100**, 032328 (2019).
- [73] Richard Kueng, David M. Long, Andrew C. Doherty, and Steven T. Flammia, “Comparing experiments to the fault-tolerance threshold,” *Phys. Rev. Lett.* **117**, 170502 (2016).
- [74] Daniel C. Murphy and Kenneth R. Brown, “Controlling error orientation to improve quantum algorithm success rates,” *Phys. Rev. A* **99**, 032318 (2019).
- [75] Christopher N. Varney, Kai Sun, Victor Galitski, and Marcos Rigol, “Kaleidoscope of exotic quantum phases in a frustrated xy model,” *Phys. Rev. Lett.* **107**, 077201 (2011).
- [76] Zhenyue Zhu, David A. Huse, and Steven R. White, “Unexpected z -direction ising antiferromagnetic order in a frustrated spin-1/2 $J_1 - J_2$ xy model on the honeycomb lattice,” *Phys. Rev. Lett.* **111**, 257201 (2013).
- [77] Haiyuan Zou, Erhai Zhao, Xi-Wen Guan, and W. Vincent Liu, “Exactly solvable points and symmetry protected topological phases of quantum spins on a zig-zag lattice,” *Phys. Rev. Lett.* **122**, 180401 (2019).
- [78] Frank Pollmann, Erez Berg, Ari M. Turner, and Masaki Oshikawa, “Symmetry protection of topological phases in one-dimensional quantum spin systems,” *Phys. Rev. B* **85**, 075125 (2012).
- [79] M. Born and V. Fock, “Beweis des adiabatsatzes,” *Zeitschrift für Physik* **51**, 165–180 (1928).

- [80] Alán Aspuru-Guzik, Anthony D. Dutoi, Peter J. Love, and Martin Head-Gordon, “Simulated quantum computation of molecular energies,” *Science* **309**, 1704–1707 (2005).
- [81] *Quantum Computer Datasheet*, Google Quantum AI (2021).
- [82] Quantum AI team and collaborators, “qsim,” (2020).
- [83] Cirq Developers, “Cirq,” (2021), See full list of authors on Github: <https://github.com/quantumlib/Cirq/graphs/contributors>.
- [84] Akel Hashim, Ravi K. Naik, Alexis Morvan, Jean-Loup Ville, Bradley Mitchell, John Mark Kreikebaum, Marc Davis, Ethan Smith, Costin Iancu, Kevin P. O’Brien, Ian Hincks, Joel J. Wallman, Joseph Emerson, and Irfan Siddiqi, “Randomized compiling for scalable quantum computing on a noisy superconducting quantum processor,” *Phys. Rev. X* **11**, 041039 (2021).
- [85] Ruslan N. Tazhigulov, Shi-Ning Sun, Reza Haghshenas, Huanchen Zhai, Adrian T.K. Tan, Nicholas C. Rubin, Ryan Babbush, Austin J. Minnich, and Garnet Kin-Lic Chan, “Simulating models of challenging correlated molecules and materials on the sycamore quantum processor,” *PRX Quantum* **3**, 040318 (2022).
- [86] Tigran A. Sedrakyan, Leonid I. Glazman, and Alex Kamenev, “Spontaneous formation of a nonuniform chiral spin liquid in a moat-band lattice,” *Phys. Rev. Lett.* **114**, 037203 (2015).
- [87] Eric B. Jones, Logan E. Hillberry, Matthew T. Jones, Mina Fasihi, Pedram Roushan, Zhang Jiang, Alan Ho, Charles Neill, Eric Ostby, Peter Graf, Eliot Kapit, and Lincoln D. Carr, “Small-world complex network generation on a digital quantum processor,” *Nature Communications* **13**, 4483 (2022).