



CHORUS

This is the accepted manuscript made available via CHORUS. The article has been published as:

Experimental violation of the Leggett-Garg inequality using the polarization of classical light

Wenlei Zhang, Ravi K. Saripalli, Jacob M. Leamer, Ryan T. Glasser, and Denys I. Bondar

Phys. Rev. A **104**, 043711 — Published 28 October 2021

DOI: [10.1103/PhysRevA.104.043711](https://doi.org/10.1103/PhysRevA.104.043711)

Experimental Violation of the Leggett-Garg Inequality Using the Polarization of Classical Light

Wenlei Zhang,^{*} Ravi K. Saripalli,^{*} Jacob M. Leamer, Ryan T. Glasser,[†] and Denys I. Bondar[‡]
Department of Physics and Engineering Physics, Tulane University

(Dated: October 14, 2021)

In contrast to Bell’s inequalities which test the correlations between multiple spatially separated systems, the Leggett-Garg inequalities test the temporal correlations between measurements of a single system. We experimentally demonstrate the violation of the Leggett-Garg inequality in a classical optical system using only the polarization degree-of-freedom of a laser beam. Our results show maximal violations of the Leggett-Garg inequality.

I. INTRODUCTION

In their seminal paper [1], Leggett and Garg formulated a test for violations of the postulates of macrorealism based on the temporal correlations between measurements of a single system. The original postulates are stated as (A1) macroscopic realism per se: a macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states, and (A2) non-invasive measurability: it is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics. For a macrorealistic system and some dichotomic observable Q (with realizations ± 1), the correlation function $C_{ij} \equiv \langle Q(t_i)Q(t_j) \rangle$, where $Q(t_i)$ is the measurement of Q at time t_i , must satisfy a class of inequalities, which are now referred to as the Leggett-Garg inequalities (LGIs). In the simplest scenario, the LGI is given by [2]

$$C_{21} + C_{32} - C_{31} \leq 1. \quad (1)$$

Leggett and Garg originally proposed experimental tests using superconducting quantum interference devices [1]. Since then, violations of the LGIs have been observed in experiments based on superconducting transmon qubits [3, 4], nuclear spin qubits [5–10], and light-matter interaction [11]. Experimental violations of the LGIs have also been demonstrated in optical systems, most of which measure properties of single photons [12–16]; whereas, Ref. [17] has experimentally demonstrated the violation of an LGI with the polarization of classical light. However, the authors of Ref. [17] utilize a particular initial state, namely the eigenstate of $Q(t_1) = +1$, to test a modified version of the LGI shown in Eq. (1). They employed a non-invasive measurement at t_2 , which consists of a CNOT operation with the orbital angular momentum as the controlling bit and polarization as the controlled bit. Effectively, the portions of the classical light beam with horizontal and vertical polarizations are split

into two different paths. In a recent theoretical proposal [18], the authors considered the possibility of violating an LGI with coherent states in a random walk setup, which utilizes the path degree-of-freedom. All the previous examples of violations of the LGIs, with the exception of [17] and [18], are based on measurements of microscopic systems. While the study of violations of the LGIs in microscopic systems is interesting in its own right, we feel this is not sufficient evidence to refute macrorealism in the eye of a stubborn macrorealist.

In this work, we present an experimental violation of the LGI in Eq. (1) in a classical optical system using only the polarization degree-of-freedom of a laser beam. Unlike Ref. [17], we demonstrated the violation of the LGI in Eq. (1) using different initial states of polarization. In the quantum optical sense, a classical state of light is defined as a convex (i.e., incoherent) sum of coherent states, and a laser beam is the closest approximation to a coherent state experimentally. The power of the laser beam used in our experiment is ~ 100 mW, which corresponds to $\sim 10^{17}$ photons per second. A system with such a large number of particles should be considered a macroscopic system. Throughout the paper, the classical state of polarization (or simply “state”) of the laser beam is described using the coherency matrix generalization of the Jones calculus [19–22] and quantities with the hat symbol ($\hat{\cdot}$) denote matrices within this context. Our results show maximal violations of the LGI in Eq. (1).

II. THE JONES CALCULUS AND THE COHERENCY MATRIX

The Jones calculus [19] is commonly used to describe the interaction between polarized light and linear optical components. In the Jones calculus, the state of polarized light is represented by a Jones vector, and linear optical components are represented by Jones matrices. For a monochromatic transverse wave travelling in the $+z$ -direction, at a particular point in the transverse profile, the Jones vector is given by a column vector,

$$|\psi\rangle = \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}, \quad (2)$$

^{*} These authors contributed equally.

[†] rglasser@tulane.edu

[‡] dbondar@tulane.edu

where $E_x(t)$ and $E_y(t)$ are the complex amplitudes of the electric field in the x and y directions, respectively. After passing through a linear optical component represented by a 2×2 Jones matrix \hat{J} , the initial Jones vector $|\psi\rangle$ is transformed into $\hat{J}|\psi\rangle$.

While the Jones vector represents polarized light, it can be generalized to describe partially polarized and unpolarized light. In this generalization, the state of polarization is represented by a coherency matrix [20–22]. The coherency matrix is defined as

$$\hat{\rho} = \begin{pmatrix} \langle E_x(t)E_x^*(t) \rangle & \langle E_y(t)E_x^*(t) \rangle \\ \langle E_x(t)E_y^*(t) \rangle & \langle E_y(t)E_y^*(t) \rangle \end{pmatrix}, \quad (3)$$

where $\langle \cdot \rangle$ represents the infinite time average. Physically, the trace of the coherency matrix is equal to the intensity of the wave, averaged over time. After passing through a component with the Jones matrix \hat{J} , the initial coherency matrix is transformed into $\hat{J}\hat{\rho}\hat{J}^\dagger$. The coherency matrix can also be given by the Stokes parameters s_0 , s_1 , s_2 , and s_3 [23, 24],

$$\hat{\rho} = \frac{1}{2} \begin{pmatrix} s_0 + s_1 & s_2 + is_3 \\ s_2 - is_3 & s_0 - s_1 \end{pmatrix}. \quad (4)$$

If we consider the normalized Stokes parameters s_1/s_0 , s_2/s_0 , and s_3/s_0 as coordinates in the 3-dimensional Stokes space, then the Jones vectors represent the points on the Poincaré sphere, while the coherency matrices represent the points on and inside the Poincaré sphere.

III. EXPERIMENTAL SETUP

Figure 1 shows the experimental setup for testing the LGI in Eq. (1). The output of a laser can be approximated by the fundamental transverse Gaussian mode (TEM₀₀). For such a mode with initial state $\hat{\rho}$, the time evolution is equivalent to its propagation along the optical axis [Fig. 1 (a)]. The unitary time evolution \hat{U} of $\hat{\rho}$ is generated by two half-wave plates (HWP) as shown in Fig. 1 (b), where θ_1 and θ_2 are the angles between the fast axes of the respective HWP and the horizontal axis. The time evolutions in each time step are chosen to be the same for simplicity. The correlations functions C_{21} , C_{32} , and C_{31} are measured using the respective setups shown in Fig. 1 (c)-(e). In these setups, the measurement on the coherency matrix is realized via a polarizing beamsplitter with realizations +1 for horizontal polarization and -1 for vertical polarization. Under such realizations, the observable being measured is the Pauli matrix $\hat{\sigma}_z$, i.e. $\hat{Q}(t_i) = \hat{\sigma}_z$ for $i = 1, 2, 3$. The measured intensities $I_{ij}(m, n)$ at the intensity detectors represent the portion of light after measuring m and n at t_i and t_j , respectively. The measured correlation functions are given by the intensity measurements in Fig. 1 (c)-(e) as

$$C_{ij} = \frac{\sum_{m,n \in \{-1,1\}} mn I_{ij}(m, n)}{\sum_{m,n \in \{-1,1\}} I_{ij}(m, n)}. \quad (5)$$

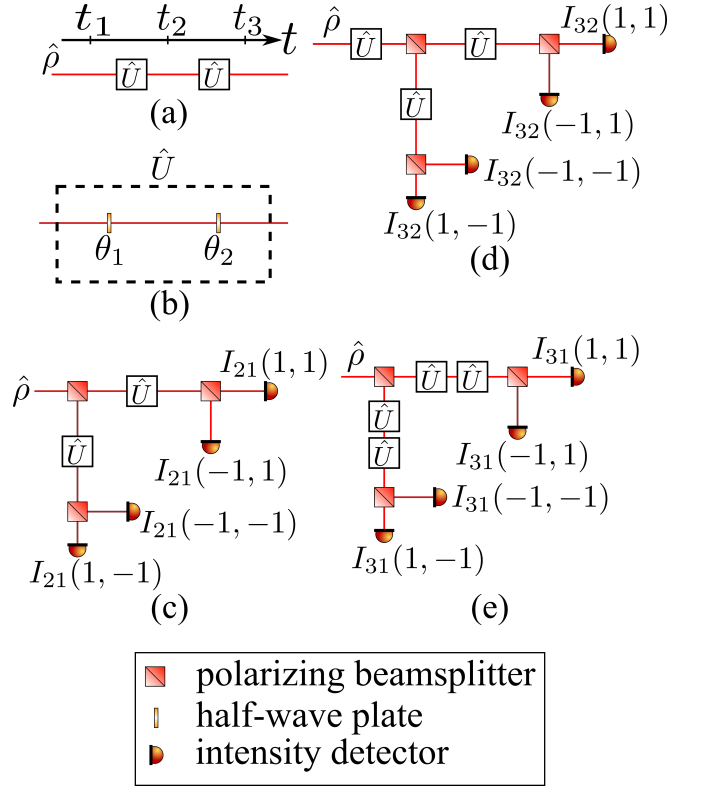


FIG. 1. (a) Time evolution of the coherency matrix $\hat{\rho}$. (b) Realization of a unitary operation \hat{U} on the polarization matrix using two half-wave plates (H). Experimental setups for measuring (c) C_{21} , (d) C_{32} , and (e) C_{31} .

Let us denote the left-hand side of the LGI in Eq. (1) as $\mathcal{K} \equiv C_{21} + C_{32} - C_{31}$. To theoretically calculate \mathcal{K} within the coherency matrix representation, we use the following equations to obtain the theoretical values of the intensities,

$$I_{21}(m, n) = \text{Tr}(\hat{P}(m)\hat{U}\hat{P}(n)\hat{\rho}\hat{P}(n)\hat{U}^\dagger\hat{P}(m)), \quad (6)$$

$$I_{32}(m, n) = \text{Tr}(\hat{P}(m)\hat{U}\hat{P}(n)\hat{U}\hat{\rho}\hat{U}^\dagger\hat{P}(n)\hat{U}^\dagger\hat{P}(m)), \quad (7)$$

$$I_{31}(m, n) = \text{Tr}(\hat{P}(m)\hat{U}\hat{U}\hat{P}(n)\hat{\rho}\hat{P}(n)\hat{U}^\dagger\hat{U}^\dagger\hat{P}(m)), \quad (8)$$

$$\hat{P}(m) = \frac{1+m}{2}\hat{P}_H + \frac{1-m}{2}\hat{P}_V, \quad (9)$$

where \hat{U} represents the total Jones matrix of the two HWPs in Fig. 1 (b), \hat{P}_H and \hat{P}_V are the respective Jones matrices of a horizontal and a vertical linear polarizer, and $\text{Tr}(\cdot)$ denotes the trace of a matrix. The total Jones matrix of the two HWPs is determined by θ_1 and θ_2 . The intensities in Eqs. (6)-(8) can be used to calculate the theoretical values of the correlations functions in Eq. (5). From the above equations, we can obtain the following expression for the theoretical value of \mathcal{K} in terms of the HWP angles θ_1 and θ_2 ,

$$\mathcal{K} = 2 \cos(4\theta_1 - 4\theta_2) - \cos(8\theta_1 - 8\theta_2). \quad (10)$$

Note that \mathcal{K} is independent of the initial state $\hat{\rho}$ [16].

IV. RESULTS

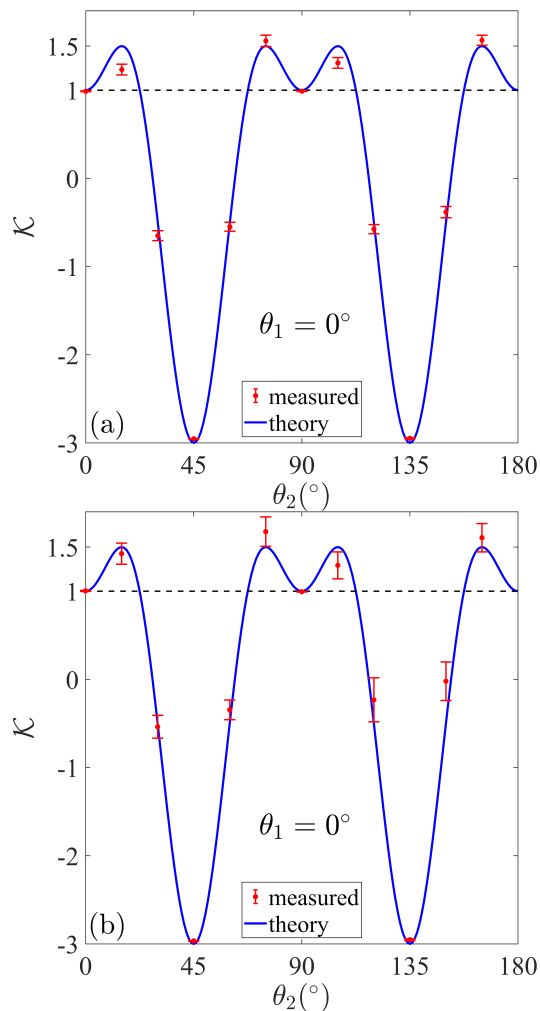


FIG. 2. Measured and theoretical values of \mathcal{K} for initial states of (a) diagonal polarization and (b) a partially-polarized state with degree of polarization of ~ 0.8 . In both plots, the data points above the dashed line are the violations of the LGI in Eq. (1). The theoretical values are calculated using Eq. (10).

Figure 2 shows the experimentally measured and theoretically calculated values of \mathcal{K} for different initial states. The diagonal polarization state is generated by rotating the polarization of the linearly polarized laser beam with a HWP. The partially-polarized initial state is generated by combining two orthogonal polarizations [25]. For both initial states, θ_1 is fixed at 0° and θ_2 is varied from 0° to 180° . The measurements were repeated five times. Both plots in Fig. 2 show violations of the LGI in Eq. (1). The main source of experimental error is the laser intensity fluctuations during measurement, as twelve intensity measurements are needed to calculate one data point in

Fig. 2. The detector noise is comparatively small and negligible ($< 1\%$). Some of the violations shown in Fig. 2 are higher than the theoretical maximum, which is mostly a consequence of the intensity fluctuations between measurements. The method used to generate the partially-polarized polarization state introduces more fluctuations, which is reflected by the error bars in Fig. 2.

V. DISCUSSION

We presented an experimental demonstration of violations of the Leggett-Garg inequality using only the polarization degree-of-freedom of a laser beam. Our results show that it is possible to violate the LGI in Eq. (1) using only classical linear polarization optics. The possibility of violating the LGI with the polarization of classical light is attributed to the superposition of temporally coherent fields [2, 17]. It has been shown that the LGI can no longer be violated if the temporal degree of coherence is below a certain value [17]. However, as we did not quantify the invasiveness of our measurement apparatus, our results are still subject to the “clumsiness” loophole [1, 26], which says that the violation of the LGI is not due to the fact that the system is non-macrorealistic, but because the system was subjected to a measurement technique that happens to disturb it. This loophole can be closed by using ideal negative-result measurements [1] or adroit measurements [26]. For ideal negative-result measurements, we would need a device that interacts with the laser beam for only one particular state of polarization, and does not interact at all otherwise. For adroit measurements, we would need to measure an additional observable, $\hat{\sigma}_\theta$, which can be realized by placing a series of retarders before the polarizing beamsplitter. However, even with these improvements, the stubborn macrorealist can still reject the violation results due to other weaker loopholes [8, 26].

ACKNOWLEDGMENTS

D.I.B. thanks Kurt Jacobs for insightful discussions. This work was supported by the Defense Advanced Research Projects Agency (DARPA) grant number D19AP00043 under mentorship of Dr. Joseph Altepeter. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of DARPA, or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation herein. RTG acknowledges that this material is based upon work supported by, or in part by, the Army Research Laboratory and the Army Research Office under contract/grant numbers W911NF-19-2-0087 and W911NF-20-2-0168. J.M.L. was supported by the Louisiana Board of Regents Graduate Fellowship Program.

-
- [1] A. J. Leggett and A. Garg, Quantum mechanics versus macroscopic realism: Is the flux there when nobody looks?, *Physical Review Letters* **54**, 857 (1985).
- [2] C. Emary, N. Lambert, and F. Nori, LeggettGarg inequalities, *Reports on Progress in Physics* **77**, 016001 (2014).
- [3] A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. N. Korotkov, Experimental violation of a Bells inequality in time with weak measurement, *Nature Physics* **6**, 442 (2010).
- [4] J. P. Groen, D. Riste, L. Tornberg, J. Cramer, P. C. de Groot, T. Picot, G. Johansson, and L. DiCarlo, Partial-Measurement Backaction and Nonclassical Weak Values in a Superconducting Circuit, *Physical Review Letters* **111**, 090506 (2013).
- [5] G. Waldherr, P. Neumann, S. F. Huelga, F. Jelezko, and J. Wrachtrup, Violation of a Temporal Bell Inequality for Single Spins in a Diamond Defect Center, *Physical Review Letters* **107**, 090401 (2011).
- [6] V. Athalye, S. S. Roy, and T. S. Mahesh, Investigation of the Leggett-Garg Inequality for Precessing Nuclear Spins, *Physical Review Letters* **107**, 130402 (2011).
- [7] A. M. Souza, I. S. Oliveira, and R. S. Sarthour, A scattering quantum circuit for measuring bell's time inequality: a nuclear magnetic resonance demonstration using maximally mixed states, *New Journal of Physics* **13**, 053023 (2011).
- [8] G. C. Knee, S. Simmons, E. M. Gauger, J. J. L. Morton, H. Riemann, N. V. Abrosimov, P. Becker, H.-J. Pohl, K. M. Itoh, M. L. W. Thewalt, G. A. D. Briggs, and S. C. Benjamin, Violation of a LeggettGarg inequality with ideal non-invasive measurements, *Nature Communications* **3**, 606 (2012).
- [9] R. E. George, L. M. Robledo, O. J. E. Maroney, M. S. Blok, H. Bernien, M. L. Markham, D. J. Twitchen, J. J. L. Morton, G. A. D. Briggs, and R. Hanson, Opening up three quantum boxes causes classically undetectable wavefunction collapse, *Proceedings of the National Academy of Sciences* **110**, 3777 (2013).
- [10] H. Katiyar, A. Shukla, K. R. K. Rao, and T. S. Mahesh, Violation of entropic Leggett-Garg inequality in nuclear spins, *Physical Review A* **87**, 052102 (2013).
- [11] Z.-Q. Zhou, S. F. Huelga, C.-F. Li, and G.-C. Guo, Experimental Detection of Quantum Coherent Evolution through the Violation of Leggett-Garg-Type Inequalities, *Physical Review Letters* **115**, 113002 (2015).
- [12] M. E. Goggin, M. P. Almeida, M. Barbieri, B. P. Lanyon, J. L. O'Brien, A. G. White, and G. J. Pryde, Violation of the Leggett-Garg inequality with weak measurements of photons, *Proceedings of the National Academy of Sciences* **108**, 1256 (2011).
- [13] J.-S. Xu, C.-F. Li, X.-B. Zou, and G.-C. Guo, Experimental violation of the Leggett-Garg inequality under decoherence, *Scientific Reports* **1**, 101 (2011).
- [14] J. Dressel, C. J. Broadbent, J. C. Howell, and A. N. Jordan, Experimental Violation of Two-Party Leggett-Garg Inequalities with Semiweak Measurements, *Physical Review Letters* **106**, 040402 (2011).
- [15] Y. Suzuki, M. Iinuma, and H. F. Hofmann, Violation of LeggettGarg inequalities in quantum measurements with variable resolution and back-action, *New Journal of Physics* **14**, 103022 (2012).
- [16] K. Wang, M. Xu, L. Xiao, and P. Xue, Experimental violations of Leggett-Garg inequalities up to the algebraic maximum for a photonic qubit, *Physical Review A* **102**, 022214 (2020).
- [17] X. Zhang, T. Li, Z. Yang, and X. Zhang, Experimental observation of the Leggett-Garg inequality violation in classical light, *Journal of Optics* **21**, 015605 (2019).
- [18] H. Chevalier, A. J. Paige, H. Kwon, and M. S. Kim, Violating the Leggett-Garg inequalities with classical light, *Physical Review A* **103**, 043707 (2021).
- [19] R. C. Jones, A New Calculus for the Treatment of Optical Systems I. Description and Discussion of the Calculus, *JOSA* **31**, 488 (1941).
- [20] E. Wolf, Coherence properties of partially polarized electromagnetic radiation, *Il Nuovo Cimento (1955-1965)* **13**, 1165 (1959).
- [21] N. Wiener, Coherency Matrices and Quantum Theory, *Journal of Mathematics and Physics* **7**, 109 (1928).
- [22] J. W. Goodman, *Statistical Optics*, 2nd ed. (Wiley, Hoboken, New Jersey, 2015).
- [23] G. G. Stokes, On the Composition and Resolution of Streams of Polarized Light from different Sources, *Transactions of the Cambridge Philosophical Society* **9**, 399 (1851).
- [24] G. G. Stokes, On the composition and resolution of streams of polarized light from different sources, in *Mathematical and Physical Papers*, Cambridge Library Collection - Mathematics, Vol. 3 (Cambridge University Press, 2009) p. 233258.
- [25] D. Barberena, G. Gatti, and F. De Zela, Experimental demonstration of a secondary source of partially polarized states, *Journal of the Optical Society of America A* **32**, 697 (2015).
- [26] M. M. Wilde and A. Mizel, Addressing the Clumsiness Loophole in a Leggett-Garg Test of Macrorealism, *Foundations of Physics* **42**, 256 (2012).