Resonant frequency ratios for the dynamical Casimir effect

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Novel frequency ratios for the dynamical Casimir effect

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The dynamical Casimir effect (DCE) is the production of photons by the amplification of vacuum fluctuations. In this paper we demonstrate new resonance conditions in DCE that potentially allow the production of optical photons when the mechanical frequency is smaller than the lowest frequency of the cavity field. We consider a cavity with one mirror fixed and the other allowed to oscillate. In order to identify the region where production of photons takes place, we do a stability analysis of the Heisenberg–Langevin equations of motion, and investigate the dynamic stability of the system under small fluctuations. By using a numerical solution of the master equation, the time evolution of the mean number of photons produced in the unstable region is studied.

I. INTRODUCTION

The dynamical Casimir effect (DCE) is the generation of photons from the quantum vacuum due to a time-dependent boundary condition of the electromagnetic field. Because of the structure of the vacuum, which is different in a cavity than in free space, time-changing boundary conditions change the field structure inside the cavity and in some cases leads to the production of photons in the cavity. This was first analyzed by Moore [1]. A clear way to accomplish this is by changing the cavity length [1, 2], as for instance when one of the mirrors undergoes harmonic oscillations (see Fig. 1). The experimental demonstration of DCE at microwave frequencies was recently reported in [3, 4]. However, no optical frequency photons produced by DCE have been seen yet. A fundamental limitation is that the periodic modulation of the cavity imposes a definite ratio of photon-to-mechanical frequencies as a resonance condition for photon generation [5–7]: When the mechanical frequency Ω of the moving mirror is twice the fundamental frequency ω_1 of the unperturbed cavity, the effect of parametric resonance is the largest and the number of photons in a perfect cavity grows exponentially with time [8]; this has been the traditional resonance condition in the study of DCE. This unfavorable frequency ratio makes it experimentally extremely hard to see DCE in, for example, the visible domain, i.e., production of optical photons would require a massive mirror moving at relativistic frequencies. It is the main goal of this work to uncover new resonances that lift this stringent frequency condition and, in principle, open the door for DCE in the higher frequency domain. We only explicitly show the "instantaneous" cavity photons, similar to the mechanism in [9], but with cavity leakage a photon number measurement could be feasible.

The simplest description of DCE can be pictured as a cavity with one fixed and one oscillating mirror (Fig. 1), which is equivalent to the simplest picture of optomechanical systems [3, 10, 11]. The coupling between the photon and mechanical modes is comprised by the modulation of the cavity length by the mechanical motion; the photon frequency thus depends on time through the changing length of the resonator, via the Hamiltonian

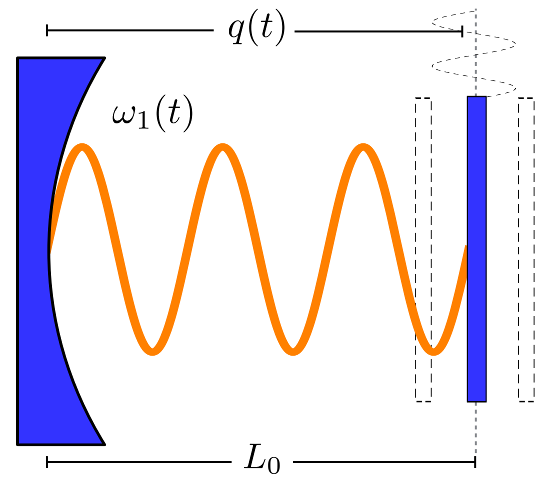

FIG. 1. Schematic representation of a cavity with a moving mirror. Equilibrium length and photon frequency are L_0 and $\omega_1(t)$, respectively. Both are periodically modified by small oscillations $q(t)$ of one of the mirrors.

$$H_{\text{photon}} = \hbar\omega(t)\hat{a}^{\dagger}(t)\hat{a}(t), \tag{1}$$

where the photon frequency ω and photon operators $\hat{a}^{\dagger}$ ($\hat{a}$) become time dependent. The traditional perturbative treatment is to assume that the maximum displacement of the mirror $x_m \equiv q_{\max} - L_0$ with respect to the unperturbed cavity length L_0 is a small parameter ($\epsilon \equiv x_m/L_0 \ll 1$). Thus the time–dependent photon frequency is typically expanded to lowest order in ϵ [9, 12, 13]. (Quadratic orders of ϵ are necessary for the "membrane-in-the-middle" optomechanical set-up [14].) A multi-mode expansion for the photon operators and expanding the photon frequency to first order in ϵ gives then, e.g., the traditional DCE description, such as in

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[5, 6, 15], where the terms responsible for DCE are proportional to $(\hat{b}+\hat{b}^\dagger)(\hat{a}^{\dagger 2}+\hat{a}^2)$ with phonon operators $\hat{b}+\hat{b}^\dagger \propto x_m$, which accounts for the resonance at $\Omega \approx 2\,\omega$.

In this work, we obtain higher order resonances that allow the production of Casimir photons when the mechanical frequency of the moving mirror is smaller than the lowest cavity frequency. We study the implications of keeping higher orders of ϵ in the emergence of such resonances. While these terms can be very small, we show that they can be important as they give rise to new resonance conditions that could help to create experimentally feasible frequency ratios for the demonstration of DCE in the optical domain. In order to identify which parameter regime can lead to DCE at such frequencies, we do a stability analysis of the Heisenberg–Langevin equations of motion. We then calculate the mean number of photons produced in the stable and unstable regions when resonant and non-resonant terms are included in the numerical solution of the master equation. The organization of the paper is as follows: in Sec. II we discuss the model. The method to find new resonance conditions in DCE and set of parameters that lead to photon growth is formulated in Sec. III. Full numerical solution of the master equation in the presence of cavity losses is discussed in Sec. IV. Conclusions and a comparison of our results to traditional DCE treatments with current experimental parameters is presented in Sec. V.

II. MODEL

In our model, the mirror's motion is treated classically, since we assume a strongly driven mechanical oscillator. We use Law's effective Hamiltonian [9] (dropping the operator hats on the photon operators):

$$H_{\text{eff}} = \sum_k \left[\omega_k(t) a_k^\dagger(t) a_k(t) - \frac{\dot{q}(t)}{q(t)} f(a_k(t), a_k^\dagger(t))\right], \quad (2)$$

where

$$f(a_k(t), a_k^\dagger(t)) = \frac{i}{4}\left[\left(a_k^{\dagger 2}(t) - a_k^2(t)\right) - 2\sum_{j(\neq k)} g_{kj}\sqrt{\frac{\omega_k(t)}{\omega_j(t)}} \times \right.$$
$$\left. \times \left(a_k^\dagger(t) a_j^\dagger(t) + a_k^\dagger(t) a_j(t) - a_j(t) a_k(t) - a_j^\dagger(t) a_k(t)\right)\right],$$

$$\text{and} \qquad g_{kj} = \begin{cases} (-1)^{k+j}\frac{2kj}{j^2-k^2}, & k \neq j \\ 0, & k = j. \end{cases}$$

The effective Hamiltonian (2) contains terms characterized by $a_k^\dagger a_j$ that are responsible for the scattering of photons between modes.

A multimode expansion of the cavity field annihilation operator in general is given by

$$a(t) = \sum_{k=1}^{\infty}\left[c_k(t)\, a_k + d_k(t)\, a_k^\dagger\right],$$

where $c_k(t), d_k(t)$ are c-numbers and a_k ($a_k^\dagger$) is the annihilation (creation) operator of the kth mode of the equilibrium-length cavity. An effective Hamiltonian thus will contain this series expansion as well as extra terms, due to the explicit time dependence of the cavity frequency and mode coefficients [9, 16–18].

The exact form of the expansions now depends on the mirror's trajectory, chosen in what follows as nearly harmonic [9, 19]

$$q(t) = L_0 \exp\left[\epsilon \sin\Omega t\right],$$

such that $\dot{q}(t)/q(t)$ is simplified without introducing a qualitative difference to a pure harmonic motion. Thus, the photon frequency $\omega_k(t) = k\pi c/q(t)$ of mode k, where c is the speed of light, is expanded in a series in ϵ as

$$\omega_k(t) = \omega_{k0}\left[1 - \epsilon\sin\Omega t + \frac{1}{2}\left(\epsilon\sin\Omega t\right)^2 - \dots\right], \quad (3)$$

where ω_{k0} denotes the photon frequency associated with the unperturbed cavity length L_0. Similarly, for the time-dependent annihilation operator of mode k we get

$$a_k(t) = \sum_{\ell \text{ even}} \frac{\left(\epsilon\sin\Omega t\right)^\ell}{2^\ell \ell!}\, a_{k0} - \sum_{\ell \text{ odd}} \frac{\left(\epsilon\sin\Omega t\right)^\ell}{2^\ell \ell!}\, a_{k0}^\dagger, \quad (4)$$

where we have used the typical expression of the annihilation operator in terms of time–dependent cavity frequency, and generalized cavity field position and momentum operators. Note that the operator expansion above does not mix other modes, and the time–dependent expansion coefficients depend on odd and even powers of the expansion. We should point about that expansion (4) preserves bosonic commutation relations between cavity field operators at a considered order of the expansion. In the rest of the article we will omit the subscript 0 in the unperturbed photon frequency and field operators.

The equations of motion are then given by introducing Heisenberg-Langevin formalism, with noise operators that account for cavity decay κ_k of mode k, and fluctuations arising from the coupling of the photon modes to an external bath at zero temperature. It is worth mentioning that we start with zero photons in the cavity, hence only photon vacuum fluctuations are initially present. We are, in particular, interested in the regime where the mechanical frequency is smaller than the optical frequency of the lowest cavity mode ($\Omega < \omega_1$).

III. METHOD

A. Shifted photon frequency

The series expansion produces a shift in the photon frequency $\tilde{\omega}_k$, that is determined by

$$\tilde{\omega}_k = \frac{1}{2}\left[1 + \sum_{l=0}^{\infty}\frac{(-2)^l \epsilon^l}{l!}\left\langle \sin^l \Omega t\right\rangle\right]\omega_k = \frac{1}{2}\left[1 + \sum_{p=0}^{\infty}\frac{\epsilon^{2p}}{(p!)^2}\right]\omega_k, \quad (5)$$

where $\langle \dots \rangle$ is the average value of a function and k is the mode number. Eq. (5) can be resolved to

$$\tilde{\omega}_k = \frac{1}{2}\left[1 + I_0(2\epsilon)\right]\omega_k,$$

where $I_0(2\epsilon)$ is the modified Bessel function of the first kind. Since $\epsilon \ll 1$, a series expansion of $I_0(2\epsilon)$ produces a small shift on the photon frequency. This renormalization is due to higher orders of ϵ, which can be analytically added up.

B. General equations of motion

Using the effective Hamiltonian (2) for k modes of the field and expanding up to terms in ϵ^n, we get Heisenberg-Langevin equations of motion for the operators in a frame rotating at frequency $\tilde{\omega}_k$, e.g., for the annihilation operator we get

$$\frac{d\tilde{a}_k}{dt} = \frac{i}{2}\left[1 - \sum_{l=0}^{n}\sum_{m=0}^{l}(-1)^{l+m}\frac{\epsilon^l}{i^l\, l!}\binom{l}{m}e^{-i\Omega t(2m-l)}\right]\omega_k \tilde{a}_k^\dagger e^{2i\tilde{\omega}_k t} - \frac{1}{2}\Omega\epsilon\cos\Omega t\,\tilde{a}_k^\dagger e^{2i\tilde{\omega}_k t} - \\ - \Omega\epsilon\cos\Omega t\sum_{\substack{j=1\\k\neq j}}(-1)^{k+j}\left[\frac{\sqrt{\omega_k\omega_j}}{\omega_j+\omega_k}\tilde{a}_j^\dagger e^{2i\tilde{\omega}_j t} - \frac{\sqrt{\omega_k\omega_j}}{\omega_j-\omega_k}\tilde{a}_j e^{-2i\tilde{\omega}_j t}\right] - \frac{\kappa_k}{2}\tilde{a}_k + f_{\tilde{a}_k}(t), \tag{6}$$

where κ_k is the damping rate of mode k of the cavity field, and $f_{\tilde{a}_k}(t)$ is noise operator that satisfies

$$\left\langle f_{\tilde{a}}(t) f_{\tilde{a}}^\dagger(t')\right\rangle_{\mathrm{R}} = \kappa\delta\left(t-t'\right), \qquad \left\langle f_{\tilde{a}}^\dagger(t) f_{\tilde{a}}(t')\right\rangle_{\mathrm{R}} = 0,$$

where $\langle \dots \rangle_{\mathrm{R}}$ is the average over the reservoir. Such equations of motion take into account all resonant and non-resonant terms, including counter-rotating terms which are responsible for DCE.

C. Resonance conditions

Resonances in Eq. (6) are determined by

$$\Omega = \frac{2\,\tilde{\omega}_k}{2m-l},$$

where $l > 0$ and $m \in \mathbb{N}$ are fixed values such that $m \leq l$. The shifted photon frequency of mode k is given by mode number times the shifted frequency of the fundamental mode ($\tilde{\omega}_k = k\,\tilde{\omega}_1$). Note that when $m = l$, then our regime of interest in terms of the resonance condition is minimized, and the minimum order of the expansion that one has to take to satisfy $\Omega < \tilde{\omega}_1$ is $l = 3$. As an example, we choose the initial mode number ($k = 1$), hence, the minimum order of the expansion that one has to take to satisfy such condition is $l = 3$, which occurs in the term of third order in the expansion parameter ϵ, thus $\Omega = 2\,\tilde{\omega}_1/3$ is a new resonance condition for DCE. In general, we can choose as resonant any pair of frequencies that are given by

$$\Omega = \frac{2}{n}\,\tilde{\omega}_1, \tag{7}$$

which occurs in the term of nth order in the expansion parameter ϵ, therefore for $\Omega < \tilde{\omega}_1$ we take orders higher than 2. Thus, for consistency, for the general resonance in expression (7), we need to at least carry orders in ϵ up to nth order, and along with that, an expansion to the nth cavity mode. Note that in an experiment this resonance would be chosen by selecting the correct photon and mechanical frequency ratios.

D. Stability analysis

A trivial steady-state solution of this system of equations is the situation when no photons are present in the cavity, i.e., $\langle a_k^\dagger a_k\rangle = 0$, for all mode numbers k. When this steady-state solution is stable it means that small perturbations (i.e., photon fluctuations) decay in time and hence no photons are produced. Opposite, when this “zero-solution” is dynamically unstable, any noise (e.g., vacuum fluctuations) drives the system away from steady-state resulting in photon build-up. To determine the region where photon production takes place, we do a stability analysis.

The first step is to find the approximate boundary between stable and unstable region. The second is the stability analysis. A system is dynamically stable if $\mathrm{Re}\,(\lambda) < 0$ for all eigenvalues λ of the system. For a simple estimate of the border and to easily identify the parameters where photon growth could occur, we neglect all non-resonant terms for a chosen resonance condition in this step. With this approximation, the remaining equations of motion simplify significantly

$$\frac{d\tilde{a}_k}{dt} = \frac{i}{2}f_{nk}(\epsilon,k)\,\omega_k\,\tilde{a}_k^\dagger - \frac{1}{2}\kappa_k\tilde{a}_k + f_{\tilde{a}_k}(t),$$

and

$$\frac{d\tilde{a}_k^\dagger}{dt} = -\frac{i}{2}f_{nk}^*(\epsilon,k)\,\omega_k\,\tilde{a}_k - \frac{1}{2}\kappa_k\tilde{a}_k^\dagger + f_{\tilde{a}_k}^\dagger(t),$$

where

$$f_{nk}(\epsilon, k) = \begin{cases} i^{nk} I_{nk}(2\epsilon), & \text{when} \quad nk = \text{odd} \\ (-1)^{\frac{nk}{2}+1} I_{nk}(2\epsilon), & \text{when} \quad nk = \text{even}, \end{cases}$$

and $I_{nk}(2\epsilon)$ is the modified Bessel function of the first kind. For a series expansion with $n > 2$, the eigenvalue with the largest real part is given by

$$\lambda_{\max} = \frac{1}{2} I_{n1}(2\epsilon)\,\omega_1 - \frac{1}{2}\kappa_1, \tag{8}$$

where κ_1 is the damping rate of the fundamental mode of the cavity field. The general eigenvalue condition Eq. (8) follows from the characteristic equation of the resonant equations of motion for a chosen resonant condition.

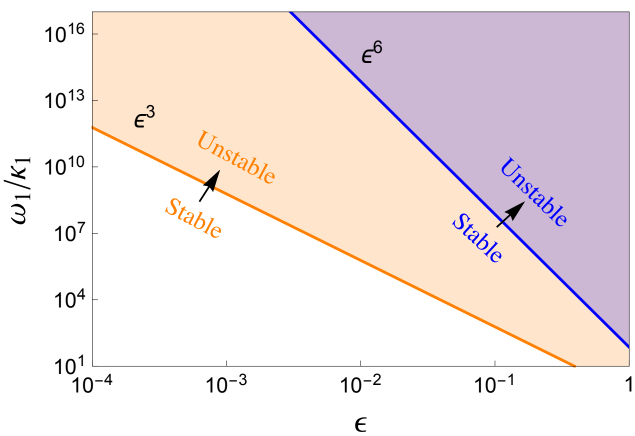

FIG. 2. (Color online) Dimensionless frequency vs. coupling parameter ϵ. The estimated stability boundary of the vacuum state is determined by the maximum eigenvalue $\lambda_{\max}$ for 3 modes (orange/lower solid line) and 6 modes (blue/upper solid line) when the resonance condition is $\Omega = 2\,\tilde{\omega}_1/3$ and $\Omega = 2\,\tilde{\omega}_1/6$, respectively.

As an example, consider three modes of the cavity field and $n = 3$ in Eq. (7); the lowest order where the mechanical oscillation frequency is lower than the photon frequency, thus leading to the resonant condition $\Omega = 2\,\tilde{\omega}_1/3$. In order to determine the region of operation for DCE, we first find the boundary where $\lambda_{\max}$ in Eq. (8) goes through zero, in the parameter space of ω_1/κ_1 vs. ϵ, see Fig. 2. (In this figure, we also plot the boundary for six modes of the cavity field and $n = 6$.)

IV. FULL NUMERICAL SOLUTION

Having approximately identified the region of operation of our model, we then determine the exact photon generation, where all the resonant and non-resonant terms are included. We solve numerically the master equation using the effective Hamiltonian Eq. (2) for 3 (6) modes of the field, and expanding up to terms in ϵ^3

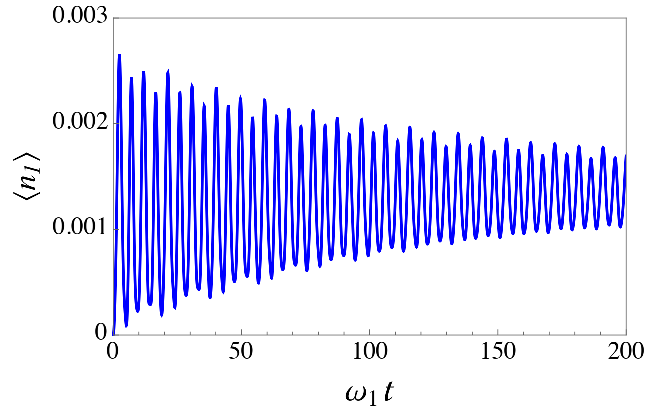

FIG. 3. Time evolution of the mean photon number $\langle n_1 \rangle$ in the fundamental cavity mode vs dimensionless time when expanding up to third order in ϵ and with the resonance condition $\Omega = 2\,\tilde{\omega}_1/3$. With $\omega_1/\kappa_1 = 1 \times 10^2$ and $\epsilon = 0.05$ the system operates in the stable region resulting in no photon growth.

(ϵ^6), respectively. By inspecting the equations of motion and using the general resonant condition Eq. (7), we note that the contribution of the terms for order higher than 3 (6) in ϵ is small and thus neglected.

We investigate the full time evolution of the mean number of photons for $n = 3$ in Eq. (7). With $\omega_1/\kappa_1 = 1 \times 10^2$ and $\epsilon = 0.05$, the phase diagram in Fig. 2 corresponds to the stable region with such parameters, and thus no photons are produced in the long-term (see Fig. 3). In Fig. 4 we plot the time evolution of the average number of resonant photons $\langle n_k \rangle$ in the unstable region for the first three modes k of the cavity field, for the same resonant condition $\Omega = 2\,\tilde{\omega}_1/3$, with $\omega_1/\kappa_1 = 5 \times 10^2$ and $\epsilon = 0.45$. The number of photons in all modes considered is resonantly excited. The number of Casimir photons in the fundamental cavity mode $\langle n_1 \rangle$ is stronger and saturates due to the presence of cavity decay. The number of scattered photons from mode 1 to 2 and from mode 1 to 3 grow at a slower rate and also saturate.

In Fig. 5 we show the time evolution for 6 modes and $n = 6$ in the general resonant condition Eq. (7). Taking $\omega_1/\kappa_1 = 1 \times 10^5$ and $\epsilon = 0.75$, which correspond to the unstable region in Fig. 2 (blue/upper solid line), thus DCE at optical frequencies takes place when $\Omega < \tilde{\omega}_1$. In order to guide the eye we plotted a time–average over a number of cycles (orange dotted line), and one can see that the mean number of photons tends to saturate due to the presence of cavity damping.

Here, we have, to a large degree, adopted the theoretical methodology of Law's original paper [9]. One important point in his paper concerns the caveat of "instantaneous" vs. "measurable" photons, i.e., photons that should be at any instant inside the cavity but cannot be measured without interrupting the process. Our treatment/figures also indicate just the instantaneous photon number in Figs. 3–5. We do assume, however, cavity

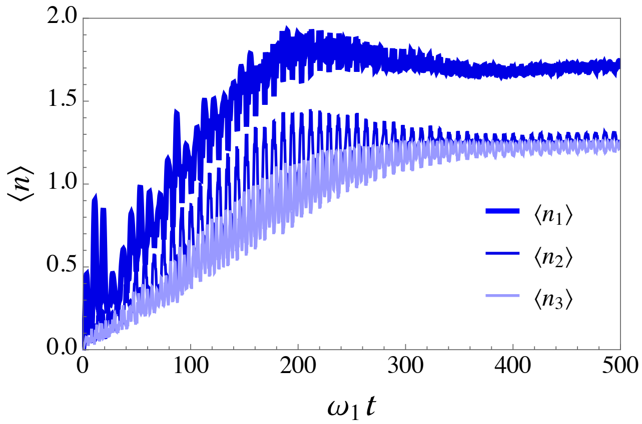

FIG. 4. Time evolution of the mean photon number $\langle n \rangle$ in modes 1, 2 and 3 vs. dimensionless time for three modes and expanding up to third order. With $\omega_1/\kappa_1 = 5 \times 10^2$ and $\epsilon = 0.45$ the system operates in the unstable region resulting in photon build-up. The fundamental cavity mode $\langle n_1 \rangle$ is stronger. The number of photons in the scattered modes 2 and 3 also grow at a slower rate.

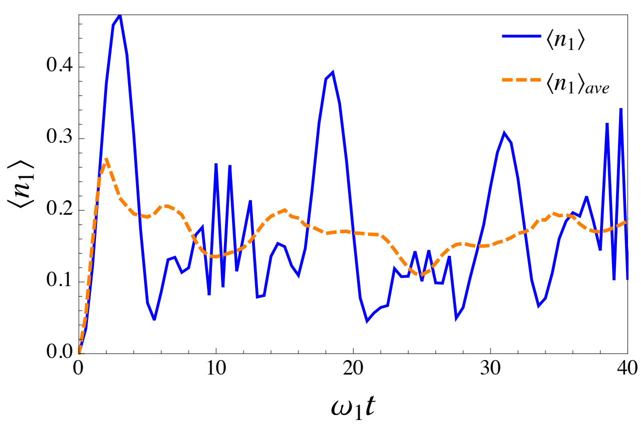

FIG. 5. Time evolution of the mean photon number (blue solid line) in the fundamental mode and time–average over a number of cycles (orange dotted line) vs. dimensionless time for six modes and expanding up to sixth order. With $\omega_1/\kappa_1 = 1 \times 10^5$ and $\epsilon = 0.75$, the system operates in the unstable region and DCE at optical frequencies takes place. The mean number of photons saturates due to the presence of cavity damping

leakage. The obvious measurable quantity in this context would thus be the number of photons leaked from the cavity, which saturates the photon build-up process inside the cavity. The steady-state value reached by the mean photon number in mode 1 inside the cavity at optical frequencies is $\langle n_1 \rangle_{ss} \approx 0.2$ (see in Fig. 5), thus a stedy-state output photon flux of $\Phi = \kappa_1 \langle n_1 \rangle_{ss}$ can be measured.

V. CONCLUSIONS

To conclude, we have shown that the presence of higher orders of the small parameter ϵ in the expansions of the photon frequency and field operators, gives rise to resonances in DCE that have not been explored before. This constitutes novel frequency ratios of DCE with the potential for the situation where the mechanical frequency of the moving mirror is smaller than the photon frequency. While we here have only discussed photon production by the traditional method of a moving cavity mirror, other means of creating time-changing cavity modes could be employed, e.g., by absorptive or dispersive properties of the cavity reached through placing radiators inside, such as shown in [20–23].

The small parameter ϵ is related to the single-photon coupling strength g_0 for opto-mechanical setups allowing a linear coupling (cavity with a moving mirror) as $g_0^{(1)} = \epsilon\,\omega_1$, and for quadratic coupling as $g_0^{(2)} = \epsilon^2\,\omega_1/2$; experiments exhibiting this type of coupling include: "membrane-in-the-middle" systems [14, 24, 25], trapped cold atoms [26], trapped microspheres [27] or double-disk structures [28]. The upper limit of the small parameter ϵ can be determined as follows: since the maximum amplitude of the mirror's motion is determined by $x_m = v_m/\Omega$, where v_m is the maximum speed of the mirror, and for the fundamental cavity mode the unperturbed photon frequency is $\omega_1 = \pi c/L_0$, then $\epsilon = \frac{v_m}{c}\frac{\omega_1}{\pi\Omega}$. This quantity can be compared to the number of photons n_1 produced in the more traditional context of DCE, e.g., in one-dimensional free space [29] of $n_1 = \frac{\Omega\, T}{6\pi}\left(\frac{v_m}{c}\right)^2$, where T is the period of a mechanical oscillation (for 3-D cavities see [15, 30, 31]): Eq. (8) gives the approximate photon growth rate in our case. With the approximation of $I_n(2\epsilon) \approx \frac{1}{n!}\epsilon^n$, then the rate is $\frac{\pi}{n!}\left(\frac{v_{\rm m}}{c}\right)^n\left(\frac{\omega_1}{\pi\Omega}\right)^{n+1}\Omega$; we get a comparable value, but for $n > 2$. When including cavity damping, we get a photon growing rate similar to [32]. For experimental parameters, like in [22], with a cavity length $L_0 = 110$ mm, mechanical frequency $\Omega/2\pi = 2.5$ GHz and effective mechanical displacement $x_m = 0.3$ mm, this gives $\epsilon_{\max} \approx 0.003$. However our resonance condition $\Omega = 2\,\tilde{\omega}_1/n$ in Eq. (7) gives a maximum value $\epsilon_{\max} = nv_m/c$, which for a series expansion up to $n = 6$, gives $\epsilon_{\max} \approx 0.02$, hence the maximum value of the small parameter can be increased by considering higher orders of ϵ in the expansions. For a situation with much weaker mechanical diving field, this motion should be quantized into phonons [33, 34].

As an outlook to improve the effective coupling quantity ϵ further, one could envision "coupling amplification" using additional sources, e.g, like in [35], i.e., introducing a strongly detuned parametric drive to the mechanics.

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