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Calculations of Neutron Reflectivity in the eV Energy Range from Mirrors made of Heavy Nuclei with Neutron-Nucleus Resonances

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We evaluate the reflectivity of neutron mirrors composed of certain heavy nuclei which possess strong neutron-nucleus resonances in the eV energy range. We consider possible corrections to the usual formulae of neutron optics for this case and find that dispersion and multiple scattering effects are negligible but one must take Doppler broadening into account. Effects from surface roughness, and phonon scattering are small compared to the Doppler corrections and are either the same as or smaller than those for mirror reflection with meV neutrons. We show that the reflectivity of such a mirror for some nuclei can in principle be high enough near energies corresponding to compound neutron-nucleus resonances to be of interest for certain scientific applications in non-destructive evaluation of subsurface material composition.

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I. INTRODUCTION

Neutron optics based on mirror reflection [1, 2] from the neutron optical potential of matter, which is derived from the spatial average of the individual neutron-nucleus scattering amplitudes from the nuclei in the medium, has been widely used in slow neutron instrumentation now for decades. The ability to conduct measurements using slow neutrons far from the radiation backgrounds and noisy environment near the neutron source without $1/r^2$ intensity falloff revolutionized the field and greatly expanded its range of scientific applications [3, 4]. Early slow neutron mirrors were made from nuclei with relatively large neutron optical potentials. ^{58}Ni was (and still is in many cases) a common choice.

Supermirror neutron guides [5–7] also employ the neutron optical potential to reflect neutrons. Neutron supermirrors are engineered to realize a reflectivity whose critical angle for near-total external reflection is larger than that from the neutron optical potential of a uniform medium. This is done using multilayer coatings of materials with a large contrast in the neutron scattering length density. Incident neutrons diffract from the one-dimensional crystal created by these stacked multilayers, and the constructive interference from this diffraction scattering produces a peak in the neutron reflectivity for specific values of the transverse momentum which match the diffraction condition [8, 9]. If one deposits a set of layers with a continuous distribution of bilayer separations which span the transverse momentum phase space in a neutron beam, one can efficiently transport all of the neutrons of interest. In a typical beam from a cold neutron source, one can use supermirrors to efficiently transport a much larger fraction of the beam phase space from the source over distances as long as 100 meters or more compared to a mirror made of a single material. By convention the transverse phase space acceptance of a supermirror guide is typically characterized by a dimensionless number m which is the ratio of the critical angle for near-total reflection from the guide to the same critical angle for a neutron mirror composed of natural nickel. In these units ^{58}Ni corresponds to $m = 1.2$. Supermirror guides with $m = 7$ are now commercially available.

eV neutron beams are used in some measurements in neutron scattering [10, 11] and neutron imaging for condensed matter and materials studies [12]. The most heavily used neutron scattering application for eV neutrons are measurements of the longitudinal momentum distribution of hydrogen, deuterium, and other light atoms in materials [13] and vibrational spectroscopy [14]. Neutron resonance imaging [15] to get nondestructive quantitative information on the local environment of specific isotopes embedded in materials from the shapes of the Doppler broadening of the resonance linewidths also of course operates in the eV neutron energy regime where resonances are abundant. With the development of a growing number of intense neutron sources in the eV energy range based on proton spallation [16], where the incompletely-moderated $1/E$ high energy tail of the neutron energy spectrum from a hydrogenous moderator is much richer in eV neutrons than is the high energy tail of the spectrum from a research reactor, it is well worth considering the possibility of developing other scientific applications of eV neutrons.

Both normal neutron mirrors and neutron supermirrors typically employ the real part of the (in general complex) neutron optical potential of the medium. This is because, in the meV energy range emitted by slow neutron sources, the neutron-nucleus scattering amplitude has a real part that is typically much larger than the imaginary part. This follows in turn from probability conservation as embodied in the optical theorem

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of nonrelativistic scattering theory, $Im[f(\theta=0)] = \frac{k\sigma}{4\pi}$ where k is the neutron wave vector, σ is the total cross section, and $f(\theta=0)$ is the forward scattering amplitude. The slow neutron energy regime corresponds to $kR \approx 10^{-4}$ where R is the potential scattering range from the neutron-nucleus interaction, which implies that the scattering is dominated by the s-wave component of the partial wave expansion of the scattering amplitude. For this case, $f(\theta=0) = f$ is of order R in magnitude and $\sigma = 4\pi|f|^2$, so the optical theorem implies $Im[f] = k[Re(f)^2 + Im(f)^2] \approx kR^2 \approx 10^{-4}|f|$.

This argument breaks down however for inelastic scattering and for resonance scattering. For the case of resonance scattering of most interest in this work, f can be much larger than R. The simplest case of elastic scattering with resonances will serve to make the point clear. One can decompose the partial wave expansion in nonrelativistic scattering theory into terms associated with ingoing and outgoing spherical waves [17]

$$f(\theta) = \sum_{l=0}^{\infty} \frac{(2l+1)P_l}{2ikr}[(1+2ikf_l(k))e^{ikr} - e^{-i(kr-l\pi)}] \quad (1)$$

where $f_l(k)$ is the partial wave scattering amplitude in orbital angular momentum channel l and $P_l(\cos\theta)$ is the l^{th} Legendre polynomial. As elastic scattering merely redirects the incident wave packet, the optical theorem demands that $|1 + 2ikf_l(k)| = 1$. This condition can be written in terms of the elastic scattering phase shift δ_l as $1 + 2ikf_l(k) = \exp 2i\delta_l$, which implies

$$kf_l = \frac{\exp(2i\delta_l) - 1}{2i} = \frac{1}{\cot\delta_l - i}\,. \quad (2)$$

In resonance scattering near the resonance energy E_r the phase shift increases rapidly from 0 to π. On resonance $\delta_l = \pi/2$ and $\cot\delta_l = 0$ so $f_l = i/k$ is purely imaginary and also takes the maximum possible value consistent with unitarity. In particular f_l need not be of order R. Expanding $\cot\delta_l$ near the resonance energy as a power series in $E - E_r$ and keeping only the linear term, $\cot\delta_l(E) = 0 + (\Gamma/2)(E - E_r)$ gives the well-known Breit-Wigner form

$$kf_l = \frac{\Gamma/2}{E - E_r + i\Gamma/2}\,. \quad (3)$$

f_l then both possesses a large imaginary component and also can be large compared to R as long as $(E - E_r) \approx \Gamma$.

Similar to visible light reflection from a metallic mirror, neutrons can coherently reflect from the imaginary part of the optical potential of the medium as well as from the real part. In this paper we evaluate the neutron reflectivity in the eV energy range of mirrors composed of heavy nuclei. The idea is to take advantage of the large imaginary component of the scattering amplitude present at and near neutron-nucleus resonance energies to enhance the reflectivity of neutrons. The large imaginary component of the scattering amplitude is only present for neutrons with energies within the width Γ of the resonance energy, and since both $\Gamma/E_r \ll 1$ and $\Gamma \ll \Delta E$ where ΔE is the typical separation between neighboring resonances for the great majority of low-lying n-A resonances, no single nucleus can be used to reflect a broad range of neutron energies using this mechanism. As the neutron optical potential for coherent reflection is simply a weighted linear sum of the amplitudes from all of the scatterers in the medium, one can imagine making a mirror out of a "cocktail"of different nuclei. Although as we will see below this idea does not seem practical, we will see that the reflectivity is large enough for some nuclei at the resonance energies to be of interest for certain applications. As long as the thickness of the mirror is great enough (a few microns of solid is usually enough) that one can neglect the transmission through the mirror for all of the relevant energies in the beam, one can apply the well-known formulae from the theory of neutron optics and use the very extensive set of measured data on n-A scattering resonances to calculate the reflectivity. It is also important to take into account the fact that the neutron-nucleus resonances in the eV energy range typically posses a large inelastic component, usually dominated by (n, γ) reactions. Therefore the fraction of the coherently scattered neutrons on resonance is proportional to $\frac{\Gamma_n}{\Gamma_{tot}}$ where Γ_n and Γ_{tot} are the neutron and total widths of the resonance, respectively. Typically $\frac{\Gamma_n}{\Gamma_{tot}} \ll 1$, which further suppresses the reflection probability for many nuclei.

We were not able to find any previous quantitative analysis of this idea in the neutron optics literature. We found a qualitative comment on the possibility of interesting resonance effects in eV neutron optics in a theoretical treatment of neutron multiple scattering in neutron optics [18] which we will discuss in more detail below. In the case of ultracold neutrons with energies of hundreds of neV such calculations were actually done a very long time ago for the case of nuclei with large absorption cross sections near threshold [19]. It is well-known that ultracold neutrons bouncing from a mirror made of nuclei with a large neutron absorption cross section can possess a very large reflection probability, and this was demonstrated experimentally long ago [20]. Frank and collaborators [21] analyzed the resonance component of the neutron optical potential of Gd for the interpretation of some ultracold neutron transmission experiments designed to investigate the $1/v$ neutron absorption law. By contrast our interest is in the eV neutron energy range. The contribution of neutron resonances to the neutron optics phase shift of moving matter, the so-called neutron Fizeau effect [22], was predicted theoretically [23] and has been observed experimentally [24] but involve s neutron transmission in the several meV neutron energy range, not neutron reflection at eV energies. The effects of resonances in the theory of xray optics are by contrast

well-developed in both theory and experiment [25, 26].

The rest of this paper is organized as follows. In section II we review the relevant parts of the theory of neutron optics and of resonance scattering. Effects from surface roughness, and phonon scattering. We consider possible corrections to the usual formulae in the kinematic limit of neutron optics, roughness, phonon scattering, and Doppler broadening of the resonances. We present and discuss the results of our calculations in section III. In section IV we discuss some possible applications of our results. We conclude in section V.

II. NEUTRON OPTICS THEORY AND CORRECTIONS TO THE REFLECTIVITY FORMULA

In this section we give a brief review of the relevant results from the theory of neutron optics that are needed to understand the foundation for the formulae used in this work. For a more detailed treatment see Sears [27]. Neutron optics is based on the existence of the "coherent wave" which is the coordinate representation of the coherent state formed by the incident wave and the forward scattered wave in a scattering medium [28]. It is determined by the solution of a one-body Schrodinger equation

$$\left[\frac{-\hbar^2}{2m}\Delta + v(r)\right]\psi(r) = E\psi(r) \tag{4}$$

where $\psi(r)$ is the coherent wave and $v(r)$ is the optical potential of the medium. The coherent wave satisfies the Lippmann-Schwinger equation

$$\psi(r) = |k\rangle + gv(r)\psi(r) \tag{5}$$

where $|k\rangle$ is the incident wave, g is the one-body Green's function for nonrelativistic motion of a neutron and $v(r)$ is the optical potential. The optical potential is related to the one body t matrix by

$$t = v(r) + tgv(r) \tag{6}$$

and this combination forms the usual coupled system of equations of nonrelativistic scattering theory from a medium of a large number of scatterers. Given a form for the t matrix one can determine the optical potential and then solve the one-body Schrodinger equation for the coherent wave.

One must make an approximation for the t matrix of the neutron in a medium of scatterers. The usual approximation is essentially the Born approximation in which $v = t$ and

$$t = \sum_l t_l. \tag{7}$$

Finally one must approximate the one-body t matrix t_l. Using the impulse approximation for scattering, one gets

$$t_l = (2\pi\hbar^2/m)\sum_l b_l\delta(\mathbf{r} - \mathbf{R}_l). \tag{8}$$

Here, l denotes the elemental species, b_l is the coherent scattering length for element l, $\mathbf{r}$ is a random spatial coordinate and $\mathbf{R}_l$ defines the coordinate of each atom the neutron can scatter from. Since the neutron-nucleus interaction is much stronger and much shorter range than the binding forces of the atoms in matter, it is reasonable to neglect the effects of chemical binding during the neutron-nucleus collision. In addition, the short-range of the interaction means that the timescale of the collision is much shorter than the timescales associated with the motion of the atom in the potential well. For both of these reasons, the t operator is usually approximated by the t operator for a free atom. This is known as the impulse approximation in scattering theory. In this approximation, the neutron optical properties of a medium depend only on the coherent scattering length of the atoms and not at all on the details of the binding of the atoms.

From Eq. 8 we then arrive at an expression for the optical potential

$$v_{opt}(r) = (2\pi\hbar^2/m)\sum_l N_l b_l = v_0, \tag{9}$$

where N_l is the number density of scatterers. In general the scattering amplitude, and therefore the optical potential, is complex to account for incoherent scattering, absorption, and resonance contributions to the scattering amplitude. The neutron index of refraction is then

$$n^2 = 1 - \frac{v_0}{E} \tag{10}$$

This relation between the neutron optical potential and the scattering amplitude is an approximation which neglects effects due to dispersion and multiple scattering in the medium. The approximation fails in Eqs. 7 and 8, due to effects from atomic binding, and in Eqn. 6 due to multiple scattering. Both these effects as well as the intrinsic energy dependence of the scattering amplitude b_l itself render the neutron optical potential energy dependent in general, so that

$$n^2 = 1 - \frac{v(k)}{E}. \tag{11}$$

In Eq. 7 the t matrix of the bound system of N scatterers is expressed as the sum of the (impulse approximation) one-body t matrix. But this is known in exact treatments of scattering theory to be an approximation [28]. The next order of approximation for the t matrix of the system is

$$t = \sum_l t_l + \sum_{l,l',l\neq l'} t_l G t_{l'} + \ldots \tag{12}$$

where G is the Green's function. To calculate the full t operator of the system, which is what is required to obtain the optical potential in Eq. 4 one must take multiple scattering also into account. Then finally the optical potential, which is now a function of the neutron momentum, must be solved from Eq. 6.

The calculations of these additional terms in the theory of dispersion in neutron optics which leads to a modified expression for the index of refraction n' presented below was conducted years ago using many different techniques. The calculations performed in the 80s [27, 29–31] built upon much earlier work [32–38] and were conducted within the framework of the traditional multiple scattering theory outlined above. A different calculational method [18] based on resummation of dominat subclasses of diagrams important for backscattering was seen to give equivalent results. Yet a third approach motivated by a desire to understand decoherence in neutron optics [39] used a Lindblad operator treatment and also agrees with the results of Sears presented below. All of these calculations restore consistency with the optical theorem and reduce in appropriate limits to the usual kinematic limit.

Here we outline the derivation presented in Sears and just state the final results of the other two approaches mentioned above. Sears employs the static approximation, which assumes that each atom in the system is bound to a fixed position in space, which is true for the solid materials under consideration in this paper as candidates for practical neutron mirrors. In that case, the coupled equations can be written in terms of the scattering wave function ψ and the local field χ_i

$$\psi = |k\rangle + \sum_i gT_i\chi_i \tag{13}$$

$$\chi_i = |k\rangle + \sum_{j\neq i} gT_j\chi_j \tag{14}$$

These two equations simply say that the neutron wave function is the sum of the incident plane wave plus a scattered spherical wave from each atom in the system, and that the local field χ_i that generates the scattered wave from location i is, in turn, the sum of the incident plane wave plus the scattered spherical wave from all of the other atoms. The dispersion theory of neutron optics must relate the coherent wave to the local field χ. Using an approach directly analogous to the relation $\vec{D} = \epsilon\vec{E}$ in electrodynamics in a medium, one derives the constitutive relation $\chi(r) = c\psi(r)$ where c, like the dielectric constant in light optics, depends on the medium properties. Defining $c = 1/(1 - J')$, the dispersion corrections to the neutron optical potential can be written in the form

$$n' = 1 - \frac{2\pi\rho b'}{k^2}[1 + J' + \frac{\pi\rho b'}{k^2}] \tag{15}$$

where the (dominant) real part of $J' = \frac{2\pi\rho b}{k}\int \sin(2kr)[1 - g(r)]dr$ for an isotropic medium, where $g(r)$ is the pair correlation function for the atoms in the material, n' is the real part of the neutron index of refraction with the multiple scattering correction, b' is the neutron scattering length with the multiple scattering correction, ρ is the number density of atoms in the material, and k is the incident neutron wave vector.

The correction terms in the parentheses are very small even in the presence of n-A resonances. Sears shows that J' is of order 10^{-4} for slow neutrons and is smaller for eV energy neutrons due to the $1/k$ factor. As for the second term, even if we take the extreme on-resonance case at a resonance energy E_r one gets $kb = \Gamma_n/\Gamma_{tot}$ and furthermore consider the ideal case of total elastic scattering where $kb = 1$, the factor $\frac{\pi\rho}{k^3}$ is less than 10^{-5} for neutrons with energies above 1eV reflecting from solid materials of normal densities. Both of these terms are therefore negligible for the case of interest in this paper, since almost all of the n-A resonances are at or above eV energies. Table 1 in the Appendix shows the product $kb = \Gamma_n/\Gamma_{tot}$ for all of the resonances considered at the resonance peaks, few of which are even close to one, thereby further suppressing the second correction term above.

The results from the other two approaches mentioned above are similar and consistent. Warner [18] derives the nonlinear expression for n

$$n^2 - 1 = \frac{4\pi\rho b}{k^2}/[1 + (\frac{4\pi\rho b}{nk^2})\int_0^\infty dy e^{iy}\sin(ny)[g(y/k) - 1] \tag{16}$$

which reduces to the expression of Sears for small n. The Lindblad operator treatment by Lanz and Vacchini [39] gives the neutron optical potential

$$v_{opt} = \frac{2\pi\hbar^2\rho b}{m}[1 - \frac{kb}{4\pi}\int d\Omega_q S_c(q)] \tag{17}$$

which also reduces to the Sears result upon writing the static structure factor $S_c(q) = 1 + \rho\int[g(r) - 1]d^3\vec{r}\exp i\vec{q}\cdot\vec{r}$. We therefore conclude that we can use the usual expression for the index of refraction in the kinematical theory of neutron optics to calculate the reflectivity even in the presence of n-A resonances.

If we write out the real and imaginary parts of the optical potential $U = V - iW$

$$U = V - iW = (2\pi\hbar^2/m)\sum_l N_l(b_{l,r} - ib_{l,i}), \tag{18}$$

and use this complex optical potential to calculate the reflection probability $|R^2|$ of a neutron incident on a uniform medium, one gets the result [42, 43]

$$|R^2| = \frac{E_\perp - \sqrt{E_\perp(2\alpha - 2(V - E_\perp)} + \alpha}{E_\perp + \sqrt{E_\perp(2\alpha - 2(V - E_\perp)} + \alpha} \tag{19}$$

where

$$\alpha = \sqrt{(V - E_\perp)^2 + W^2} \tag{20}$$

and $E_\perp = \hbar^2 k_z^2/2m$ is the fraction of the incident neutron kinetic energy associated with the momentum normal to the material boundary.

In our case we want to consider the situation in which the neutron-nucleus scattering possesses both a potential scattering term as usual and also a resonance scattering term. The resonance both absorbs neutrons and also adds a large imaginary component to the total n-A neutron scattering amplitude. In the presence of n-A resonances the expression for the resonant part b_{res} of the total scattering amplitude $b = b_{pot} + b_{res}$ becomes [44]

$$b_{res} = \sum_j \frac{g_{\pm,j}}{2k_j^{'}} \frac{\Gamma_{n,j}}{[(E' - E_j) + i\Gamma_j/2]} \tag{21}$$

where $\Gamma_{n,j}$ and Γ_j are the neutron width and total width of the resonance at energy E_j and $k^{'} = \mu k/m$ is the wave vector in the n-A center of mass system of reduced mass μ, $E^{'}$ is the associated energy in the COM frame, and $g_{+,j} = (I+1)/(2I+1)$ and $g_{-,j} = I/(2I+1)$ are the statistical weight factors for a resonance at energy E_j in the total angular momentum channel $J = I \pm 1/2$. b_{res} is purely imaginary on resonance, as can easily be seen explicitly by writing out the real and imaginary parts of b_{res}

$$b_{res,real} = \sum_j \frac{g_{\pm,j}}{2k_j^{'}} \frac{\Gamma_{n,j}(E^{'} - E_j)}{[(E' - E_j)^2 + \Gamma_j^2/4]} \tag{22}$$

$$b_{res,im} = -\sum_j \frac{g_{\pm,j}}{4k_j^{'}} \frac{\Gamma_{n,j}\Gamma_j}{[(E' - E_j)^2 + \Gamma_j^2/4]} \tag{23}$$

There are some additional corrections to the formula presented above which we now consider which are important for practical measurement conditions.

As the atoms in the mirror will be moving in the solid due to zero point motion and thermal effects, the resonance scattering will undergo Doppler broadening. We can evaluate this effect for a neutron mirror at room temperature in a reasonably general way assuming a simple Debye model for the phonon spectrum in the solid [49]. We include the effect of the Doppler broadening of the resonances due to the motion of the atoms in the mirror at finite temperature approximately using the convolution

$$R(E) = \int dE' S(E' - E) R(E') \tag{24}$$

where the Doppler shifts come from the convolution with the energy distribution of the atomic motions

$$S(E' - E) = \sqrt{\frac{M}{4\pi k_B m T_{eff} E_j}} \times \exp\left[-\sqrt{\frac{M}{4k_B m T_{eff} E_j}}(E' - E)^2\right] \tag{25}$$

where $T_{eff} = \frac{<h\nu>_T}{k_B T}$ is set by the mean oscillator energy $< h\nu >_T$ at temperature T. We evaluate T_{eff} using the Debye model, where

$$k_B T_{eff} = \frac{3}{2}\int_0^{k_B\Theta_D} E^3 \coth\frac{E}{2k_B T} dE/(k_B\Theta_D) \tag{26}$$

where θ_D is the Debye temperature of the material.

$$T_{eff} \approx \frac{3}{8}\Theta_D \coth\left(\frac{3\Theta_D}{8T}\right) \tag{27}$$

This approach gives the leading term in an approximate treatment of the Doppler broadening effects which gets the second moment of the broadening function $R(E)$ correct but neglects the effects of higher order moments. The theory required for a more detailed treatment of the resonance broadening for individual nuclei in specific material environments than the one used here has been developed [50–52] and could be adapted to specific cases for our calculation as needed. Later theoretical work [53] has developed this calculation to the point that detailed measurements of the resonance lineshape can be used to get information on low-order moments of the phonon energy spectra in the material. Evaluating these higher-order correction to the resonance lineshape are again beyond the scope of this paper but are doable given sufficient knowledge of the local structural environment of the atoms with resonant nuclei.

Another effect which lowers the reflectivity of the mirror somewhat is due to inelastic scattering of the neutrons from phonons. This is a very small effect of order 10^{-5} per bounce for slow neutron mirror reflection as the penetration depth of the evanescent neutron wave into the mirror material is only on the order of 100 nm for almost all neutron transverse momenta which are not right at the edge of the $V_{opt} = p_z^2/2m$ critical angle condition. The fractional loss in neutron reflectivity from thermal phonons is so small that it is typically only measurable at all using very low energy neutrons, called ultracold neutrons (UCN) [48], which can bounce several times from material surfaces. The expression for the total inelastic scattering cross section σ_{tot}^{in} is

$$\sigma_{tot}^{in} = \frac{4\pi m a^2}{M k_i}\int d\omega \exp -2W(k_f)\frac{|\gamma|^2 g(\omega)\sqrt{(2m\omega/\hbar)}}{(\exp\hbar\omega/kT - 1)} \tag{28}$$

where k_i and k_f are the incident and scattered neutron wave vectors, m is the neutron mass, M is the atom mass,

$W(k_f)$ is the Debye-Waller factor, $g(\omega)$ is the phonon density of states, and γ is the amplitude of a phonon mode. For the case of eV neutrons, this cross section is even smaller than for slow neutrons as k_i is larger and the larger k_f leads to a smaller Debye-Waller factor with all other factors in the cross section expression the same.

The reduction of the reflectivity of the mirror surface due to surface roughness can be important in the slow neutron regime. Its evaluation is beyond the scope of this paper as the calculation depends in detail on many specific, non-universal features of the surface roughness that will vary greatly among different practical materials. Mirrors for slow neutrons typically possess a loss of specular reflectivity which ranges from 10^{-4} to 10^{-5} per bounce, which is negligible for most purposes. However we can show that, for the same surface roughness profile, one expects eV neutrons to possess a lower nonspecular loss probability per bounce than for meV neutrons. The reason for this expectation can be seen if one adopts a simple model of a mirror surface with a Gaussian distribution of surface heights relative to the mean surface location. In this case [47, 48] one can show that the angular distribution of the nonspecular component of the mirror reflection is proportional to the Fourier transform of the roughness autocorrelation function $F(q) = \exp -w^2q^2/2$ where w^2 is the variance in surface heights away from the mean flat surface and q is the momentum transfer along the surface of the mirror. As the longitudinal momentum transfer q is larger for eV neutrons than meV neutrons by a factor of $\sqrt{1000}$ for the same value of the transverse momentum, the angular distribution of the roughness scattering from an eV neutron mirror is much more sharply peaked than for cold neutrons.

Our conclusion from the analysis presented above is that, in the presence of n-A resoances, the calculation of the reflectivity is accurately described by Eq. 19 using the kinematic theory of neutron optics, the potential and resonance contributions to the scattering amplitude with their contributions to the real and imaginary parts of the neutron optical potential, and resonance broadening from the Doppler effect of nuclei moving in a mirror at finite temperature.

III. ANALYSIS, CORRECTIONS, AND RESULTS

Extensive data on n-A resonances exists. Table 1 in the Appendix lists the nuclei and the resonances which we considered as in principle available as possible eV neutron mirror materials. Figures 1 and 2 show the total resonance widths, resonance energies, and associated isotopes in our energy range of interest. The density of n-A resonances in this regime along with their overlapping total widths encouraged us to pursue the calculation. We consider here heavy stable nuclei with n-A resonances with energies below 10 eV. This data is taken from the National Nuclear Data Center (NNDC) [45] and the Japanese Evaluated Nuclear Data Library (JENDL) [46]. We use these measured resonance parameters and substitute them into Eq. 19.

We show the results of our calculations in a series of plots in which we simply superpose all of the reflectivity results from mirrors made of the relevant pure substances. The reflectivity of any compound material would be the appropriate weighted sum according to equation 9. We chose to use as variables the total neutron energy E and the incident neutron angle θ. We chose a typical value of $\theta = 1$ mrad for all of the plots: at constant E the reflectivity is a sharply decreasing function of θ according to Eq.14.

In the regime of unit reflectivity shown in Figure 3, all of the heavy nuclei fall below natural nickel and therefore there is no special advantage that we can see to use heavy nuclei in a eV neutron guide. The imaginary parts of the optical potential change the shape of the reflectivity curve away from the usual shape given by the Fresnel formula.

The $1/k$ factor in the resonance amplitude eventually reduces the reflectivity at high energy. Nevertheless as can be seen in Figure 4 certain nuclei possess a high enough reflectivity on resonance in the 1-10 eV energy range to be clearly visible for an incident angle of 1 mrad. This observation is in our judgement the most interesting result of our calculation.

We included the effects of the real part of the scattering amplitudes as well in the formula. One must take care not to "double count" the resonance scattering contribution, since the scattering amplitudes $b_{measured}$ that are reported in the literature from measurements using slow neutrons are in fact a sum of the potential scattering contribution and also the tails of all of the other resonances in the limit $E \to 0$:

$$b_{measured} = b_{pot} + \sum_j \frac{g_{\pm,j}}{2k'_j} \frac{\Gamma_{n,j}}{[(E_j) + i\Gamma_j/2]} \cdot \qquad (29)$$

We took this effect into account but it is typically rather small and makes a negligible change in the reflectivity of order 10^{-3} in the worst case. This is because the meV neutron energies where the scattering amplitudes are measured are almost always several resonance widths away from the resonances at eV and higher neutron energies. Certain nuclei which possess subthreshold n-A resonances very close to threshold are an exception, but none of the nuclei identified with visible peaks in the reflectivity curve and for which the neutron scattering data exists at thermal energies possess significant subthreshold effects. This can be seen by inspecting the experimental data on the relevant n-A cross sections at low (meV) energies.

Finally we plot the reflectivity for a few special nuclei (^{155}Gd, ^{157}Gd, ^{113}Cd, and ^{10}B) which have especially large absorption cross sections close to zero neutron energy from either a subthreshold resonance (in the case of

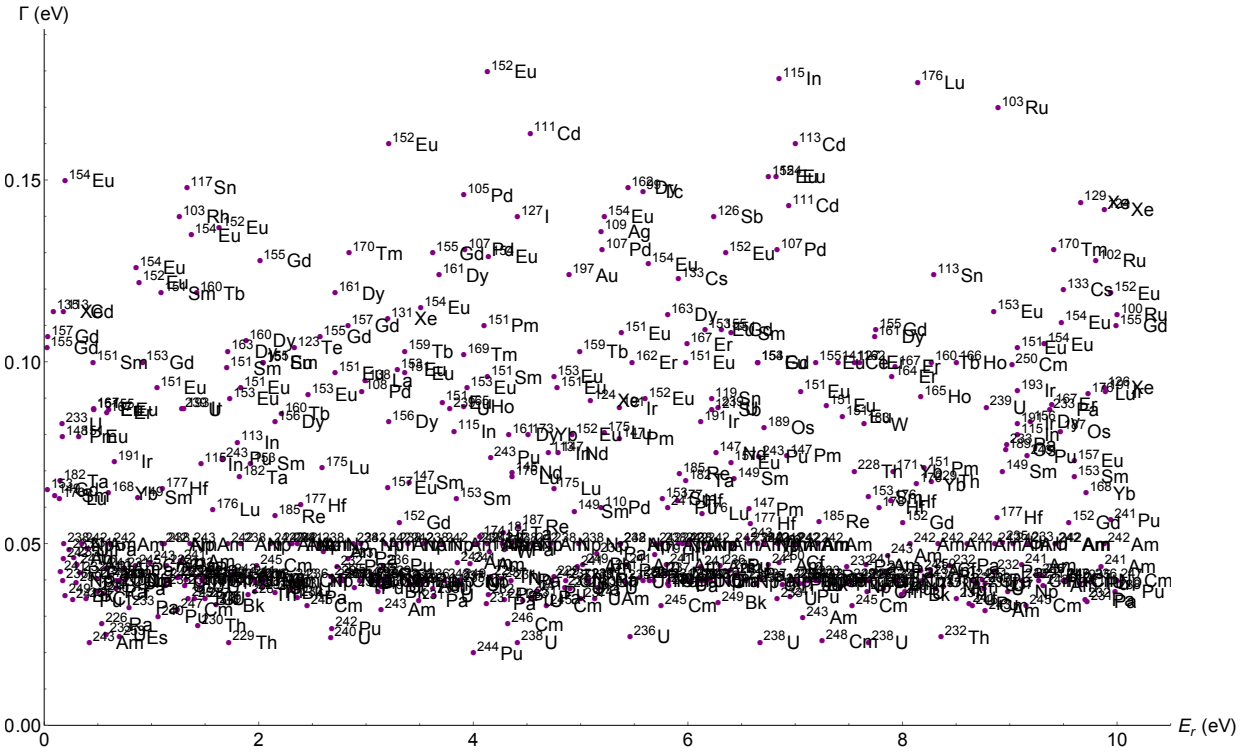

FIG. 1: Plot of total resonance widths of neutron resonances versus resonance energy with identification of the relevant isotopes.

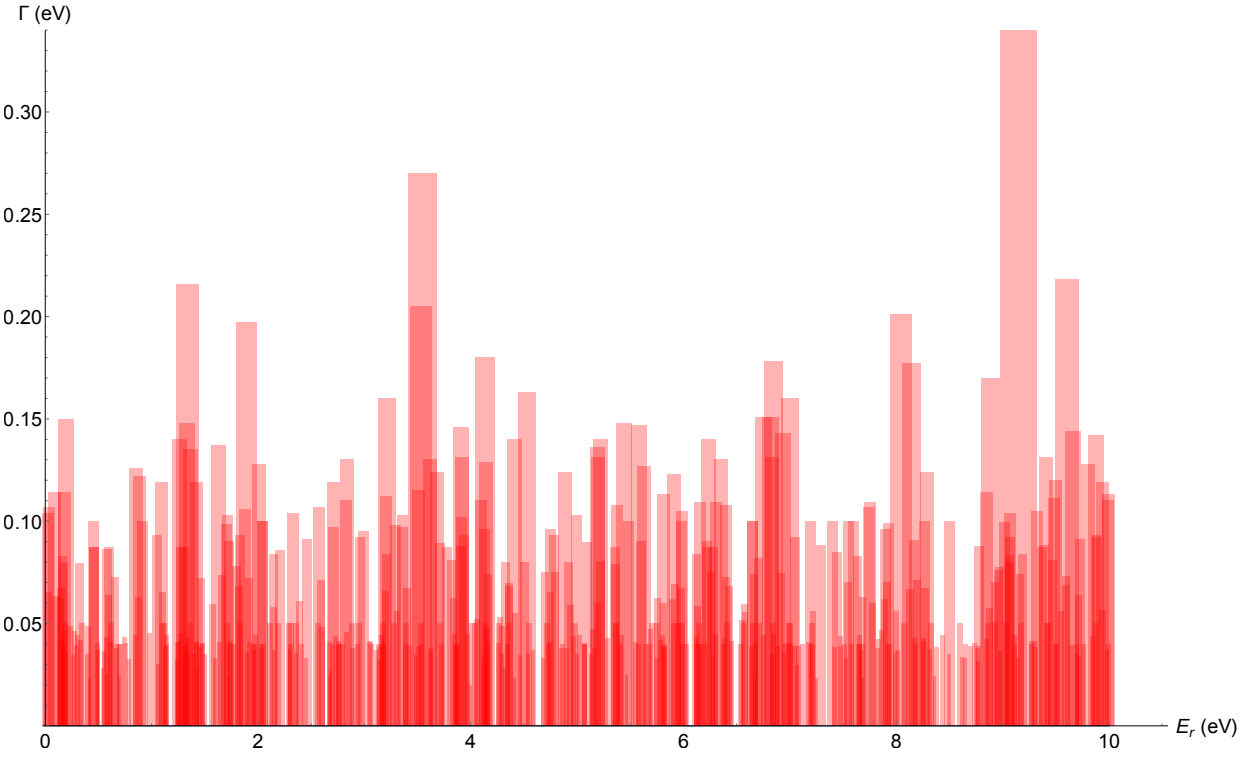

FIG. 2: Plot of total resonance widths of neutron resonances versus resonance energy for heavy nuclei in a representation that illustrates the "forest". The horizontal dimension of the bars is 2Γ.

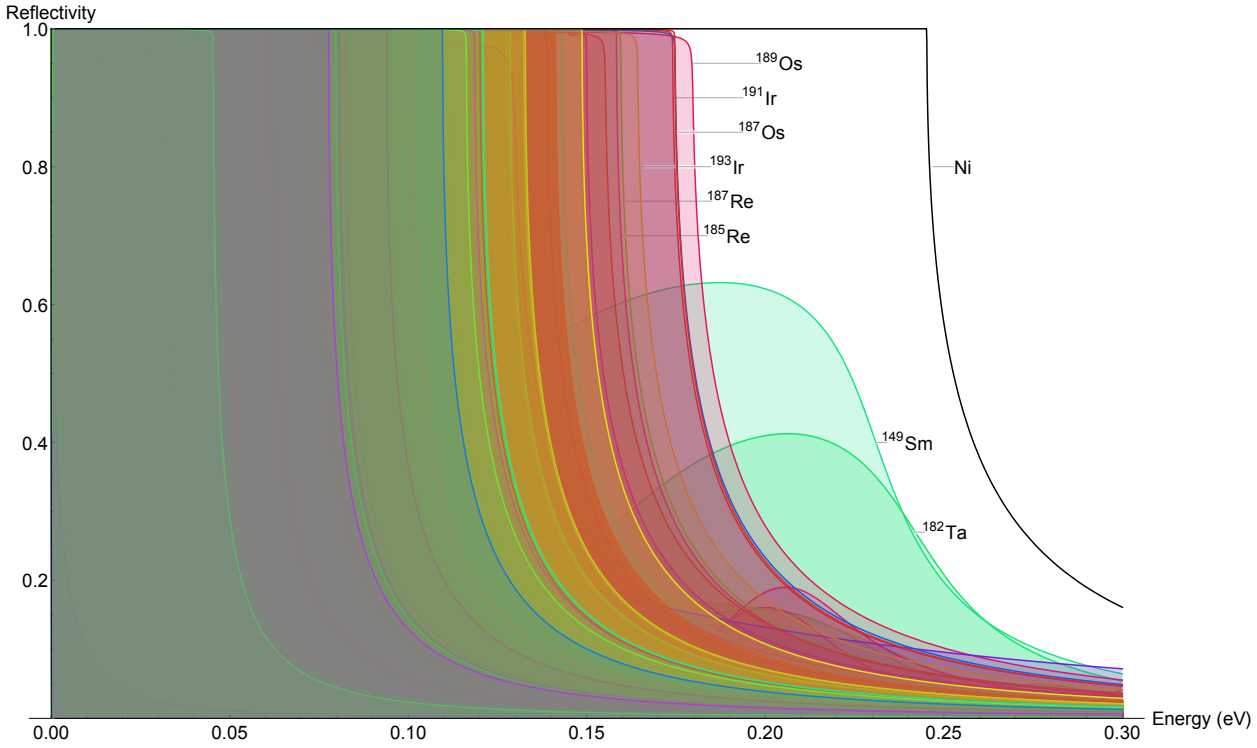

FIG. 3: Reflectivity for neutron energies between 0-0.3 eV for an incident angle of 1mrad. All of the heavy nuclei fall below natural nickel even after including the resonance contributions. The nuclei with reflectivity curves closest to natural nickel are labeled explicitly.

^{10}B) or from an above-threshold but especially low energy resonance (as in ^{155}Gd, ^{157}Gd, and ^{113}Cd). One can see that the reflectivity is reasonably large up to about 100 meV. This fact could concievably be of interest for neutron beam transport in thermal and epithermal neutron beams in upstream sections of the beamline close to the neutron source. Such a guide would both reflect some of the neutrons in the epithermal energy range and also efficiently absorb many of the neutrons incident on the guide above the critical angle.

IV. POSSIBLE APPLICATIONS

Figure 4 shows that there is a reasonably large number of heavy nuclei which possess resonances whose properties can produce a noticeably large neutron reflectivity on resonance. The relatively high neutron reflectivities for these cases opens the possibility for new applications in nondestructive subsurface analysis. One can exploit the relatively shallow penetration of the neutron wave amplitude into the medium upon mirror reflection to detect the presence of certain subsurface isotopes from the neutron energy dependence of the reflectivity from the flat surface of a material. As the reflected neutrons can be separated from the incident beam and this enhanced reflectivity is completely absent for all other nuclei, this method could possess a high signal to background ratio in a practical apparatus. One could also exploit the fact that the nuclear resonance will also emit a gamma cascade upon neutron capture to detect both this gamma cascade along with the delayed coincidence with the neutron reflectivity peak at the resonance energy from the large collection of nuclei in the mirror to further suppress environmental backgrounds. This latter mode of operation can easily be realized at a pulsed spallation neutron source where the neutron energy is tightly correlated with the neutron time of flight.

V. CONCLUSION

We present a calculation of the reflectivity of neutron mirrors in the eV energy range which takes into account the large imaginary parts of neutron-nucleus resonance scattering amplitudes based on the kinematic theory of neutron optics and the known properties of neutron-nucleus resonances. We find that the reflectivity is large enough to be visible for several nuclei on different resonances in the 0 to 10 eV neutron energy range. We include the largest relevant correction to the idealized reflectivity formula, which comes from the Doppler effect from relative motion of the neutron and nucleus of the atoms bound in the mirror. Other effects from multiple scattering corrections to the neutron optical potential, surface roughness, and phonon scattering are small com-

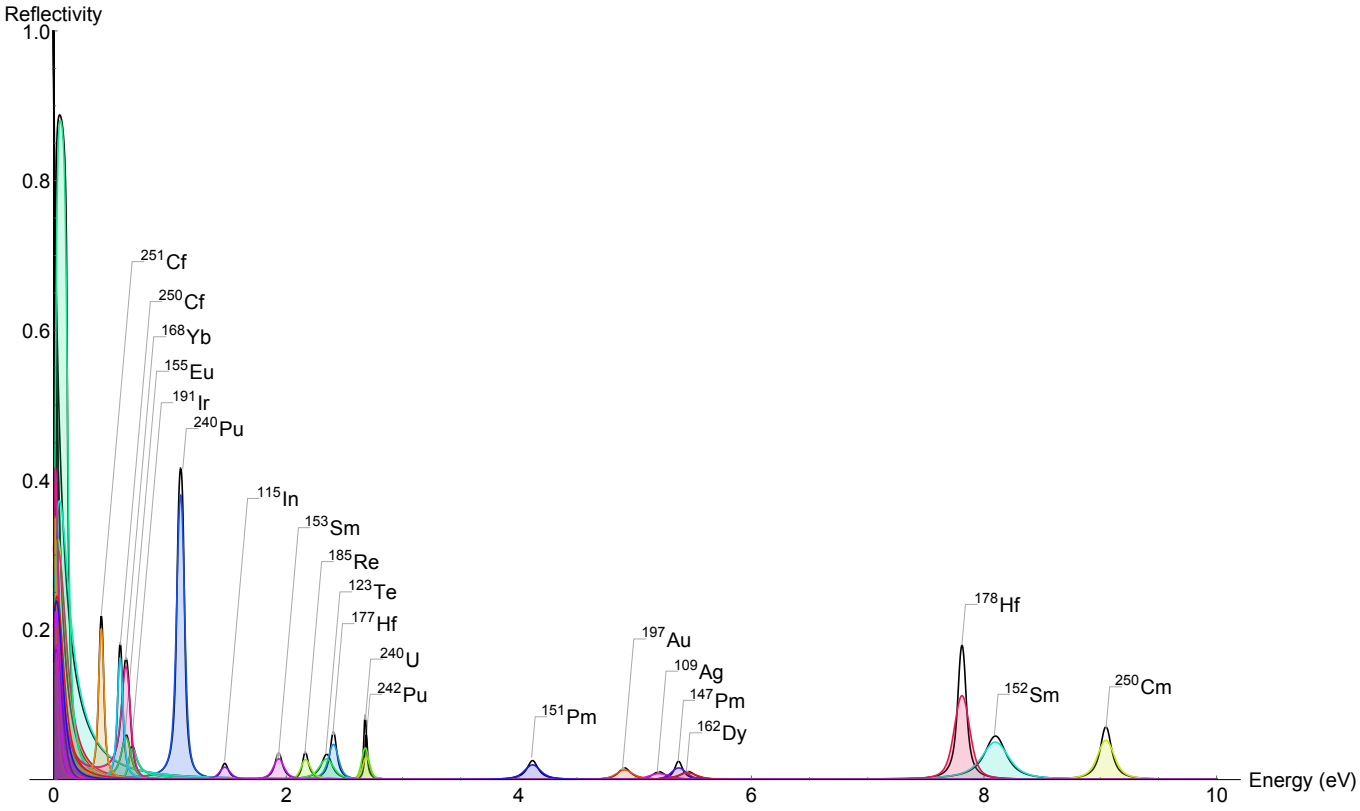

FIG. 4: Reflectivity envelope for the resonance guide for neutron energies between 0-10 eV for an incident angle of 1mrad. The lower amplitude curves for each nucleus show the approximate effects from Doppler broadening at 300K for the associated pure materials.

pared to the Doppler corrections and are either the same as or smaller than those for mirror reflection with meV neutrons. The relatively high neutron reflectivities for the cases of the specific nuclei we identify opens the possibility for new applications in nondestructive subsurface analysis, especially at pulsed spallation neutron sources where neutron energy is directly correlated with time of flight from the source to the sample and detector.

VI. ACKNOWLEDGEMENTS

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VII. APPENDIX

TABLE I: A list of neutron-nucleus resonances considered in this paper with their respective resonance parameters, the product kb of the resonance wave vector, the scattering amplitude on resonance, and the Debye temperature at 298 K where available. Debye temperatures were taken from [55], unless otherwise noted on the first instance of the element.

Isotope	**Resonance Energy (eV)**	**Γ (meV)**	**Γ_n (meV)**	**$kb_{res} = \Gamma_n/\Gamma$**	**Debye Temperature (K)**
^{87}Sr	3.53	205	0.618	3.01E-03	148
^{99}Tc	5.58	147	3.45	2.35E-02	422
^{100}Ru	10	113	0.138	1.22E-03	415

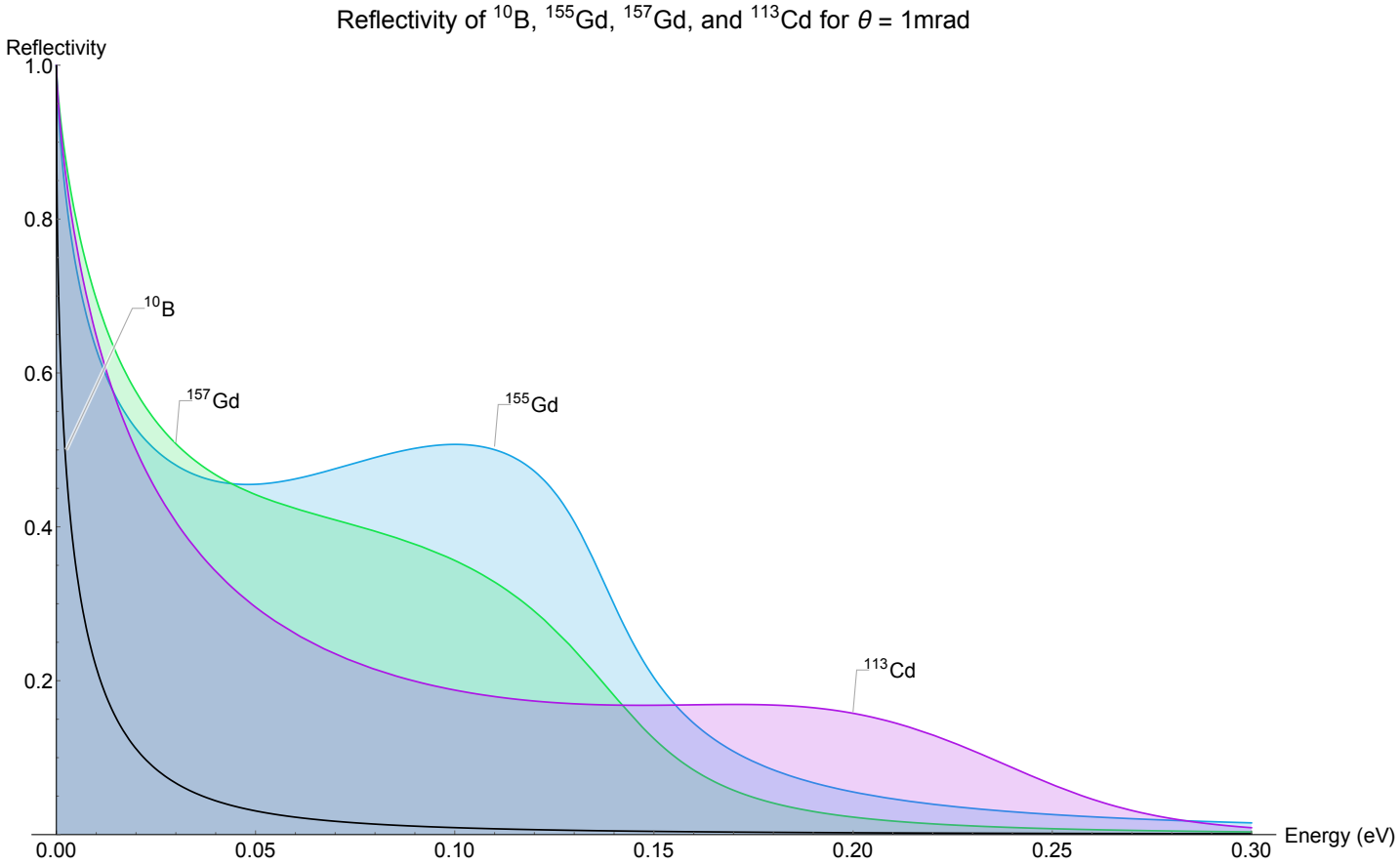

FIG. 5: Reflectivity of ^{155}Gd, ^{157}Gd, ^{113}Cd, and ^{10}B between 0-0.3 eV for an incident angle of 1mrad.

Isotope	**Resonance Energy (eV)**	**Γ (meV)**	**Γ_n (meV)**	**$kb_{res} = \Gamma_n/\Gamma$**	**Debye Temperature**
^{102}Ru	9.8	128	0.0093	7.27E-05	415
^{103}Ru	8.89	170	0.134	7.88E-04	415
^{103}Rh	1.26	140	0.52	3.71E-03	350
^{105}Pd	3.91	146	0.000613	4.20E-06	275
^{107}Pd	3.92	131	0.00171	1.31E-05	275
	5.2	131	0.0197	1.50E-04	275
	6.83	131	0.0588	4.49E-04	275
^{108}Pd	2.96	92	0.01	1.09E-04	275
^{110}Pd	5.19	60	0.0024	4.00E-05	275
^{109}Ag	5.19	136	12.7	9.34E-02	221
^{111}Cd	4.53	163	0.0014	8.59E-06	221
	6.94	143	0.00086	6.01E-06	221
^{113}Cd	0.179	114	0.64	5.61E-03	221
	7	160	0.000413	2.58E-06	221
^{113}In	1.8	78	0.223	2.86E-03	129
	4.7	75	0.106	1.41E-03	129
^{115}In	1.46	72	2.98	4.14E-02	129
	3.82	81	0.38	4.69E-03	129
	6.85	178	0.000418	2.35E-06	129
	9.07	80	1.5	1.88E-02	129
^{113}Sn	8.29	124	4.47	3.60E-02	254
^{117}Sn	1.33	148	0.00011	7.43E-07	254
^{119}Sn	6.22	90	0.00148	1.64E-05	254
^{121}Sb	6.22	87	1.88	2.16E-02	200
^{126}Sb	6.24	140	2.23	1.59E-02	200

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
^{123}Te	2.33	104	10.4	1.00E-01	
^{127}I	4.41	140	0.0001	7.14E-07	
	7.55	100	0.000483	4.83E-06	
^{124}Xe	5.09	89.5	12.9	1.44E-01	
	9.88	142	42.3	2.98E-01	
^{126}Xe	9.88	93	1.35	1.45E-02	
^{129}Xe	9.66	144	7.76	5.39E-02	
^{131}Xe	3.2	112	0.000256	2.29E-06	
^{135}Xe	0.0842	114	20.4	1.79E-01	
^{133}Cs	5.91	123	7.38	6.00E-02	43
	9.5	120	0.00217	1.81E-05	43
^{138}La	2.99	95	1.19	1.25E-02	135
^{139}La	0.758	40.1	0.0000996	2.48E-06	135
^{141}Ce	7.4	100	1.01	1.01E-02	138
^{145}Nd	4.36	69.6	1.18	1.70E-02	148
^{147}Nd	4.79	75	0.309	4.12E-03	148
	6.26	75	0.651	8.68E-03	148
^{147}Pm	5.36	78.9	40.1	5.08E-01	
	6.57	59.6	1.4	2.35E-02	
	6.92	74.3	4.75	6.39E-02	
^{148}Pm	0.169	79.6	0.545	6.85E-03	
^{151}Pm	4.1	110	41.4	3.76E-01	
	8.2	70.9	1.87	2.64E-02	
^{147}Sm	3.4	67	1.35	2.01E-02	184
^{149}Sm	0.0973	63.4	0.525	8.28E-03	184
	0.872	62.7	0.75	1.20E-02	184
	4.94	59	2.14	3.63E-02	184
	6.43	68	1.2	1.76E-02	184
	8.93	70	11.8	1.69E-01	184
^{151}Sm	0.456	100	0.0223	2.23E-04	184
	1.09	119	0.948	7.97E-03	184
	1.7	98.4	0.444	4.51E-03	184
	2.04	99.9	0.744	7.45E-03	184
	4.13	95.9	1.26	1.31E-02	184
	6.4	108	6.71	6.21E-02	184
^{152}Sm	8.05	201	135	6.72E-01	184
^{153}Sm	1.92	72.1	10.1	1.40E-01	184
	3.84	62.4	0.42	6.73E-03	184
	5.76	62.5	0.523	8.37E-03	184
	7.68	62.9	0.917	1.46E-02	184
	9.6	68.6	6.61	9.64E-02	184
^{151}Eu [56]	0.32	79.5	0.071	8.93E-04	120
	0.46	87	0.665	7.64E-03	120
	1.05	93	0.19	2.04E-03	120
	1.83	93	0.0324	3.48E-04	120
	2.71	97	0.223	2.30E-03	120
	3.36	97	1.1	1.13E-02	120
	3.71	89	0.694	7.80E-03	120
	4.78	93	0.146	1.57E-03	120
	5.38	108	0.228	2.11E-03	120
	5.98	100	0.408	4.08E-03	120
	7.05	92	0.0429	4.66E-04	120
	7.29	88	1.89	2.15E-02	120
	7.44	85	2.03	2.39E-02	120
	9.07	104	1.3	1.25E-02	120
^{152}Eu	0.884	122	0.187	1.53E-03	120
	1.34	216	0.146	6.76E-04	120
	1.63	137	0.104	7.59E-04	120
	1.89	197	0.48	2.44E-03	120
	3.21	160	0.0035	2.19E-05	120

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
	3.55	270	0.791	2.93E-03	120
	4.13	180	0.391	2.17E-03	120
	4.92	80	0.285	3.56E-03	120
	5.6	90	0.199	2.21E-03	120
	6.35	130	0.551	4.24E-03	120
	6.75	151	0.543	3.60E-03	120
	9.61	218	2.57	1.18E-02	120
	9.94	119	0.665	5.59E-03	120
^{153}Eu	1.73	90	0.0489	5.43E-04	120
	2.46	91	1.03	1.13E-02	120
	3.29	98	1.09	1.11E-02	120
	3.94	93	0.943	1.01E-02	120
	4.75	96	0.0429	4.47E-04	120
	6.16	109	0.54	4.95E-03	120
	8.85	114	3	2.63E-02	120
^{154}Eu	0.195	150	0.0717	4.78E-04	120
	0.857	126	0.056	4.44E-04	120
	1.37	135	0.461	3.41E-03	120
	3.51	115	0.222	1.93E-03	120
	4.14	129	0.642	4.98E-03	120
	5.22	140	0.359	2.56E-03	120
	5.63	127	0.227	1.79E-03	120
	6.65	100	0.0303	3.03E-04	120
	6.82	151	1.08	7.15E-03	120
	9.32	105	3.16	3.01E-02	120
	9.48	111	1.53	1.38E-02	120
^{155}Eu	0.602	87	4.08	4.69E-02	120
	2.04	100	0.0394	3.94E-04	120
	7.19	100	0.18	1.80E-03	120
^{157}Eu	3.2	65.5	0.504	7.69E-03	120
	6.4	72.5	7.54	1.04E-01	120
	9.6	72.9	7.92	1.09E-01	120
^{152}Gd	3.31	56	0.018	3.21E-04	155
	8	56	5.1	9.11E-02	155
	9.55	56	0.093	1.66E-03	155
^{153}Gd	0.0297	65	0.041	6.31E-04	155
	0.917	100	0.0208	2.08E-04	155
	6.65	100	1.04	1.04E-02	155
^{155}Gd	0.0252	104	0.097	9.33E-04	155
	2.01	128	0.4	3.13E-03	155
	2.57	107	1.71	1.60E-02	155
	3.62	130	0.05	3.85E-04	155
	6.31	109	2.2	2.02E-02	155
	7.75	109	1.16	1.06E-02	155
	9.99	110	0.2	1.82E-03	155
^{157}Gd	0.032	107	0.428	4.00E-03	155
	2.83	110	0.377	3.43E-03	155
^{159}Tb	3.36	103	0.337	3.27E-03	158
	4.99	103	0.0834	8.10E-04	158
^{160}Tb	1.42	119	3	2.52E-02	158
	2.21	85.8	0.583	6.79E-03	158
	8.27	100	8.72	8.72E-02	158
^{156}Dy	2.15	83.6	0.2	2.39E-03	158
	3.21	83.6	0.8	9.57E-03	158
	9.19	83.6	0.6	7.18E-03	158
^{160}Dy	1.88	106	0.2	1.89E-03	158
^{161}Dy	2.71	119	0.561	4.71E-03	158
	3.68	124	2.14	1.73E-02	158
	4.33	80	1.38	1.73E-02	158
	7.74	107	0.514	4.80E-03	158
^{162}Dy	5.44	148	21	1.42E-01	158

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
^{163}Dy	1.71	103	2.04	1.98E-02	158
	5.81	113	0.0231	2.04E-04	158
^{165}Ho	3.91	87.8	2.13	2.43E-02	161
	8.17	90.5	0.187	2.07E-03	161
^{166}Ho	8.5	100	11	1.10E-01	161
^{162}Er	5.48	100	0.33	3.30E-03	163
	7.6	100	0.66	6.60E-03	163
^{164}Er	7.9	96	0.71	7.40E-03	163
^{167}Er	0.46	87.1	0.269	3.09E-03	163
	0.583	86.2	0.247	2.87E-03	163
	5.99	105	20.7	1.97E-01	163
	7.93	98.8	0.16	1.62E-03	163
	9.39	88.3	9.2	1.04E-01	163
^{169}Tm	3.91	102	7.47	7.32E-02	167
^{170}Tm	2.84	130	0.375	2.88E-03	167
	9.41	131	1.25	9.54E-03	167
^{168}Yb	0.597	64	2.2	3.44E-02	Yb
	9.71	64	0.08	1.25E-03	Yb
^{170}Yb	8.13	66.5	1.64	2.47E-02	Yb
^{171}Yb	7.91	70	2.2	3.14E-02	Yb
^{173}Yb	4.51	80	0.18	2.25E-03	Yb
^{175}Lu	2.59	71	0.188	2.65E-03	116
	4.75	65.3	0.267	4.09E-03	116
	5.22	80.6	1.6	1.99E-02	116
^{176}Lu	0.141	62.4	0.0927	1.49E-03	116
	1.57	59.5	0.485	8.15E-03	116
	4.36	68.4	0.403	5.89E-03	116
	6.13	58.4	1.37	2.35E-02	116
	8.14	177	0.0308	1.74E-04	116
	9.73	91.3	1.28	1.40E-02	116
^{174}Hf	4.06	52	0.015	2.88E-04	213
^{176}Hf	7.89	61.8	10.1	1.63E-01	213
^{177}Hf	1.1	65.2	2.22	3.40E-02	213
	2.39	60.7	8.04	1.32E-01	213
	5.9	62	5.32	8.58E-02	213
	6.58	55.6	8.21	1.48E-01	213
	8.88	57.3	5.89	1.03E-01	213
^{178}Hf	7.78	60	50	8.33E-01	213
^{179}Hf	5.69	47	4.27	9.09E-02	213
^{181}Ta	4.28	53	3.2	6.04E-02	225
^{182}Ta	0.147	67.3	0.315	4.68E-03	225
	1.82	68.4	1.35	1.97E-02	225
	5.98	67.4	0.406	6.02E-03	225
^{182}W	4.15	48	1.55	3.23E-02	312
^{183}W	7.64	83	1.66	2.00E-02	312
^{185}Re	2.15	57.7	2.83	4.90E-02	275
	5.92	69.3	0.264	3.81E-03	275
	7.22	56.2	1.19	2.12E-02	275
^{187}Re	4.42	54.9	0.318	5.79E-03	275
^{187}Os	9.47	81	1.74	2.15E-02	400
^{189}Os	6.71	82	4.53	5.52E-02	400
	8.96	76	9.04	1.19E-01	400
^{191}Ir	0.653	72.6	0.44	6.06E-03	228
	5.36	87.4	5.44	6.22E-02	228
	6.12	83.7	0.707	8.45E-03	228
	9.07	83.1	3.12	3.75E-02	228
	9.89	92	1	1.09E-02	228
^{193}Ir	1.3	87.2	0.73	8.37E-03	228
	9.07	92.2	2.24	2.43E-02	228
^{197}Au	4.89	124	15.2	1.23E-01	178

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
^{226}Ra	0.537	28	0.021	7.50E-04	
^{228}Th	1.9	36	0.785	2.18E-02	100
	7.55	70	1.21	1.73E-02	100
^{229}Th	0.609	41	0.158	3.85E-03	100
	1.25	41	0.16	3.90E-03	100
	1.42	41	0.01	2.44E-04	100
	1.72	23	0.0643	2.80E-03	100
	1.96	37	0.223	6.03E-03	100
	2.67	41	0.0137	3.34E-04	100
	3.18	41	0.12	2.93E-03	100
	4.16	41	1.17	2.85E-02	100
	4.75	41	0.0317	7.73E-04	100
	5.58	40	0.497	1.24E-02	100
	6.95	40	0.912	2.28E-02	100
	8.27	67.1	0.137	2.04E-03	100
	9.15	341	0.432	1.27E-03	100
^{230}Th	1.43	27.6	0.39	1.41E-02	100
^{232}Th	8.36	24.4	0.000249	1.02E-05	100
^{231}Pa	0.4	48.6	0.0592	1.22E-03	262
	0.497	36.9	0.0107	2.90E-04	262
	0.744	43.1	0.0125	2.90E-04	262
	1.24	40.8	0.0242	5.93E-04	262
	1.96	40	0.0144	3.60E-04	262
	2.79	40	0.013	3.25E-04	262
	3.49	34.4	0.0386	1.12E-03	262
	4.12	33.6	0.0709	2.11E-03	262
	4.35	39.9	0.0557	1.40E-03	262
	4.54	35.3	0.0202	5.72E-04	262
	5.07	40.2	0.418	1.04E-02	262
	5.29	42.7	0.0915	2.14E-03	262
	5.64	40	0.0506	1.27E-03	262
	5.82	40	0.0515	1.29E-03	262
	6.21	40	0.052	1.30E-03	262
	6.54	40	0.0707	1.77E-03	262
	6.88	38.8	0.21	5.41E-03	262
	7.58	40	0.147	3.68E-03	262
	7.83	42.4	0.261	6.16E-03	262
	8.74	40.1	1.08	2.69E-02	262
	9.27	40	0.056	1.40E-03	262
	9.72	34.1	0.349	1.02E-02	262
^{232}Pa	0.33	42	0.0676	1.61E-03	262
	0.67	37.8	0.0371	9.81E-04	262
	1.14	44	0.0348	7.91E-04	262
	1.42	41	0.0223	5.44E-04	262
	2.73	43.7	0.457	1.05E-02	262
	3.06	41.5	0.344	8.29E-03	262
	4.14	36.1	0.484	1.34E-02	262
	4.9	37.7	0.179	4.75E-03	262
	5.55	40.8	0.0462	1.13E-03	262
	5.82	38.5	0.138	3.58E-03	262
	6.45	41.5	0.505	1.22E-02	262
	7.48	43.8	0.144	3.29E-03	262
	8.44	44.1	0.338	7.66E-03	262
	8.85	42.5	0.507	1.19E-02	262
	9.7	34.8	0.433	1.24E-02	262
^{233}Pa	0.789	32.6	0.00144	4.42E-05	262
	1.34	43.3	0.108	2.49E-03	262
	1.64	40.6	0.309	7.61E-03	262
	2.36	35.2	0.00744	2.11E-04	262
	2.83	45.9	0.156	3.40E-03	262
	3.39	40	0.526	1.32E-02	262

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
	4.29	48.4	0.0876	1.81E-03	262
	5.15	47.3	0.399	8.44E-03	262
	7.18	41	0.139	3.39E-03	262
	8.26	41.2	0.0401	9.73E-04	262
	8.97	77.3	0.158	2.04E-03	262
	9.37	87	1.05	1.21E-02	262
^{232}U	5.98	40	1.5	3.75E-02	300
^{233}U	0.166	83	0.0001	1.20E-06	300
	0.232	48.5	0.0000318	6.56E-07	300
	0.577	25.2	0.000629	2.50E-05	300
	1.45	38.3	0.201	5.25E-03	300
	1.77	40	0.258	6.45E-03	300
	2.3	37.4	0.152	4.06E-03	300
	3.51	39	0.181	4.64E-03	300
	3.63	35.9	0.0545	1.52E-03	300
	4.46	34.5	0.379	1.10E-02	300
	5.81	39	0.1	2.56E-03	300
	6.64	39	0.343	8.79E-03	300
	6.83	34.9	0.689	1.97E-02	300
	7.48	39	0.0334	8.56E-04	300
	8.7	39	0.019	4.87E-04	300
	8.77	39	0.329	8.44E-03	300
	9.17	50.2	0.0677	1.35E-03	300
^{234}U	5.16	38.1	3.64	9.55E-02	300
^{235}U	0.274	46.2	0.00425	9.20E-05	300
	1.13	38.6	0.0145	3.76E-04	300
	1.31	38.6	0.000195	5.05E-06	300
	2.03	38	0.009	2.37E-04	300
	2.76	41.4	0.000738	1.78E-05	300
	3.14	38.2	0.0238	6.23E-04	300
	3.62	37.7	0.0421	1.12E-03	300
	3.87	38.9	0.000445	1.14E-05	300
	4.85	38.1	0.0525	1.38E-03	300
	5.41	39.8	0.022	5.53E-04	300
	6.16	39.8	0.0533	1.34E-03	300
	6.39	42.7	0.242	5.67E-03	300
	6.99	40.1	0.00156	3.89E-05	300
	7.08	39	0.112	2.87E-03	300
	7.66	39.8	0.00202	5.08E-05	300
	8.76	37.7	0.966	2.56E-02	300
	8.93	50.5	0.0829	1.64E-03	300
	9.27	41.4	0.12	2.90E-03	300
	9.7	39.8	0.0356	8.94E-04	300
^{236}U	5.46	24.5	2.3	9.39E-02	300
^{237}U	1.5	35	0.533	1.52E-02	300
	5	35	1.04	2.97E-02	300
	8.5	35	1.36	3.89E-02	300
^{238}U	4.41	23	0.0000555	2.41E-06	300
	6.67	23	1.48	6.43E-02	300
	7.68	23	0.00000942	4.10E-07	300
^{239}U	1.28	87.1	0.282	3.24E-03	300
	3.78	87.3	0.486	5.57E-03	300
	6.28	87.4	0.627	7.17E-03	300
	8.78	87.6	0.742	8.47E-03	300
^{240}U	2.67	24.3	2.33	9.59E-02	300
^{241}U	8.02	36.8	1.88	5.11E-02	300
^{236}Np	0.171	40	0.0218	5.45E-04	163
	0.705	40	0.0432	1.08E-03	163
	2.02	40	0.152	3.80E-03	163
	2.41	40	0.0553	1.38E-03	163
	2.7	40	0.0971	2.43E-03	163

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
	2.77	40	0.071	1.78E-03	163
	3.13	40	0.0991	2.48E-03	163
	3.54	40	0.115	2.88E-03	163
	5.07	40	0.354	8.85E-03	163
	6.03	40	0.195	4.88E-03	163
	7.01	40	0.579	1.45E-02	163
	7.92	40	0.179	4.48E-03	163
	9.76	40	1.85	4.63E-02	163
^{237}Np	0.49	40.5	0.047	1.16E-03	163
	1.32	40.3	0.0323	8.01E-04	163
	1.48	40.5	0.184	4.54E-03	163
	1.97	39.5	0.0141	3.57E-04	163
	3.05	40.8	0.00000171	4.19E-08	163
	3.86	39.7	0.212	5.34E-03	163
	4.26	40.4	0.0326	8.07E-04	163
	4.86	40	0.0418	1.05E-03	163
	5.78	41.9	0.528	1.26E-02	163
	6.38	39.6	0.0789	1.99E-03	163
	6.68	40.1	0.0132	3.29E-04	163
	7.19	40	0.00888	2.22E-04	163
	7.42	38.4	0.122	3.18E-03	163
	7.68	40	0.00216	5.40E-05	163
	8.31	37.6	0.0902	2.40E-03	163
	8.98	37	0.102	2.76E-03	163
	9.3	41.4	0.602	1.45E-02	163
^{238}Np	0.181	50	0.118	2.36E-03	163
	1.12	50	0.308	6.16E-03	163
	1.83	50	0.149	2.98E-03	163
	2.29	50	0.0675	1.35E-03	163
	2.56	50	0.0621	1.24E-03	163
	2.88	50	0.205	4.10E-03	163
	3.28	50	0.132	2.64E-03	163
	3.53	50	0.134	2.68E-03	163
	4.04	50	0.0439	8.78E-04	163
	4.36	50	0.115	2.30E-03	163
	4.72	50	0.137	2.74E-03	163
	4.98	50	0.115	2.30E-03	163
	5.37	50	0.0388	7.76E-04	163
	5.75	50	0.0949	1.90E-03	163
	5.99	50	0.275	5.50E-03	163
	6.57	50	0.247	4.94E-03	163
^{236}Pu	3.16	44	1.56	3.55E-02	176
	6.3	44	2.14	4.86E-02	176
^{238}Pu	2.89	38	0.0747	1.97E-03	176
	9.98	37	0.208	5.62E-03	176
^{239}Pu	0.296	39.3	0.08	2.04E-03	176
	7.82	37.7	0.792	2.10E-02	176
^{240}Pu	1.06	30	2.45	8.17E-02	176
^{241}Pu	0.15	42.3	0.000384	9.08E-06	176
	0.264	34.6	0.0437	1.26E-03	176
	1.73	40.3	0.00207	5.14E-05	176
	4.29	34.7	0.576	1.66E-02	176
	4.59	36.8	0.479	1.30E-02	176
	5.81	60.1	2.77	4.61E-02	176
	6.95	35.5	0.62	1.75E-02	176
	8.62	33.6	0.78	2.32E-02	176
	9.65	39.1	0.592	1.51E-02	176
	9.94	56.6	1.89	3.34E-02	176
^{242}Pu	2.68	26.8	2	7.46E-02	176
^{243}Pu	1.66	73.4	0.556	7.57E-03	176
	4.16	73.8	0.879	1.19E-02	176

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
	6.66	74	1.11	1.50E-02	176
	9.16	74.2	1.31	1.77E-02	176
^{244}Pu	4	20	0.026	1.30E-03	176
^{241}Am [57]	0.305	44.4	0.079	1.78E-03	153
	0.572	43.3	0.134	3.09E-03	153
	1.27	45.3	0.325	7.17E-03	153
	1.92	41	0.144	3.51E-03	153
	2.36	50	0.0688	1.38E-03	153
	2.6	48	0.13	2.71E-03	153
	3.97	44.5	0.257	5.78E-03	153
	4.97	43.8	0.157	3.58E-03	153
	5.42	44.2	0.653	1.48E-02	153
	5.8	44.2	0.00247	5.59E-05	153
	6.12	43.8	0.112	2.56E-03	153
	6.75	44.2	0.0408	9.23E-04	153
	7.66	44.2	0.0423	9.57E-04	153
	8.17	42.7	0.0926	2.17E-03	153
	9.11	44.2	0.449	1.02E-02	153
	9.85	43.9	0.494	1.13E-02	153
^{242}Am	0.178	46	0.192	4.17E-03	153
	0.35	50	0.18	3.60E-03	153
	0.615	50	0.111	2.22E-03	153
	1.1	50	0.423	8.46E-03	153
	1.71	50	0.0504	1.01E-03	153
	2.11	50	0.181	3.62E-03	153
	2.19	50	0.286	5.72E-03	153
	2.31	50	0.152	3.04E-03	153
	2.95	50	0.0821	1.64E-03	153
	3.18	50	0.273	5.46E-03	153
	3.39	50	0.242	4.84E-03	153
	3.71	50	0.6	1.20E-02	153
	4.01	50	0.266	5.32E-03	153
	4.13	50	0.0019	3.80E-05	153
	4.27	50	0.234	4.68E-03	153
	4.55	50	0.231	4.62E-03	153
	5.37	50	0.526	1.05E-02	153
	5.7	50	0.0468	9.36E-04	153
	5.91	50	0.0806	1.61E-03	153
	5.95	50	0.356	7.12E-03	153
	6.15	50	0.0805	1.61E-03	153
	6.41	50	0.0128	2.56E-04	153
	6.65	50	0.214	4.28E-03	153
	6.71	50	0.13	2.60E-03	153
	6.84	50	0.038	7.60E-04	153
	6.99	50	1.74	3.48E-02	153
	7	50	0.0361	7.22E-04	153
	7.21	50	0.104	2.08E-03	153
	8.07	50	0.131	2.62E-03	153
	8.33	50	0.896	1.79E-02	153
	8.6	50	0.0733	1.47E-03	153
	8.86	50	0.00284	5.68E-05	153
	9.03	50	0.412	8.24E-03	153
	9.42	50	0.19	3.80E-03	153
	9.43	50	0.0569	1.14E-03	153
	9.88	50	0.159	3.18E-03	153
^{243}Am	0.419	22.9	0.000671	2.93E-05	153
	0.983	45	0.02	4.44E-04	153
	1.36	50	1.1	2.20E-02	153
	1.74	41.5	0.35	8.43E-03	153
	3.14	32	0.0136	4.25E-04	153
	3.42	38.7	0.235	6.07E-03	153

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
	3.85	44.9	0.0172	3.83E-04	153
	5.13	35.1	0.344	9.80E-03	153
	6.55	51.5	0.782	1.52E-02	153
	7.07	29.7	0.0545	1.84E-03	153
	7.86	46.9	1.48	3.16E-02	153
	8.38	38.4	0.00989	2.58E-04	153
	8.77	31.7	0.101	3.19E-03	153
	9.31	38.6	0.178	4.61E-03	153
^{243}Cm [58]	0.671	40	0.0528	1.32E-03	125
	1.14	40	0.0588	1.47E-03	125
	1.47	40	0.0249	6.23E-04	125
	2.05	40	0.0163	4.08E-04	125
	2.31	40	1.93	4.83E-02	125
	2.76	40	0.0137	3.43E-04	125
	3.07	40	0.816	2.04E-02	125
	3.73	40	0.54	1.35E-02	125
	5.68	40	0.497	1.24E-02	125
	6.15	40	1.73	4.33E-02	125
	7.21	40	1.67	4.18E-02	125
	8.18	40	0.394	9.85E-03	125
	9.11	40	3.74	9.35E-02	125
^{244}Cm	7.67	37	9.6	2.59E-01	125
^{245}Cm	0.85	44	0.09	2.05E-03	125
	1.98	44	0.2	4.55E-03	125
	2.45	33	0.126	3.82E-03	125
	4.68	33	1.87	5.67E-02	125
	5.75	33	0.22	6.67E-03	125
	7.53	33	2.8	8.48E-02	125
	8.65	33	0.606	1.84E-02	125
	9.15	33	0.347	1.05E-02	125
^{246}Cm	4.32	28	0.311	1.11E-02	125
^{247}Cm	1.24	32	0.665	2.08E-02	125
	2.94	40	0.13	3.25E-03	125
	3.18	40	1.06	2.65E-02	125
	4.73	40	1.47	3.68E-02	125
	6.12	40	0.105	2.63E-03	125
	7.12	40	0.45	1.13E-02	125
	7.65	40	0.115	2.88E-03	125
	7.94	40	0.4	1.00E-02	125
	9.5	40	0.72	1.80E-02	125
	10	40	0.15	3.75E-03	125
^{248}Cm	7.25	23.3	1.89	8.11E-02	125
^{250}Cm	9.02	99.3	79.3	7.99E-01	125
^{249}Bk	0.195	35.9	0.117	3.26E-03	
	1.34	35.1	0.198	5.64E-03	
	1.6	33.2	0.573	1.73E-02	
	2.15	36.7	0.107	2.92E-03	
	3.11	37	0.145	3.92E-03	
	5.02	44.3	0.231	5.21E-03	
	6.28	33.8	0.147	4.35E-03	
	7.04	39	0.165	4.23E-03	
	7.99	36.1	1.41	3.91E-02	
^{249}Cf	0.7	40	0.65	1.63E-02	
	3.88	40	0.291	7.28E-03	
	5.07	40	0.653	1.63E-02	
	7.51	40	0.147	3.68E-03	
	8.65	40	0.444	1.11E-02	
	9.51	40	1.61	4.03E-02	
^{250}Cf	0.553	36.4	1.03	2.83E-02	
	6.85	45	0.425	9.44E-03	
	8.26	43	7.15	1.66E-01	

Isotope	Resonance Energy (eV)	Γ (meV)	Γ_n (meV)	$kb_{res} = \Gamma_n/\Gamma$	Debye Temperature
^{251}Cf	0.389	35	0.877	2.51E-02	
^{252}Cf	1.4	35	0.0192	5.49E-04	
^{253}Es	0.7	24.4	0.65	2.66E-02	

[1] E. Fermi and W. N. Zinn, Phys. Rev. **70**, 103 (1946).
[2] E. Fermi and L. Marshall, Phys. Rev. **71**, 666, (1947).
[3] H. Maier-Leibnitz and T. Springer, J. Nucl. Ener. A/B **17**, 217 (1963).
[4] H. Maier-Leibnitz, Nucleonik **8**, 5 (1966).
[5] V. F. Turchin, At. Energy **22**, (1967).
[6] F. Mezei, Communications on Physics **1**, 81 (1976).
[7] F. Mezei and P. A. Dagleish, Communications on Physics **2**, 41 (1977).
[8] J. B. Hayter and H. A. Mook, J. Appl. Cryst. **22**, 35 (1989).
[9] O. Schaerpf, Physica B **156/157**, 631 (1989).
[10] S. W. Lovesey, *Theory of Neutron Scattering from Condensed Matter*, volumes 1 and 2, Oxford University Press (1984).
[11] J. M. Carpenter and C. -K. Loong, *Elements of Slow Neutron Scattering*, Cambridge University Press (2015).
[12] L. G. Windsor, *Pulsed Neutron Scattering*, Taylor and Francis (1981).
[13] C. Andreani, D. Colognesi, J. Mayers, G. Reiter, and R. Senesi, Adv. Phys. **54**, 377 (2005).
[14] P. C. H. Mitchell, S. F. Parker, A. J. Ramirez-Cuesta, and J. Tomkinson *Vibrational Spectroscopy with Neutrons: with applications in Chemistry, Materials Science and Catalysis*, World Scientific (2005).
[15] J. W. Behrens, R. A. Schrack, and C. D. Bowman, Trans. Am. Nucl. Soc. **32** ,207 (1979).
[16] J. M. Carpenter, Nucl. Inst. Meth. **145**, 91 (1977).
[17] J. J. Sakurai, *Modern Quantum Mechancis*, Benjamin/Cummings (1985).
[18] M. Warner and J. E. Gubernatis, Phys. Rev. B **32**, 6347 (1985).
[19] I. I Gurevich and P. E. Nemiovskii, Zhurnal Eksp. and Theo. Fiz. **41**, 1175 (1961).
[20] V. I. Morozov, M. I. Novopoltsev, Yu. N. Panin, Yu. N. Pokotilovskii, and E. V. Rogov, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 301 (1987).
[21] A. I. Frank, V. I. Bodnarchuk, P. Geltenbort, I. L. Karpikhin, G. V. Kulin, and O. V. Kulina, Phys. Atom. Nucl. **66**, 1831 (2003).
[22] A. G. Klein, G. I. Opat, A. Cimmino, A. Zeilinger, W. Treimer, and R. Gahler, Phys. Tev. Lett. **46**, 1551 (1981).
[23] M. A. Horne, A. Zeilinger, A. G. Klein, and G. I. Opat, Phys. Rev. A **28**, 1 (1983).
[24] M. Arif, H. Kaiser, R. Clothier, S. A. Werner, W. A. Hamilton, A. Cimmino, and A. G. Klein, Phys. Rev. A **39**, 931 (1989).
[25] D. Attwood, *Soft X-Rays and Extreme Ultraviolet Radiation*, Cambridge University Press (2000).
[26] J. Als-Nielsen and D. McMorrow, *Elements of Modern X-Ray Physics*, Wiley (2001).
[27] V. F. Sears, Phys. Rep. **82**, 1(1982).
[28] M. L. Goldberger and K. M. Watson, *Collision Theory*, Wiley, New York (1964).
[29] E. Nowak, Z. Phys. **B 49**, 1 (1982).
[30] H. D. Dietze and E. Nowak, Z. Phys. **B44**, 245 (1981).
[31] E. Nowak, Z. Phys **B45**, 265 (1982).
[32] L. L. Foldy, Phys. Rev. **67**, 107 (1945).
[33] M. Lax, Rev. Mod. Phys. **23**, 287 (1951).
[34] H. Ekstein, Phys. Rev. **83**, 721 (1951).
[35] M. Lax, Phys. Rev. **85**, 621 (1952).
[36] Phys. Rev. **89**, 490 (1953).
[37] R. Lenk, Phys. Stat. Sol. B **69**, 369 (1975).
[38] P. Blaudeck and R. Lenk, Phys. Stat. Sol. B **77**, 601 (1976).
[39] L. Lanz and B. Vacchini, Phys. Rev. A **56**, 4826 (1997).
[40] V. F. Sears, Phys. Rev. A **32**, 2524 (1985).
[41] V. F. Sears, Phys. Rep. **141**, 281 (1986).
[42] A. V. Antonov, A. I. Isakov, M. V. Kazarnovskii, and U. E. Solidov, Sov. Phys. JETP Lett. **10**, 241 (1969).
[43] R. Golub, D. J. Richardson, and S. K. Lamoreaux, *Ultracold Neutrons*, Adam Hilger (1991).
[44] S. F. Mughabghab, M. Divadeenam, and N.E. Holden, *Neutron Cross Sections*, Vol.1, Academic Press, New York (1981).
[45] National Nuclear Data Center, Brookhaven National Lab, http://www.nndc.bnl.gov
[46] K. Shibata, O. Iwamoto, T. Nakagawa, N. Iwamoto, A. Ichihara, S. Kunieda, S. Chiba, K. Furutaka, N. Otuka, T. Ohsawa, T. Murata, H. Matsunobu, A. Zukeran, S. Kamada, and J. Katakura, J. Nucl. Sci. Technol. **48**, 1 (2011).
[47] A. Steyerl, Z. Physik **254**, 169 (1972).
[48] R. Golub, D. Richardson, and S. K. Lamoreaux, *Ultracold Neutrons*, Adam-Hilger (1991).
[49] W. E. Lamb, Phys. Rev. **55**, 190 (1939).
[50] M. S. Nelkin and D. E. Parks, Phys. Rev. **119**, 1060 (1960).
[51] G. T. Trammell, Phys. Rev. **126**, 1045 (1962).
[52] R. E. Word and G. T. Trammell, Phys. Rev. B **24**, 2430 (1981).
[53] J. E. Lynn and W. J. Trela, Nucl. Inst. Meth. B **108**, 147 (1996).
[54] A. K. Freund, Nucl. Inst. Meth. **213**, 495 (1983).
[55] knowledgedoor, Debye Temperature — The Elements Handbook at KnowledgeDoor, http://www.knowledgedoor.com/2/elements_handbook/debye_temperature.html, KnowledgeDoor
[56] N. G. Nereson, C. E. Olsen, and G. P. Arnold, Phys. Rev. **135**, A176 (1964).
[57] A. A. Povzner, A. N. Filanovich, V. A. Oskina, and A. G. Volkov, Zhurnal Tekhnicheskoi Fiz. **83**, 141 (2013).
[58] A. A. Povzner, A. N. Filanovich, and V. A. Oskina, Journal of Nucl. Mat. **443**, 17 (2013).