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# Probing Stark-induced nonlinear phase variation with opto-optical modulation

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We extend the recently developed technique of opto-optical modulation (OOM) to probe stateresolved AC-Stark-induced phase variations of a coherently excited ensemble of helium atoms. In a joint experimental and theoretical study, we find that the spatial redirection of the resonant emission from the OOM process is different for the low-lying 1s2p state as compared with the higher-lying Rydberg states, and that this redirection can be controlled through the spatial characteristics of the infrared (IR) probe beam. In particular, we observe that the intensity dependence of the IR-induced Stark phase on the 1s2p emission is nonlinear, and that the phase accumulation changes sign for moderate intensities. Our results suggest that OOM, combined with precise experimental shaping of the probe beam, could allow future measurements of Stark-induced phase shifts of excited states.

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### I. INTRODUCTION

Light-matter interactions can be addressed from two 17 complementary points of view [1]. Just as light can be 18 used as a tool to probe and control matter [2-7], atoms 19 can be exploited to probe and control light [8–14]. The 20 recently demonstrated technique of opto-optical modula-21 tion (OOM) [8, 9] is an example of this duality in the 22 realm of ultrafast extreme ultraviolet (XUV) sources. 23 OOM relies on the combination of two coherent fem-24 tosecond pulses with different properties. First, an XUV 25 pump pulse resonantly excites an atomic target produc-26 ing a coherent superposition of ground and excited states. 27 This triggers a long-lived emission of coherent XUV light 28 at the resonant transition frequencies. Subsequently, a 29 strong, infrared (IR) probe pulse arrives and modifies 30 the XUV emission, altering its spatio-temporal profile. 31

The effect of the IR probe pulse on the coherent XUV 32 emission is mediated by the AC-Stark shift [15]. This 33 IR-induced shift of the excited state energies yields an 34 additional state-dependent phase that is imprinted on the 35 dipole and thus on the emitted XUV light [2, 16, 17]. The 36 OOM technique translates the spatial intensity variation 37 of the IR beam into a state-specific spatial phase gradient 38 that results in the redirection of the XUV emission. 39

Previously OOM has been used to redirect ultrafast  $^{\scriptscriptstyle 53}$ 40 XUV light pulses in an argon gas, from both Rydberg and  $^{\rm 54}$ 41 autoionizing states [8, 9]. Further details of the technique <sup>55</sup> 42 using also helium and neon gases can be found in refer-<sup>56</sup> 43 ence [9]. The direction of emission in these experiments <sup>57</sup> 44 was explained via the known, approximately linear AC-<sup>58</sup> 45 Stark shift of high lying Rydberg states. For these states <sup>59</sup> 46 the AC-Stark shift approaches the average kinetic energy  $^{60}$ 47 of a free electron oscillating in an electric field, namely  $^{\rm 61}$ 48 the ponderomotive energy  $U_{\rm p} = e^2 F^2 / 4m_e \omega^2$ , where  $e^{62}$ 49 and  $m_e$  are the electron charge and mass, and F and  $\omega$  <sup>63</sup> 50 are the electric field amplitude and angular frequency. <sup>64</sup> 51



FIG. 1. Example of XUV spatial control using OOM from a manifold of excited np states in helium. The unperturbed 2p-7p energies are indicated in white, and the ionization energy in red. The 2p emission is redirected both up and down by the 800 nm IR pulse, whereas the high-lying np emission is only redirected up. States pertinent to later discussions are shown in black. He state energy levels are taken from [18].

The ponderomotive Stark phase depends linearly on the IR intensity, which acts as a control parameter on the XUV spatio-temporal properties.

In this article we demonstrate that the OOM technique can be used to probe unknown, non-linear Stark phases. In particular, we reveal the intensity dependence of the Stark phase for the low-lying 1s2p state in helium (hereafter we omit the passive 1s occupation label). We coherently excite the manifold of higher energy np Rydberg states as a reference and observe that the spatial redirection of the XUV light from the 2p transition is different relative to the higher-lying np states. Significantly, we find that the 2p energy shift changes sign as a function of intensity, so that if the 2p emission is redirected down at low intensity, it will be redirected up at high intensity. In practice we observe 2p emission in both directions at higher peak intensities, because both high and low intensity regions of the IR beam contribute to

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the redirection, as illustrated in Fig. 1. Solutions of the 70 coupled Maxwell wave equation and the time-dependent 71 Schrödinger equation (MWE-TDSE) reproduce the main 72 features of the experimental results. They allow us to 73 understand the observed 2p state behavior in terms of 74 a transition from a regime of strong near-resonant cou-75 pling with nearby states at low intensity, to a regime of 76 non-resonant free-electron-like behavior at high intensity. 77

# II. PRINCIPLE

The principle of OOM [8, 9] and how it may be used to 79 probe the intensity dependence of the Stark phase is illus-80 trated in Fig. 2. A broadband, coherent XUV pump pulse 81 excites a time-dependent dipole moment, which leads to 82 coherent emission in the forward direction at a number 83 of resonant frequencies [19, 20]. The long lifetime of the 84 resonances is reflected as sharp absorption features in 85 the spectral domain. An IR probe pulse following the 86 XUV excitation interacts with the target and produces 87 a spatial phase gradient through the intensity-dependent 88 Stark phase, thereby modifying the XUV wavefront and 89 redirecting the emission. This happens because the phase ٩n gradient yields a transverse contribution to the wave vec-91 tor,  $k_{\perp} = d\phi_s/dr$ , where  $\phi_s$  is the accumulated Stark 92 phase, which alters the direction of wave-vector phase 93 matching. Since the AC Stark shift is state specific, the 94 emission associated with different excited states can be 95 redirected in different ways by the IR interaction. 96

To understand the expected behavior of the OOM redirection we consider the spatial dependence of the accumulated Stark phase in the limit where ionization can be ignored [8]:

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$$\phi_s(\mathbf{r}) = \frac{1}{\hbar} \int_{\mathcal{T}_{\rm IR}} \Delta E(\mathbf{r}, t) \, dt, \qquad (1)$$

where  $\Delta E(\mathbf{r}, t)$  is the intensity-dependent Stark shift of 102 a specific resonance,  $\hbar$  is the reduced Planck constant, 103 and  $\tau_{\text{\tiny IR}}$  is the total duration of the IR probe pulse. For 104 Rydberg states the shift in energy with increasing field 105 intensity is positive and close to linear. Spatially off-106 setting a smaller pump beam and a larger probe beam<sub>124</sub> 107 imprints an approximately linear phase gradient  $across_{125}$ 108 the pump beam so that all the np emission is redirected<sub>126</sub> 109 in the same direction, as observed in [8, 9] [upward  $in_{127}$ 110 Fig. 2(a)]. If, however, the intensity-dependent  $\text{phase}_{128}$ 111 shift for a state as a function of intensity is nonlinear,  $as_{129}$ 112 in Fig. 2(b), the phase front of the emission can be al- $_{130}$ 113 tered in a more complex way. In particular, if the  $\mathrm{Stark}_{\scriptscriptstyle 131}$ 114 phase decreases for low intensity and increases at  $high_{132}$ 115 intensity, the XUV emission can be redirected through 116 both negative and positive divergence angles, resulting 117 in an effective beamsplitter for XUV light. 118

With the pump and probe beams offset as in Fig. 2,<sub>134</sub> the redirected light can be adjusted via the focal over-<sub>135</sub> lap between the pump and probe beams, and the spatial<sub>136</sub> intensity profile of the probe pulse at the target. For<sub>137</sub> resonances long-lived with respect to the duration of the<sub>138</sub>



FIG. 2. Illustration of OOM redirection for (a) linear and (b) nonlinear Stark phase behavior. A small (blue in color version online) pump XUV beam excites the atoms. (a) Following interaction with a spatially offset, larger (red in color version online) probe IR beam, the XUV emission phase front can become tilted if the Stark phase response is approximately linear, as for the np states. (b) A nonlinear Stark response can result in the phase front being tilted in one direction at low intensity, and the other direction at high intensity. The amount of phase accumulation, and consequently the phase gradient spatial profile, is determined by the IR intensity and spatial distribution across the XUV pump focus.

pulses, redirection can occur many tens or hundreds of fs after the excitation pulse has passed, allowing this measurement to be performed outside of temporal overlap of the pump and probe pulses. For the OOM technique, the lifetime of the excited state must be sufficient for an appreciable Stark-shifting to occur, enabling redirection. Redirection from short-lived states could require shorter pulse durations to satisfy the condition for IR-free XUV excitation of the excited ensemble.

#### III. EXPERIMENTAL SETUP

The experimental setup is a pump-probe scheme where both pulses are derived from the same 1 kHz repetition rate, 800 nm titanium-sapphire laser system producing pulses of  $\sim 20$  fs duration. Annular mirrors are used to spatially separate and recombine the pump and probe

beam-paths. The outer, annular part of the IR beam 139 is focused into a pulsed gas jet of argon atoms to pro-140 duce the pump XUV light through high-order harmonic 141 generation (HHG) [21–25]. To shift the 13th harmonic 142 into resonance with the 1s-2p transition in the helium 143 target gas, the HHG process is driven at sufficient inten-144 sity to induce blue-shifting of the generated harmonics 145 [26]. This blue-shifting, along with overlaid second order 146 diffraction components from the diffraction grating, pro-147 duces the observed near continuous harmonic spectrum 148 detected on axis (Fig. 1). An iris is positioned down-149 stream in the HHG beam-path to limit the divergence of 150 the XUV beam and thereby suppress any off-axis emis-151 sion in the far field that is not due to the IR interaction. 152 This iris also acts to reduce the residual fundamental 153 light from the HHG process. The inner part of the IR 154 beam bypasses the HHG gas and serves as the probe. 155 Both pump and probe beams are focused into the tar-156 get helium gas using a toroidal mirror. Through imaging 157 we measure the probe focus to be  $\sim 160 \,\mu \text{m}$  full width 158 at half maximum (FWHM). From the ability to redirect 159 the XUV np emission either up (as in Fig. 1) or down 160 by adjusting the XUV-IR spatial offset, we deduce that 161 the XUV focus is smaller than this. The beams are re-162 combined at a small angle, and the probe is offset spa-163 tially from the pump in the interaction region to capture  $_{196}$ 164 the steepest slope of the IR spatial intensity distribu-165 tion. The delay between the pump and probe pulses is  $^{\scriptscriptstyle 197}$ 166 controlled using a precision translation stage, and the  $^{198}\,$ 167 delay of the IR probe used in the following measure-199 168 ments is several tens of fs after temporal overlap. The  $^{\rm 200}$ 169 helium pressure has been adjusted to optimize the  $2p^{201}$ 170 emission and avoid effects of resonant pulse propagation<sup>202</sup> 171 (RPP) [27, 28]. The spectrally resolved spatial profile of  $^{203}$ 172 the XUV light is recorded in the far field using a flat-<sup>204</sup> 173 field spectrometer, with a micro-channel plate detector,<sup>205</sup> 174 imaged by a CCD camera. The probe intensity in the  $^{\rm 206}$ 175 interaction region is controlled using a motorized, zero-<sup>207</sup> 176 208 aperture iris after the focus in the IR beam path. 177 209

## IV. RESULTS

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#### A. Experiment

Fig. 3(a) shows the evolution of the XUV emission from 213 180 a narrow energy region around the 2p-state excitation214 181 energy with an iris-opening parameter that varies from  $0_{215}$ 182 (fully closed) to 1 (fully open). Note that the exact map-216 183 ping between this opening parameter and the actual iris217 184 diameter is not perfectly known. The estimated IR peak<sub>218</sub> 185 intensity for the fully open iris is  $9 \times 10^{12}$  W/cm<sup>2</sup>. The<sub>219</sub> 186 effect of the iris is two-fold since it changes both the to-220 187 tal energy in the probe beam and its confocal parameter.221 188 The figure shows that at low intensity (up to iris opening<sub>222</sub> 189  $\approx 0.35$ ), the 2p emission is redirected only downward (op-223 190 posite to the np emission), whereas at higher intensities<sub>224</sub> 191 it splits and is redirected both up and down. This indi-225 192 cates that the intensity dependence of the accumulated<sub>226</sub> 193 Stark phase changes sign, or, equivalently, that the shift<sub>227</sub> 194 in energy changes from being negative to being positive.<sup>228</sup> 195



FIG. 3. Far field divergence of the 2p emission in (a) the experiment, and (b) the calculation. (c) shows the calculated spatio-spectral profile of the 2p emission for a fully open iris.

#### B. Theory

For comparison with experiment, Fig. 3(b) shows the 2p emission calculated by solving the coupled MWE-TDSE equations for a He gas interacting with two spatially offset XUV and IR fields [29]. The 2p-resonant XUV pump pulse duration is 4 fs, with a focus of  $28 \,\mu m$ FWHM and a peak intensity of  $10^{11}$  W/cm<sup>2</sup>, and the 800 nm probe pulse duration is 27 fs, with a focus of 56  $\mu$ m FWHM and a peak intensity of 10<sup>13</sup> W/cm<sup>2</sup> when the iris is fully opened. The two pulses are delayed with respect to each other by 40 fs and spatially offset by  $35 \,\mu\text{m}$ . We use a thin 10  $\mu\text{m}$  He gas medium with a density of  $5 \times 10^{18} \text{cm}^{-3}$  to avoid effects of RPP. To account for the non-cylindrical symmetry, the MWE calculations were performed in one transverse direction (1D). This means that the iris in the calculations, which is applied before focusing the IR beam, does not exactly replicate the effect of the experimental iris on the two-dimensional (2D) beam. In particular, the intensity of the 1D beam increases too slowly as the 1D iris diameter is increased as compared to the experiment. To compensate for this, we multiply the intensity after the aperture,  $I_a$ , by the square of the intensity loss,  $I_a/I_0$ , where  $I_0$  is the intensity before the aperture. The two factors of  $I_a/I_0$  mimic the extra drop in intensity due to the energy loss and the increased confocal parameter.

The calculations can also provide further insight into the 2p emission. Fig. 3(c) shows the calculated far-field, spatio-spectral profile of the XUV light near the 2p state for a fully open iris, clearly exhibiting both up- and downdirected emission. In the calculation, we can block out selected parts of the near field interaction region, which alters the far-field signal. From this we confirm the pic-



FIG. 4. (a) TDSE calculation of the total IR-induced phase<sub>272</sub> accumulation for the different np states in helium after inter-<sub>273</sub> action with a resonant 4 fs, 10<sup>11</sup> W/cm<sup>2</sup> pump and a subse-<sub>274</sub> quent 40 fs delayed, 800 nm, 27 fs IR probe pulse for a range<sub>275</sub> of different probe peak intensities. The lower solid line corresponds to the 2p state, while the upper solid lines correspond<sup>276</sup> to the higher p states, and  $U_p$ . (b) 2p population at the end<sup>277</sup> of the IR pulse normalized by the 2p population at the end<sup>278</sup> of the XUV pulse. The dashed lines correspond to the 2p (a)<sup>279</sup> phase and (b) population in the same conditions but with an<sup>280</sup> 829 nm IR pulse which drives near-resonant two-photon Rabi<sup>281</sup> oscillations between the 2p and the 5f states. Rapid phase<sup>282</sup> variations are observed at intensities matching near zeros in<sub>283</sub> the 2p state populations.

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ture illustrated in Fig. 2(b): the downward 2p emission<sup>287</sup> 229 comes from the upper part of the probe beam where the<sup>288</sup> 230 intensity is low, and the upward 2p emission comes from<sup>289</sup> 231 the lower part of the probe beam where the intensity<sup>290</sup> 232 is high. Fig. 3(b) shows the calculated behavior as  $a^{291}$ 233 function of iris opening. Allowing for the differences be-<sup>292</sup> 234 tween the experiment and theory discussed above, the<sup>293</sup> 235 general features of the calculated behavior agree very<sup>294</sup> 236 well with those of the experiment, both in terms of the<sup>295</sup> 237 down-only redirection at low intensity, and the asym-<sup>296</sup> 238 metry between up and down-directed emission at higher<sup>297</sup> 239 intensities. From the calculations we find that the de-  $^{298}$ 240 tailed behavior as a function of iris opening, especially<sup>299</sup> 241 in terms of the up/down emission asymmetry, is sensi-300 242 tive to the peak IR intensity, the relative sizes of the<sup>301</sup> 243 pump and probe beams, and in particular to the spatial<sup>302</sup> 244 offset of the pump-probe foci. This suggests that more<sup>303</sup> 245 precise experimental control over the probe spatial pro-246

file, for example through the use of spatial light modula-<sup>304</sup>
tors [30, 31], could allow for future reconstruction of the<sub>305</sub>
intensity-dependence of state-resolved Stark shifts from<sub>306</sub>
the experimental results, and to finely control and tailor<sub>307</sub>
the XUV emission in space and time.

Finally, to understand the observed intensity depen-309 dence of the 2p-emission redirection, Fig. 4(a) shows the310 TDSE-calculated accumulated Stark phase for each of311

the excited states discussed in this paper. The intensity axis denotes the peak intensity of the same 800 nm, 27 fs IR pulse used in Fig. 3(b), and the phase is extracted at the end of the IR pulse by projecting onto the field free states. The accumulated phase due to a Stark shift equal to the ponderomotive energy  $\Delta E = U_{\rm p}$ is shown for comparison, and marks the simplest possible linear Stark phase. This figure shows that the accumulated phase increases approximately as  $U_{\rm p} \tau_{\rm IR}$  for the 3p and higher-lying np states (upper solid lines). The phase of the 2p state (lower solid line), however, exhibits a completely different behavior. It drops rapidly at low intensity, below approximately  $1.9 \times 10^{12}$  W/cm<sup>2</sup>, then reverses and increases almost linearly at higher intensity, although slower than the higher np states. These general trends are in good agreement with the results discussed above, and can be understood with the following considerations. At low intensity, the 2p state couples strongly to the 3s and 3d states, which are in close to one-photon resonance with it. Indeed, we find that the low intensity behavior of the 2p phase can be accurately reproduced with a three-level model using only the 2p, 3s and 3d states (not shown). We also find, as expected for near-resonant interactions, that the sign of the 2p-3s and 2p-3d detuning controls the sign of the low intensity phase shift. The 2s state, which is below the 2p state by about half an IR photon, is too far detuned to play a significant role. Conversely, at high intensities, the electric field strongly distorts the potential felt by the electron so that it behaves increasingly like a free electron in an oscillating field, and the 2p state presents a near-linear phase more similar to the higher-lying np states.

At low and moderate intensities, the IR field also enables near-resonant two-photon coupling between the 2p and higher-lying nf states [32] that drives Rabi oscillations between these states, as can be seen in the 2p population shown in Fig. 4(b). These oscillations are highly sensitive to the IR wavelength and are best observed at a slightly longer wavelength (829 nm) than the one used in the experiment. The longer wavelength 2p population and phase are shown as dashed lines in Fig. 4(a) and (b). Note that the minima in the 2p population are associated with rapid variations of the phase (near 1.5 and  $3.5 \times 10^{12}$  W/cm<sup>2</sup>), as expected for Rabi flopping [33]. This provides another interesting perspective on XUV spatial control through OOM: in the resonant case, both the phase and the amplitude of the XUV field can be modulated through IR-control of the Stark shift and the population of the resonant state.

#### V. SUMMARY

In summary, we have used the all-optical OOM technique to probe the Stark-induced phase change of excited states in matter. We have experimentally observed the change of sign of the 2p-state phase accumulation as the intensity of the non-resonant IR field is increased, in good agreement with MWE-TDSE based calculations. This result opens the possibility for the future study of Stark

phases in more complicated atoms or molecules, where 326 312 the states and/or their dipole couplings may be less well<sub>227</sub> 313 known, and even allow for reconstruction of the  $\text{phase}_{328}$ 314 accumulation from the experimental result given tighter<sub>329</sub> 315 control over the experimental parameters. We also  $em_{330}$ 316 phasize the potential for the OOM technique to be  $used_{331}$ 317 to probe unknown Stark phases of states embedded in<sub>332</sub> 318 the continuum, which although beyond the scope of the  $_{333}$ 319 work presented here, would be interesting to study in fu- $_{334}$ 320 ture experiments. This work also highlights the potential 321 for the OOM technique to control XUV frequency light  $in_{336}$ 322 different ways, such as by creating variable beam-splitters<sub>337</sub> 323 in the XUV by exploiting the nonlinear response of states  $_{\scriptscriptstyle 338}$ 324 to IR intensity changes. 325 339

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- [1] F. Calegari, G. Sansone, S. Stagira, C. Vozzi, and 381
   M. Nisoli, J. Phys. B: At. Mol. Opt. Phys. 49, 062001382
   (2016).
- [2] M. Wu, S. Chen, S. Camp, K. J. Schafer, and M. B.384
   Gaarde, Journal of Physics B: Atomic, Molecular and 385
   Optical Physics 49, 062003 (2016).
- [3] W. Cao, E. R. Warrick, A. Fidler, D. M. Neumark, and 387
   S. R. Leone, Physical Review A 94, 053846 (2016). 388
- [4] W. Cao, E. R. Warrick, A. Fidler, S. R. Leone, and D. M.<sup>389</sup>
   Neumark, Physical Review A **94**, 021802(R) (2016).
- [5] S. Baker, J. S. Robinson, C. A. Haworth, H. Teng, R. A.391
   Smith, C. C. Chiril, M. Lein, J. W. G. Tisch, and J. P.392
   Marangos, Science **312**, 424 (2006).
- [6] M. Chini, X. Wang, Y. Cheng, Y. Wu, D. Zhao, D. A.<sup>394</sup>
   Telnov, S.-I. Chu, and Z. Chang, Scientific Reports 3<sup>395</sup>
   (2013), 10.1038/srep01105. <sup>396</sup>
- [7] N. Shivaram, H. Timmers, X.-M. Tong, and A. Sandhu, 397
   Physical Review Letters 108, 193002 (2012). 398
- [8] S. Bengtsson, E. W. Larsen, D. Kroon, S. Camp, M. Mi-399
   randa, C. L. Arnold, A. L'Huillier, K. J. Schafer, M. B.400
   Gaarde, L. Rippe, and J. Mauritsson, Nature Photonics401
   11, 252 (2017). 402
- [9] S. Bengtsson and J. Mauritsson, J. Phys. B: At. Mol.<sup>403</sup>
   Opt. Phys. 52, 063002 (2019).
- L. Drescher, O. Kornilov, T. Witting, G. Reitsma,<sup>405</sup>
   N. Monserud, A. Rouze, J. Mikosch, M. J. J. Vrakking,<sup>406</sup>
   and B. Schütte, Nature 564, 91 (2018).
- <sup>367</sup> [11] A. Kaldun, C. Ott, A. Blättermann, M. Laux, K. Meyer,<sup>408</sup>
   <sup>368</sup> T. Ding, A. Fischer, and T. Pfeifer, Phys. Rev. Lett.<sup>409</sup>
   <sup>369</sup> **112**, 103001 (2014).
- [12] A. Blättermann, C. Ott, A. Kaldun, T. Ding, V. Stoss, 411
   M. Laux, M. Rebholz, and T. Pfeifer, Opt. Lett. 40, 412
   3464 (2015). 413
- <sup>373</sup> [13] A. Fleischer, O. Kfir, T. Diskin, P. Sidorenko, and O. Co-414
   <sup>374</sup> hen, Nature Photonics 8, 543 (2014).
- [14] O. Kfir, P. Grychtol, E. Turgut, R. Knut, D. Zusin,416
  D. Popmintchev, T. Popmintchev, H. Nembach, J. M.417
  Shaw, A. Fleischer, H. Kapteyn, M. Murnane, and418
  O. Cohen, Nature Photonics 9, 99 (2015). 419
- [15] N. B. Delone and V. P. Krainov, Physics-Uspekhi 42, 669420
   (1999). 421

- [16] C. Ott, A. Kaldun, P. Raith, K. Meyer, M. Laux, J. Evers, C. H. Keitel, C. H. Greene, and T. Pfeifer, Science 340, 716 (2013).
- [17] H. Mashiko, T. Yamaguchi, K. Oguri, A. Suda, and H. Gotoh, Nature Communications 5 (2014), 10.1038/ncomms6599.
- [18] W. C. Martin, Physical Review A **36**, 3575 (1987).
- [19] R. G. Brewer and R. L. Shoemaker, Physical Review A 6, 2001 (1972).
- [20] F. A. Hopf, R. F. Shea, and M. O. Scully, Physical Review A 7, 2105 (1973).
- [21] A. McPherson, G. Gibson, H. Jara, U. Johann, T. S. Luk, I. A. McIntyre, K. Boyer, and C. K. Rhodes, JOSA B 4, 595 (1987).
- [22] M. Ferray, A. L'Huillier, X. F. Li, L. A. Lompre, G. Mainfray, and C. Manus, Journal of Physics B: Atomic, Molecular and Optical Physics 21, L31 (1988).
- [23] K. J. Schafer, B. Yang, L. F. DiMauro, and K. C. Kulander, Physical Review Letters 70, 1599 (1993).
- [24] P. B. Corkum, Physical Review Letters 71, 1994 (1993).
- [25] M. Lewenstein, P. Balcou, M. Y. Ivanov, A. LHuillier, and P. B. Corkum, Physical Review A 49, 2117 (1994).
- [26] W. M. Wood, C. W. Siders, and M. C. Downer, Physical Review Letters 67, 3523 (1991).
- [27] U. van Bürck, Hyperfine Interactions 123/124, 483 (1999).
- [28] C.-T. Liao, A. Sandhu, S. Camp, K. J. Schafer, and M. B. Gaarde, Phys. Rev. Lett. **114**, 143002 (2015).
- [29] M. B. Gaarde, C. Buth, J. L. Tate, and K. J. Schafer, Physical Review A 83, 013419 (2011).
- [30] S. Hasegawa, H. Ito, H. Toyoda, and Y. Hayasaki, Optics Express 24, 18513 (2016).
- [31] B. sun, P. S. Salter, C. Roider, A. Jesacher, J. Strauss, J. Heberle, M. Schmidt, and M. J. Booth, Light: Science and Applications 7, 17117 (2018).
- [32] M. Fushitani, C.-N. Liu, A. Matsuda, T. Endo, Y. Toida, M. Nagasono, T. Togashi, M. Yabashi, T. Ishikawa, Y. Hikosaka, T. Morishita, and A. Hishikawa, Nature Photonics **10**, 102 (2016).
- [33] M. Wu, S. Chen, K. J. Schafer, and M. B. Gaarde, Physical Review A 87, 013828 (2013).