MINIMAL THEORY OF ULTRASONIC ATTENUATION IN SUPERCONDUCTORS*

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Abstract

The conventional formula for ultrasonic attenuation of longitudinal waves in superconductors is usually derived from the coherence factors of the BCS theory. It is noted that this formula has in actuality a much more general foundation. With an alternative deformation potential approach, the detailed properties of the superconductor do not enter and the result depends only upon the minimum energy in the spectrum of the elementary excitations.

SHORTLY after the development of the theory of superconductivity by Bardeen, Cooper and Hieffler [1] it was applied with considerable success to the interpretation of the decrease in the attenuation of longitudinal ultrasound below the transition temperature of superconductors. The application of the BCS theory makes use of the following combination of the coherence factors (using standard notation with the unprimed and primed quantities, referring to the initial and final states of a quasi-particle, respectively):

$$\begin{aligned} uu' - vv' &\approx u^2 - v^2 \\ &= \frac{1}{2} \left(1 + \frac{\epsilon}{E} \right) - \frac{1}{2} \left(1 - \frac{\epsilon}{E} \right) \\ &= \frac{\epsilon}{E} \end{aligned} \tag{1}$$

This ratio of the normal state single-electron energy to the corresponding quasi-particle energy must be included as a modification of the matrix element for the coupling of the sound quanta to the elementary excitations in the superconductor. The result of including this factor is to yield a formula which does exhibit the observed drop in the attenuation below the superconducting transition temperature. Although the decrease at temperatures well below the transition temperature follows mainly from the depletion of the quasi-particles by the Boltzmann factor, the suppression of the absorption of the sound quanta by the effect of equation (1) on the strength of the basic interaction plays an especially essential role just below the transition temperature. It is therefore often stated that this success constitutes an important verification of the details of the BCS theory and that it is particularly a crucial test of the coherence

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factors which are involved in equation (1). The purpose of this brief note is to point out that such is not at all the case. We sketch below an alternative derivation of equation (1) which makes no use of the BCS coherence factors.

Because of the very long wavelength of the sound waves in question it is permitted to use the deformation potential approach. The spirit of the deformation potential approach is that the presence of the elastic crystal strain associated with the passage of the sound wave through a certain portion of the metal results in a small shift in the single electron energy, say $d\epsilon$. Now in the superconducting state the actual quasi-particle energy has a functional dependence upon ϵ , usually taken to be of the simple BCS form

$$E = (\epsilon^2 + \Delta^2)^{1/2}, \quad (2)$$

where Δ is the energy gap. Therefore in order to find the effective deformation potential coupling to the quasi-particles in the superconductor we must form the derivative

$$\frac{dE}{d\epsilon} = \frac{\epsilon}{E}, \quad (3)$$

which is identical to the coupling strength expressed above in equation (1). Thus we see that quite general arguments suffice to yield the modification in coupling strength when passing from the normal to the superconducting state.

Although in obtaining this result we have explicitly employed the BCS formula of equation (2), it is by no means necessary to do so. This is seen by leaving the coupling in its general function form $dE/d\epsilon$, which can be regarded heuristically as the difference between the fractional extents to which the quasi-particle is particle-like and hole-like. (The case of zero coupling at the minimum of the spectrum $dE/d\epsilon = 0$ corresponds to equally strong particle-like and hole-like character). This coupling strength in the superconducting state enters as the square in the time-dependent perturbation formula for the sound attenuation and completely cancels the factor $d\epsilon/dE$ in the initial and final density of states per unit energy. Thus there only remain the thermal Fermi occupation functions which contribute to the power absorption the factor

$$\begin{aligned} f(1 - f') - f'(1 - f) &= f - f' \\ &= -\hbar\omega \frac{df}{dE}. \end{aligned} \quad (4)$$

$\hbar\omega$ is the energy of the sound quantum being absorbed by the gas of quasi-particles. Including a factor of two (for the two branches of the quasi-particle spectrum), and integrating over all quasi-particles available for transition in the superconductor we obtain the general result for the ratio of the attenuation in the superconducting state to that in the normal state,

$$\begin{aligned} -2 \int_{E_{\min}}^{\infty} \frac{df}{dE} dE &= 2 f(E_{\min}) \\ &= 2(1 + e^{E_{\min}/kT})^{-1}. \end{aligned} \quad (5)$$

Thus it is seen that the theoretical formula for the rate of attenuation of longitudinal ultrasound in a superconductor follows from very general arguments and is not in actuality a test of

the detailed coherence features of the BCS theory.

Reference

J. BARDEEN, L.N. COOPER and J.R. SCHRIEFFER, *Phys. Rev.* **108**, 475 (1957).