PLASMON EXCITATIONS IN NUCLEAR MAGNETIC RESONANCE*

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Abstract

A theory is presented to account for the recently observed anomalous peak in the nuclear magnetic relaxation rate ($1/T_1$) in *n*-doped InSb. The magnitude of the magnetic field at the peak has been found to satisfy the equation $\omega_s = \omega_p$, where ω_s and ω_p is the spin-flip energy and plasma frequency of the electrons, respectively. Furthermore, the peak occurs at a field such that only the spin-up, lowest Landau level is occupied (at $T = 0^\circ\text{K}$). Our theory discusses the anomalous peak as being due to plasmon absorption and emission by electrons as an intermediate step in the mutual electron-nucleus spin-flip process and predicts a logarithmic singularity in ($1/T_1$) at $\omega_s = \omega_p$.

1. Introduction

RECENTLY, oscillations in the nuclear spin-lattice relaxation rate ($1/T_1$) of In^{115} , Sb^{121} , and Sb^{123} as a function of magnetic field have been observed in *n*-type InSb at low temperatures [1,2]. Electron concentrations were from $10^{14}/\text{cm}^3$ to $10^{16}/\text{cm}^3$. The low field oscillations are directly attributable to the de Haas-van Alphen (henceforth dHvA)-like oscillations in the electron density of states as a function of the field. The last such peak corresponds to the Fermi energy passing through the last spin-down Landau level, leaving only the $n = 0$, spin-up level occupied at $T = 0^\circ\text{K}$. At higher fields there occurs still another peak at a field $B = B_A$ (labeled "Type A" by Bridges) [3] which has the following characteristics:

$$(a) \quad |g| \mu_B B_A \approx \hbar \omega_p,$$

where g is the spin gyromagnetic ratio of the carrier electrons ($g \approx -48$ for InSb), μ_B is the Bohr magneton, and ω_p is the plasma frequency. If we denote by e , n , and m the electron charge, density, and effective mass ($m \approx 0.0137$ electron mass for InSb) respectively and by K the static dielectric constant ($K \approx 16$ for InSb) [4], then $\omega_p^2 = 4\pi n e^2 / K m$.

$$(b) \quad B_A \text{ and } (1/T_1)|_{B=B_A} \text{ are slightly temperature dependent.}$$

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(c) $(1/T_1)|_{B=B_A}$ is the same for Sb^{121} and Sb^{123} .

(d) While the low field, dHvA-like peaks in $(1/T_1)$ are coincidental with peaks in the resistivity, the Type A peaks are absent in the resistivity [5].

George Feher has suggested that plasmons are playing a dominant role in producing the "Type-A" peaks. The theory which we will present below elaborates this suggestion. We first recall that in a magnetic field, the electron energy spectrum is split up into an infinite set of Landau levels, with energies:

$$\epsilon_n(p_z) = \hbar \omega_c (n + \frac{1}{2}) + \hbar^2 p_z^2 / 2m, \quad n = 0, 1, 2, \dots \quad (1)$$

where $\omega_c \equiv eB/mc$ is the cyclotron frequency. If we take into account the electron spin, each Landau level (originally doubly-degenerate) is split in two, with the resulting energy spectrum

$$\epsilon_{n\uparrow}(p_z) = \hbar \omega_c (n + \frac{1}{2}) + \hbar^2 p_z^2 / 2m - \hbar \omega_s / 2 \quad (2)$$

$$\epsilon_{n\downarrow}(p_z) = \hbar \omega_c (n + \frac{1}{2}) + \hbar^2 p_z^2 / 2m + \hbar \omega_s / 2,$$

where $\hbar \omega_s \equiv |g| \mu_B B$. The energy spectrum is not discrete. However, the energy density of states has square-root singularities at the energies $\epsilon = \epsilon_{n\uparrow,\downarrow}(p_z = 0)$. Restricting ourselves now, for simplicity, to $T = 0^\circ\text{K}$, we find that as the field is raised, fewer Landau levels remain occupied. In the region of the field B_A , only the lowest level ($n = 0, \uparrow$) is occupied.

FIGURE 1

Electron density of states. μ is the Fermi energy. $g_{\uparrow,\downarrow}(\epsilon)$, the up/down density of states, respectively, are plotted as ordinates versus the energy ϵ .

We have also illustrated the two-step process whereby an electron absorbs a plasmon, gaining energy $\hbar \omega_p$, and then has its spin flipped via the electron spin-nuclear spin contact interaction V_{en} .

Figure 1 illustrates the electron density of states in this case, including only the two lowest levels: $(n = 0, \uparrow)$ and $(n = 0, \downarrow)$. Our theory can be understood in terms of the Bohm-Pines description of the high-density electron gas. Namely, the gas of electrons is described as a system of *quasi-particles* ("states" having particle-like characteristics) plus a system of collective excitations, *plasmons*. The former interact with each other via a screened Coulomb interaction V_{sc} and interact via the interaction Hamiltonian H_{ep} with the plasmons as any charged particle would interact with a charge density fluctuation. We will neglect V_{sc} since we are dealing with high frequencies, comparable to the Fermi energy. We will also neglect the plasmon-plasmon interaction. We thus concentrate on H_{ep} as being responsible for the "Type-A" peak. There are two lowest order processes whereby H_{ep} contributes to the nuclear spin-relaxation rate. The first, illustrated in Fig. 1, consists of an electron in the level $(n = 0, \uparrow)$ absorbing a plasmon, thus gaining energy $\hbar\omega_p$, and subsequently having its spin flipped by a nucleus (In or Sb) via the electron-nucleus Fermi contact interaction V_{en} into the level $(n = 0, \downarrow)$, with the negligible change in energy $\hbar\omega$. The other equally prevalent process involving plasmons is the spin flip of an electron from the level $(n = 0, \downarrow)$ into the level $(n = 0, \uparrow)$, with the subsequent emission of a plasmon. At $B = B_A$, both processes have probabilities proportional to the Boltzmann factor $\exp(-\hbar\omega_p/k_B T)$ in the temperature range of the experiments.

Our theory accounts for the peak at the plasma frequency. Why do we get a sharp peak in the presence of a non-discrete density states? The square-root singularities in the density of states, $g_{\uparrow,\downarrow}(\epsilon)$, lead, in the absence of damping, to a logarithmic singularity in $(1/T_1)$ at $\omega_s = \omega_p$. The singularity is weaker than usual but is nevertheless there. In the framework of the present work, we cannot hope to account quantitatively for the temperature dependence of A and of $(1/T_1)|_{B=B_A}$. We only take note that all the parameters (e.g. m , g) are temperature dependent. Since the background and tails of others peaks were not subtracted from $(1/T_1)|_{B=B_A}$, we cannot at this time attribute any significance to the equality of $(1/T_1)|_{B=B_A}$ for Sb¹²¹ and Sb¹²³. Finally, the absence of a "Type-A" peak in the resistivity is easily understood: The resistivity need involve only no-spin-flip processes which can always take place at and around the Fermi level. Plasmon processes, with small Boltzmann factors, would thus be unobservable. However, since $(1/T_1)$ necessarily involves a spin-flip process, the non-plasmon, direct transition and the indirect (via plasmons) processes have Boltzmann factors of comparable magnitude.

In § 2, we formulate the basic problem in mathematical terms, relating the transition rate to an auto-correlation function and subsequently to the analytic continuation of a thermal Green's function. The Green's function method, though seemingly complicated, is, nevertheless, a straightforward way of automatically taking into account the interference terms and appropriate Fermi factors in the intermediate states. In § 3, we present the detailed calculation of the transition rate, discussing the manner by which plasmons enter the problem, first by using a direct electron-electron interaction approach and second by considering an electron-plasmon interaction approach. Lowest order diagrams involving plasmons (but neglecting finite lifetimes) reveal a logarithmic singularity in the transition rate at a field $\omega_s = \omega_p$. Finally, in § 4, we present a summary of our results and conclusions thereof.

2. Formulation of the Problem

We consider a simplified model of one nucleus with spin one-half embedded in an electron gas, which is electrically neutralised by a background of positive charge. The nuclear relaxation time T_1 is inversely proportionally to the transition rate W for the nucleus to flip its spin, and we assume that only the electron-nucleus, Fermi contact interaction [6], V_{en} , is operative.

To the lowest order in V_{en} the transition rate is given by the "golden rule" (with $\hbar = 1$)

$$W \equiv W(\downarrow \rightarrow \uparrow) = 2\pi \overline{\sum_I} \sum_F |\langle \uparrow; F | V_{en} | I; \downarrow \rangle|^2 \delta(E_I - E_F + \omega), \quad (3)$$

where only nuclear "down" to "up" transitions are accounted for. In equation (3), I and F denote the initial and final states of the electrons with energies E_I and E_F respectively and ω is the nuclear spin-flip energy. The $\overline{\sum_I}$ is the thermal average over initial states, while $\sum_F$ runs over all final states. Remark that the states $|I\rangle$ and $|F\rangle$ are those of the complete interacting (via Coulomb interaction) many electron system in an external constant magnetic field. It is convenient to rewrite equation (3) as a time correlation function of the electron system, that is

$$W = A \int_{-\infty}^{+\infty} dt e^{i\omega_n t} \langle\langle S^+(t) S(0) \rangle\rangle, \quad (4)$$

where $A = (\frac{4\pi}{3} g \mu_B \gamma_n)^2$ (with γ_n - the nuclear gyromagnetic ratio). The electron spin-flip operator is

$$S(0) = \sum_i \delta(\mathbf{r}_i - \mathbf{R}) s^i, \quad (5)$$

and

$$S(t) = e^{iHt} S(0) e^{-iHt} \quad (6)$$

in the Heisenberg representation. The expectation value

$$\langle\langle \dots \rangle\rangle = Tr \left\{ e^{\beta(\Omega - H + \mu N)} \dots \right\} \quad (7)$$

is the grand-canonical one, with $\beta = 1/k_B T$ - the inverse temperature (k_B is the Boltzmann constant), Ω is the thermodynamic potential, H is the many-electron Hamiltonian, μ is the chemical potential and N is the electron number operator. In equation (5), the summation runs over all the electron coordinates: $\mathbf{r}_i$ is the position of the i -th electron and s^i is the spin lowering operator of the i -th electron; the nucleus position $\mathbf{R}$ will be taken as the origin of our coordinate system ($\mathbf{R} = 0$).

Our main problem is the evaluation of W from equation (4). Since we are concerned with the effects of plasma excitations, which are of a many body nature, i.e. require perturbations in the electron-electron interaction to all orders, we choose the temperature dependent Green's function technique [7] to attack the problem. Remark that this complicated approach is not needed for the calculation of the dHvA type oscillations.

We define an appropriate Green's function for our problem

$$G(u) = (-i)^2 \langle\langle T S^+(u) S(0) \rangle\rangle; \quad -\beta < u < \beta, \quad (8)$$

where T is the Dyson ordering operator, and

$$S(u) = e^{uH} S(0) e^{-uH}. \quad (9)$$

We introduce the "Fourier Transform" with respect to u

$$G_v = \frac{1}{2} \int_{-\beta}^{+\beta} du e^{x_v u} G(u) \quad (10)$$

$$x_v = i 2\pi v/\beta \quad v = 0, \pm 1, \pm 2, \dots,$$

which can be written as

$$G_v = - \sum_{nm} e^{\beta(\Omega - E_n + \mu N_n)} | \langle m | S | n \rangle |^2 \frac{e^{\beta(E_n - E_m)} - 1}{E_n - E_m + 2\pi i v/\beta}, \quad (11)$$

where $|n\rangle$ and $|m\rangle$ represent the eigenstates of the many-electron Hamiltonian and the number operator:

$$H |n\rangle = E_n |n\rangle, \quad N |n\rangle = N_n |n\rangle. \quad (12)$$

Let that in equation (11) $N_m = N_n$, since S does not change the particle number. We now introduce a spectral function

$$\Phi(u) = 2\pi \sum_{nm} e^{\beta(\Omega - E_n + \mu N_n)} | \langle m | S | n \rangle |^2 \delta(E_n - E_m + \omega) \quad (13)$$

and an auxiliary function,

$$M(z) = \frac{1}{2\pi} \int d\omega' \frac{e^{-\beta\omega'} - 1}{\omega' - z} \Phi(\omega'). \quad (14)$$

The motivation for the latter definitions are, that on the one hand $M(z)$ is the analytical continuation of G_v (equation (11)), and, on the other hand, the transition rate, equation (4), is

$$W = A \Phi(\omega). \quad (15)$$

These two are related via

$$\Phi(\omega) = -i \frac{M^+(\omega) - M^-(\omega)}{e^{-\beta\omega} - 1} \quad (16)$$

where

$$M^\pm(\omega) = \lim_{\eta \rightarrow +0} M(z \rightarrow \omega \pm i\eta). \quad (17)$$

Thus, our program is as follows: we start with a diagrammatic representation of G_v , evaluate it in terms of x_v , continue the resulting expressions analytically (i.e. replace x_v by the complex number z), and finally we make use of equation (16) to obtain the transition probability W .

3. The Transition Rate

We find it convenient to calculate the Green's function, equation (8), in the presence of a

constant external magnetic field, in terms of the second quantization operators of the Landau states. Denote by $a_{\alpha, \sigma}$ ($a_{\alpha, \sigma}^+$) the annihilation (creation) operator of the Landau one electron state $|\alpha; \sigma\rangle$, which is the eigenfunction of the Schrodinger equation

$$\left\{ \frac{1}{2m} (\mathbf{P} - \frac{e}{c} \mathbf{A})^2 - \frac{\omega_s}{2} \sigma \right\} |\alpha; \sigma\rangle = \epsilon_{\alpha\sigma} |\alpha; \sigma\rangle. \quad (18)$$

In equation (18), the vector potential is given by $\mathbf{A} \equiv \{0, Bx, 0\}$, the electron spin-flip energy is ω_s and the energy values are (cf. equation (2))

$$\epsilon_{\alpha\sigma} = \omega_c \left(n + \frac{1}{2}\right) + p^2/2m - \omega_s \sigma/2, \quad (19)$$

where ω_c is the cyclotron frequency, p is the momentum component along the magnetic field and $\sigma = +1$ (-1) for up (down) spin. The eigenstates of equation (18) are

$$\varphi_{\alpha; \sigma}(r) \equiv |\alpha; \sigma\rangle \equiv |n, p, q; \sigma\rangle = \frac{1}{2\pi} e^{ipz + iqy} u_n(x - q/m\omega_c) |\sigma\rangle, \quad (20)$$

with u_n being harmonic oscillator eigenfunctions. The electron spin-flip operator, equation (5) is written as

$$\begin{aligned} S &= \sum_{\alpha_1 \sigma_1} \sum_{\alpha_2 \sigma_2} \langle \alpha_1 \sigma_1 | \delta(r) \mathbf{s} | \alpha_2 \sigma_2 \rangle a_{\alpha_1 \sigma_1}^+ a_{\alpha_2 \sigma_2} \\ &= \sum_{\alpha_1 \alpha_2} \langle \alpha_1 | \delta(r) | \alpha_2 \rangle a_{\alpha_1 \downarrow}^+ a_{\alpha_2 \uparrow}; \end{aligned} \quad (21)$$

and thus, our Green's function is

$$G(u) = (-i)^2 \sum_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \langle \alpha_1 | \delta(r) | \alpha_2 \rangle \langle \alpha_3 | \delta(r) | \alpha_4 \rangle \langle\langle T \{ a_{\alpha_1 \uparrow}^+(u) a_{\alpha_2 \downarrow}(u) a_{\alpha_3 \downarrow}^+ a_{\alpha_4 \uparrow} \} \rangle\rangle$$

Equation (22) is represented schematically in Fig. 2.

FIGURE 2

Schematic illustration of the Green's function $G(u)$ given by equation (22).

a) Lowest order diagram

It is instructive to evaluate first the lowest order diagram of Fig. 2, when no electron-electron interaction is considered. This diagram, in its "Fourier transform" representation, is depicted in Fig. 3(a). Here a solid line represents the propagator, $g_{\alpha\sigma}$, of a Landau electron with a given spin, i.e.

$$g_{\alpha\sigma}(\zeta_l) = [\zeta_l - \epsilon_{\alpha\sigma}]^{-1} . \quad (23)$$

$$\zeta_l = \mu + i\pi(2l + 1)/\beta \quad l = 0, \pm 1, \pm 2, \dots ,$$

where μ is the chemical potential, and the energy, $\epsilon_{\alpha\sigma}$, is given by equation (19). The dotted line represents the interaction with the nucleus and carries the "energy" $\chi_v = 2\pi i v/\beta$, and the circled vertex stands for the matrix element $\langle \alpha_1 | \delta(r) | \alpha_2 \rangle$. The contribution of this diagram to our Green's function, G^a , is

$$G_v^a = \sum_{\alpha_1 \alpha_2} |\langle \alpha_1 | \delta(r) | \alpha_2 \rangle|^2 \frac{1}{\beta} \sum_l \frac{1}{\zeta_l - \epsilon_{\alpha_1 \uparrow}} \frac{1}{\zeta_l + \chi_v - \epsilon_{\alpha_2 \downarrow}} . \quad (24)$$

The sum over l can be transformed into an integral:

$$\frac{1}{\beta} \sum_l \rightarrow \frac{1}{2\pi i} \int_C d\zeta f(\zeta) \quad (25)$$

where the contour C encloses only the poles of the Fermi function, $f(\zeta) = [1 + e^{\beta(\zeta - \mu)}]^{-1}$. Deforming the contour leads to

$$G_v^a = \sum_{\alpha_1 \alpha_2} |\langle \alpha_1 | \delta(r) | \alpha_2 \rangle|^2 (f_1 - f_2) \frac{1}{\epsilon_{1\uparrow} - \epsilon_{2\downarrow} + \chi_v} . \quad (26)$$

We make use of the value of the matrix element

$$\langle \alpha_1 | \delta(r) | \alpha_2 \rangle = (2\pi)^{-2} u_{n_1}(-q_1/m\omega_c) u_{n_2}(-q_2/m\omega_c) ,$$

sum over q_1 and q_2 (making use of the orthonormality of the functions u_{n_1}), and finally follow the prescription outlined in Section 2: we analytically continue G_v^a to the complex plane ($\chi_v \rightarrow z$). Upon utilizing equation (16) and taking the limit as $\omega \rightarrow 0$ (since $\beta\omega$ is negligible), we obtain

$$W^a = - \frac{2\pi A}{\beta} \int d\epsilon g_{\uparrow}(\epsilon) g_{\downarrow}(\epsilon) \partial f / \partial \epsilon . \quad (27)$$

This result exhibits the dHvA-like oscillations of $(1/T_1)$ and agrees with that given by Bridges [2].

b) Plasmon contribution

We now consider the diagrams of Fig. 3(b) which account for the lowest order processes involving plasmons. The wavy lines represent the plasmon lines, carried by the propagator

(a)

(b)

FIGURE 3

Diagrams in the "Fourier transform" representation.

- (a) The lowest order diagram (without electron-electron interactions). This diagram accounts for the standard result for the nuclear relaxation rate, inclusive of dHvA-like oscillations.
- (b) The leading diagrams which account for plasma excitations.

$$D_k(\omega_m) = v_k \frac{\omega_m^2}{\omega_m^2 - \omega_p^2} \quad (28)$$

$$\omega_m = 2\pi i m / \beta, \quad m = 0, \pm 1, \pm 2, \dots,$$

where ω_p is the plasma frequency (cf. Section 1) and $v_k = 4\pi e^2 / K k^2$. Associated with the electron-plasmon vertex is a factor

$$\langle \alpha_n | e^{i\mathbf{k} \cdot \mathbf{r}} | \alpha_m \rangle,$$

where $|\alpha_m\rangle$ and $|\alpha_n\rangle$ are the incoming and outgoing electron states respectively and k is the momentum of the outgoing plasmon (or negative of the momentum of the incoming plasmon). We can look at $D_k(\omega_m)$ in two different ways:

(i) First, it can be derived from the Coulomb potential divided by the frequency and wave-number dependent dielectric function, i.e.

$$D_k(\omega_m) = v_k / \epsilon_k(\omega_m), \quad (29)$$

where

$$\epsilon_k(\omega_m) \equiv 1 - v_k \sum_{\alpha_1 \alpha_2} |\langle \alpha_1 | e^{i\mathbf{k} \cdot \mathbf{r}} | \alpha_2 \rangle|^2 \frac{(f_1 - f_2)}{\epsilon_1 - \epsilon_2 + \omega_m} \quad (30)$$

the dielectric function in the random-phase approximation, and it is to be noted that v_k includes the static dielectric constant of the host medium. In the high frequency limit - high compared to the characteristic carrier-electron frequencies but small compared to the characteristic frequencies of the host InSb - we approximate

$$\epsilon_k(\omega_m) \rightarrow 1 - \omega_p^2 / \omega_m^2, \quad (31)$$

which leads to equation (28). Equation (31) is a good approximation for $k \rightarrow 0$, unless $k \perp B$.

(ii) Second it appears as the plasmon propagator if we *ab initio* take as our effective Hamiltonian

$$H = H_e + H_p + H_{ep}, \quad (32)$$

where H_e and H_p are the quasi-free electron and free plasmon Hamiltonians, respectively, i.e.:

$$H_e = \sum_{\alpha, \sigma} \epsilon_{\alpha\sigma} q_{\alpha\sigma}^+ q_{\alpha\sigma} \quad (33)$$

$$H_p = \sum_k \omega_p b_k^+ b_k,$$

with b_k and b_k^+ the annihilation and creation operators for the plasmon respectively. H_{ep} is the electron-plasmon interaction which we take in the high frequency limit to be:

$$H_{ep} = -i \sum_{\substack{\alpha_1 \alpha_2 \\ \sigma_1 = \sigma_2}} \sum_{\mathbf{k}} \sqrt{\frac{v_k \omega_p}{2}} \langle \alpha_1 | e^{i\mathbf{k} \cdot \mathbf{r}} | \alpha_2 \rangle a_{\alpha_1 \sigma_1}^+ a_{\alpha_2 \sigma_2} (b_{\mathbf{k}} - b_{-\mathbf{k}}^+). \quad (3)$$

The Hamiltonian of equation (32) should be valid in the high frequency region with which we are concerned.

We proceed with the first diagram of Fig. 3(b); its contribution to G is

$$G_V^b = \sum_{\alpha_1 \alpha_2 \alpha_4} \sum_{\mathbf{k}} |\langle \alpha_1 | \delta(r) | \alpha_2 \rangle|^2 |\langle \alpha_4 | e^{i\mathbf{k} \cdot \mathbf{r}} | \alpha_2 \rangle|^2 \frac{1}{\beta} \sum_m D_k(\omega_m) L_m^b, \quad (3)$$

where

$$L_m^b = \frac{1}{2\pi i} \int_C d\zeta f(\zeta) \frac{1}{\zeta - \epsilon_{\alpha_1 \uparrow} - x_v} \frac{1}{(\zeta - \epsilon_{\alpha_2 \downarrow})^2} \frac{1}{\zeta - \epsilon_{\alpha_4 \downarrow} - \omega_m}.$$

Deforming the contour to pick up the poles of the Green's functions, we obtain, after some algebra:

$$\begin{aligned} L_m^b = & (f_{1\uparrow} - f_{4\downarrow}) \frac{1}{(\epsilon_{\alpha_1 \uparrow} - \epsilon_{\alpha_2 \downarrow} + x_v)^2} \frac{1}{\epsilon_{\alpha_1 \uparrow} - \epsilon_{\alpha_4 \downarrow} - \omega_m + x_v} \\ & + (f_{4\downarrow} - f_{2\downarrow}) \frac{1}{\epsilon_{\alpha_1 \uparrow} - \epsilon_{\alpha_2 \downarrow} + x_v} \frac{1}{\epsilon_{\alpha_2 \downarrow} - \epsilon_{\alpha_4 \downarrow} - \omega_m} \\ & \cdot \left[\frac{1}{\epsilon_{\alpha_1 \uparrow} - \epsilon_{\alpha_2 \downarrow} + x_v} - \frac{1}{\epsilon_{\alpha_2 \downarrow} - \epsilon_{\alpha_4 \downarrow} - \omega_m} \right] \\ & - \frac{\partial f_2}{\partial \epsilon_{\alpha_2 \downarrow}} \frac{1}{\epsilon_{\alpha_1 \uparrow} - \epsilon_{\alpha_2 \downarrow} + x_v} \frac{1}{\epsilon_{\alpha_2 \downarrow} - \epsilon_{\alpha_4 \downarrow} - \omega_m}. \end{aligned} \quad (36)$$

The only important term in L_m^b is the first one [8]. To carry out the sum over m , we make use of the following identities:

$$I(x) \equiv \frac{1}{\beta} \sum_m \frac{1}{\omega_m - X} = -\frac{1}{2} \coth(\beta X/2), \quad (37)$$

from which we easily obtain

$$F(X) \equiv \frac{1}{v_k} \frac{1}{\beta} \sum_m D_k(\omega_m) \frac{1}{\omega_m - X} = \frac{X/2}{\omega_p^2 - X^2} \left\{ X \coth\left(\frac{\beta X}{2}\right) - \omega_p \coth\left(\frac{\beta \omega_p}{2}\right) \right\}. \quad (38)$$

We will retain, from now on, only the $n = 0$ states, since all others are very far removed from the lowest, occupied states. Next, we perform our sum (integral) over q_1 , q_2 , and q_4 :

$$\sum_{q_1 q_2 q_4} \left| \langle \alpha_1 | \delta(\mathbf{r}) | \alpha_2 \rangle \right|^2 \left| \langle \alpha_4 | e^{i\mathbf{k} \cdot \mathbf{r}} | \alpha_2 \rangle \right|^2 = \delta_{p_4, p_2 - k_z} \left(\frac{a}{2\pi} \right)^4 e^{-k_1^2/2a^2} \quad (39)$$

where $k_1^2 \equiv k_x^2 + k_y^2$ and $a^2 \equiv m\omega_c$.

Equation (35) (retaining only the dominant terms) then takes the form

$$M_v = \sum_k v_k \left(\frac{a}{2\pi} \right)^4 e^{-k_1^2/2a^2} \sum_{p_1 p_2 p_4} \left(\frac{1}{\epsilon_{2\downarrow} - \epsilon_{1\uparrow} + \chi_v} \right)^2 (f_1 - f_4) \delta_{p_4, p_2 - k_z} F(\epsilon_{1\uparrow} - \epsilon_{4\downarrow} + \chi_v). \quad (40)$$

Before undertaking the analytical continuation of equation (40) we must set

$$\coth \frac{\beta}{2} (\epsilon_{1\uparrow} - \epsilon_{4\downarrow} + \chi_v) = \frac{\beta}{2} (\epsilon_{1\uparrow} - \epsilon_{4\downarrow}). \quad (41)$$

In this way the ambiguity in the analytical continuation is removed, and thus we simply set $v \rightarrow z$ everywhere except in the coth.

The other diagrams in Fig. 3(b) are handled in a similar way. The sole difference being that in the last diagram the sum over the q 's, corresponding to equation (39), leads to

$$\delta_{p_3 - p_1 - k_z} \delta_{p_4, p_2 + k_z} \left(\frac{a}{2\pi} \right)^4 e^{-k_1^2/a^2}.$$

Collecting the dominant contributions of the diagrams of Fig. 3(b), we obtain, after the integration over k ,

$$M(z) = \frac{e^2 a^5 \sqrt{\pi}}{K(2\pi)^4} (\sqrt{2} - 1) \sum_{p_1 p_2} (f_1 - f_2) \left(\frac{1}{\epsilon_{1\uparrow} - \epsilon_{2\downarrow} + z} \right)^2 F(\epsilon_{1\uparrow} - \epsilon_{2\downarrow} + z). \quad (42)$$

In deriving equation (42), we have set, for convenience $k_z = 0$ in the energy differences, since the plasmon-electron momentum transfer along the direction of the field is much smaller than the average p of an electron [8]. Notice that the same could not be done for the transverse component of the momentum transfer k .

The discontinuity of $M(z)$ across the real axis leads, via equation (16), to

$$\Phi(\omega) \sim \frac{\pi}{\omega_p} \frac{1}{e^{-\beta\omega} - 1} \sum_{p_1 p_2} (f_{1\uparrow} - f_{2\downarrow}) \left[\coth \frac{\beta\Delta}{2} - \frac{\omega_p}{\Delta + \omega} \coth \frac{\beta\omega_p}{2} \right] \cdot \left[\delta(\Delta - \omega_p + \omega) - \delta(\Delta + \omega_p + \omega) \right] \quad (43)$$

with $\Delta = \epsilon_{1\uparrow} - \epsilon_{2\downarrow}$. Since the nuclear spin-flip energy, ω , is much smaller than the temperature, i.e. $\beta\omega \ll 1$, equation (43) reduces to

$$\Phi \sim \frac{\pi}{\omega_p} \frac{1}{\sinh^2 \frac{\beta\omega_p}{2}} \sum_{p_1 p_2} [f_{1\uparrow} - f_{2\downarrow}] \delta(\epsilon_{1\uparrow} - \epsilon_{2\downarrow} + \omega_p), \quad (44)$$

where the plasmon emission term [i.e. the term with $\delta(\epsilon_{1\uparrow} - \epsilon_{2\downarrow} - \omega_p)$] is left out in spirit of our approximations throughout this work. Thus, we obtain (for $\beta\omega_p \gg 1$)

$$\Phi \sim e^{-\beta\omega_p} \int dp_1 dp_2 (f_{1\uparrow} - f_{2\downarrow}) \delta(\epsilon_{1\uparrow} - \epsilon_{2\downarrow} + \omega_p), \quad (4)$$

or

$$\Phi \sim 2m e^{-\beta\omega_p} \int_{(0, \omega_s - \omega_p)} d\epsilon \frac{f(\epsilon) - f(\epsilon + \omega_p)}{\sqrt{\epsilon} \sqrt{\epsilon + \omega_p - \omega_s}}. \quad (4)$$

In this equation, the lower limit of the integral is the larger of the two $(0, \omega_s - \omega_p)$. Furthermore, all energies are measured from $(\omega_c - \omega_s)/2$. We are interested in values of ω_s close to i.e. $|\omega_s - \omega_p| \ll \mu$. Then we can neglect the Fermi function $f(\epsilon + \omega_p)$. We can also let $T \rightarrow 0$ in the integral. Our final result for the plasmon contribution to the transition rate W^b

$$W^b = \left(\frac{4\pi}{3} g \mu_B \gamma_n\right)^2 \frac{4\pi e^2}{K} \frac{(m\omega_c)^{5/2}}{(2\pi)^6} \frac{\sqrt{X}}{\omega_p} (\sqrt{2} - 1) \frac{m}{2} e^{-\beta\omega_p} \ln\left(\frac{4\mu}{|\omega_p - \omega_s|}\right), \quad (4)$$

where the chemical potential

$$\mu = 8\pi^6 n^2 / m^3 \omega_c^2 \quad (4)$$

is evaluated at absolute zero. The resonance at $\omega_s = \omega_p$ is apparent in the expression for W^b .

4. Summary and Conclusions

We have presented a simple theory to account for the anomalous peak ("Type-A" peak of Bridges) in the nuclear relaxation rate $(1/T_1)$. That the peak lies at a magnetic field $\omega_s = \omega_p$ has been shown to be due to an electron-plasmon resonance. The expression for W^b [equation (4)] reveals that the shape of the peak is quite complicated due to the dependence of W^b on the magnetic field via ω_c , μ , and ω_s . To obtain the detailed peak shape would necessitate including finite lifetime effects of both plasmons and electrons.

While W^b has the exponential damping factor $e^{-\beta\omega_p}$, the lowest order background contribution due to V_{en} has a factor $e^{-\beta(\omega_s - \mu)}$. The factor $e^{-\beta\mu}$ (relevant when $\omega_s = \omega_p$) may be small (≈ 0.01). However, provided the lifetime is long enough, the peak in W^b will be observable.

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At even higher fields, there is a peak (labeled "Type-B" by Bridges) which we will not discuss in this paper.

The static dielectric constant must be included in ω_p since the plasma frequency is much lower than the characteristic frequencies of the host material, InSb.

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This term is the only one having to do with plasmon absorption. The other terms contribute only to the self-energy of the electron. Moreover, all such terms vanish in the limit that the plasmon momentum component p_z in the direction of the magnetic field vanishes, and therefore are expected to be of order no greater than $\langle p_z^2 \rangle / p_F^2 \approx p_c^2 / p_F^2$ (where p_c is the inverse cyclotron radius and p_F is the Fermi momentum). Terms of such a small order of magnitude will be neglected throughout.

to be added in proof

The La Jolla experimental group has pointed out to us that our results have a much too strong temperature dependence which is not in accord with the experimental observations. Nonetheless the possibility of a resonance at $\omega_s \approx \omega_p$ is revealed in our work.