THE SYMMETRY BREAKING INTERACTION*†

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Abstract

The bootstrap idea is explored in terms of a non-linear equation projecting all hadron states out of one master field. The symmetries are assumed to derive from the non-strong interactions, the bootstrap's role being reduced to their regeneration as strong-interactions symmetries. The interaction responsible for the breaking of $SU(3)$ is reviewed and its non-bootstrap features are emphasized. Finally, some remarks are included with respect to an $R(8)$ extension of $SU(3)/Z(3)$ capable of yielding $\Delta S = -\Delta Q$ weak currents and the anomalous $K_2^0 \rightarrow 2\pi$ decay.

Introduction

THAT a title is something of an unexpected restriction on one's freedom of action has been noticed by many graduate students and by several members of Her Majesty's Government and of the loyal opposition. I have just gone through the same experience, having bound myself to the name of this lecture long ago in August. Unable to shed it properly, I have chosen a middle course and intend to comment on my original subject after devoting the first half of this talk to some more general features.

Interpreting the Bootstrap Idea

With about fifty firmly established hadrons, it is only natural that the term "elementary" should have been dropped from the name of our field of physics. The omega-minus experiment has shown that we cannot regard the short-lived resonances as less elementary than the stable or metastable particles; the multiplet structure implies their overall equivalence, unperturbed by the accidents of threshold and channel availability. Two answers have been supplied at this point: a choice of one minimal set, generally one or more triplets – or the bootstrap hypothesis. Considering that there is no experimental evidence for the existence of these triplets,

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and that even if they are found, it is not clear why they or any other particles should have this preferred role, it is rather appealing to explore the alternative approach. This is a theory in which no hadron is more elementary than the others; the strong interactions are regarded as the result of kinematical freedom in the S -matrix transitions, perhaps within the restrictions imposed by hadron symmetries.

We would like, at this point, to introduce an interpretation which may be regarded as a misinterpretation by the true bootstrappers, but seems to us to convey a somewhat more definite structure. We apologize for its crude and naive features; in fact, we have adopted this description because of its conceptual simplicity.

We regard strongly interacting matter as one self-interacting system, with the particles as its states. This system contains all allowed spins and unitary spins: octets with spins 0^- , $1/2^+$, 1^- , $3/2^-$, etc., decuplets with spin $3/2^+$ and so forth. It satisfies an equation of the type

$$(\square + j^2 \dots + k^2 \dots) \phi = 0$$

where j^2 , k^2 , etc., are Casimir operators, diagonalizing the system so as to project out the various constituents of ϕ .^{*} We thus have an expansion of ϕ into the various particle operators; these operators create the particle states when operating on the vacuum. One may hope that they will turn out to behave like our former fields when partaking in non-strong interactions, so that the expansion into $SU(3)$ and spin multiplets is then just a previously missing step before the usual expansions into momenta, number of particle representations, etc. As for the strong interactions themselves, we should be dealing with composite states, i.e., Regge trajectories; all parameters result from the equations, as we shall see. Indeed, to get back to the bootstrap, we now introduce a non-linear structure into the equation. This can be done for instance in the following way: if each particle is to be determined by any pair of particles capable of making it, within the symmetry restrictions, the term k^2 becomes $k^2(\bar{\phi} F \phi)$ with F_{bc}^a a set of coefficients relating the multiplets to each other and projecting out, for example, the baryon octet (b) and meson octet (c) as one set of constituents contributing to the making of the baryon octet (a) in the ϕ eigenfunction. The full bootstrap would have terms with 3, 4, etc., particle states; it would also deal with the couplings in the same spirit. Considering that particle multiplets are also characterized by parity, we note that the k^2 will have to emerge from the chiral symmetry, where parity has been digested and one can use true Lie group Casimir operators.

This, of course, reminds us of Heisenberg's non-linear theories; but whereas we do not know (at this stage) of a coupling whereby electrons could make themselves, we do have it in hadron matter. This is why we hope that this approach will at least have a conceptual advantage in the strong interactions, with their non-linear features.^{**} We suggest regarding the equations we wrote as an extremely simplified model of the true thing.

Generating the Symmetries

In this picture, we have introduced the symmetries *a priori*. True bootstrappers hope to get it as a result of their kinematics; we tend to doubt that this can be the true picture. The

^{*} They may also form a spin-unitary spin extended symmetry as suggested by Gursey and Radicati (*Phys. Rev. Letters* 13, 173 (1964)).

^{**} This is, of course, similar to the viewpoint of Nambu and Jona-Lasinio (*Phys. Rev.* 122, 345 (1961)).

following argument will illustrate our view. If these "extremist" views are right, we are faced with two problems:

1) To prove the emergence of the broken $SU(3)$ in the strong interactions, as a kinematical result.

2) To explain why certain particular directions in the $SU(3)$ algebra's space, at definite angles to the symmetry breaking axis, are projected into the weaker interactions, where they appear as electric charge and the weak currents. These, of course, now happen to be multiplied by some small number and to serve as the source term for non-strongly interacting fields.

To this we propose the following alternative:

1) The first problem is to explain these weaker interactions, ending up with an electromagnetic current and a weak current, etc.

2) Problem II then does not exist, since it has been solved in principle by our bootstrapist friends. For example, taking Cutkosky's [1] vector meson study, we note that he proved the emergence of a Lie group provided we have *a priori* electric charge conservation (he had to assume that a certain coupling is proportional to electricity). He did not get a definite group; if we now require another structure constant of the emerging algebra to be proportional to the almost-conserved weak vector current, and another one with strangeness, his machine yields $SU(3)$. Similarly, Gell-Mann [2] has generated the hadron algebra from observed weak and electric current matrix elements, through repeated equal time commutation (up to closure).

The Fifth Interaction

In this view, hadron matter displays the dynamical symmetry of $SU(3)$ (or of the larger chiral symmetry) as a result of the bootstrap mechanism, but only upon injection of some information by the weaker interactions. This has to be sufficient to fix the symmetry, or at least its generator algebra. The weaker interaction currents start out at certain angles to each other and widely different coupling scales; the bootstrap sees to it that the strong interactions display the same algebra, with all couplings now brought to one scale (in fact, 1 : 1) and particles appearing as representations of the group.

To achieve this result, we need one more *a priori* current, since the weak and the electric currents would close on $U(1) + SU(2)$. This is one reason to look for another set of well-defined current matrix elements; we have picked the eighth component, [3] since we do observe an effect linked to it - the mass spectrum. From $Q = (\sqrt{3} F_3 + F_8)/2$, $(aF_0 + bF_8)/(a^2 + b^2)$ and

$$W^\pm = \cos \theta (F_1 \mp iF_2) + \sin \theta (F_4 \mp iF_5)$$

we generate the whole $SU(3)$. Note that it has to be a vector current. If, for example, we take the gravitational matrix element, i.e., the Hamiltonian, its commutator with $W^\pm$ will give a different symmetry, since it involves a derivative and yields a current divergence instead of a current.

Cutkosky and Tarjanne, among others, have studied the effects of symmetry breaking in the bootstrap [4]. Provided they add strangeness conservation as an *a priori* requirement, they can get a self-consistent symmetry breaking, except that they can't make the breaking choose the F_8 direction. Considering that electromagnetism exists as a perturbation, one would almost expect particles to arrange themselves in U -spin multiplets instead of I -spin. If there is a stronger *a priori* perturbation in the F_8 direction, we understand the observed multiplet structure.

We now come to the problem of octet dominance in the symmetry breaking Hamiltonians. Cutkosky and Tarjanne noted that although the representation 27 of $SU(3)$ contains an $I = Y = 0$ component that could have therefore contributed to the mass operator, it does not yield self-consistent results. The system is stable against disturbances of the 27 type; it is unstable when perturbed by an 8-like effect, which tends to propagate and amplify itself. Dashen and Frautschi [5] have noted that this would explain the $|\Delta I| = 1/2$ rule in non-leptonic weak interactions and some results in electromagnetism. For the mass spectrum, this explains the success of the mass formula. If our fifth force is coupled to

$$aF_0 + bF_8$$

the mass will go like the square of this coupling, thus containing

$$1 + 8 + 27.$$

The larger the ratio of a to b , the less of the 27; but with octet-dominance produced by the related kinematics, we get the mass formula anyhow.

Beder, Dashen, and Frautschi [6] have made a thorough analysis of the experimental data bearing on the mass of the vector meson of the fifth force. Small masses seem to be excluded, and a two-pion mass seems to be a minimum. The coupling to F_8 has to be of the order of

$$g_x^2/4\pi \sim 0.3.$$

Both the ϕ and the ω mesons are eligible candidates; $\omega - \phi$ mixing would still result in any case. The experimental criteria identifying our meson in the purest sense are the ones that distinguish a Regge trajectory from a C.D.D. pole. The fifth force's meson is not on a Regge trajectory and would be identified through Levinson's theorem, for instance [7]. Unfortunately, these are extremely difficult criteria at the present stage, though perhaps a possibility of the future. Moreover, it is conceivable that neither ϕ nor ω are "it", and that helpful nature will provide us with a clearcut case of a new meson whose late appearance will have been due to the relative weakness of its coupling.

Beyond $SU(3)$

We now return to our original approach, in which we noted that it is the chiral symmetry that will determine the hadron states. As is well known, this is the symmetry we get when we consider the weak axial vector current as partially conserved (in the limit of vanishing baryon mass). Its algebra will contain a minimum of 8 vector (i.e., parity retaining) and 8 axial vector (parity changing) generators. In a recent study [8] with M. Gell-Mann, we found that $SU(3) \times SU(3)$ with these 16 currents [2, 9] is the only chiral symmetry satisfying the strongest universality requirement for the weak currents. The situation would be different, of course, if new types of weak transitions are observed, as could possibly occur if triplets are discovered. For a double-triplet parity retaining proto-symmetry like the $SU(3) \times SU(3)$ used by Schwinger [19] and Lee, Gursey, and Nauenberg [11], one gets an $[SU(3)]^4$ as pointed out by Nambu and Freund.

We would like to suggest an alternative outlook. If $\Delta Q = -\Delta S$ currents, however small, are observed to occur in leptonic decays, the generated algebra is $R(8)$. This is the orthogonal algebra operating on the 8-space of $SU(3)$ octets. The 28 anti-symmetric reduction matrices of the $SU(3)$ product

$$8 \times 8 = 1 + 8 + 27 + (8 + 10 + \overline{10})$$

are linearly independent and are indeed linear combinations of $R(8)$ generators; the F octet defines the $SU(3)$ content. We note that the vector currents would belong to $8 + 10 + \overline{10}$, including a $\Delta Q = -\Delta S$ generator [14]. In a recent study [15], we have shown that this could yield a weak current satisfying universality and including an out of phase term which could explain the $CP = -1$, K^0 experiment [12] as suggested by Wolfenstein [13]. The chiral symmetry would then apparently be $R(8) \times R(8)$, and the possible meson icosuplet discussed by B. Lee, Okubo, and Schecter [16] would fit into the $28 + 28$ vector and axial vector particles it predicts. Note that in this picture no triplets can occur, since $R(8)$ does not contain $SU(3)$, only $SU(3)/Z(3)$.

The fifth force coupling breaks our $R(8)$ in the right way. It is now in $(1 + 28)$ and the square of 28 contains an $SU(3)$ singlet which is a component of a large $R(8)$ representation. This term carves out $SU(3)/Z(3)$ from the larger $R(8)$, thus using the singlet in $(1 + 8)(1 + 8)$, with the octet term fulfilling its usual role of breaking $SU(3)$.

I would like to conclude this talk with an apologetic remark intended to explain my lack of enthusiasm for the most extreme bootstrap hopes. It took several hundred years for theory to produce a kinematical origin for the mass; after all, in Newton's theories, it was just a coefficient, a coupling, and a conserved quantity, accounting for the phenomenology as systematized by Kepler. We now see mass emerging out of the infinitesimal Lorentz generators in space-time; we have not yet reached this stage for electric charge, another such set of symmetry coefficients introduced 150 years ago. It seems to us we can be patient and live and use unitary spin in the same light; somehow, its emergence seems to require an inter-interaction outlook, perhaps again linked with some geometrical features of space-time.

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