Electromagnetic Impurity-Immunity Induced by Parity-Time Symmetry

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Impurities usually play an important role in modifying the bulk properties of electronic or electromagnetic materials. In this work, we demonstrate a way to break this discipline and realize the extraordinary physical property of impurity-immunity, which leads to perfect transmission irrespective of embedded impurities of almost any material and shape. This extraordinary property comes from the exceptional points of a pair of parity-time (*PT*)-symmetric metasurfaces sandwiching a slab of epsilon-near-zero medium. By systematically investigating the *PT*-symmetric metasurfaces sandwiching a slab of dielectrics or metamaterials, we obtain the two complementary solutions of exceptional points corresponding to perfect transmission. Interestingly, when the permittivity of the slab approaches zero, the two solutions of exceptional points coalesce into one exceptional point. At such a critical point, we find that the original “doping” effect of impurities in epsilon-near-zero media, proposed by Nader Engheta’s group very recently [I. Liberal *et al.*, *Science* **355**, 1058 (2017)], is significantly suppressed. Our work shows that exceptional points can be used to eliminate scattering of impurities in a bulk medium.

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Impurities usually play an important role in modifying the bulk properties of electronic or electromagnetic materials. Doping, i.e., intentionally introducing impurities into a pure material, is of vital importance in the development of the semiconductor industry. Similar phenomena have been observed in the field of electromagnetic materials. Effective medium theories [1–3] have been developed to predict the modified bulk properties of electromagnetic media embedded with impurities in different scenarios. Very recently, it has been observed that the bulk properties, as well as transmission behaviors of epsilon-near-zero (ENZ) media [4–22], are very sensitive to embedded impurities. Based on this effect, a “doping” method has been proposed to control the impedance of ENZ media [4].

Even though sensitivity on impurities is general and useful in many scenarios, the pursuit of impurity-immunity or antidoping, i.e., independence of impurities including nonlinear impurities [23,24], has always been a hot research topic in physics. This is mainly because impurity-immunity could lead to robust high transmission of energy against small disturbances, which is of vital importance in many applications. Recent investigation of topological insulators has provided an approach to realize impurity-immunity for surface waves, which are robust against impurities and defects on the surface. Tremendous effort has been devoted to realizing such intriguing impurity-immunity in the surface waves of topological insulators as well as their electromagnetic analogue [25–28]. However, the nature of topological insulators only supports edge states instead of bulk states. To the best of our knowledge, the extraordinary property of electromagnetic impurity-immunity for bulk states has not been realized yet.

In this work, we demonstrate a principle to realize impurity-immunity for bulk electromagnetic waves, which leads to perfect transmission irrespective of embedded impurities of almost any material and shape. The property of electromagnetic impurity-immunity is induced by the introduction of a pair of parity-time (*PT*)-symmetric metasurfaces that sandwich a slab of ENZ medium, which

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is originally sensitive to impurities. In the past few years, significant efforts have been devoted to exploring the properties of the optical PT -symmetric systems with balanced gain and loss [29–53]. The optical analog of PT symmetry [54] requires a refractive index profile of $n(\mathbf{r}) = n(-\mathbf{r})^*$ [29,30]. The gain and loss in the optical materials allow the deviation from a conventional Hermitian system and introduce additional physics based on PT -phase transition and the associated exceptional points (EPs). Many intriguing phenomena have been discovered, such as power oscillations [31–33], coherent perfect absorption and lasing [34–38], unidirectional transparency [39–43], etc. More recently, the concept of PT symmetry has been introduced in metasurfaces [55–57], which renders many exciting novel phenomena such as active negative refraction and imaging [46–49], unidirectional cloaking [50], electromagnetic teleportation [51], etc.

The electromagnetic impurity-immunity realized here comes from the EPs in the PT -symmetric system. In the phase diagram of PT -symmetric systems obtained by the variations of external parameters, EPs emerge as the unique phase transition points between the PT -symmetric and PT -broken phases. In the case of PT -symmetric metasurfaces, an interesting type of EP has been found at which the metasurfaces function as a pair of coherent perfect absorbers (CPAs) and lasers [46–49]. Here, by systematically investigating PT -symmetric metasurfaces, we observe a very different type of EP at which the metasurfaces function as a pair of unidirectional antireflection coatings (ARCs) for general dielectrics or metamaterials. By applying such PT -symmetric metasurfaces to ENZ media, we can compensate for the divergent impedance contrast between the ENZ media and free space. More importantly and interestingly, with such PT -symmetric metasurfaces, we find that the original “doping” effect of impurities in ENZ media [4] is significantly suppressed, and the perfect transmission becomes immune to two- and three-dimensional impurities of almost any material and shape.

First, we consider a lossy metasurface (left) and a metasurface with gain (right) separated by a slab with relative permittivity ϵ and relative permeability μ , as illustrated in Fig. 1(a). The metasurfaces can be characterized by surface admittance $\pm|Y_s|$, or relative permittivity $\epsilon_{ms} = 1 \pm [(i|Y_s|/Y_0)/k_0 d_{ms}]$, where Y_0 , k_0 , and d_{ms} are the admittance of free space, the wave number in free space, and the thickness of the metasurface, respectively. The relative permeability of the metasurfaces is unity. When the surface admittances of the metasurfaces are opposite to each other, the optical PT -symmetric condition $n(-x) = n(+x)^*$ is satisfied. In such a PT -symmetric system, the scattering matrix describing the relation between the incoming and outgoing waves can be written as

$$S = \begin{pmatrix} t & r_R \\ r_L & t \end{pmatrix}, \quad (1)$$

where $r_{L(R)}$ is the reflection coefficient for left (right) incidence and t is the transmission coefficient. Since the system is reciprocal, t is identical for both left and right incidences. By considering PT symmetry in Eq. (1), we can obtain the generalized energy-conservation relation [40]

$$\sqrt{R_L R_R} = |T - 1|, \quad (2)$$

where $R_{L(R)} \equiv |r_{L(R)}|^2$ and $T \equiv |t|^2$ are, respectively, the reflectance and transmittance for left (right) incidence.

When $T < 1$ (or $T > 1$), the eigenvalues of the scattering matrix are unimodular (or nonunimodular), indicating a PT -symmetric (or PT -broken) phase [36], in which case, the phase difference between r_L and r_R is zero (or π). The transition between the two phases happens at the EPs, at which $T = 1$ and $R_L R_R = 0$, indicating unity transmission and R_L and/or R_R is zero. As a result, at the EPs, PT -symmetric systems can exhibit the so-called unidirectional transparency [39].

By applying the requirement of $T = 1$, the conditions of EPs for metasurfaces can be derived as [58]

$$Y_s = \left(\cos \theta \pm \sqrt{\epsilon/\mu - \sin^2 \theta / \mu^2} \right) Y_0, \quad (3a)$$

for the case of transverse electric (TE, with electric fields in the z direction) polarization, and

$$Y_s = \left(\frac{1}{\cos \theta} \pm \frac{\epsilon}{\sqrt{\epsilon\mu - \sin^2 \theta}} \right) Y_0, \quad (3b)$$

for the case of transverse magnetic (TM, with magnetic fields in the z direction) polarization. Here, θ is the incidence angle in free space, which should be smaller than the critical angle of total reflection, i.e., $\arcsin \sqrt{\epsilon\mu}$. The “ $\pm$ ” sign indicates that there exist two solutions of EPs. When $Y_s > 0$, the solution gives $R_L = 0$ for incidence on the lossy metasurface, i.e., from the left-hand side. When $Y_s < 0$, the solution gives $R_R = 0$ for incidence on the gain metasurface, i.e., from the right-hand side. Interestingly, both solutions are independent of the thickness d of the sandwiched slab. As we shall see later, the two solutions are complementary to each other but have very different physical nature.

In Fig. 1(b), we plot the phase diagram of the PT -symmetric system under an incident angle of $\theta = 30^\circ$ for TE polarization. The slab sandwiched by the PT -symmetric metasurfaces is chosen as a dielectric medium with $\epsilon = 4$ and $\mu = 1$. In Fig. 1(b), the white and gray regions represent the symmetric and broken phases, respectively, while EPs are between the PT -symmetric and PT -broken phases, i.e., $Y_s = 1.07Y_0$ and $Y_s = 2.80Y_0$, both independent of d . The two values are consistent with Eq. (3), with the choice of a minus or plus sign, respectively. In Figs. 1(c) and 1(d), we plot the reflectance R_L and R_R , respectively, showing unidirectional transparency at the EPs.

We note that the additional white lines in Fig. 1(b) or the horizontal blue lines in Figs. 1(c) and 1(d) (periodic in d) in

FIG. 1. (a) Illustration of the PT -symmetric system composed of a slab with relative permittivity ϵ and relative permeability μ sandwiched by PT -symmetric metasurfaces with surface admittance $\pm|Y_s|$. (b) Phase diagram for incident angle $\theta = 30^\circ$ under TE polarization. The white and gray regions denote the symmetric and broken phases, respectively. (c,d) Reflectance as functions of d/λ_0 and Y_s/Y_0 for (c) left and (d) right incidence on the PT -symmetric system. The red dots are related to the zeros and poles of the scattering matrix of the PT -symmetric system. In panels (b)–(d), the parameters of the slab are $\epsilon = 4$ and $\mu = 1$.

the PT -broken phase are induced by Fabry-Pérot resonances of the slab and are therefore independent of Y_s . The condition can be easily derived as $d = m\pi/(k_0\sqrt{\epsilon - \sin^2\theta})$, where m is an integer. In this case, there is no reflection for either left or right incidence, as shown in Figs. 1(c) and 1(d).

Next, we demonstrate that the physical nature of the two EPs are very different. At the EP of $Y_s = 1.07Y_0$, $R_R = 0$. In the upper panel of Fig. 2(b), we show the field distribution obtained by using COMSOL multiphysics software. We see that the angle of refraction is positive in this case. The energy flux flows from the right to the left inside the slab, which is the same as those in free space. Interestingly, there is no reflection on either surface of the dielectric slab. In this sense, the functionality of PT -symmetric metasurfaces is a pair of ARCs [59], denoted as an ARC-ARC pair, as illustrated in Fig. 2(a).

On the other hand, at the EP of $Y_s = 2.80Y_0$, $R_L = 0$. In the field distribution shown in the upper panel of Fig. 2(d), we can see that the angle of refraction is negative in this case. The energy flux flows from the right to the left inside the slab, indicating that the lossy metasurface absorbs waves from both sides, while the metasurface with gain radiates toward both sides. In this sense, the functionality of PT -symmetric metasurfaces is a pair of coherent perfect absorption or lasing [60,61], denoted as a CPA-laser pair, as illustrated in Fig. 2(c).

The wave behaviors at the EPs under TM polarization also exhibit the above two types, as demonstrated in the

FIG. 2. (a) Illustration of the positive refraction in an ARC-ARC pair associated with the lower EP under the right incidence. (b) Simulated electric or magnetic fields: E_z (TE polarization, $Y_s = 1.07Y_0$) or H_z (TM polarization, $Y_s = 0.91Y_0$), and the time-averaged power flow (arrows) for the model in panel (a). (c) Illustration of the negative refraction in a CPA-laser pair associated with the upper EP under the left incidence. (d) Simulated electric or magnetic fields: E_z (TE polarization, $Y_s = 2.80Y_0$) or H_z (TM polarization, $Y_s = 3.22Y_0$), and the time-averaged power flow (arrows) for the model in panel (c). The incident angle in panels (b) and (d) is $\theta = 30^\circ$.

lower panels of Figs. 2(b) and 2(d), respectively. We note that the EPs associated with the ARC-ARC pair have not been reported before because, previously, only free space was considered between the PT -symmetric metasurfaces [46–49], in which such EPs do not exist. Here, with both types of complementary EPs given by a unified formula, i.e., Eq. (3), the solutions of the EPs are now complete.

When the relative permittivity of the slab changes from $\epsilon > 1$ to $\epsilon < 1$, an interesting transition behavior appears for the EP associated with an ARC-ARC pair of metasurfaces. By assuming a nonmagnetic slab with $\mu = 1$ and normal incidence with $\theta = 0$, Eq. (3) is simplified to $Y_s = (1 - \sqrt{\epsilon})Y_0$ for the EP associated with an ARC-ARC pair. Apparently, Y_s changes from $Y_s < 0$ to $Y_s > 0$ at the transition point of $\epsilon = 1$. This indicates a transition behavior from $R_R = 0$ to $R_L = 0$. When $\epsilon < 1$, to obtain unidirectional nonreflection, wave incidence is from the left-hand side, which is opposite from the case shown in Figs. 1 and 2 with $\epsilon > 1$. Such a transition behavior occurring at $\epsilon = 1$ is nonexistent for the EP associated with a CPA-laser pair because $Y_s = (1 + \sqrt{\epsilon})Y_0$ is always positive.

Next, we consider the interesting case of an ENZ medium slab (with $\epsilon = 0.001$ and $\mu = 1$) between the PT -symmetric metasurfaces, as shown in Fig. 3(a). In Fig. 3(b), we plot the phase diagram for normal incidence. Interestingly, the region of the PT -broken phase shrinks significantly. However, the EPs still exist, and both of them

FIG. 3. (a) Illustration of the PT -symmetric system with $\varepsilon \approx 0$. (b) Phase diagram for normal incidence. The white and gray regions denote the symmetric phase and broken phase, respectively. [(c) and (d)] Reflectance as the functions of d/λ_0 and Y_s/Y_0 for (c) left and (d) right incidence on the PT -symmetric system.

converge to the same value of $Y_s = Y_0$. In other words, the two original solutions of EPs coalesce into one EP when $\varepsilon \rightarrow 0$ [58]. Figures 3(c) and 3(d) show, respectively, the reflectance R_L and R_R . We notice that for both EPs, nonreflection is obtained for incidence from the left side because of the transition behavior for the EP associated with an ARC-ARC pair of metasurfaces.

In the following, we demonstrate that the properties of ENZ media undergo a fundamental change with the introduction of PT -symmetric metasurfaces. The well-known divergent impedance mismatch of ENZ media can be removed by the EPs. More importantly, the original sensitivity of impurities in ENZ media, which can tune the transmission behavior from total transmission to total reflection [4,12–20], can also be eliminated, leading to robust perfect transmission, irrespective of embedded impurities. In other words, the “doping” effect of ENZ media [4] is significantly suppressed, endowing the bulk system with the unprecedented property of impurity-immunity.

Through numerical simulations (using the COMSOL Multiphysics finite element software), we demonstrate the intriguing property of impurity-immunity for almost arbitrary impurities embedded in ENZ media, as illustrated in Fig. 4(a). For example, we consider a dielectric sphere of $\varepsilon_d = 4$ and radius R embedded in a slab of ENZ media (with $\varepsilon = 10^{-3} + 10^{-3}i$). In Fig. 4(b), we show the results obtained for R_L , R_R , and T ($=T_L = T_R$) as a function of R/λ_0 . Here, T_L (T_R) denotes the transmittance for incidence from the left (right). When the PT -symmetric metasurfaces are applied, $R_L \rightarrow 0$ (blue dashed lines) and $T \approx 1$ (red dashed lines) irrespective of the radius of the sphere, R . However, we also see $R_R \neq 0$ [blue dotted

FIG. 4. (a) Illustration of the PT -symmetric metasurfaces separated by an ENZ medium slab embedded with an arbitrary three-dimensional object. (b) Reflectance and transmittance for an embedded dielectric sphere of $\varepsilon_d = 4$ under the left (dashed lines) and right (dotted lines) incidence. (c)–(f) Distribution of E_z in the ENZ medium slab for (c,f) the case with PT -symmetric metasurfaces for left incidence, (d) the case with PT -symmetric metasurfaces for right incidence, and (e) the case without PT -symmetric metasurfaces. The impurity is as follows: (c)–(e) A dielectric sphere of $\varepsilon_d = 4$ and (f) a metallic cube of $\varepsilon_d = -2$. The relative permittivity of the ENZ medium is $\varepsilon = 10^{-3} + 10^{-3}i$.

lines in Fig. (b)], indicating that the perfect transmission is unidirectional. For comparison, when the PT -symmetric metasurfaces are removed, we observe that $R_{\text{Bare}} \approx 1$ and $T_{\text{Bare}} \rightarrow 0$, i.e., almost total reflection and near-zero transmission due to the divergent impedance mismatch. We note that the surface admittance $Y_s = 0.968Y_0$ associated with the EP for the ARC-ARC pair is utilized for the demonstration of the impurity-immunity effect here. Such an impurity-immunity effect can also be realized by using the EP for the CPA-laser pair, at which the surface admittance is $Y_s = 1.032Y_0$. The two EPs approach each other to the coalescence point of $Y_s = Y_0$ when $\varepsilon \rightarrow 0$.

The calculated field distributions are shown in Figs. 4(c)–4(e). In the simulations, the front and back boundaries (with the surface normal in the y direction) are perfect magnetic conductors, and the upper and lower boundaries (with the surface normal in the z direction) are perfect electric conductors so as to construct a waveguide supporting transverse electric and magnetic modes. Electromagnetic waves with electric fields polarized in

the z direction are incident from the left port boundary. The simulated electric fields E_z in Figs. 4(c)–4(e) confirm the transmittance results in Fig. 4(b) for the ENZ medium slab of $d = 1.5\lambda_0$. In Fig. 4(f), we replace the dielectric sphere by a metallic cube of $\epsilon_d = -2$ and still obtain $R_L \rightarrow 0$ and $T \approx 1$. This indicates that the perfect transmission is independent of the shape and size of the embedded impurity. Moreover, the effect is also independent of the thickness of the ENZ medium slab [58]. In this sense, PT -symmetric metasurfaces transform the ENZ media into a unidirectional impedance-matched media, which is immune to any three-dimensional impurities [21].

We have demonstrated the impurity-immunity phenomenon for isolated three-dimensional impurities like a sphere or a cube. Previously, in the case of a connected defect such as a cylindrical wire of infinite length, without the PT -symmetric metasurfaces currently considered, it has been demonstrated that these impurities in ENZ media would lead to interesting doping effects, which can be used to tune the transmission behavior from total transmission to total reflection [4,12–20]. We note that even the impedance-matched double-zero media (with both permittivity and permeability near zero) exhibit similar doping effects, resulting in impurity-dependent transmission properties [13,14]. Such doping effects can be understood as a consequence of the long-range connectivity of impurities [21,22] in zero-index materials. On the contrary, here, by applying the PT -symmetric metasurfaces to ENZ media, the doping effect of continuous impurity can be suppressed, leading to impurity-immunity even for impurities with long-range connectivity, such as cylinders with infinite length. To demonstrate this important phenomenon, we place a dielectric cylinder of relative permittivity $\epsilon_d = 4$ with a random cross section inside the ENZ medium with the PT -symmetric metasurfaces, as shown in Fig. 5(a). The orientation is horizontal, so the system becomes a two-dimensional one with TM polarization. Clearly, perfect transmission is observed. For comparison, in Fig. 5(b), we show that removal of the PT -symmetric metasurfaces leads to very low transmission. In Fig. 5(c), we show that even for an impedance-matched double-zero medium (without metasurfaces), the transmission is still very low because of the doping effect of the embedded cylinder. Moreover, in Fig. 5(d), we plot the transmittance T as a function of ϵ_d for the cases in Figs. 5(a) and 5(c). Clearly, without the PT -symmetric metasurfaces, T varies dramatically as ϵ_d increases, even for the impedance-matched double-zero medium, while for the ENZ medium with the PT -symmetric metasurfaces, we have $T \approx 1$, except for a very small region around a singular point at $\epsilon_d = 4.7$. When the ENZ medium approaches an ideal value, i.e., $\epsilon \rightarrow 0$, the transmittance $T \rightarrow 1$, even near the singular point of $\epsilon_d = 4.7$.

Such extraordinary impurity-immunity effects for the infinite-sized impurities can actually be explained from the aspect of an effective medium by applying the doping

FIG. 5. (a)–(c) Distribution of E_z for a dielectric cylinder of $\epsilon_d = 4$ with a random cross section embedded in (a) an ENZ medium ($\epsilon = 10^{-3} + 10^{-3}i$) with PT -symmetric metasurfaces, (b) an ENZ medium ($\epsilon = 10^{-3} + 10^{-3}i$) without metasurfaces, and (c) a double-zero medium ($\epsilon = \mu = 10^{-3} + 10^{-3}i$). (d) Transmittance as a function of the relative permittivity of the dielectric cylinder for the case of the double-zero medium without metasurfaces (gray solid lines) or the ENZ medium with the PT -symmetric metasurfaces (other lines).

theory [4]. In two dimensions, the ENZ medium with impurities embedded behaves as an effective medium with effective permittivity $\epsilon_{\text{eff}} \approx 0$ and a tunable effective permeability μ_{eff} . When $\mu_{\text{eff}} \rightarrow 0$, the ENZ medium turns into a double-zero medium, which enables total transmittance [4]. When μ_{eff} is a finite number, i.e., $|\mu_{\text{eff}}| \ll \infty$, Eq. (3) can be rewritten as $Y_s = (1 \pm \sqrt{\epsilon_{\text{eff}}/\mu_{\text{eff}}})Y_0$ [58]. From this equation, it can be seen that, as long as $\epsilon_{\text{eff}} \rightarrow 0$, the EP solutions are $Y_s \approx Y_0$ for any finite-valued μ_{eff} . This indicates that the EP is almost independent of the impurities, leading to impurity-immune perfect transmission. In Fig. 5(d), we can see that, even near the singular point of $\epsilon_d = 4.7$, where $|\mu_{\text{eff}}| \rightarrow \infty$ [4,19], the transmission can be restored to near unity when the ENZ medium has $\epsilon = 10^{-6} + 10^{-6}i$.

In the following, we apply field analysis to understand the physical mechanism of the impurity-immunity phenomenon. Without loss of generality, we consider the two-dimensional case under TM-polarized incidence (the same case as in Fig. 5) in which the magnetic field is in the out-of-plane direction, as schematically shown in Fig. 6. The origin of the doping effect is the tunable resonant magnetic field in the impurities. Because of the near-zero permittivity of the ENZ medium, the magnetic field H_{ENZ} is almost a constant in the ENZ medium, as shown in Fig. 6(a). Therefore, $H_L \approx H_R \approx H_{\text{ENZ}}$, where H_L and H_R are the magnetic fields on the left and right boundaries of the ENZ medium, respectively. On the other hand, by using Faraday's law $\oint \mathbf{E} \cdot d\mathbf{l} = i\omega \int \mathbf{B} \cdot d\mathbf{S}$, E_L and E_R , i.e., the tangential electric fields on the left and right boundaries of the ENZ

FIG. 6. (a,b) Schematic graphs of the electric- and magnetic-field analysis for (a) the doping model without PT -symmetric metasurfaces and (b) the impurity-immunity model with PT -symmetric metasurfaces. The dashed lines denote the integral area for the calculation of magnetic flux. (c,d) The distributions of calculated electric fields (arrows) and magnetic fields (color) for the doping model and the impurity-immunity model with PT -symmetric metasurfaces, respectively. A triangular impurity of $\epsilon_d = 5$ and a square impurity of $\epsilon_d = 2$ exist in both cases.

medium, can be linked by the magnetic flux through the whole region bounded by the ENZ medium, including the ENZ medium itself and the embedded impurities, i.e.,

$$E_R - E_L = \frac{i\omega}{h} \int \mathbf{B} \cdot d\mathbf{S}, \quad (4)$$

where h is the height of the ENZ medium and $\mathbf{B}$ is the magnetic field. We note that when impurities are absent, the electric field decays linearly with the thickness of the ENZ medium slab (see, e.g., Ref. [62]). However, when impurities exist in the ENZ medium, there are resonant magnetic fields inside the impurities, i.e., H_{imp} , as shown in Fig. 6(c). Note that H_{imp} will change the total magnetic flux $\int \mathbf{B} \cdot d\mathbf{S}$ of the whole area and the electric fields on the boundary of the ENZ medium, $E_R - E_L$, which will further change the transmittance through the whole system. Since H_{imp} strongly depends on the parameters of the impurities, the transmittance, as well as the effective medium of the system (see, e.g., Ref. [4]), can be tuned efficiently by the impurities, leading to the doping effect.

Interestingly, when the PT -symmetric metasurfaces are applied, the electric field still exists in the ENZ media, but the magnetic field is dramatically reduced to almost zero, as shown in Fig. 6(b). We note that across the ultrathin PT -symmetric metasurfaces with unity permeability, the tangential electric field is almost a constant, but the tangential magnetic field will experience a sharp change due to surface currents on the metasurfaces (see, e.g., Fig. 2 in the case of H_z polarization). Specifically, at the EP where the metasurfaces satisfy Eq. (3), the left metasurface will work as a CPA or an ARC, so there is no reflection on the metasurface. In this case, considering the constant electric

fields across the metasurface, the magnetic field inside the ENZ medium can be derived as $H_{\text{ENZ}} = \sqrt{\epsilon} H_L$. Here, ϵ is the permittivity of the ENZ medium. When $\epsilon \rightarrow 0$, the magnetic fields in the ENZ medium and impurities are also near zero, i.e., $H_{\text{ENZ}} \sim 0$ and $H_{\text{imp}} \sim 0$. This is a consequence of the dramatic impedance difference between free space and the ENZ medium and the unidirectional transmission at the EP. Therefore, we obtain $E_R - E_L = [(i\omega)/(h)] \int \mathbf{B} \cdot d\mathbf{S} \sim 0$, i.e., $E_R \approx E_L$. Since the tangential electric field across the metasurface is constant, the transmittance through the system will be restored to unity, and this property is independent of the embedded impurities. The above analysis explains the physical origin of the impurity-immunity phenomenon.

To verify the above field analysis, in Figs. 6(c) and 6(d), we plot the calculated electric- and magnetic-field distributions, for the doping model with a triangular impurity of $\epsilon_d = 5$ and a square impurity of $\epsilon_d = 2$, as well as the impurity-immunity model with PT -symmetric metasurfaces, respectively. Clearly, in the doping case without metasurfaces, H_{imp} is large inside the impurities, which can thus tune the total magnetic flux and lead to the doping effect. However, when the PT -symmetric metasurfaces are applied, H_{ENZ} and H_{imp} are both reduced to almost zero, which guarantees perfect transmission and turns off the doping effect of the original impurities.

The origin of impurity-immunity lies in the near-zero magnetic flux in the region containing both ENZ medium and impurities, which occurs at $\epsilon \rightarrow 0$. When $\epsilon \rightarrow 0$, we also find that the region of the PT -broken phase diminishes, and the two EPs approach each other and coalesce into one EP. In this sense, the impurity-immunity phenomenon is associated with the coalescence of EPs. The coalescence and high order of EPs [63–66] are interesting phenomena with many important applications such as enhanced sensitivity. Here, we have demonstrated another interesting consequence of the coalescence of EP in the transmission behavior of a PT -symmetric system with ENZ medium. We have plotted and discussed the coalescence process of the EPs in Ref. [58].

The electric fields throughout the ENZ media serve as a bridge linking the incident and transmitted waves. Such electric fields are also nonvariant across both metasurfaces because of the ultrathin thickness and $\mu = 1$. More interestingly, the electric fields can link unparallelled PT -symmetric metasurfaces, enabling more general-shaped configurations beyond the above planar configurations, as we have shown in Ref. [58].

One may notice that in our simulations, the impurity-immunity phenomenon is demonstrated in the existence of the material loss of the ENZ media. A detailed analysis of the effects of loss in both the ENZ media and the impurities is presented in Ref. [58].

The above results have demonstrated the impurity-immunity phenomenon under normal incidence. The condition of normal incidence can usually be realized by using

wide-aperture coherent incident beams or waveguide structures. In practice, the permittivity of the ENZ medium has a small but finite value. In such cases, the impurity-immunity phenomenon can be extended to slightly oblique incidence [58]. From Eq. (3), it can be seen that weak nonlocality is required in the ideal parameters of metasurfaces under oblique incidence. The concept and design methods of such nonlocal metasurfaces have previously been developed in Refs. [48,49].

Practical implementation of the PT -symmetric metasurfaces can be realized in the microwave and optical frequency regimes. The lossy metasurfaces can be simply achieved by conductive films [59–61]. In THz and optical designs, the conductive films can be composed of conductive materials like chromium [67], gold [68], graphene [69], indium tin oxide [70], etc. On the other hand, the metasurfaces with gain can be achieved by employing gain structures in metasurface or metamaterial designs. For instance, a grating of wide metallic strips periodically loaded with lumped elements has been proposed to design the metasurfaces with gain for microwave cloaking [50]. In addition, subwavelength structures with a large amount of gain can be realized by using multilayered structures [48], microwave and optical metastructures with gain [71–75], etc. In particular, the effective gain media with pure permittivity or permeability have also been demonstrated in core-shell composites [76]. These works manifest the possibility of realizing the metasurfaces with the required effective gain parameters.

Finally, we have also investigated the stability of our PT -symmetric system by plotting the poles of the scattering matrix on the complex frequency plane, as reported in Ref. [58]. By applying appropriate Lorentz and anti-Lorentz dispersion models to the metasurfaces and a Drude dispersion model to the ENZ medium, we find that all poles can be confined to the lower half-plane, indicating that our system can be unconditionally stable for arbitrary temporal excitations [48–50]. Moreover, the position change of the poles caused by the embedded impurity is almost negligible, confirming the stability and the feasibility of the proposed PT -symmetric system, as well as the robustness of the impurity-immunity functionality.

To conclude, we have analytically given the complete set of EPs for PT -symmetric metasurfaces sandwiching a slab of dielectric-metamaterial slab. A new type of EP at which the metasurfaces function as ARCs is revealed. By applying such metasurfaces to a slab of ENZ medium, the original two types of EPs coalesce into one EP. Interestingly, at this critical point of $\epsilon \rightarrow 0$, not only can the significant issue of a huge impedance mismatch with free space be solved, but the doping effect of ENZ medium can also be significantly suppressed, leading to the unprecedented property of electromagnetic impurity-immunity for bulk electromagnetic waves. This physical mechanism of the impurity-immunity phenomenon lies in the near-zero

magnetic flux in the region bounded by the ENZ medium; thus, the principle applies for impurities of almost arbitrary shapes and materials. Our work demonstrates that, with proper design, PT -symmetry with EPs can be utilized to realize the rare and valuable phenomenon of impurity-immunity for bulk states.

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