Giant Spin-Orbit Interaction Due to Rotating Magnetic Fields in Graphene Nanoribbons

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We study graphene nanoribbons theoretically in the presence of spatially varying magnetic fields produced, e.g., by nanomagnets. We show, both analytically and numerically, that an exceptionally large Rashba spin-orbit interaction of the order of 10 meV can be produced by a nonuniform magnetic field. As a consequence, helical modes exist in armchair nanoribbons that exhibit nearly perfect spin polarization and are robust against boundary defects. This result paves the way for realizing spin-filter devices in graphene nanoribbons in the temperature regime of a few kelvins. If a nanoribbon in the helical regime is in proximity contact to an *s*-wave superconductor, the nanoribbon can be tuned into a topological phase that sustains Majorana fermions.

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I. INTRODUCTION

In the last decade, we have seen remarkable progress in the physics and fabrication of graphene-based systems [1,2]. Recently developed growing techniques now allow the production of graphene nanoribbons (GNRs) with well-defined edges, in particular, those of the armchair type [3,4]. Transport experiments on GNRs of several micrometer lengths demonstrate finite conductance in spite of disorder coming, e.g., from irregular edges [5]. Moreover, it has been shown that the presence of adatoms can significantly increase the strength of the Rashba type of spin-orbit interaction (SOI) [6]. All this progress makes nanoribbons promising candidates for spintronics effects. In particular, the generation of helical states (conducting modes that transport opposite spins in opposite directions) is of great interest. Such modes have been proposed in semiconducting nanowires [7], carbon nanotubes [8,9], and bilayer graphene [10], and have been reported experimentally for quantum wires in GaAs hole gases [11]. These modes find applications in spin filters [7] and Cooper-pair splitters [12], and, in contact with an *s*-wave superconductor, they provide a platform for Majorana fermions with non-Abelian braiding statistics [13].

In the present work, we propose a novel way to generate a giant effective SOI in GNRs by spatially varying magnetic fields that can be produced by nanomagnets [14]. This approach has an advantage over using adatoms because the surface of graphene is not in tunnel contact with other atoms, which usually leads to high disorder with strong intervalley scattering. As we show in this paper, large values of SOIs result in helical modes of nearly perfect polarization. Moreover, nanoribbons, in stark contrast to semiconducting

nanowires, have considerably larger subband splittings, allowing for superior control of the number of propagating modes and of the gaps that are characteristic for the helical regime. Furthermore, our proposal is a next step in bringing topological features to graphene systems. Topological states proposed by Kane and Mele [15] have turned out to be experimentally undetectable due to the small intrinsic SOI of graphene. In contrast, we show, in this paper, that, if a GNR in the helical regime is brought into proximity with an *s*-wave superconductor, the system can be tuned into a topological phase that supports Majorana fermions. This opens up the possibility of using GNRs for topological quantum computing.

The low-energy physics of armchair GNRs is characterized by broken-valley degeneracy enforced by the boundary effects [16]. To generate helical states, we also need to lift the spin degeneracy, which can be achieved by using magnetic fields in two ways: with a uniform magnetic field and Rashba SOI or with a spatially varying magnetic field. The chemical potential should be tuned inside the gap that is opened, leading to a helical regime. We study these two scenarios both analytically and numerically. Moreover, we show numerically that the presence of helical states is robust against small nonidealities of the GNR edges, which shows that our proposal is realistic and experimentally feasible.

II. GRAPHENE NANORIBBONS

GNRs are strips of graphene, a honeycomb lattice defined by translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ and composed of two types of atoms, *A* and *B* (see Fig. 1). The GNR axis is chosen along the *z* axis and has a finite width in the *x* direction. GNRs are usually characterized by a width *W* and a chiral angle θ , which is the angle between the GNR axis and $\mathbf{a}_1$. We consider only those armchair nanoribbons defined by $\theta = \pi/2$.

Graphene can be analyzed in the framework of the tight-binding approach. The effective Hamiltonian includes hoppings of electrons between neighboring sites:

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FIG. 1. An armchair GNR formed by a finite strip of graphene aligned along the z axis and of the width $W = Na$ in the x direction. The GNR is composed of two types of atoms, A (blue dot) and B (green dot), and is characterized by hexagons in real space with translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. The low-energy physics is determined by the momenta $\mathbf{k} = (k_x, k_z)$ around the two valleys $\mathbf{K} = -\mathbf{K}' = (4\pi/3a, 0)$. Nanomagnets (green slabs) placed with period λ_n on the sides of the GNR provide a spatially varying magnetic field $\mathbf{B}_n$ (red arrows).

$$\bar{H}_0 = \sum_{\langle ij \rangle, \lambda, \lambda'} t_{ij, \lambda \lambda'} c_{i\lambda}^\dagger c_{j\lambda'}, \quad (1)$$

where $c_{i\lambda}$ are the standard electron operators, i and j are nearest-neighbor sites, and λ, λ' are spin projections on the z axis. Without SOI, the spin is conserved and the hopping amplitude becomes $t_{ij, \lambda \lambda'} = t_{ij} \delta_{\lambda \lambda'}$, where t_{ij} is spin independent. It is more convenient to treat $\bar{H}_0$ in momentum space (k_x, k_z) . The low-energy physics of graphene is determined by two valleys around $\mathbf{K} = -\mathbf{K}' = (4\pi/3a, 0)$, where $a = |\mathbf{a}_1|$ is the lattice constant. Wave functions can be represented in the form $\psi = \sum_{\tau\sigma} \phi_{\sigma\tau} e^{i\tau K_x x}$, where $\tau = \pm 1$ corresponds to K/K' and $\sigma = \pm 1$ to the A/B sublattice. The Hamiltonian for the slowly varying wave functions $\phi_{\sigma\tau}(x, z)$ is written in terms of the Pauli matrices σ_i (τ_i), acting on the sublattice (valley) degrees of freedom, as

$$H_0 = \hbar v_F (\tau_3 k_x \sigma_1 + k_z \sigma_2), \quad (2)$$

where k_z (k_x) is the longitudinal (transverse) momentum calculated from a Dirac point, and v_F is the Fermi velocity. In the rest of this paper, we work in the basis $\Phi = (\phi_{AK}, \phi_{BK}, \phi_{AK'}, \phi_{BK'})$.

A GNR, in contrast to a graphene sheet, is of finite width W , leading to well-gapped subbands. In order to impose open boundary conditions on a GNR consisting of N unit cells in the transverse direction, we effectively extend the GNR by two unit cells [so that the width is equal to $W' = (N + 2)a$] and impose vanishing boundary conditions on these virtual sites, $\psi(0, z) = 0$ and $\psi(W', z) = 0$ [16]. This procedure leads to quantization of the transverse momentum k_x , $K_x + k_x = \pi n / W'$, where n is an integer.

If the width of the GNR is such that $N = 3M + 1$, where M is a positive integer, the GNR is metallic [16]. The low-energy spectrum is linear and is given by $E_\sigma = \gamma \hbar v_F k_z$, where the isospin $\gamma = \pm 1$, corresponding to the eigenvalues of the Pauli matrix σ_2 . The higher levels are twofold

FIG. 2. The spectrum of an armchair GNR obtained by numerical diagonalization of the tight-binding Hamiltonian $\bar{H}_0 + \bar{H}_{SO} + \bar{H}_Z$. The low-energy spectrum is linear for metallic [(a),(b) with $N = 82$] and quadratic for semiconducting [(c),(d) with $N = 81$] GNRs. The SOI [(b) $\Delta_{SO} = 1$ meV and (d) $\Delta_{SO} = 5$ meV] lifts the spin degeneracy, so the spectrum consists of (b) two Dirac cones or (d) two parabolas shifted by k_{SO} from zero (shown by green lines). For a metallic GNR, each branch is characterized not only by the spin projection s , but also by the isospin γ . The solid (dashed) lines correspond to $\gamma = 1$ ($\gamma = -1$) [see (b)]. Whereas for a semiconducting GNR, a magnetic field ($\Delta_Z = 0.1$ meV) alone opens a gap $2\Delta_g \approx 0.2$ meV [(d)], we also need to include intervalley scattering (modeled by fluctuations in on-site energies) for the metallic GNR [(b)]. If the chemical potential μ is tuned inside the gap, the system is in the helical regime with nearly perfect polarization, $\langle s_x \rangle \approx 0.99$, in both semiconducting and metallic cases.

degenerate (apart from spin; see below) and gapped by $\pi \hbar v_F / 3(M + 1)a$ [see Fig. 2(a)].

If the GNR width is such that $N = 3M$ (or $N = 3M + 2$), where M is a positive integer, then the GNR is semiconducting [16]. The gap at $k_z = 0$ is given by $2\hbar v_F |k_x^{\min}|$, with $|k_x^{\min}| = \pi/3(N + 2)a$. In the case of a semiconducting GNR, all orbital states are nondegenerate [see Fig. 2(c)], which means that the boundaries induce intervalley coupling and break the valley degeneracy [16–18].

If the chemical potential μ crosses only the lowest level of the spectrum, then two states are propagating in opposite directions with opposite isospins σ . Further, taking spin into account, there will be four states at the Fermi level in total. As we show next, this degeneracy can be lifted if we include Rashba SOI and a uniform magnetic field or, equivalently, a spatially varying magnetic field.

III. ROTATING MAGNETIC FIELD AND THE RASHBA SOI

The Rashba SOI arises from breaking inversion symmetry. This inversion symmetry breaking can be caused by an electric field E_{ext} applied perpendicular to the GNR plane or, alternatively, by adatoms, which produce local electric

fields. In the electric-field case, the SOI is quite small, $\Delta_{\text{so}} = eE_{\text{ext}}\xi$, with $\xi = 4 \times 10^{-5}$ nm [19,20], for realistic fields $E_{\text{ext}} \sim 1$ V/ μm . In the adatom case, the strength of the SOI is significantly increased by doping, and values for Δ_{SO} of 10–100 meV have been observed [6]. The general form of the Rashba SOI Hamiltonian can be obtained from symmetry considerations [15],

$$H_{\text{SO}} = \Delta_{\text{SO}}(\tau_3 s_z \sigma_1 - s_x \sigma_2), \quad (3)$$

where the Pauli matrices s_i act on the spin.

An alternative approach to generate helical modes is to apply a spatially varying magnetic field [21–23]. Such a field can be produced by nanomagnets [14] (e.g., using nanopatterning methods [24]) or by vortices of a superconductor. We emphasize that this scheme does not require perfect periodicity of the field; rather, it just needs a substantial weight of the Fourier component at twice the Fermi wave vector. Moreover, this mechanism is valid for both rotating and linearly oscillating fields. For simplicity, we assume in this section that the Rashba SOI discussed above is negligible. The case in which both a spatially varying magnetic field and a Rashba SOI are present has been discussed recently in the context of nanowires [23], and this case was shown to lead to a number of striking effects, such as fractionally charged fermions [23].

First, we consider a field rotating in a plane perpendicular to the GNR plane, leading to the Zeeman term

$$H_n^\perp = \Delta_Z[s_y \cos(k_n z) + s_z \sin(k_n z)], \quad (4)$$

where $\Delta_Z = g\mu_B B/2$, μ_B is the Bohr magneton, g is the g factor, and $\lambda_n = 2\pi/k_n$ is the period of the rotating field. It is convenient to analyze the position-dependent Hamiltonian $H_0 + H_n^\perp$ in the spin-dependent rotating basis [21,25]. The unitary transformation $U_n = \exp(ik_n z s_x/2)$ brings the Hamiltonian back to a GNR with an in-plane Rashba SOI and a uniform field perpendicular to the GNR,

$$H^\perp = U_n^\dagger (H_0 + H_n^\perp) U_n = H_0 + \Delta_Z s_y + \Delta_{\text{so}}^n s_x \sigma_2. \quad (5)$$

In a similar way, a field rotating in the plane of a GNR,

$$H_n^\parallel = \Delta_Z[s_x \cos(k_n z) + s_z \sin(k_n z)], \quad (6)$$

is equivalent to a GNR with an out-of-plane Rashba SOI, together with a uniform field along the x axis,

$$H^\parallel = U_n^\dagger (H_0 + H_n^\parallel) U_n = H_0 + \Delta_Z s_x + \Delta_{\text{so}}^n s_y \sigma_2. \quad (7)$$

The induced SOI favors the direction of spin perpendicular to the applied rotating magnetic field, and its strength is given by $\Delta_{\text{so}}^n = \hbar v_F k_n/2$, independent of the amplitude Δ_Z . For example, Δ_{so}^n is equal to 10 meV for nanomagnets placed with a period of 200 nm. We remark once more that this mechanism is valid not only for rotating magnetic fields, but also for fields that oscillate only in one direction [21–23], as shown in Fig. 1. Such field configurations have

been modeled for realistic nanomagnets, and they have been shown to be experimentally feasible [22].

IV. HELICAL MODES

The spectrum of $H^\perp$ (or, by analogy, of $H^\parallel$) can be easily found by using perturbation theory. Taking into account that, realistically, $\Delta_Z \ll \Delta_{\text{so}}^n$, we treat the Zeeman term as a small perturbation. The induced SOI, given by $\Delta_{\text{so}}^n s_x \sigma_2$, leads to spin-dependent shifts of the k_z momenta by $k_{\text{SO}} = \Delta_{\text{so}}^n/\hbar v_F = k_n/2$, both for the metallic and for the semiconducting GNRs [see Figs. 2(b) and 2(d)]. Every level is characterized by the spin projection $s = \pm 1$ on the x axis, so the spin part of the wave functions, $|s\rangle$, is an eigenstate of the Pauli matrix s_x . The corresponding spectrum and wave functions that satisfy the vanishing boundary conditions (for ψ) are given for a metallic GNR by

$$\Phi_{\gamma,s}^{E,k_z} = e^{iz(k_z + sk_{\text{SO}})}(-i\gamma, 1, i\gamma, -1)|s\rangle, \quad (8)$$

$$E_{\gamma,s} = \gamma \hbar v_F (k_z + sk_{\text{SO}}), \quad (9)$$

and for a semiconducting GNR by

$$\begin{aligned} \Phi_{\pm,s}^{E,k_z} = e^{iz(k_z + sk_{\text{SO}})} & (\pm e^{i\varphi_s + ixk_x^{\min}}, e^{ixk_x^{\min}}, \mp e^{i\varphi_s - ixk_x^{\min}}, \\ & - e^{-ixk_x^{\min}})|s\rangle, \end{aligned} \quad (10)$$

$$E_{\pm,s} = \pm \hbar v_F \sqrt{(k_x^{\min})^2 + (k_z + sk_{\text{SO}})^2}. \quad (11)$$

Here we use the notation $e^{i\varphi_s} = [k_x^{\min} - i(k_z + sk_{\text{SO}})]/\sqrt{(k_x^{\min})^2 + (k_z + sk_{\text{SO}})^2}$.

A uniform magnetic field that is perpendicular to the spin-quantization axis defined by the SOI results in the opening of a gap $2\Delta_g$ at $k_z = 0$. Using the wave functions given by Eq. (10), we show that $\Delta_g = \Delta_Z k_x^{\min}/\sqrt{(k_x^{\min})^2 + k_{\text{SO}}^2} \approx \Delta_Z$ for a semiconducting GNR. The spin polarization in this state is given by $|\langle s_x \rangle| \approx 1 - (\Delta_Z k_x^{\min}/4\hbar v_F k_{\text{SO}}^2)^2$. In contrast, a metallic GNR possesses an additional symmetry. Each branch is characterized not only by spin ($s = \pm 1$), but also by isospin ($\sigma = \pm 1$) [see Fig. 2(b)]. Thus, a magnetic field alone cannot lift the degeneracy at $k_z = 0$. However, if we also include terms that break the sublattice symmetry, such as intervalley scattering described by $H_{KK'} = \Delta_{KK'} \tau_1$, a gap is opened. Here, $\Delta_{KK'}$ is the strength of the intervalley scattering, which can be caused by impurities or fluctuations in the on-site potential. Assuming $\Delta_Z, \Delta_{KK'} \ll \Delta_{\text{so}}$, the gap becomes $2\Delta_g = 2\Delta_{KK'} \Delta_Z/\Delta_{\text{so}}$ in leading order. The spin polarization of the helical states is given by $|\langle s_x \rangle| \approx 1 - (\Delta_Z/\Delta_{\text{so}})^2$. We note that, for both semiconducting and metallic GNR, Δ_Z limits the size of the gap Δ_g , which, in turn, determines the temperature regime, $k_B T < \Delta_g$, where k_B is the Boltzmann constant.

We note that $H^\perp$ is equivalent to the Hamiltonian describing a GNR in the presence of the Rashba SOI and a uniform magnetic field applied in the perpendicular y direction, $H_{\text{tot}} = H_0 + H_{\text{SO}} + H_Z$ [see Eqs. (2) and (3)], in first-order perturbation theory in the SOI. Here, the Zeeman term is given by $H_Z = \Delta_Z s_y$. The wave functions given by Eqs. (8) and (10) are eigenstates of the Pauli matrix τ_1 , so the diagonal matrix element of τ_3 is zero. The result is that the term $\tau_3 s_z \sigma_1$ in the Rashba Hamiltonian H_{SO} averages out in first-order perturbation theory, and H_{tot} is indeed equivalent to $H^\perp$. Thus, the effect of the SOI is a spin-dependent shift of k_z by $k_{\text{SO}} = \Delta_{\text{SO}}/\hbar v_F$. Similarly, the uniform magnetic field opens a gap at $k_z = 0$, which can be as big as 10 K for a field of about 10 T.

An alternative approach to the perturbation theory mentioned above is to analyze analytically the GNR with the Rashba SOI. For graphene, the spectrum of the effective Hamiltonian $H_0 + H_{\text{SO}}$ is given by $E_{j,\pm} = \pm(\Delta_{\text{SO}} + j\sqrt{(\hbar v_F k_x)^2 + (\hbar v_F k_z)^2 + \Delta_{\text{SO}}^2})$, where the index j is equal to 1 (−1) for the highest (lowest) electron level, and the $\pm$ sign distinguishes between electrons and holes. The SOI lifts the spin degeneracy; however, the valley degeneracy is maintained, and τ_3 is a good quantum number. Analogously to Ref. [16], we search for a sum over the eigenstates $\psi_{\tau,q}(x)$ of $H_0 + H_{\text{SO}}$, $\psi^{E,k_z}(x, z) = \sum_{\tau,q} b_{\tau,q} \psi_{\tau,q}^{E,k_z}(x)$, such that the boundary conditions are satisfied. The index $q = (j, \pm)$ distinguishes between four wave vectors satisfying $E_{j,\pm}(k_x = \pm k_j) = E$, $\hbar v_F k_{1,2} = \sqrt{E^2 - (\hbar v_F k_z)^2 \pm 2E\Delta_{\text{SO}}}$. We also introduce new variables θ and γ , via $\cos\theta = \hbar v_F k_z/E$ and $\sqrt{2}\sin\theta\sin\gamma = \hbar v_F k_1/E$. We allow for real as well as imaginary values of $k_{1,2}$, θ , and γ . The spectrum of a metallic GNR is then given implicitly by

$$\tan^2\theta \left[\sin\left(\frac{k_1 W}{2}\right) \cos\left(\frac{k_2 W}{2}\right) + \cos\left(\frac{k_1 W}{2}\right) \sin\left(\frac{k_2 W}{2}\right) \times \sin(2\gamma) \right]^2 = -\sin(k_1 W) \sin(k_2 W) \sin(2\gamma). \quad (12)$$

The exact solution defined by Eq. (12) can be analyzed analytically by means of Taylor expansion. For example, if $\Delta_{\text{SO}} \ll \hbar v_F k_z$, we get $E = \pm \hbar v_F k_z \pm \Delta_{\text{SO}}$, which is in agreement with previous perturbative calculations.

V. NUMERICS WITH EDGE DEFECTS

To check our analytical results numerically, we extend the tight-binding Hamiltonian $\bar{H}_0$ by allowing for hoppings with spin flip,

$$\bar{H}_{\text{SO}} = \sum_{\langle ij \rangle, \lambda, \lambda'} i c_{i\lambda}^\dagger \mathbf{u}_{ij} \cdot \mathbf{s}_{\lambda\lambda'} c_{j\lambda'} + \text{H.c.}, \quad (13)$$

in such a way that $\bar{H}_g$ is equivalent to the Rashba SOI in the low-energy sector. In Eq. (13), $\mathbf{s}_{\lambda\lambda'}$ is a vector composed of

FIG. 3. Defects on the edges of a metallic armchair GNR ($N = 82$), resulting in the opening of a gap at zero energy. In the numerical diagonalization, the defects are modeled by omitting two atoms on the edges, which is assumed to be periodic with period $l_d = 5\sqrt{3}a$. We see that the spectrum changes only slightly, and the qualitative features of a metallic armchair GNR are maintained.

the Pauli matrices, and spin-dependent hopping elements are defined as $\mathbf{u}_{ij} = -(3\Delta_{\text{SO}}/4)\mathbf{z} \times \mathbf{e}_{ij}$. A unit vector $\mathbf{e}_{ij}$ points along the bond between two sites i and j . The results of the numerical diagonalization of the Hamiltonian $\bar{H}_0 + \bar{H}_{\text{SO}} + H_Z$ are presented in Fig. 2, where the Zeeman term corresponding to a magnetic field $\mathbf{B}$ is modeled as

$$\bar{H}_Z = \sum_{i,\lambda,\lambda'} c_{i\lambda}^\dagger \mathbf{B} \cdot \mathbf{s}_{\lambda\lambda'} c_{i\lambda'}. \quad (14)$$

As shown in Fig. 2, the numerical results fully confirm the analytical calculations.

The spectrum of GNRs is known to be sensitive to the specific form of the edges. For example, the linear spectrum of a metallic GNR becomes parabolic for nonideal armchair boundaries (see Fig. 3). In contrast, subband gaps are only slightly modified for semiconducting GNRs. We conclude that the valley degeneracy is, in general, lifted due to strong intervalley mixing induced by the boundaries, and note that this is a property of all armchair GNRs [17,18]. We emphasize that, for the scenario of helical modes developed above, we do not need any specific symmetries. Thus, our proposal is robust against edge defects.

The scenario with a rotating magnetic field is even more universal. The only criterion is that the Fermi wave vector k_F is not too large; typically, k_F/K_x should be smaller than 10^{-2} . This natural limit results from the fact that the period of rotation of a magnetic field should be much larger than the lattice constant.

For convenience, we summarize the typical parameter values needed to observe the effects proposed in this work. The nanomagnets placed with a period of 400 nm ($\Delta_{\text{SO}} \approx 5$ meV) produce a spatially varying magnetic field of strength $B = 0.1\text{--}1$ T ($\Delta_Z \approx 6\text{--}60$ μeV). The temperature required then lies in the range of 50–500 mK.

VI. MAJORANA FERMIONS

Proximity effects in graphene have been studied extensively over the years [26,27], in particular, for GNRs [28]. Motivated by this extensive study, we consider, as a next

step, GNRs in the helical regime brought close to a superconductor. We show that this system can be tuned into a topological phase containing Majorana bound states at the ends of the ribbon. Such Majorana fermions have attracted much attention recently due to their non-Abelian statistics, which could be exploited for topological quantum computation [13].

If μ is tuned inside the gap that is opened by the field, the two propagating modes would be helical. The proximity-induced superconductivity in the GNR leads to the coupling between such states and gaps the spectrum. The system can be effectively described in the linearized model for the exterior branches ($\chi = e$, states with momenta close to the Fermi momentum, $k_e = k_F$) and the interior branches ($\chi = i$, states with nearly zero momenta, $k_i = 0$) [29]. The electron operator is represented as $\Psi(z) = \sum_{\rho=\pm 1, \chi=e,i} e^{i\rho k_\chi z} \Psi_{\rho\chi}$, where the sum runs over the right movers (R , $\rho = 1$) and left movers (L , $\rho = -1$). The effective Hamiltonian becomes

$$H = -i\hbar v_F \rho_3 \chi_3 \partial_x + \frac{\Delta_g}{4} \eta_3 \rho_1 (1 + \chi_3) + \Delta_s \eta_2 \rho_2, \quad (15)$$

where the Pauli matrices χ_i (η_i) act in the interior-exterior branch (electron-hole) space, and $\tilde{\Psi} = (\Psi_{Re}, \Psi_{Le}, \Psi_{Re}^\dagger, \Psi_{Le}^\dagger, \Psi_{Ri}, \Psi_{Li}, \Psi_{Ri}^\dagger, \Psi_{Li}^\dagger)$. Following Refs. [23,29], we find that the criterion for the topological phase transition is given by $\Delta_g > \sqrt{\mu^2 + \Delta_s^2}$. In terms of Zeeman energy, we obtain $\Delta_Z > \Delta_{SO} \sqrt{\mu^2 + \Delta_s^2} / \Delta_{KK'}$ [$\Delta_Z > \sqrt{\mu^2 + \Delta_s^2} \times \sqrt{(k_x^{\min})^2 + k_{SO}^2 / k_x^{\min}}$] for a metallic [semiconducting] GNR. The localization length of the Majorana fermion is determined by the wave function, which can be obtained explicitly for various parameter regimes (see Refs. [23,29]). We have shown elsewhere that the Majorana fermions discussed above are stable against electron-electron interactions [23,30]. Finally, Majorana fermions can be detected experimentally in several ways [13], the most straightforward one being transport measurements searching for zero-bias peaks [31,32]. Similar experiments can be performed in GNRs, which offer the additional advantage of being cleaner than semiconducting nanowires.

VII. CONCLUSIONS

We have shown that helical modes can be generated in graphene nanoribbons by using a spatially varying magnetic field or a Rashba spin-orbit interaction with a uniform magnetic field. We have demonstrated that the opening of the gap is universal for both semiconducting and metallic graphene armchair nanoribbons, independent of the mechanism that induces the spin-orbit interaction, leading to a helical regime with nearly perfect spin polarization. Moreover, we have checked numerically that the helical regime is robust against boundary defects. In addition, the

experimental state of the art in nanoribbon fabrication [3,4], transport measurements [5], inducing the proximity regime [26,27], and nanomagnetism [14] makes the present proposal realistic and experimentally feasible. Thus, graphene nanoribbons are promising candidates for spin effects, spintronics applications, and Majorana fermions for topological quantum computation.

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