Controlled Generation and Manipulation of Vortex Dipoles in a Bose-Einstein Condensate

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We propose methods to generate and manipulate vortex dipoles in an atomic Bose-Einstein condensate using Gaussian beams of red- or blue-detuned laser. Vortex dipoles with controlled velocities are shown to be created and launched by a red-detuned beam and by two blue-detuned beams. Critical beam velocities for the vortex nucleation are investigated. The launched vortex dipoles can be trapped, curved, accelerated, and decelerated by using Gaussian laser beams. Collisions between vortex dipoles are demonstrated.

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I. INTRODUCTION

A vortex dipole is a pair of vortices with opposite circulations, which is a fundamental excitation in an effectively two dimensional (2D) fluid and plays a pivotal role in the thermodynamic behavior of 2D systems [1–7]. In superfluids, vortices are topological defects of the macroscopic wave function and a vortex dipole consists of a quantized vortex and antivortex, whose circulations are quantized to $\pm h/m$ with h and m being Planck's constant and the mass of a particle [8,9]. Such a topological structure of a quantized vortex dipole has been observed in a Bose-Einstein condensate (BEC) of an atomic gas [10] and in an exciton-polariton condensate in a semiconductor cavity [11,12]. Recently, dynamics of vortex dipoles have directly been observed in oblate BECs [13–15].

In the experiments reported in Refs. [10,13], vortex dipoles were created by stirring an atomic BEC with a blue-detuned laser beam, which acted effectively as a repulsive potential for the atoms. When the velocity of the beam exceeds a critical velocity, vortex-antivortex pairs are released from the density dip produced by the laser beam. The created vortex dipoles then move through the BEC and exhibit various interesting dynamics [13–16], which can be observed in real time using the technique developed in Refs. [14,15]. Theoretical study on a moving potential in superfluids has so far been focused on drag force [17–20], vortex formation and shedding dynamics [21–23], critical velocity [24,25], scaling laws [26], and potential oscillation [27].

The objective of the present paper is to establish a “vortex-dipole launcher” that creates and launches vortex dipoles with controlled velocities and directions. We also

aim to manipulate their trajectories and velocities. An optical stirrer that has so far been used for vortex-dipole nucleation is a blue-detuned Gaussian laser beam [10,13], which produces a repulsive Gaussian potential moving in the condensate. In the present paper, we investigate the vortex-dipole nucleation by a red-detuned Gaussian laser beam, which produces an attractive Gaussian potential. In this case, the atomic density increases at the potential, which acts as a different type of optical stirrer from a blue-detuned beam. Vortex creation by this kind of stirrer has not been studied for superfluids including liquid helium. We will show that vortex dynamics for a red-detuned beam are quite different from those for a blue-detuned beam. We also consider a stirrer produced by multiple Gaussian laser beams. After establishing the method of controlled generation of vortex dipoles, we will study the manipulation of the created vortex dipoles. This is simply done by deploying blue- and red-detuned Gaussian laser beams in spatially and temporally controlled ways. Thus, controlled generation and manipulation of quantized vortex dipoles in an atomic gas BEC can be achieved by making versatile use of Gaussian laser beams.

The present paper is organized as follows. Section II provides the formulation and numerical method. Section III studies vortex-dipole generation by a red-detuned beam and shows that it can launch a vortex dipole in a controlled manner. Section IV investigates dynamics of vortex generation using two laser beams. Section V proposes methods to manipulate launched vortex dipoles and demonstrates their collisions. Section VI shows that the launch and manipulation of vortex dipoles are possible in a realistic trapped atomic BEC. Section VII gives conclusions to this study.

II. FORMULATION OF THE PROBLEM

Bosonic atoms with mass m are described by the Hamiltonian

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$$\hat{H} = \int d\mathbf{r} \hat{\Psi}^\dagger \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}, t) \right] \hat{\Psi} + \int d\mathbf{r} d\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') V_{\text{int}}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}), \quad (1)$$

where $\hat{\Psi}$ is the bosonic field operator, V_{ext} is the external potential, and V_{int} is the interatomic interaction potential. In the zero-temperature mean-field approximation, we assume that all the atoms occupy the same single-particle wave function Ψ obeying the Gross-Pitaevskii (GP) equation given by [28]

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V_{\text{ext}}(\mathbf{r}, t) \Psi + \frac{4\pi\hbar^2 a}{m} |\Psi|^2 \Psi, \quad (2)$$

where we have used the Fermi pseudopotential with a s -wave scattering length a . The wave function is normalized as $\int |\Psi|^2 d\mathbf{r} = N$ with N being the number of atoms.

We assume that the system is tightly confined in the z direction by a potential $V_z(z)$ and the confinement energy is much larger than other energy scales of the system. The wave function is thus frozen into $\Psi(x, y, z, t) = \psi(x, y, t) e^{-iE_0 t/\hbar} \psi_0(z)$, where $\psi_0(z)$ and E_0 are the ground state and its energy for $V_z(z)$. Multiplying $\psi_0(z)$ to Eq. (2) and integrating it with respect to z , we reduce the 3D GP equation to an effective 2D form as

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla_\perp^2 \psi + V(x, y, t) \psi + g |\psi|^2 \psi, \quad (3)$$

where $\nabla_\perp^2$ is the 2D Laplacian, $V(x, y, t) = V_{\text{ext}}(\mathbf{r}, t) - V_z(z)$, and $g = 4\pi\hbar^2 a m^{-1} \int |\psi_0|^4 dz$ is the effective interaction parameter in the 2D space. We normalize the 2D wave function as $\tilde{\psi} = n_0^{-1/2} \psi$, where n_0 is the 2D atomic density $|\psi|^2$ in the absence of V . Normalizing the length and time by $\xi = \hbar/(mgn_0)^{1/2}$ and $t_0 = \xi/v_s = \hbar/(gn_0)$, where $v_s = (gn_0/m)^{1/2}$ is the sound velocity, Eq. (3) is rewritten as

$$i \frac{\partial \tilde{\psi}}{\partial \tilde{t}} = -\frac{1}{2} \tilde{\nabla}_\perp^2 \tilde{\psi} + \tilde{V} \tilde{\psi} + |\tilde{\psi}|^2 \tilde{\psi}, \quad (4)$$

where the tilde represents normalized quantity. From this normalization, we find that the scaling law holds for the mean-field approximation. For example, the dynamics for the interaction strength gn_0 with a Gaussian potential $V = V_0 \exp\{-(x - vt)^2 + y^2\}/d^2\}$ and that for αgn_0 with $V = \alpha V_0 \exp\{-(x - \sqrt{\alpha}vt)^2 + y^2\}/(d/\sqrt{\alpha})^2\}$ obey the same equation (4) for an arbitrary constant $\alpha > 0$.

In such 2D atomic systems, the healing length and the sound velocity at the trap center are on the order of $\xi \sim 0.3 \mu\text{m}$ and $v_s \sim 1.7 \text{ mm/s}$, respectively [13]. The unit of time is then $t_0 \sim 0.18 \text{ ms}$. The Thomas-Fermi radius of the pancake-shaped quasi-2D BEC in Ref. [13] is $52 \mu\text{m} \sim 173\xi$.

A far-off-resonance laser beam produces the external potential V that stirs the condensate to create vortices

[10,13]. The laser-atom coupling is characterized by $\Omega = E_0 d_e/\hbar$, where E_0 is the electric-field amplitude of the laser and d_e is the matrix element of the electric-dipole transition. When the detuning $\delta = \omega_L - \omega_A$ (ω_L : laser frequency, ω_A : atomic-transition frequency) is much larger than both Ω and the spontaneous-decay rate of the excited state, the potential produced by the laser is given by $V = \hbar\Omega^2/(4\delta)$. The potential is therefore proportional to the intensity of the laser and repulsive for blue detuning ($\delta > 0$) and attractive for red detuning ($\delta < 0$). The detuning is large enough so that the heating of atoms by the spontaneous emission is negligible.

In the numerical simulations, we first prepare the ground state for $V(x, y, t = 0)$ by the imaginary-time propagation of Eq. (4), i.e., i is replaced by -1 on the left-hand side of Eq. (4). We then add a small white noise (a complex random number with maximum magnitude 10^{-3} on each mesh) to the ground state to break the exact numerical symmetry. The dynamics are not affected by the white noise qualitatively. Starting from this initial state, we obtain the time evolution by solving Eq. (4). The imaginary- and real-time propagation is calculated by the pseudospectral method [29], and therefore the periodic boundary condition is imposed. We take a large enough space in the simulations so that the boundary does not affect the dynamics.

III. VORTEX GENERATION BY A RED-DETUNED GAUSSIAN BEAM

A. Flow around a moving potential and the critical velocity for vortex generation

First, we consider a single Gaussian laser beam moving through the condensate, which produces a potential as

$$V(x, y, t) = V_0 \exp\left[-\frac{(x + vt)^2 + y^2}{d^2}\right], \quad (5)$$

where d is the $1/e$ width of the laser beam. We assume that the laser beam moves in the $-x$ direction with the velocity v . The constant V_0 is proportional to the intensity of the laser beam, which is positive for blue detuning and negative for red detuning.

Frisch *et al.* [17] showed that quantized vortex dipoles are created around a moving rigid disk ($V = +\infty$ inside a disk) above a critical velocity. The vortex and antivortex are created at the lateral sides of the disk [30], and they are released into the wake (see Fig. 1 of Ref. [17]). The Gaussian potential (5) for $V_0 > 0$ also exhibits similar behavior [19,21].

By contrast, we investigate the dynamics for a red-detuned ($V_0 < 0$) moving Gaussian beam, which is shown in Fig. 1. First, a density dip appears at the head of the potential [Fig. 1(a)], which splits into vortex and antivortex [Fig. 1(b)]. The vortex pair then passes through the potential avoiding the high density region [Fig. 1(c)], and it is

FIG. 1. Dynamics of the density $|\tilde{\psi}|^2 = |\psi|^2/n_0$ and phase $\phi = \arg \psi$ profiles for the Gaussian potential with $V_0/(gn_0) = -5$, $v/v_s = 0.6$, and $d/\xi = 4.5$ in the frame comoving with the potential. The phase profiles magnify the region in the dashed square. The arrows indicate the directions of circulations. The field of view of the density profiles is $160\xi \times 67\xi$. The time is normalized as $\tilde{t} = t/t_0 = tv_s/\xi$. (See also Supplemental Material [31].)

released behind the potential. The released vortex dipole moves with the velocity v_{vd} in the $-x$ direction. Here $v_{vd} \simeq \hbar/(ml) = v_s \xi/l$, where l is the distance between the vortex and antivortex. Such vortex dynamics are repeated and vortex dipoles are shed in the wake periodically. The train of vortex dipoles is dynamically unstable and eventually sinuous destabilization spreads it in the $\pm y$ directions [23,32]. In contrast to the case of a blue-detuned Gaussian beam, the Bénard-von Kármán vortex street [23] was not found for a red-detuned beam to the best of our investigation.

As shown in Fig. 1, a vortex dipole is nucleated at the head of a moving potential for an attractive potential ($V_0 < 0$), whereas a vortex dipole is nucleated at the lateral sides for a repulsive potential ($V_0 > 0$). To understand these behaviors, we consider the irrotational flow of an inviscid and incompressible fluid around a disk. The velocity field $\mathbf{u}(x, y)$ in the frame comoving with a repulsive rigid disk ($V = \infty$ inside a disk) is given by [33]

$$u_x - iu_y = v \left(1 - \frac{R^2}{r^2} e^{-2i\theta} \right), \quad (6)$$

where v and R are the velocity and radius of the disk, $r = (x^2 + y^2)^{1/2}$, and $\theta = \arg(x + iy)$. The velocity becomes maximum at the lateral sides of the disk, $(r, \theta) = (R, \pm\pi/2)$, which agree with the locations at which

FIG. 2. Critical velocities for vortex-dipole creation versus beam waist d of a moving red-detuned Gaussian beam. When the velocity of the beam is adiabatically increased, the vortex-dipole creation occurs at v_{cr1} (circles). When the velocity of the beam is adiabatically decreased, the vortex-dipole creation stops at v_{cr2} (squares), and thus the system exhibits hysteresis behavior. The intensity of the Gaussian potential (5) is fixed to $V_0/(gn_0) = -5$.

vortices are nucleated. This is because vortices are nucleated when the velocity exceeds the local Landau critical velocity. Moreover, the density is low where the velocity is large due to Bernoulli's law and a quantized vortex with a density hole is easily nucleated.

For a disk-shaped strong attractive potential (V is a large negative inside a disk), where the density inside the disk n_{in} is much larger than the density outside n_{out} , the velocity inside must vanish in the limit of $n_{in}/n_{out} \rightarrow \infty$ since the normal component of the flow must be continuous across the boundary, $n_{in}u_{in}^\perp = n_{out}u_{out}^\perp$. Therefore, the tangential component of the velocity $u^\parallel$ must vanish at the boundary since it must be continuous. Such velocity field outside the disk is given by

$$u_x - iu_y = v \left(1 + \frac{R^2}{r^2} e^{-2i\theta} \right). \quad (7)$$

The velocity becomes maximum at $r = R$ and $\theta = 0, \pi$, and thus a vortex dipole is nucleated at the head of a potential. Perturbations at the tail of a potential ($r = R$ and $\theta = 0$) do not grow because they go downstream away.

It is interesting to note that the system exhibits hysteresis behavior with respect to the velocity v . When the velocity v of the beam is adiabatically increased from zero, the successive vortex-dipole creation starts at a critical velocity v_{cr1} , which is shown by the circles in Fig. 2. However, when the velocity of the beam is decreased from $v > v_{cr1}$, the vortex-dipole generation does not stop at $v = v_{cr1}$, but at v_{cr2} , a velocity lower than v_{cr1} . Thus, for the velocity $v_{cr2} < v < v_{cr1}$, the system exhibits bistability: the stationary flow remains to be stationary, and the successive vortex-dipole creation is kept once it is triggered by perturbations.

B. Controlled generation of a single vortex dipole

Using a red-detuned Gaussian laser beam, we can launch vortex dipoles in a controlled manner. The initial state is the ground state for the Gaussian potential in Eq. (5) with $V_0 < 0$ and $v = 0$. The potential then starts to move in the $-x$ direction at $t = 0$ and stops at $t = T$ as

$$v = \begin{cases} 0 & (t < 0, t > T) \\ v_0 & (0 \leq t \leq T). \end{cases} \quad (8)$$

Figures 3(a)–3(h) show launching dynamics of a vortex dipole in the rest frame. After the stop of the potential, the created vortex dipole is released from the potential and continues to move in the $-x$ direction. Figures. 3(e)–3(h) show the case of a smaller potential, in which a vortex dipole is bound so tight that the individual vortices cannot be discerned [Fig. 3(h)]. Such a solitonic structure is referred to as a rarefaction pulse [34], which has been shown to be formed by a local depletion [35] and an oscillating potential [27]. Since a vortex dipole propagates

faster for smaller distance l between its vortex and antivortex, the rarefaction pulse in Fig. 3(g), which corresponds to the limit of $l \rightarrow 0$, propagates faster than the well separated ($l \neq 0$) vortex dipole in Fig. 3(c). Figure 3(i) shows the dependence of the velocity v_{vd} of the launched vortex dipole on the beam waist d and the velocity of the potential v_0 . Since the distance l between the created vortex and antivortex increases with an increase in d , the velocity of the vortex dipole is a decreasing function of d . The velocity of the launched vortex dipole v_{vd} is higher for smaller v_0 , since a vortex dipole is launched before it is pushed and l is broadened by the potential.

Vortex dipoles can also be launched by a blue-detuned Gaussian beam as in Ref. [13], where the beam was switched off after the launch of the vortex dipole. For a blue-detuned beam, vortex dipoles are released backward and propagate in the same direction as that of the potential. The vortex dipole must therefore overtake the potential if the potential is stopped. Another difference between the generation of a vortex dipole by a blue-detuned beam and that by a red-detuned beam is that a vortex and antivortex are created at the lateral sides of the potential for a blue-tuned beam, and then a blue-detuned beam is suitable for generating a slow vortex dipole with large l .

IV. VORTEX GENERATION BY MULTIPLE GAUSSIAN BEAMS

A. Wake of double beams and the critical velocity for vortex generation

We next consider the case of multiple Gaussian potentials. For simplicity, we assume that two identical Gaussian potentials move at the same velocity as

$$V = V_0 \sum_{j=1}^2 \exp\left[-\frac{(x - x_j + vt)^2 + (y - y_j)^2}{d^2}\right]. \quad (9)$$

The positions of the potentials are, without loss of generality, $(x_j, y_j) = (\mp \frac{d}{2} \sin \chi, \pm \frac{d}{2} \cos \chi)$ with the upper signs for $j = 1$ and the lower signs for $j = 2$, where the angle χ determines the alignment of the potentials.

Figure 4 shows a typical dynamics for two blue-detuned ($V_0 > 0$) beams with $\chi = 0$. The initial state is the ground state for $v = 0$ and the potentials start to move in the $-x$ direction at $t = 0$. After the initial disturbance, the system exhibits periodic behavior. A vortex dipole is created between the potentials and is released in the wake [circle in the left panel of Fig. 4(a)] with its counterpart remaining in the potentials [topological defects indicated by the circles in the right panel of Fig. 4(a)]. Then, these topological defects remaining in the potential are released from both ends [circles in Fig. 4(c)], and this dynamics is repeated. We note that the small vortex dipoles released from between the potentials propagate in the x direction, whereas the vortices in the outer rows propagate in the $-x$ direction [arrows in Fig. 4(e)].

FIG. 3. (a)–(h) Snapshots of the density $|\psi|^2/n_0$ and phase $\phi = \arg \psi$ profiles in the rest frame. A Gaussian potential with $V_0/(gn_0) = -5$ is moved as in Eq. (8) with $v_0/v_s = 0.47$ and $T/t_0 = 17$. The beam waist is $d/\xi = 3.54$ in (a)–(d) and $d/\xi = 2.67$ in (e)–(h). The arrows in (c) and (g) indicate the traveling direction of the vortex dipoles. The phase profiles (d) and (h) magnify the regions in the dashed squares in (c) and (g). The field of view of (a)–(c) and (e)–(g) is $78\xi \times 33\xi$. (See also Supplemental Material [31].) (i) Velocity of the launched vortex dipole versus beam waist d for $v_0/v_s = 0.47$ (circles), 0.44 (squares), and 0.41 (triangles). The other parameters are the same as those in (a)–(h).

FIG. 4. Dynamics of the density $|\tilde{\psi}|^2 = |\psi|^2/n_0$ and phase $\phi = \arg \psi$ profiles for two Gaussian potentials with $V_0/(gn_0) = 5$, $d/\xi = 3$, $v/v_s = 0.32$, and $D/\xi = 14$ in the frame comoving with the potential. The phase profiles magnify the region in the dashed square. The circle in the left panel of (a) indicates a vortex dipole released from between the potentials, and those in the right panel indicate its counterpart remaining in the potentials. The circles in (c) indicate the counterpart released from the potentials. The arrows in (e) indicate the directions in which the vortices propagate. The field of view of the density profiles is $160\xi \times 67\xi$. (See also Supplemental Material [31].)

In order to understand the behavior shown in Fig. 4, we consider the irrotational flow of an inviscid and incompressible fluid around two rigid disks with radius $R < D/2$, which is located at $(x, y) = (\mp \frac{D}{2} \sin \chi, \pm \frac{D}{2} \cos \chi)$. The velocity field is given by [36–38]

$$u_x - iu_y = \frac{4v \sinh^2 \beta}{(z/R)^2 + \sinh^2 \beta} \left[e^{i\chi} \wp(i\tau - i\beta) + e^{-i\chi} \wp(i\tau + i\beta) + \frac{2}{\pi} \zeta(\pi) \cos \chi \right], \quad (10)$$

where $\beta = \cosh^{-1}[D/(2R)]$, $z = x + iy$, $\tau = \beta + \log(ie^{-i\chi}z/R + \sinh \beta) - \log(ie^{-i\chi}z/R - \sinh \beta)$, and $\wp$ and ζ are Weierstrass's elliptic functions with half periods π and $i \cosh^{-1}[(D^2/R^2 - 2)/2]$. $u_x - iu_y$ approaches v for $|z| \rightarrow \infty$. For $\chi = 0$, which corresponds to the case in Fig. 4, the velocity field in Eq. (10) is maximum at the inner edges of the disks, which we denote as $u_{\max} \equiv u_x(x=0, y = \pm D/2 \mp R)$. The contour plot in Fig. 5

FIG. 5. Contour lines show the maximum flow velocity $u_{\max}$ as a function of D/R and v/v_s , obtained from Eq. (10). The contour lines indicate $u_{\max}/v_s = 0.05, 0.1, 0.15, 0.2, \dots$, from below. The plots show the critical velocity v_{cr} for vortex-dipole nucleation versus D/R_{eff} , which is obtained by simulations as in Fig. 4. Here R_{eff} is the radius of the density hole produced by the potential. The beam waist and the effective radius are $(d/\xi, R_{\text{eff}}/\xi) = (5, 9.5)$ for the circles and $(10, 18)$ for the squares.

shows $u_{\max}$ as a function of the distance D between the disks and the unperturbed velocity v . The maximum velocity $u_{\max}$ monotonically decreases with an increase in D and linearly increases with an increase in v .

In Fig. 5, we superimpose the plots of the critical velocity v_{cr} for vortex-dipole creation, which is obtained from the GP equation, Eq. (4), numerically. In the time evolution, we adiabatically increase the velocity v of the potential from $v = 0$ until a vortex dipole is nucleated as in Fig. 4. The point of nucleation determines the critical velocity v_{cr} . From Fig. 5, we find that the plots roughly fit the contour lines, implying that a vortex dipole is nucleated when the maximum flow velocity between the disks exceeds a local critical velocity. The deviation of the plots from the lines in Fig. 5 indicates that the local critical velocity weakly depends on D .

B. Controlled generation of a single vortex dipole

Using two blue-detuned Gaussian beams, we can launch a vortex dipole, as in the case of a red-detuned beam in Sec. III B. The initial state is the ground state in the presence of two Gaussian potentials (9) with $V_0 > 0$, $v = 0$, and $\chi = 0$. The potentials are then displaced as in Eq. (8). Figures 6(a)–6(c) show the launching dynamics in the rest frame. A vortex dipole is generated between the potentials and moves in the direction opposite to that of the displacement of the potentials. A vortex dipole is thus launched by “pulling” the potential, which is in contrast to the case of a red-detuned beam, in which a vortex dipole is launched by “pushing” the potential. When a vortex-antivortex pair is created in each potential, one vortex of each vortex-antivortex pair will form a new vortex dipole and be launched. After the launch, the counterpart of the launched vortex dipole remains at the potential [dashed circle in Fig. 6(d)], which prevents the successive launch.

FIG. 6. (a)–(c) Dynamics of the density profiles $|\psi|^2/n_0$ for two Gaussian potentials with $V_0/(gn_0) = 5$, $d/\xi = 4.5$, and $D/\xi = 22$ in the rest frame. The potentials are moved as in Eq. (8) with $v_0/v_s = 0.24$ and $T/t_0 = 150$. The arrows in (a) and (c) indicate the direction of the displacement of the potentials and that of the propagation of the vortex dipole, respectively. (d) Phase profile $\arg\psi$ at $\tilde{t} = t/t_0 = 140$. The topological defects in the dashed circle are the counterpart of the launched vortex dipole. The field of view of each panel is $90\xi \times 65\xi$. (See also Supplemental Material [31].) (e) Velocity of the launched vortex dipole versus distance D between the two Gaussian potentials. The parameters except D are the same as those in (a)–(d).

By contrast, the successive launch is possible for a red-detuned beam.

Figure 6(e) shows the dependence of the velocity v_{vd} of a launched vortex dipole on the distance D between the two Gaussian potentials. The velocity v_{vd} decreases with an increase in D , since the distance l between the launched vortex and antivortex increases.

C. Trapping of a vortex dipole

We consider the dynamics for two red-detuned beams as shown in Fig. 7. The initial state is the ground state for two Gaussian potentials in Eq. (9) with $V_0 < 0$, $v=0$, and $\chi=0$ [Fig. 7(a)] and the potentials start to move in the $-x$ direction at $t=0$. Initially, the sudden increase in the velocity disturbs the system and several vortex dipoles are generated near the potential [Fig. 7(b)]. The vortex generation takes place only at the beginning, and the generated vortex dipoles depart from the potential, except for the one between the potentials. Interestingly, this vortex dipole actually is trapped between the potentials and oscillates around the center in the moving frame [Figs. 7(c)–7(e)].

FIG. 7. (a)–(e) Dynamics of the density $|\tilde{\psi}|^2 = |\psi|^2/n_0$ profiles for the potential in Eq. (9) with $V_0/(gn_0) = -5$, $v/v_s = 0.6$, $d/\xi = 4.5$, and $D/\xi = 31.6$ in the frame comoving with the potential. The field of view is $160\xi \times 67\xi$. (f) The phase profile $\phi = \arg\psi$ of the region in the dashed square in (e). (See also Supplemental Material [31].)

In order to understand the trapping of a vortex dipole between moving potentials as shown in Fig. 7, we perform a linear-stability analysis assuming inviscid, incompressible, and irrotational flow. We consider the problem in the frame comoving with the potentials. Two point vortices with circulations $h/m \equiv \kappa$ and $-\kappa$ are located at $(x_{\pm}, y_{\pm}) = (\xi_{\pm}, l/2 \mp \eta_{\pm})$, where $\xi_{\pm}$ and $\eta_{\pm}$ are small deviations from the stationary state and l is the distance between the vortex and antivortex for the stationary state. Equations of motion for the deviations are given by

$$\dot{\xi}_{\pm} = u_x(x_{\pm}, y_{\pm}) \mp u_x^v(x_{\pm}, y_{\pm}; x_{\mp}, y_{\mp}) \equiv U_x^{\pm}(x_{\pm}, y_{\pm}; x_{\mp}, y_{\mp}), \quad (11a)$$

$$\dot{\eta}_{\pm} = u_y(x_{\pm}, y_{\pm}) \mp u_y^v(x_{\pm}, y_{\pm}; x_{\mp}, y_{\mp}) \equiv U_y^{\pm}(x_{\pm}, y_{\pm}; x_{\mp}, y_{\mp}), \quad (11b)$$

where $\mathbf{u}$ is the velocity field without the point vortices and $\mathbf{u}^v(x_1, y_1; x_2, y_2)$ is the velocity field at (x_1, y_1) produced by a point vortex with circulation κ located at (x_2, y_2) . We impose the same boundary condition as in Eq. (7) at $x^2 + (y \pm D/2)^2 = R^2$. The velocity field without the point vortices is given by

$$u_x - iu_y = \frac{4v\sinh^2\beta}{(z/R)^2 + \sinh^2\beta} [\varphi(i\tau - i\beta) - \varphi(i\tau + i\beta)], \quad (12)$$

where β and τ are defined in Sec. IV A. The velocity field at (x_1, y_1) produced by a point vortex located at (x_2, y_2) has the form [39]

$$u_x^v - iu_y^v = -\frac{\kappa}{2\pi Ri} \frac{2\sinh\beta}{(z_1/R)^2 + \sinh^2\beta} \left[\zeta(i\tau_1 - i\tau_2) + \zeta(i\tau_1 - i\tau_2^*) - \zeta(i\tau_1 - i\beta) - \zeta(i\tau_1 + i\beta) - \frac{2\zeta(\pi)}{\pi} \text{Im}\tau_2 \right], \quad (13)$$

where $z_j = x_j + iy_j$ and $\tau_j = \beta + \log(iz_j/R + \sinh\beta) - \log(iz_j/R - \sinh\beta)$ with $j = 1, 2$. In the limit of $D \rightarrow \infty$,

FIG. 8. Stability of a trapped vortex dipole with respect to the distance D between the potentials and the normalized velocity $\tilde{v} = 2\pi Rv/\kappa$. When λ in Eq. (15) is real, the system is dynamically unstable. The contour plot shows $2\pi R^2|\lambda|/\kappa$ where λ is purely imaginary. The circle corresponds to the case in Fig. 7.

Eq. (13) reduces to the velocity field produced by a point vortex in a uniform system $u_x^v - iu_y^v = \kappa/[2\pi i(z_1 - z_2)]$. In order for the point vortices to be stationary for $\xi_{\pm} = \eta_{\pm} = 0$, we set $\mathbf{U}^{\pm} = 0$ at $(x_{\pm}, y_{\pm}) = (0, \mp l/2)$, which determines l for given v , D , and R . Substituting Eqs. (12) and (13) into Eq. (11) and expanding the velocity fields up to the first order of $\xi_{\pm}$ and $\eta_{\pm}$, we have

$$\dot{\xi}_{\pm} = \mp b\eta_{\pm} \pm c\eta_{\mp}, \quad (14a)$$

$$\dot{\eta}_{\pm} = \mp b(\xi_{\pm} - \xi_{\mp}), \quad (14b)$$

where $b = -\partial_{y_1} U_x^+(x_1, y_1; x_2, y_2)$ and $c = \partial_{y_2} U_x^+(x_1, y_1; x_2, y_2)$ at $(x_1, y_1) = (0, -l/2)$ and $(x_2, y_2) = (0, l/2)$. Equations (14a) and (14b) have the solutions of the form $\xi_j, \eta_j \propto e^{\lambda t}$ and the eigenvalues λ are obtained as

$$\lambda = \pm \sqrt{2(b^2 - bc)}. \quad (15)$$

If the eigenvalues are purely imaginary, the solutions are oscillatory and the two point vortices are stable. If the eigenvalues are real, the mode with a positive eigenvalue grows exponentially and the system is dynamically unstable. For Fig. 7, the effective radius of the potentials is $R_{\text{eff}}/\xi \approx 8$ and the oscillation frequency ω of the trapped vortex dipole is $2\pi R_{\text{eff}}^2 \omega/\kappa \approx 2.4$. This is in good agreement with the value of Eq. (15), $2\pi R^2 \lambda/\kappa \approx 2.37$.

Figure 8 shows a contour plot of $|\lambda|$ in Eq. (15) for the stable region. The parameters used in Fig. 7 correspond to the circle in Fig. 8. From Fig. 8, we find that $|\lambda|$ increases with an increase in v and the trapping is tight for large v . However, if v exceeds the critical velocity, vortex dipoles are successively generated from the potentials as in Fig. 1, which disturbs the trapping. Therefore, there is an appropriate region of D and v for the most efficient trapping.

V. MANIPULATION OF VORTEX DIPOLES

We next show that the velocity and trajectory of a launched vortex dipole can be changed after the launch.

We use a static Gaussian potential to curve trajectories of vortex dipoles, and use a time-dependent Gaussian potential to accelerate and decelerate them. Figure 9 shows stroboscopic images of such dynamics. A vortex dipole is launched in the same manner as in Fig. 6. In Fig. 9(a), a weak repulsive Gaussian potential is located near the trajectory of the vortex dipole and the trajectory bends toward the potential. In Fig. 9(b), the obstacle potential is attractive and the trajectory bends in the opposite direction. In Fig. 9(c), a repulsive Gaussian potential is switched on at the moment that the vortex dipole passes through, which accelerates the vortex dipole. In Fig. 9(d), on the other hand, an attractive potential is switched on and the vortex dipole is decelerated.

The behaviors in Figs. 9(a)–9(d) can be understood by the Magnus effect. The Magnus force on each vortex is given by $\mathbf{F}_M = m\mathbf{n}\boldsymbol{\kappa} \times \Delta\mathbf{v}_v$, where $\boldsymbol{\kappa}$ is the vector of circulation and $\Delta\mathbf{v}_v$ is the velocity of a vortex relative to the unperturbed velocity of the vortex dipole. When force $\mathbf{F}$ is exerted on a vortex, the velocity changes in such a way that $\mathbf{F}$ balances with $\mathbf{F}_M$, and hence

$$\Delta\mathbf{v}_v = \frac{1}{m\mathbf{n}\kappa^2} \boldsymbol{\kappa} \times \mathbf{F}. \quad (16)$$

In Fig. 9, the origin of the force is the density gradient produced by the obstacle Gaussian potential. The energy of

FIG. 9. Stroboscopic images of the density profile $|\psi|^2/n_0$ in the rest frame. A vortex dipole is launched by the two Gaussian potentials as in Fig. 6 with $V_0/(gn_0) = 5$, $d/\xi = 4.5$, and $D/\xi = 19$. The stroboscopic images are taken at time intervals of $\Delta t/t_0 = 20$. (a) A Gaussian potential with $V_0/(gn_0) = 1$ and $d/\xi = 4.5$ is located at $y = 1.6$. (b) A Gaussian potential with $V_0/(gn_0) = -1$ and $d/\xi = 4.5$ is located at $y = 1.6$. (c) A Gaussian potential with $V_0/(gn_0) = 1$ and $d/\xi = 4.5$ is switched on at $t/t_0 = 330$. (d) A Gaussian potential with $V_0/(gn_0) = -1$ and $d/\xi = 4.5$ is switched on at $t/t_0 = 330$. The field of view of each panel is $224\xi \times 80\xi$. (See also Supplemental Material [31].) (e), (f) Schematic illustrations for explaining the Magnus effect.

a vortex is roughly given by $E \sim \pi \hbar^2 n/m$ and the force on a vortex is $\mathbf{F} \sim -\pi \hbar^2 \nabla n/m$, which is in the direction opposite to the density gradient. Two examples of the effect of density gradient on a vortex dipole are shown in Figs. 9(e) and 9(f). When the density gradient is perpendicular to the traveling direction of a vortex dipole [rightward for Fig. 9(e)], its trajectory is curved toward the direction of $-\nabla n$, which explains the dynamics in Figs. 9(a) and 9(b). When a vortex dipole propagates in the direction of ∇n as in Fig. 9(f), the distance between the two vortices decreases and the vortex dipole accelerates. This corresponds to the situation in Fig. 9(c), where a density dip suddenly appears just behind the vortex dipole. The deceleration in Fig. 9(d) can also be understood in a similar manner.

Using the method to launch vortex dipoles, we demonstrate collisions between two counter-propagating vortex dipoles. The two vortex dipoles are launched in the $\pm x$ directions by red-detuned Gaussian beams as in Fig. 3. Figures 10(a) and 10(b) show collisions between two vortex dipoles launched with the same velocity. When the “impact parameter” σ is small, the vortices in the two vortex dipoles [1, 2 and 3, 4 in Fig. 10(a)] exchange places with each other, and the two vortex dipoles with new pairs [1, 3 and 2, 4 in Fig. 10(a)] go away with the same scattering angle. The scattering angle is $\pi/2$ for a frontal collision ($\sigma = 0$, data not shown). In Fig. 10(b), the impact parameter is large and the two vortex dipoles pass each other without a vortex exchange. Figure 10(c) shows the collision between vortex dipoles with different velocities, where the vortex exchange occurs and the new vortex dipoles scatter with scattering

angles φ and $\pi - \varphi$. The behavior in Fig. 10(c) is understood from the momentum conservation. Since a slower vortex dipole (larger distance between vortices) has a larger momentum [24], the total momentum before the collision is in the $+x$ direction [left panel in Fig. 10(c)]. This is the reason why the two vortex dipoles scatter slightly rightward [right panel in Fig. 10(c)]. A more complicated collision dynamics is presented in Supplemental Material [31], where two vortex dipoles turn around each other. Collisions between point vortex dipoles in classical fluids have been studied in Refs. [40,41].

VI. GENERATION AND MANIPULATION OF VORTEX DIPOLES IN A THREE-DIMENSIONAL TRAP

We have so far considered 2D infinite systems. In this section, in order to examine the experimental reality, we perform full 3D numerical simulations of a trapped system. We assume that $N = 2 \times 10^6$ ^{87}Rb atoms are confined in an axisymmetric trapping potential $V_{\text{trap}} = m[\omega_{\perp}^2(x^2 + y^2) + \omega_z^2 z^2]/2$ with frequencies $(\omega_{\perp}, \omega_z) = 2\pi \times (8, 200)$ Hz. The s -wave scattering length is assumed to be $a = 100a_B$ with a_B being the Bohr radius. To suppress the disturbance in the finite system, the Gaussian potentials for generating vortex dipoles are gradually accelerated and decelerated as

$$V_G(x, y, t) = V_0 \sum_j \exp\left\{-\frac{[x - X(t)]^2 + (y - y_j)^2}{d^2}\right\}, \quad (17a)$$

$$X(t) = \begin{cases} x_0 + A \sin^2 \frac{\pi t}{2T} & (0 \leq t \leq T) \\ x_0 + A & (t > T), \end{cases} \quad (17b)$$

where x_0 is the initial x coordinate of the potentials and A is their displacement. For a single Gaussian beam, $y_1 = 0$, and for a double beam, $y_1 = D/2$ and $y_2 = -D/2$. The initial state is the ground state for $V = V_{\text{trap}} + V_G(x, y, 0)$.

Figure 11 demonstrates the launch of a vortex dipole and its manipulation. In Fig. 11(a), a vortex dipole is launched by a red-detuned Gaussian laser beam using the method in Fig. 3. The launched vortex dipole propagates in the $-x$ direction, which is curved by a blue-detuned beam in a manner similar to Fig. 9(a). The distance between the pair of vortices increases as it approaches the edge of the condensate due to the Magnus effect [Fig. 9(f)]. The trajectory of a vortex dipole can also be curved by a red-detuned beam as in Fig. 9(b) (data not shown). Figure 11(b) shows the launching of a vortex dipole by two blue-detuned beams using the method in Fig. 6. After the launch of the vortex dipole, a red-detuned beam is suddenly switched on at the moment when the vortex dipole passes through, and the vortex is decelerated by the same mechanism as in Fig. 9(d). The acceleration indicated in Fig. 9(c) is also possible (data not shown). Thus, the generation and manipulation of vortex dipoles are experimentally feasible using a pancake-shaped BEC.

FIG. 10. Collisions between vortex dipoles launched by the method in Fig. 3. The directions in which the vortex dipoles propagate are indicated by the arrows. The numbers identify the vortices. (a) Collision with the impact parameter $\sigma/\xi = 3.16$. Vortex exchange occurs. (b) Collision with $\sigma/\xi = 6.32$. The two vortex dipoles pass each other. (c) Collision of vortex dipoles with different velocities. The beam waist is $d/\xi = 3.54$ for the left-hand side in (c) and $d/\xi = 3.16$ for the others. The other parameters are the same as those in Fig. 3. The field of view of each panel is $78\xi \times 33\xi$. (See also Supplemental Material [31].)

FIG. 11. Stroboscopic images of the column density showing generation and manipulation of a vortex dipole in a 3D trap. (a) A vortex dipole is launched by a red-detuned beam using the method in Fig. 3 and curved using the method in Fig. 9(a). (b) A vortex dipole is launched by a blue-detuned beam using the method in Fig. 6 and decelerated using the method in Fig. 9(d). The launching parameters in Eq. (17) are $V_0 = -200\hbar\omega_\perp$, $d = 1.5 \mu\text{m}$, $x_0 = 20 \mu\text{m}$, $A = -7.5 \mu\text{m}$, and $T = 10 \text{ ms}$ in (a), and $V_0 = 200\hbar\omega_\perp$, $d = 1.5 \mu\text{m}$, $D = 7 \mu\text{m}$, $x_0 = 10 \mu\text{m}$, $A = 4.5 \mu\text{m}$, and $T = 10 \text{ ms}$ in (b). In (a) a repulsive Gaussian potential with $V_0 = 200\hbar\omega_\perp$ and $d = 1.5 \mu\text{m}$ is located at $(x, y) = (7, 4.5) \mu\text{m}$, and in (b) an attractive Gaussian potential with $V_0 = -50\hbar\omega_\perp$ and $d = 1.5 \mu\text{m}$ is switched on at $(x, y) = (7, 0) \mu\text{m}$ at $t = 37 \text{ ms}$. The stroboscopic images are taken at time intervals of 5 ms in (a) and 8 ms in (b). The unit of the density is $1.4 \times 10^{13} \text{ cm}^{-2}$. The field of view is $80 \times 80 \mu\text{m}$. (See also Supplemental Material [31].)

VII. CONCLUSIONS

We have investigated dynamics of a 2D BEC stirred by far-off-resonance Gaussian laser beams. When a red-detuned beam (attractive potential) moves through the BEC exceeding a critical velocity, a vortex dipole is nucleated at the head of the moving potential (Fig. 1). This is in contrast to the case where a blue-detuned beam is used, in which a vortex dipole is generated on both sides of the potential. The vortex shedding from a red-detuned beam exhibits the hysteresis behavior with respect to the velocity (Fig. 2). When a red-detuned beam is displaced, a created vortex dipole propagates toward the direction of the displacement. Such a beam can thus be used as a “vortex-dipole launcher” (Fig. 3). When two repulsive potentials are moved, a vortex dipole is generated between the potentials, which propagates in the direction opposite to the motion of the potential (Fig. 4). A vortex dipole can also be launched by two blue-detuned beams (Fig. 6). The velocity of vortex dipoles can be controlled in both launching methods. For two red-detuned beams, we have found that a vortex dipole is trapped between two moving potentials (Fig. 7). Launched vortex dipoles can be manipulated by an external potential (Fig. 9). A trajectory of a vortex dipole is curved by a Gaussian potential located near the trajectory. A Gaussian potential with time-dependent intensity located on the trajectory of a vortex dipole can change its velocity. The full 3D numerical simulations of a system confined in a tight pancake-shaped trap have shown that the proposed phenomena can be realized in experiments (Fig. 11).

We have thus shown that we can create and launch vortex dipoles in a controlled manner and also manipulate their trajectories and velocities after the launch. These are achieved only by Gaussian laser beams. This technique enables us to investigate a variety of dynamics of quantized vortices in superfluids, such as collisions of vortex dipoles (Fig. 10). The controlled generation and manipulation of vortex dipoles proposed in the present paper increase the range of experiments and lead to a precise understanding of dynamical properties of vortex dipoles.

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