Regenerative multipass beam breakup in two dimensions

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In this paper, a formula, describing the threshold of regenerative multipass beam breakup (BBU) for a single dipole higher-order mode with arbitrary polarization in a two-pass accelerator with a general-form, 4×4 recirculation matrix, is derived. A new two-dimensional BBU code is also introduced. To illustrate specifics of the BBU in two dimensions, the formula is used to calculate the threshold in several cases including two-dimensional uncoupled optics, reflecting optics, and rotating optics. The analytical results are compared to results of simulations obtained with the new code. A mathematical relation between transfer matrices between cavities of the accelerating structure and recirculation matrices for each cavity, which must be satisfied in order to successfully suppress the BBU by reflection or rotation in several cavities, is presented.

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I. INTRODUCTION

Regenerative multipass BBU arises from interaction of an intense recirculated beam with higher-order modes (HOM) of the accelerating structures. In a two-pass machine, the beam deflected off axis by the HOM magnetic field on the first pass comes back to the cavity with a displacement on the second pass. The recirculated beam induces an HOM voltage, depending on the magnitude and direction of the beam displacement. Thus, the beam constitutes a feedback that can become unstable if the beam current reaches the threshold. In the JLab FEL upgrade [1], the regenerative, multipass BBU was observed at a beam current of approximately 5 mA. Because BBU imposes an immediate threat to operations of the machine, suppression of regenerative BBU is a task of paramount importance.

As shown by Rand and Smith in [2], the threshold of regenerative BBU strongly depends on a form of the 4×4 recirculation matrix. In their paper, the authors investigated the possibility of suppressing the BBU by means of reflection or 90° rotation of the beam displacement on the second pass. In the summer of 2004, an optical reflector was installed in the recirculation pass of the JLab FEL upgrade. The insertion consists of 5 skew quadrupoles and reflects the beam displacement respective to the $x = y$ plane [3]. The first experience with the reflector showed an increase of the BBU threshold current from 5 mA to a level above 8.5 mA. The maximum current was limited by the injector performance at the time of the experiment.

This paper is aimed at analytical and numerical studies of two-dimensional aspects of regenerative BBU. In Sec. II, a formula for the BBU threshold for a single dipole HOM with arbitrary polarization in a two-pass machine with a general-form, 4×4 recirculation matrix is derived. Section II also contains formulas describing evolution of the HOM voltage above and below the threshold. To illus-

trate specifics of BBU in two dimensions, the threshold formula is used to calculate the starting current of the instability in several simple cases including uncoupled recirculator optics, reflecting optics, and 90° -rotating optics. These results are presented in Sec. III and compared to simulation results obtained with a new two-dimensional BBU tracking code which is briefly described in the appendix.

To suppress BBU by optical transformations in a machine consisting of several or many accelerating cavities, the recirculation matrix for each cavity has to be of a form that provides effective BBU suppression. To preserve the form of the recirculation matrix from cavity to cavity, transfer matrices between the cavities have to satisfy a condition that is discussed in Sec. IV.

II. BBU THRESHOLD IN TWO-DIMENSIONAL SYSTEMS AND HOM VOLTAGE BEHAVIOR ABOVE AND BELOW THE THRESHOLD

A. Single-mode approximation

Simulation results and experimental data presented elsewhere (see, for example, [4,5]) suggest that dipole TM HOMs with a quality factor of 10^5 – 10^6 will limit the maximum recirculated current to the order of tens to hundreds of milliamps. Assuming a quality factor of the order of 10^5 – 10^6 and a typical HOM frequency of the order of 2 GHz, one concludes that a typical bandwidth of dangerous HOMs is of the order of 1 to 10 kHz. Interference between HOMs can be neglected and each mode can be considered independently if the bandwidth of dangerous modes is smaller than the separation between frequencies of the HOMs.

Imperfections and asymmetrical features of cavities such as couplers violate two-dimensional symmetry of cavity geometry and split frequencies of degenerate dipole modes. A typical separation between two orthogonal polarizations of the same dipole mode in a superconducting rf

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cavity is of the order of several hundred kHz to several MHz. Besides, imperfections cause the frequency of HOMs to vary from cavity to cavity. The variation of the HOM frequency from cavity to cavity is also of the order of several hundred kHz to several MHz.

The probability of overlapping of HOM frequencies is small if the number of cavities is significantly smaller than the ratio of the spread of HOM frequencies to the typical bandwidth of the modes. For example, assuming an HOM bandwidth of 10 kHz and a frequency spread of a few MHz, one can consider modes separately if the number of cavities is of the order of a few tens.

B. Two-dimensional formula for the BBU threshold

A bunch traveling through a cavity off axis interacts with cavity dipole HOMs. It changes the voltage of the HOMs and the energy stored in the modes. Steering and misalignment errors lead to a constant offset of the beam centroid relative to the cavity axis on the first pass. On the second pass, the transverse beam displacement consists of two terms: a constant offset due to steering/misalignment errors and an oscillatory term induced by the HOM voltage on the first pass. The constant offset induces the HOM voltage, which is always limited in amplitude. The oscillatory term, on the contrary, constitutes a feedback that can cause regenerative beam breakup and, in theory, an infinite growth of the HOM voltage.

In calculating the threshold of the regenerative BBU, we treat the voltage induced by the oscillatory term on the second pass as a perturbation and assume that the HOM voltage is a harmonic function of time. In a one-dimensional case, when the bunch displacement x and the vector of the HOM polarization are collinear, a variation of the energy stored in a dipole TM HOM produced by a pointlike bunch, ΔU , is given, to the first order in the bunch charge, by [6]

$$\Delta U = -qV_a \cos(\phi) \frac{x}{a}, \quad (1)$$

where V_a is the accelerating HOM voltage at the radius of the beam pipe a induced by previous bunches, ϕ is the phase of the pointlike bunch with respect to the peak of the HOM electric field, and q is the bunch charge.

In two dimensions, the projection of the beam displacement on the vector of mode polarization, $\vec{n} = \cos(\alpha)\hat{x} + \sin(\alpha)\hat{y}$, has to be substituted for the transverse displacement x . Therefore,

$$x \rightarrow \vec{d} \cdot \vec{n} = x \cos(\alpha) + y \sin(\alpha), \quad (2)$$

where $\vec{d}$ is the displacement vector (x, y) and α is the mode polarization angle in the xy coordinate system. The energy deposited by the bunch in the HOM on the first and second passes can be written as

$$\Delta U_1 = -q \frac{V_a}{a} \cos(\phi) [x_1 \cos(\alpha) + y_1 \sin(\alpha)], \quad (3)$$

$$\Delta U_2 = -q \frac{V_a}{a} \cos(\phi + \omega T_r) [x_2 \cos(\alpha) + y_2 \sin(\alpha)], \quad (4)$$

where T_r is the recirculation time. Here, subscripts 1 and 2 denote the first and second passes, respectively. It is assumed in (3) and (4) that the voltage amplitude V_a does not change significantly during recirculation. As will be shown later, a limited range of beam current values where this assumption is valid always exists around the threshold current. Exactly at threshold, V_a is constant. Coordinates of the recirculated bunch, x_2 and y_2 , can be expressed via the bunch coordinates before deflection and the HOM accelerating voltage on the first pass as

$$x_2 = m_{11}x + m_{12}x'_1 + m_{13}y_1 + m_{14}y'_1 - \frac{qV_a}{\omega a p} \sin(\phi) [m_{12} \cos(\alpha) + m_{14} \sin(\alpha)], \quad (5)$$

$$y_2 = m_{31}x + m_{32}x'_1 + m_{33}y_1 + m_{34}y'_1 - \frac{qV_a}{\omega a p} \sin(\phi) [m_{32} \cos(\alpha) + m_{34} \sin(\alpha)], \quad (6)$$

where p is the beam momentum. In the last two equations, the deflection angle produced by the HOM was expressed via the accelerating HOM voltage, V_a , and its phase as [6]

$$x' = -\frac{qV_a}{\omega a p} \sin(\phi) \cos(\alpha), \quad (7)$$

$$y' = -\frac{qV_a}{\omega a p} \sin(\phi) \sin(\alpha). \quad (8)$$

The average power deposited by the beam in the HOM is equal to the average energy deposited by individual bunches times the bunch repetition frequency:

$$\dot{U}_{\text{beam}} = \langle \Delta U_1 + \Delta U_2 \rangle \cdot f_b. \quad (9)$$

The averaging is done, respectively, to the phase of the HOM, ϕ , taken at moments when bunches pass through the cavity on the first pass. Ohmic losses in the cavity can be expressed as [6]

$$P_c = \frac{V_a^2}{(\frac{\omega}{c})^2 a^2 (\frac{R_a}{Q}) Q}. \quad (10)$$

The energy balance equation for the HOM stored energy is

$$\dot{U} = \dot{U}_{\text{beam}} - P_c = \langle \Delta U_1 + \Delta U_2 \rangle \cdot f_b - P_c. \quad (11)$$

Terms proportional to $\cos(\phi)$, $\sin(\phi)$, $\cos(\phi + \omega T_r)$, and $\sin(\phi + \omega T_r)$ yield zero after averaging if x_1 , x'_1 and y_2 , y'_2 in (5) and (6) are steering/misalignment errors. If the HOM frequency is not equal to a harmonic of the bunch repetition rate, terms proportional to $\cos(\phi) \cdot \sin(\phi)$ also yield

zero and the average value of the $\sin^2(\phi)$ is equal to $1/2$. Thus, Eq. (11) can be rewritten as

$$\begin{aligned}\dot{U} &= I_b \frac{qV_a^2}{\omega p a^2} m_{12}^* \langle \sin(\phi) \cos(\phi + \omega T_r) \rangle \\ &\quad - \frac{V_a^2}{(\omega/c)^2 a^2 (R_d/Q) Q} \\ &= -I_b \frac{qV_a^2}{\omega p a^2} m_{12}^* \frac{\sin(\omega T_r)}{2} - \frac{V_a^2}{(\omega/c)^2 a^2 (R_d/Q) Q},\end{aligned}\quad (12)$$

where

$$\begin{aligned}m_{12}^* &= m_{12} \cos^2(\alpha) + (m_{14} + m_{32}) \sin(\alpha) \cos(\alpha) \\ &\quad + m_{34} \sin^2(\alpha).\end{aligned}\quad (13)$$

At threshold, $\dot{U}$ is equal to zero. Thus, the threshold current is given by

$$I_{th} \frac{m_{12}^*}{2p} \frac{q}{\omega} \sin(\omega T_r) + \frac{1}{(\omega/c)^2 (R_d/Q) Q} = 0, \quad (14)$$

which yields the threshold current as

$$I_{th} = - \frac{2pc}{q \frac{\omega}{c} (R_d/Q) Q m_{12}^* \sin(\omega T_r)}. \quad (15)$$

Precise knowledge of the phase ϕ in (12) is required to find the threshold if the HOM frequency is equal to a harmonic of the bunch repetition rate. Because the presented method does not provide this information, in the case of a resonance, one has to calculate the phase, using other methods, or proceed, using computer simulations. Note that the resonance between potentially dangerous HOMs and beam harmonics should be avoided by a proper choice of cavity and beam parameters at the design stage.

For negative values of the product $m_{12}^* \sin(\omega T_r)$, the threshold given by (15) perfectly agrees with simulation results presented later in this paper. Equation (15) yields absolute beam stability if $m_{12}^* \sin(\omega T_r)$ is positive. The simulations results, however, show that the beam can be unstable at extremely high values of the beam current even if $m_{12}^* \sin(\omega T_r) > 0$. The ratio of the threshold in the “pseudostable” region to the threshold for $m_{12}^* \sin(\omega T_r) < 0$ is of the order of 10^3 – 10^4 , corresponding to strong, but not complete, suppression of BBU. This discrepancy is caused by the assumption that the voltage induced by the beam on the second pass is a small perturbation to the HOM voltage, which fails at high beam intensities. In one-dimensional cases, similar dependence of the threshold on $m_{12} \sin(\omega T_r)$ was predicted analytically and observed in simulations in early works by Krafft, Bisognano, and Laubach [7], Sereno [8], and later by Hoffstaetter and Bazarov [9].

C. Evolution of the HOM voltage below and above the threshold

Equation (12) can be used to describe evolution of the HOM voltage below and above the threshold. Because the energy U stored in the HOM can be expressed via the accelerating voltage V_a/a as [6]

$$U = \frac{V_a^2}{a^2 (\frac{\omega}{c})^2 (R_d/Q)}, \quad (16)$$

Equations (12) and (16) yield an equation for the HOM stored energy

$$\frac{dU}{U} = -dt \frac{\omega}{Q} \frac{I_{th} - I}{I_{th}}. \quad (17)$$

The solution of the last equation is

$$U = U_0 \exp\left(-t \frac{\omega}{Q} \frac{I_{th} - I}{I_{th}}\right). \quad (18)$$

The HOM voltage depends on time as

$$V = V_0 \exp\left(-t \frac{\omega}{2Q} \frac{I_{th} - I}{I_{th}}\right). \quad (19)$$

Equations (18) and (19) describe the evolution of the stored energy and HOM voltage below and above the threshold. Using Eqs. (18) and (19), one can introduce the effective quality factor Q_{eff} for the beam-HOM system, given by the equation

$$Q_{eff} = Q \frac{I_{th}}{I_{th} - I}. \quad (20)$$

At zero current, Q_{eff} is equal to the HOM quality factor Q . As the beam current increases, Q_{eff} becomes larger and turns into infinity at the threshold. That is, oscillations of the HOM voltage and beam coordinates do not decay. If the beam current exceeds the threshold current, the beam-HOM system becomes unstable. The amplitude of oscillations grows exponentially. Equation (20) can be also rewritten in terms of the decay time as

$$\tau_{eff} = \tau \frac{I_{th}}{I_{th} - I}. \quad (21)$$

Above the threshold, the decay time given by (21) changes its sign and becomes the growth time of the instability.

Equations (18)–(21) are also valid for negative values of the threshold current given by (15) for $m_{12}^* \sin(\omega T_r) > 0$ as long as the beam current is not too high and the assumptions made during the derivation of (15) are valid. For negative values of the BBU threshold given by (15), Equations (20) and (21) can be rewritten as

$$Q_{eff} = Q \frac{-|I_{th}|}{-|I_{th}| - I} = Q \frac{|I_{th}|}{|I_{th}| + I}, \quad (22)$$

$$\tau_{\text{eff}} = \tau \frac{|I_{\text{th}}|}{|I_{\text{th}}| + I}. \quad (23)$$

The last two equations show that for $m_{12}^* \sin(\omega T_r) > 0$ the effective quality factor and the decay time of beam oscillations decrease with the beam current.

Equations (18) and (19) were derived within the assumption that the recirculation time is much smaller than the instability decay/growth time, τ_{eff} . This assumption is valid if

$$\frac{|\delta I|}{I_{\text{th}}} \ll \frac{\tau}{T_r}, \quad (24)$$

where δI is the difference between the threshold current and the beam current, $I_{\text{th}} - I$. Assuming an HOM decay time, τ , of the order of $10^{-5} - 10^{-4}$ sec, which corresponds to an HOM frequency of 2 GHz and a quality factor Q of the order of $10^5 - 10^6$, and a kilometer-long recirculation path, one can easily calculate that the current range where (18) and (19) are valid is approximately equal to a doubled value of the threshold current.

III. EXAMPLES OF REGENERATIVE BBU IN TWO-DIMENSIONAL SYSTEMS

To illustrate specifics of regenerative BBU in two dimensions, Eq. (15) was used to calculate the BBU threshold in a two-pass machine consisting of a recirculator and a single cavity containing a dipole HOM. HOM parameters were set to values typical for TM110 modes in the third cryomodule of the JLab FEL upgrade [4]: the HOM frequency was 2113.9 MHz, R/Q was 30 Ω , and Q was 5×10^6 . The recirculation time and the bunch repetition frequency were 433.2 nsec and 74.85 MHz, respectively, identical to those in the JLab FEL upgrade. [Note that, for this HOM frequency and the recirculation time, $\sin(\omega T_r)$ is approximately equal to -1 .] The beam energy was 70 MeV.

Three cases of recirculation optics were examined: an uncoupled matrix, a matrix interchanging x and y motion (pseudoreflexion), and a matrix rotating the vector of the beam displacement by 90° . Although values of the matrix elements were randomly chosen, their magnitude roughly corresponded to that of recirculation matrix elements of the JLab FEL upgrade.

The analytical results were compared to results of simulations obtained with a new tracking code developed at Jefferson Lab. The code is briefly described later in this paper.

A. Uncoupled recirculation matrix

For an uncoupled recirculation matrix, the threshold given by (15) is inversely proportional to

$$m_{12}^* = m_{12} \cos^2(\alpha) + m_{34} \sin^2(\alpha). \quad (25)$$

Figure 1 shows the BBU threshold calculated from (15)

FIG. 1. BBU threshold for the recirculation matrix given by (26), calculated from (15) and simulated by the new code, vs HOM rotation angle. For $m_{12}^* \sin(\omega T_r) < 0$, analytical and simulation results practically coincide with each other. For $m_{12}^* \sin(\omega T_r) > 0$, the simulation results show strong BBU suppression instead of the infinite threshold predicted by (15).

and simulated by the new code vs the HOM rotation angle for the recirculation matrix

$$M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} = \begin{bmatrix} 1.1 & 2 & 0 & 0 \\ 0.325 & 1.5 & 0 & 0 \\ 0 & 0 & 0.9 & m_{34} \\ 0 & 0 & -0.19/m_{34} & 0.9 \end{bmatrix}, \quad (26)$$

where m_{34} is equal to -4 , -2 , -1 , and 4 meters.

The simulation results perfectly agree with the analytical results for $m_{12}^* \sin(\omega T_r) < 0$. For negative values of m_{34} , the threshold current given by (15) becomes infinite if the mode angle exceeds the critical angle given by

$$\alpha_{\text{st}} = \arctan\left(\sqrt{-\frac{m_{12}}{m_{34}}}\right). \quad (27)$$

However, the code finds the threshold even for $m_{12}^* \sin(\omega T_r) > 0$. As follows from Fig. 1, the threshold for $m_{12}^* \sin(\omega T_r) > 0$ is of the order of 10–100 A. The code was not able to find any threshold for α equal to α_{st} given by (27), indicating an infinite threshold current at this point.

B. $x - y$ -motion interchange (pseudoreflexion)

The 4×4 transfer matrix

$$M = \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}, \quad (28)$$

where A and B are 2×2 block matrices, interchanges the vertical and horizontal betatron planes. If A and B are the

same, the matrix (28) reflects the beam respective to the $x = y$ plane. According to (15), the threshold for the recirculation matrix (28) is inversely proportional to

$$m_{12}^* = (m_{14} + m_{32}) \cdot \sin(2\alpha)/2 \quad (29)$$

for $m_{12}^* \sin(\omega T_r) < 0$ and becomes infinite for $m_{12}^* \sin(\omega T_r) > 0$.

Figure 2 shows the BBU threshold for the recirculation matrix

$$M = \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}, \quad (30)$$

where A is the same as in (26), calculated from (15) and simulated by the new code. The simulated results are in better than 0.1% agreement with the analytical results for $0^\circ < \alpha < 90^\circ$. For negative values of $\sin(2\alpha)$, the threshold simulated by the code is on the order of 20–200 A for $95^\circ < \alpha < 175^\circ$. The code was not able to find the threshold for α equal to 90° and 180° degrees that indicates an infinite threshold at these points.

C. 90° rotation

A 4×4 matrix of the form

$$M = \begin{bmatrix} 0 & A \\ -A & 0 \end{bmatrix} \quad (31)$$

rotates the beam displacement by 90° . The expression

$$m_{12}^* = m_{12} \cos^2(\alpha) + (m_{14} + m_{32}) \sin(\alpha) \cos(\alpha) + m_{34} \sin^2(\alpha) \quad (32)$$

is zero for (31), yielding an infinite threshold current in the case of 90° rotation. Attempts to find the BBU threshold for A the same as in (26) using the new code also failed. The simulations were stopped at a beam current of 10^6 A.

FIG. 2. BBU threshold for the reflecting matrix (30), calculated from (15) and simulated by the new code, vs HOM rotation angle.

IV. SUPPRESSION OF REGENERATIVE BBU BY OPTICAL TRANSFORMATIONS IN SEVERAL CAVITIES OF A TWO-PASS ACCELERATOR

To suppress BBU by optical transformations in several accelerating cavities simultaneously, the recirculation matrix for each cavity has to be of a form that provides effective BBU suppression. To preserve the form of the recirculation matrix from cavity to cavity, transfer matrices between the cavities have to satisfy a condition that is discussed below.

Let us consider a two-pass accelerator with two accelerating cavities and denote recirculation matrices for the first and the second cavities as M_1 and M_2 , respectively. Let us also assume that both M_1 and M_2 are of a form that provides effective suppression of the BBU. The recirculation matrices are related to each other via transfer matrices between the cavities on the first and the second pass, T_1 and T_2 respectively, as

$$M_2 = T_2 M_1 T_1^{-1}. \quad (33)$$

Equation (33) yields a condition on the form of the transfer matrices T_1 and T_2 , which has to be satisfied for a particular form of the recirculation matrices M_1 and M_2 . Equation (33) can be transformed into a simpler form. The matrix T_1 is symplectic, as well as the other matrices, and satisfies the equation

$$T_1^T S T_1 = S, \quad (34)$$

where S is the 4×4 matrix

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}. \quad (35)$$

Using Eq. (34), one can express the inverse matrix T^{-1} via T as

$$T_1^{-1} = -S T_1^T S. \quad (36)$$

Equations (33) and (36) yield the relation between T_1 and T_2 which does not require calculation of the inverse matrix:

$$M_2 = -T_2 M_1 S T_1^T S. \quad (37)$$

The last equation also allows one to express the condition (33) as a set of equations for 2×2 block submatrices of $M_{1,2}$, $T_{1,2}$, and S . To illustrate the method let us consider two examples.

A. Uncoupled transfer matrices $T_{1,2}$, recirculation matrices $M_{1,2}$ providing 90° rotation

Equation (33) can be written as

$$M_2 = T_2 M_1 T_1^{-1} = \begin{bmatrix} X_2 & 0 \\ 0 & Y_2 \end{bmatrix} \begin{bmatrix} 0 & A \\ -A & 0 \end{bmatrix} \begin{bmatrix} X_1^{-1} & 0 \\ 0 & Y_1^{-1} \end{bmatrix} \\ = \begin{bmatrix} 0 & X_2 A Y_1^{-1} \\ -Y_2 A X_1^{-1} & 0 \end{bmatrix}. \quad (38)$$

A condition that M_2 has to have the same form as M_1 yields the relation between 2×2 block submatrices of T_1 and T_2

$$X_2 A Y_1^{-1} = Y_2 A X_1^{-1}, \quad (39)$$

which can be rewritten as

$$X_2^{-1} Y_2 = A Y_1^{-1} X_1 A^{-1}. \quad (40)$$

Designing optics with specific transfer matrices for the first and the second passes that satisfy Eq. (40) can be difficult. One can notice that Eqs. (33) and (40) are automatically satisfied if the transfer matrices $T_{1,2}$ are axisymmetric with the diagonal blocks X and Y equal to each other. Thus, axisymmetric elements with no coupling preserve the rotational form of the recirculation matrix from cavity to cavity. For example, two identical nearby solenoids of opposite polarity constitute an optical element with an axisymmetric transfer matrix that does not change its form with the beam energy.

B. Uncoupled transfer matrices $T_{1,2}$, $x - y$ -exchanging recirculation matrices $M_{1,2}$

According to Eq. (29) and Fig. 2, pseudoreflexion, provided by recirculation matrices of the form (28), effectively suppresses the BBU if HOMs are bound to the horizontal and vertical planes. It is trivial to show that all uncoupled transfer matrices $T_{1,2}$ satisfy Eq. (33) for recirculation matrices of the form (28)

$$M_2 = T_2 M_1 T_1^{-1} = \begin{bmatrix} X_2 & 0 \\ 0 & Y_2 \end{bmatrix} \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix} \begin{bmatrix} X_1^{-1} & 0 \\ 0 & Y_1^{-1} \end{bmatrix} \\ = \begin{bmatrix} 0 & X_2 A Y_1^{-1} \\ Y_2 B X_1^{-1} & 0 \end{bmatrix}. \quad (41)$$

Thus, an optical insertion interchanging the vertical and horizontal planes installed in the recirculation path can simultaneously suppress HOMs in different cavities if the HOMs are bound to the horizontal and vertical planes.

V. CONCLUSIONS

Equation (15), describing the BBU threshold for a single dipole HOM with arbitrary polarization and a general-form, 4×4 recirculation matrix in a two-pass accelerator, has been derived. The equation can be used to estimate the

BBU threshold in the case of several high- Q HOMs if the separation between frequencies of the HOMs exceeds the bandwidth of the modes.

Reflection or rotation produced by the recirculation optics can significantly suppress or completely eliminate the regenerative BBU. This result was first predicted by Rand and Smith in [2] and confirmed in this work analytically and by simulations. Additionally, it was discovered in this work that reflection suppresses BBU by a factor of 10^3 – 10^4 for a wide range of values of the mode polarization angle.

To provide BBU suppression by reflection or rotation simultaneously in several cavities of the accelerating structure, the transfer matrices between the cavities have to satisfy Eq. (33).

Suppression of BBU by 90° rotation in several cavities simultaneously can be difficult if axisymmetric elements, such as quadrupoles and dipoles, are used between cavities.

If HOMs are bound to the horizontal and vertical planes, BBU can be simultaneously suppressed in several cavities by an optical transformation with a matrix of the form

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}, \quad (42)$$

where A and B are 2×2 block matrices. An optical insertion providing the $x - y$ motion exchange can consist of solenoids and/or skew quadrupoles and can be located in the recirculation path of a two-pass machine.

The analysis presented in this paper is based on the assumption that the number of cavities is sufficiently small to neglect interference between HOMs situated in different cavities. To determine the threshold of regenerative BBU in machines with a large number of accelerating cavities, one has to resort to computer simulations. Also note that the beam in large recirculating machines can be susceptible to single-pass, cumulative BBU. This type of BBU can limit the ultimate performance of high-current recirculating accelerators after regenerative BBU is suppressed by optical techniques similar to those discussed above.

A new fast, two-dimensional tracking code for simulations of regenerative BBU in recirculating machines has been developed at Jefferson Lab. The formula for the BBU threshold derived in this paper has been used to validate the code. The new code is faster than existing codes TDBBU and MATBBU by an order of magnitude or more depending on the particular problem.

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APPENDIX: NEW TWO-DIMENSIONAL TRACKING BBU CODE

A new two-dimensional tracking BBU code has been developed at Jefferson Lab. In the present configuration, the code simulates the beam dynamics in two-pass recirculating machines.

In a two-pass machine, bunches that are accelerated through a linac for the first time are interspaced with recirculated bunches. Time intervals between bunches depend on the bunch repetition frequency and the recirculation time. The code assumes that the distance between bunches does not change in the linac. The code also assumes that bunches are pointlike and cavities are short. The number of cavities and the number of HOMs per cavity are not limited in the code.

The tracking algorithm consists of four main steps:

- (i) A freshly injected bunch propagates through the entire linac. During its passage through the linac, the bunch induces the voltage of HOMs and gets deflected by the HOM voltage excited by previous bunches. After its passage through the linac, the bunch is stored in an array. The array contains all bunches present in the linac on the first pass and in the recirculation pass.
- (ii) The code updates the voltage of all HOMs to the moment of arrival of the recirculated bunch that immediately follows the injected bunch.
- (iii) The code extracts the first recirculated bunch from the array, multiplies the bunch coordinates by the recirculation matrix, and runs the bunch through the entire linac.
- (iv) The code updates the voltage of all HOMs in the linac to the moment of arrival of the next injected bunch.

Steps (i)–(iv) are repeated until the simulation time exceeds the run time specified in the input.

The voltage of each HOM has two components: real and imaginary. The real and imaginary parts of the voltage are proportional to the electric and magnetic HOM fields, respectively. A bunch modifies the real part of the HOM voltage when it passes through a cavity according to

$$\delta V_r = q \frac{c}{2} \left(\frac{\omega}{c} \right)^2 \left(\frac{R_d}{Q} \right) [x \cos(\alpha) + y \sin(\alpha)], \quad (\text{A1})$$

where α is the mode polarization angle. The imaginary part of the voltage deflects bunches according to

$$\delta x' = \frac{V_i \cos(\alpha)}{V_b}, \quad \delta y' = \frac{V_i \sin(\alpha)}{V_b}, \quad (\text{A2})$$

where V_b is the beam voltage, pc/q . To calculate evolution of the HOM voltage between bunch passages through the cavity, the code uses the transformation

$$\begin{bmatrix} V_r \\ V_i \end{bmatrix}_{t+dt} = \exp\left(-\frac{\omega dt}{2Q}\right) \begin{bmatrix} \cos(\omega dt) & -\sin(\omega dt) \\ \sin(\omega dt) & \cos(\omega dt) \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}_t, \quad (\text{A3})$$

where ω and Q are the angular frequency and the quality factor of the HOM, respectively, and dt corresponds to the time interval between bunches. The transformation given by (A3) is equivalent to rotation of a complex vector with exponentially decaying amplitude. The code uses 4×4 transfer matrices to calculate transverse coordinates of bunches.

The first version of the code has been developed and validated. The code has been used to simulate the regenerative beam breakup in several one-dimensional cases including the JLab FEL upgrade. The results of simulations were within a 3% agreement with results obtained with TDBBU [10] and MATBBU [11]. For one-dimensional cases, the new code gave results identical to those produced by the code “bi” [12], developed by Bazarov of Cornell. Simulation results for several two-dimensional cases were presented earlier in this paper. For all simulated cases, the new code was faster than TDBBU and MATBBU by an order of magnitude or more depending on a particular problem. The new code and the code bi showed similar performance for small accelerators, consisting of a few cavities. For larger machines, the new code was faster than bi by a factor of up to 3.

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