Erratum: Centroid theory of transverse electron-proton two-stream instability in a long proton bunch **[Phys. Rev. ST Accel. Beams 6, 014204 (2003)]**

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(Received 27 April 2004; published 1 June 2004)

DOI: 10.1103/PhysRevSTAB.7.069901

PACS numbers: 29.27.Bd, 41.75.-i, 52.40.Mj, 52.35.Qz, 99.10.Cd

We have identified a few algebraic errors in our recent paper [1]. Although the overall qualitative results in the original paper are still correct, an error in Eq. (42) does cause some minor changes in the quantitative results and alternations in many equations. The most significant change is in the required frequency spread for stability given in Eq. (68), which is now found to be twice that given in the original paper [see Eq. (68) below]. The details of the revisions are listed in the following.

First, the following four sentences should be inserted after Eq. (12) to comment on the procedure of obtaining the centroid equations and the applicability of Eq. (12) in the case of nonuniform proton line density: “Using the same procedure and the full solutions of Eqs. (3) and (4) one can derive the identical results in Eqs. (11) and (12). Note that Eq. (6) and the procedure of deriving Eq. (12) are valid for uniform λ_p . For nonuniform proton line density, the ω_e^2 in Eq. (4) is replaced by a function proportional to λ_p , and $y_e = A(t) \exp[i \int \omega_e(t) dt] + A^*(t) \exp[-i \int \omega_e(t) dt]$. Equation (12) should be a good approximation for describing the motion of the electron centroid when $\Delta_e \ll \omega_{eo}$ and $|A^{-1} dA/dt| \ll \omega_e$.”

Next, in Eqs. (34), (36), and (38), the equal sign ($=$) should be replaced by the approximate sign ($\approx$) to indicate the omission of the initial conditions; i.e., these equations should read

$$\tilde{Y}(z', \omega) \approx \frac{G(z') e^{(\Delta_p - \Delta_e - i\omega)z'/v}}{v^2 W(\omega_\beta^2 - \omega^2)} \int_0^{z'} \Omega^2(x) [\Phi(z') \Psi(x) - \Psi(z') \Phi(x)] \tilde{Y}(x, \omega) e^{-(\Delta_p - \Delta_e - i\omega)x/v} dx, \quad (34)$$

$$\hat{Y}(z', \omega) \approx \frac{1}{v^2 W(\omega_\beta^2 - \omega^2)} \int_0^{z'} \Omega^2(x) G(x) [\Phi(z') \Psi(x) - \Psi(z') \Phi(x)] \hat{Y}(x, \omega) dx, \quad (36)$$

and

$$\zeta(z', \omega) \approx \frac{1}{v^2 W(\omega_\beta^2 - \omega^2)} \int_0^{z'} \Omega^2(x) G(x) \Phi(x) \Psi(x) \left[1 - \frac{\Psi(z') \Phi(x)}{\Phi(z') \Psi(x)} \right] \zeta(x, \omega) dx. \quad (38)$$

The most consequential revision is in Eq. (42). The solution given in Eq. (42) should read

$$\zeta(z', \omega) \approx \left(\frac{1}{\omega^2 - \omega_\beta^2} \right) \exp \left[\frac{i \omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2} \right], \quad (42)$$

where the factor $1/(\omega^2 - \omega_\beta^2)$ before the exponential function is needed for the inverse Fourier transformation to yield the correct betatron-oscillation solution for Y_p . Accordingly, the normalization factors in Eqs. (37) and (48) should be changed from ω_β^{-3} to ω_β^{-1} in order to make the dimension of Y_p the same as Φ and Ψ . Therefore

$$\hat{Y}(z', \omega) = \omega_\beta^{-1} \zeta(z', \omega) \Phi(z'), \quad (37)$$

and

$$\hat{Y}(z', \omega) = \omega_\beta^{-1} \zeta(z', \omega) \Psi(z'). \quad (48)$$

The corrections in Eqs. (37), (42), and (48) lead to the following revisions in Eqs. (43), (45), (46), (49)–(62), (65), (66), (68), (73), (77), and (78):

$$\tilde{Y}(z', \omega) \approx \frac{\Phi(z') G(z')}{\omega_\beta (\omega^2 - \omega_\beta^2)} \exp \left[(\Delta_p - \Delta_e - i\omega) \frac{z'}{v} + \frac{i \omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2} \right], \quad (43)$$

$$Y_p(z', t) \sim \frac{G(z')}{\omega_\beta \sqrt{2\pi}} \Phi(z') e^{-\Delta_p(t - z'/v)} e^{-\Delta_e z'/v} \int_{-\infty}^{\infty} \left(\frac{1}{\omega^2 - \omega_\beta^2} \right) \exp \left[i \omega(t - z'/v) + \frac{i \omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2} \right] d\omega, \quad (45)$$

$$Y_p(z', t) \sim -i\sqrt{\frac{\pi}{2}} \xi(z') \Phi(z') \exp\left[-\Delta_p\left(t - \frac{z'}{v}\right) - \Delta_e\left(\frac{z'}{v}\right) - \frac{iJ(z')}{4}\right] \\ \times \left\{ e^{-i\omega_\beta(t-z'/v)} \left[I_0(u) - \frac{iJ(z')I_1(u)}{2u} \right] - e^{i\omega_\beta(t-z'/v)} \left[J_0(u) - \frac{iJ(z')J_1(u)}{2u} \right] \right\}, \quad (46)$$

$$Y_p(z', t) \sim i\sqrt{\frac{\pi}{2}} \xi(z') \Psi(z') \exp\left[-\Delta_p\left(t - \frac{z'}{v}\right) - \Delta_e\left(\frac{z'}{v}\right) + \frac{iJ(z')}{4}\right] \\ \times \left\{ e^{i\omega_\beta(t-z'/v)} \left[I_0(u) + \frac{iJ(z')I_1(u)}{2u} \right] - e^{-i\omega_\beta(t-z'/v)} \left[J_0(u) + \frac{iJ(z')J_1(u)}{2u} \right] \right\}, \quad (49)$$

$$Y_p(z', t) \approx C_p M_p(z') e^{-\Delta_p t} \left\{ \left[I_0^2(u) + \frac{J^2(z')I_1^2(u)}{4u^2} \right]^{1/2} \cos T_p - \left[J_0^2(u) + \frac{J^2(z')J_1^2(u)}{4u^2} \right]^{1/2} \cos S_p \right\}, \quad (50)$$

$$M_p(z') = \xi(z') R(z') \exp[(\Delta_p - \Delta_e)z'/v], \quad (51)$$

$$T_p = T_p(z', t) = \sigma_p + \psi_p - \omega_\beta(t - z'/v) + \Theta(z') - J(z')/4, \quad (52)$$

$$S_p = S_p(z', t) = \sigma_p + \phi_p + \omega_\beta(t - z'/v) + \Theta(z') - J(z')/4, \quad (53)$$

$$Y_p(z, t) \sim -i\sqrt{\frac{\pi}{2}} \xi(z + vt) \Phi(z + vt) \exp\left[-\Delta_p\left(\frac{z}{v}\right) - \Delta_e\left(t + \frac{z}{v}\right) - \frac{iJ(z + vt)}{4}\right] \\ \times \left\{ e^{-i\omega_\beta z/v} \left[I_0(\tilde{u}) - \frac{iJ(z + vt)I_1(\tilde{u})}{2\tilde{u}} \right] - e^{i\omega_\beta z/v} \left[J_0(\tilde{u}) - \frac{iJ(z + vt)J_1(\tilde{u})}{2\tilde{u}} \right] \right\}, \quad (54)$$

$$Y_p(z', t) \approx C_p M_p(z') e^{-\Delta_p t} \left[I_0^2(u) + \frac{J^2(z')I_1^2(u)}{4u^2} \right]^{1/2} \cos T_p, \quad (55)$$

$$Y_p(z', t) \approx \frac{C'_p \xi(z') R(z')}{[2\omega_\beta J(z')(t - z'/v)]^{1/4}} \cos[\sigma_p + \Theta(z') - \omega_\beta(t - z'/v) - J(z')/4] \exp\left[-\Delta_p\left(t - \frac{z'}{v}\right) - \Delta_e\left(\frac{z'}{v}\right) + \sqrt{2\omega_\beta J(z')(t - z'/v)}\right], \quad (56)$$

$$Y_e(z', t) \sim \sqrt{2\pi} J(z') \Phi(z') \exp\left[-\frac{iJ(z')}{4} - \Delta_p\left(t - \frac{z'}{v}\right) - \Delta_e\left(\frac{z'}{v}\right)\right] \\ \times \left\{ e^{-i\omega_\beta(t-z'/v)} \left[\frac{I_1(u)}{u} - \frac{J^2(z')I_2(u)}{8u^2} \right] - e^{i\omega_\beta(t-z'/v)} \left[\frac{J_1(u)}{u} - \frac{J^2(z')J_2(u)}{8u^2} \right] \right\}, \quad (57)$$

$$Y_e(z', t) \approx C_e M_e(z') e^{-\Delta_p t} \left[\frac{I_1(u)}{u} - \frac{J^2(z')I_2(u)}{8u^2} \right] \cos T_e, \quad (58)$$

$$M_e(z') = J(z') R(z') \exp[(\Delta_p - \Delta_e)z'/v], \quad (59)$$

$$T_e = T_e(z', t) = \psi_e + \sigma_e - \omega_\beta(t - z'/v) + \Theta(z') - J(z')/4, \quad (60)$$

$$\psi_e = 0, \quad (61)$$

$$Y_e(z', t) \approx \frac{C'_e J(z') R(z')}{[2\omega_\beta J(z')(t - z'/v)]^{3/4}} \cos[\sigma_e + \Theta(z') - \omega_\beta(t - z'/v) - J(z')/4] \exp\left[-\Delta_p\left(t - \frac{z'}{v}\right) - \Delta_e\left(\frac{z'}{v}\right) + \sqrt{2\omega_\beta J(z')(t - z'/v)}\right], \quad (62)$$

$$\Gamma_p(z', t) \approx -\Delta_p + \frac{\omega_\beta J(z') I_1(u) [4u^2 I_0(u) + J^2(z') I_2(u)]}{u [4u^2 I_0^2(u) + J^2(z') I_1^2(u)]}, \quad (65)$$

$$\Gamma_e(z', t) \approx -\Delta_p + \frac{\omega_\beta J(z') [8u I_2(u) - J^2(z') I_3(u)]}{u [8u I_1(u) - J^2(z') I_2(u)]}, \quad (66)$$

$$\left(\frac{\Delta_p}{\omega_\beta}\right)_t \approx \max \left\{ \frac{J(z')}{2} \left[\frac{1 + J^2(z')/32}{1 + J^2(z')/16} \right] \right\}, \quad (68)$$

$$\tilde{Y}_p(z', \omega) \approx G \hat{Y}(z', \omega - i\Delta_p) e^{-i(\omega - i\Delta_e)z'/v} = \frac{G \Phi(z')}{\omega_\beta (\omega^2 - \omega_\beta^2)} \exp \left[-i(\omega - i\Delta_e) \left(\frac{z'}{v} \right) + \frac{i \omega_\beta^2 J(z')}{(\omega - i\Delta_p)^2 - \omega_\beta^2} \right], \quad (73)$$

$$Y_p(z', t) \approx C e^{-\Delta_p t + (\Delta_p - \Delta_e)z'/v} \left\{ \left[I_0^2(u) + \frac{J^2(z') I_1^2(u)}{4u^2} \right]^{1/2} \cos T - \left[J_0^2(u) + \frac{J^2(z') J_1^2(u)}{4u^2} \right]^{1/2} \cos S \right\}, \quad (77)$$

and

$$Y_p(z', t) \approx \frac{\hat{C}}{[\omega_0 \omega_\beta (z'/v)(t - z'/v)]^{1/4}} \cos \left[\sigma + (\omega_0 + \omega_\beta - \xi \omega_0/8)(z'/v) - \omega_\beta t \right] \exp \left[-\Delta_p \left(t - \frac{z'}{v} \right) - \Delta_e \left(\frac{z'}{v} \right) + \sqrt{\xi \omega_0 \omega_\beta \left(\frac{z'}{v} \right) \left(t - \frac{z'}{v} \right)} \right]. \quad (78)$$

In Eq. (53), $\psi_p = \cos^{-1} \{ 2u I_0(u) [4u^2 I_0^2(u) + J^2(z') I_1^2(u)]^{-1/2} \}$, and $\phi_p = \cos^{-1} \{ 2u J_0(u) [4u^2 J_0^2(u) + J^2(z') J_1^2(u)]^{-1/2} \}$. The solution given in Eq. (46) is the same as that obtained from retaining the $n = 0$, $m = 0$, and $m = 1$ terms in the exact solution given in Eq. (A13) in Appendix A. The approximate solution for $Y_e(z', t)$ to lowest order in $J(z')/u$ shown in Eq. (57) is obtained by retaining the $n = 0$, $m = 1$ and $m = 2$ terms in Eq. (A17). Note that the growth rate given in Eq. (68) is twice as large as that given in the original paper. As a result, the maximum of $J(z')$ presented in the numerical example is about 0.057, and therefore according to Eq. (68), the instability threshold is near $(\Delta_p/\omega_\beta)_t \approx 2.9\%$ instead of 1.4% obtained previously [1].

Because of the revisions in Eqs. (55), (58), (65), and (67), Figs. 3–5 in the original paper have to be corrected according to the following (there are no changes to Figs. 1 and 2):

FIG. 3. The growth rates at the tail ($z' = L$) and the center ($z' = L/2$) of the proton bunch are shown as functions of time. Here, Γ_p and Γ_e have been normalized to the betatron frequency ω_β . The solid curve and the dashed curve are computed according to Eqs. (65) and (66), respectively. The dotted curve is computed using the asymptotic approximation in Eq. (67). The parameter values are described in the text.

FIG. 4. The growing portion of the perturbed proton motion Y_p as a function of z'/v at time $t = 40, 70$, and $100 \mu s$. Here, the value of Y_p is computed using Eq. (55) with $\sigma_p = 0$ and the parameter values described in the text. The proton bunch is 260 ns long. The vertical axis has arbitrary units.

The corrections in Eqs. (37), (42), and (48) also cause some changes in Eqs. (A1), (A3), (A11)–(A13), (A16), and (A17) in Appendix A. These equations should read

$$\hat{Y}(z', \omega) \approx \frac{\Phi(z')}{\omega_\beta(\omega^2 - \omega_\beta^2)} \exp\left[\frac{i\omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2}\right], \quad (\text{A1})$$

$$\hat{Y}(z', \omega) = \frac{Q(z', \omega)\Phi(z')}{\omega_\beta(\omega^2 - \omega_\beta^2)} \exp\left[\frac{i\omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2}\right], \quad (\text{A3})$$

$$\hat{Y}(z', \omega) = \Phi(z') \exp\left[\frac{i\omega_\beta^2 J(z')}{\omega^2 - \omega_\beta^2}\right] \sum_{n=0}^{\infty} \frac{a_n(z')}{\omega_\beta(\omega^2 - \omega_\beta^2)^{n+1}}, \quad (\text{A11})$$

FIG. 5. The growing portion of the perturbed electron motion Y_e as a function of z'/v at time $t = 40, 70$, and $100 \mu\text{s}$. Here, the value of Y_e is computed using Eq. (58) with $\sigma_e = \pi/2$ and the parameter values described in the text. The proton bunch is 260 ns long. The vertical axis has arbitrary units.

$$Y_p(z', t) = \frac{\omega_\beta}{\sqrt{2\pi}} \xi(z') \Phi(z') e^{-\Delta_p(t-z'/v) - \Delta_e(z'/v)} \sum_{n=0}^{\infty} a_n \vartheta_{n+1}, \quad (\text{A12})$$

$$Y_p(z', t) = -i \sqrt{\frac{\pi}{2}} \xi(z') \Phi(z') e^{-\Delta_p(t-z'/v) - \Delta_e(z'/v)} \sum_{n=0}^{\infty} \frac{a_n}{(2\omega_\beta)^{2n}} \sum_{m=-n}^{\infty} \frac{1}{(m+n)!} \left(\frac{iu}{4\alpha} \right)^m \\ \times [e^{-i\alpha} I_m(u) - e^{i\alpha} J_m(u)] \frac{d^{m+n}}{dp^{m+n}} \left[\frac{1}{p^{n+1}} \exp\left(\frac{-iu^2}{8\alpha p} \right) \right] \Big|_{p=1}, \quad (\text{A13})$$

$$Y_e(z', t) = \frac{-1}{\sqrt{2\pi}} \Phi(z') e^{-\Delta_p(t-z'/v) - \Delta_e(z'/v)} \sum_{n=0}^{\infty} \frac{a_n}{\omega_\beta} \vartheta_n, \quad (\text{A16})$$

and

$$Y_e(z', t) = i2\sqrt{2\pi}\Phi(z')e^{-\Delta_p(t-z'/v)-\Delta_e(z'/v)} \sum_{n=0}^{\infty} \frac{a_n}{(2\omega_\beta)^{2n}} \sum_{m=1-n}^{\infty} \frac{1}{(m+n-1)!} \left(\frac{iu}{4\alpha}\right)^m \\ \times [e^{-i\alpha}I_m(u) - e^{i\alpha}J_m(u)] \frac{d^{m+n-1}}{dp^{m+n-1}} \left[\frac{1}{p^n} \exp\left(\frac{-iu^2}{8\alpha p}\right) \right] \Big|_{p=1}. \quad (\text{A17})$$

In Appendix B, there were some typographical errors in Eqs. (B4), (B6), (B8), and (B9). These equations should read

$$\vartheta_n = 2\pi(-1)^n(t - z'/v)^{2n-1} \text{Res}_{\theta=-i\alpha} \left\{ \frac{1}{(\theta^2 + \alpha^2)^n} \exp\left[\theta - \frac{u^2}{4(\theta - i\alpha)} + \frac{u^2}{4(\theta + i\alpha)}\right] \right\} \\ + 2\pi(-1)^n(t - z'/v)^{2n-1} \text{Res}_{\theta=i\alpha} \left\{ \frac{1}{(\theta^2 + \alpha^2)^n} \exp\left[\theta - \frac{u^2}{4(\theta - i\alpha)} + \frac{u^2}{4(\theta + i\alpha)}\right] \right\}, \quad (\text{B4})$$

$$\frac{1}{(\theta^2 + \alpha^2)^n} \exp\left[\theta - \frac{u^2}{4(\theta - i\alpha)} + \frac{u^2}{4(\theta + i\alpha)}\right] = \frac{e^{-i\alpha}}{s^n(s - 2i\alpha)^n} \exp\left[\frac{-u^2}{4(s - 2i\alpha)}\right] \sum_{m=-\infty}^{\infty} \left(\frac{2s}{u}\right)^m I_m(u), \quad (\text{B6})$$

$$\text{Res}_{\theta=-i\alpha} \left\{ \frac{1}{(\theta^2 + \alpha^2)^n} \exp\left[\theta - \frac{u^2}{4(\theta - i\alpha)} + \frac{u^2}{4(\theta + i\alpha)}\right] \right\} = e^{-i\alpha} \sum_{m=-\infty}^{n-1} \left(\frac{2}{u}\right)^m I_m(u) \text{Res}_{s=0} \left\{ \frac{s^{m-n}}{(s - 2i\alpha)^n} \exp\left[\frac{-u^2}{4(s - 2i\alpha)}\right] \right\} \\ = -e^{-i\alpha} \sum_{m=1-n}^{\infty} \left(\frac{-u}{2}\right)^m \frac{I_m(u)}{(m+n-1)!} \\ \times \frac{d^{m+n-1}}{dr^{m+n-1}} \left\{ \frac{1}{(r + 2i\alpha)^n} \exp\left[\frac{u^2}{4(r + 2i\alpha)}\right] \right\} \Big|_{r=0}, \quad (\text{B8})$$

and

$$\text{Res}_{\theta=i\alpha} \left\{ \frac{1}{(\theta^2 + \alpha^2)^n} \exp\left[\theta - \frac{u^2}{4(\theta - i\alpha)} + \frac{u^2}{4(\theta + i\alpha)}\right] \right\} = e^{i\alpha} \sum_{m=1-n}^{\infty} \left(\frac{-u}{2}\right)^m \frac{J_m(u)}{(m+n-1)!} \\ \times \frac{d^{m+n-1}}{dr^{m+n-1}} \left\{ \frac{1}{(r + 2i\alpha)^n} \exp\left[\frac{u^2}{4(r + 2i\alpha)}\right] \right\} \Big|_{r=0}. \quad (\text{B9})$$

Finally, we add the following equation to the end of Appendix B:

$$\vartheta_1 \approx \frac{-i\pi e^{-iJ(z')/4}}{\omega_\beta} \left\{ e^{-i\alpha} \left[I_0(u) - \frac{iu}{4\alpha} I_1(u) \right] - e^{i\alpha} \left[J_0(u) - \frac{iu}{4\alpha} J_1(u) \right] \right\}. \quad (\text{B13})$$

ACKNOWLEDGMENTS

We would like to thank Dr. D. M. Pestrikov for bringing to our attention that if the equal sign were used in Eqs. (34), (36), (38), and (41) then zero would be the only solution for $\hat{Y}$. This work was supported by the U.S. Department of Energy.

[1] T. F. Wang, P. J. Channell, R. J. Macek, and R. C. Davidson, Phys. Rev. ST Accel. Beams **6**, 014204 (2003).