Combined phenomena of beam-beam and beam-electron cloud interactions in circular e^+e^- colliders

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An electron cloud causes various effects in high intensity positron storage rings. The positron beam and the electron cloud can be considered a typical two-stream system with a certain plasma frequency. Beam-beam interaction is another important effect for high luminosity circular colliders. Colliding two beams can be considered as a two-stream system with another plasma frequency. We study the combined phenomena of the beam-electron cloud and beam-beam interactions from a viewpoint of two complex two-stream effects with two plasma frequencies.

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I. INTRODUCTION

In recent high intensity positron rings, various phenomena related to electron clouds have been observed. An electron cloud can be produced in a positron storage ring, for example, due to a multipactoring effect or due to electrons emitted from the vacuum chamber walls when hit by the synchrotron photons radiated by the positron beam. Coupled bunch instabilities have been observed at the KEK Photon Factory and IHEP-BEPC, and beam size enlargements have been observed at the B factories of SLAC (PEP-II) and KEK (KEKB). These phenomena were understood as two-stream instability of relativistic beam and slow electron clouds. The phenomena can also be understood as beam instabilities caused by wake force on the positron beam due to the electron cloud. The mechanism of the corresponding coupled bunch instability is a two-stream effect characterized by a plasma oscillation along the bunch train or is mediated by a long range wake force lasting for a range of the order of bunch spacing (~ 1 m). The mechanism of beam size enlargement, on the other hand, is a two-stream effect characterized by a plasma oscillation within a single bunch or mediated by a short range wake force with a range of the order of the bunch length (~ 1 cm). In addition to the electron cloud, the positron beam also interacts with an electron beam in a collider thought to be the beam-beam interaction. The colliding beams are regarded as another two-stream system with a plasma frequency characterized by the beam-beam force. The beam-beam interaction has a nature of a short range wake force, namely, a distortion of the head part of a beam will first induce a perturbation on the on-coming beam, which then in turn affects the tail part of the first beam.

The short range wake forces due to the electron cloud and the beam-beam interaction may couple and cause a combined phenomena of a complex nature. Such com-

bined phenomena may have been observed in KEKB. With a single positron beam, it was observed that the transverse size of the beam is enlarged beyond a certain threshold beam current. This enlargement is attributed to a beam-electron cloud interaction. With colliding beams, it was further observed that the luminosity was extremely low for bunch spacing narrower than 6 ns even below the threshold current of the beam enlargement [1].

We study combined phenomena of the two types of two-stream systems. We first discuss this instability using a linearized “one-two-particle model,” in which e^- and e^+ beams are represented by one and two macroparticles, respectively. The beam-beam force is linearized in this model. The wake force due to the electron cloud is approximated to be a constant along the longitudinal direction. A similar system has been studied in Ref. [2] for a conventional wake force. The combined effects based on the weak-strong beam-beam model have been discussed in Ref. [3].

We next discuss the phenomena using a tracking simulation in which each of the two beams is represented by a large number (~ 1000) of macroparticles (or slices) distributed in the longitudinal phase space [4]. Each macroparticle has a transverse beam size determined by the unperturbed nominal value. Nonlinearities in the interaction among the macroparticles are taken into account. The electron cloud is represented by many ($\sim 10\,000$) pointlike macroparticles. The beam-electron cloud interaction is evaluated by interaction between a sliced transverse Gaussian beam and each macroparticle in the electron cloud [4].

II. TWO-STREAM FEATURES OF BEAM-ELECTRON CLOUD AND BEAM-BEAM SYSTEMS

We discuss in the following a linear theory of the combined system of beam-beam and wake interactions.

A similar system has already been studied by Perevedentsev and Valishev [2]. We study the system using an alternative point of view, i.e., as a combined effect of two two-stream interactions. We consider a uniform beam in the region of $z = \pm \sigma_z$ interacting with an electron cloud. The electron cloud is assumed to be Gaussian with the same rms transverse size as the beam. Though the electron cloud is much larger than the beam,

the beam-cloud interactions are well approximated by the model in which the cloud has the same size as the beam [5]. The beam and the electron cloud are characterized by $\mathbf{x}_\pm(z, s)$ and $\mathbf{x}_e(s, t)$, which are their centroid coordinates. The longitudinal coordinate (s), time (t), and the relative longitudinal position in the beam (z) have a relation $z = s - ct$. The beam and cloud obey the equation of motion

$$\frac{d^2 \mathbf{x}_e(s, t)}{dt^2} = -2\lambda_+ r_e c^2 \mathbf{F}_G[\mathbf{x}_e(s, t) - \bar{\mathbf{x}}_+(s - ct, s)], \quad (1)$$

$$\frac{d^2 \mathbf{x}_\pm(z, s)}{ds^2} + \left(\frac{\omega_\beta}{c}\right)^2 \mathbf{x}_\pm(z, s) = -\frac{2\lambda_e r_e}{\gamma} \mathbf{F}_G(\bar{\mathbf{x}}_\pm(z, s) - \mathbf{x}_e[s, (s - z)/c]), \quad (2)$$

where the force $\mathbf{F}_G(\mathbf{x})$ is expressed by the Bassetti-Erskine formula [6] normalized so that $\mathbf{F}_G \rightarrow \mathbf{x}/|\mathbf{x}|^2$ as $\mathbf{x} \rightarrow \infty$, r_e is the classical radius of electrons and positrons, γ is the relativistic Lorentz factor of the positron beam, and λ_+ and λ_e are the line density of the positron beam and electron cloud, respectively. $\mathbf{F}_G$ is also characterized by the horizontal and vertical beam sizes, $\sigma_{+,x(y)} = \sqrt{\beta_{+,x(y)} \varepsilon_{+,x(y)}}$ where $\beta_{+,x(y)}(s)$ and $\varepsilon_{+,x(y)}$ are the beta function and emittance, respectively. We assume that the emittance $\varepsilon_{x(y)}$ is kept constant for the variation of s .

The electron cloud oscillates with an angular frequency due to the linear part of $\mathbf{F}_G$,

$$\omega_{e,y(x)} = \sqrt{\frac{\lambda_+ r_e c^2}{(\sigma_{+,x} + \sigma_{+,y}) \sigma_{+,y(x)}}}. \quad (3)$$

The values of $\omega_{e,y}$ and $\omega_{e,x}$ are about $1.7 \times 10^{11} \text{ s}^{-1}$ and $6.4 \times 10^{10} \text{ s}^{-1}$, respectively, with the KEKB parameters $\sigma_x = 420 \mu\text{m}$, $\sigma_y = 60 \mu\text{m}$, $\sigma_z = 5 \text{ mm}$, and $\lambda_+ = 3.3 \times 10^{12} \text{ m}^{-1}$. The quantity ω_e is considered as plasma frequency for the two-stream system of the beam-electron cloud. The phase advance of the electron motion during the interaction, $\phi_e = \omega_e \sigma_z / c$, characterizes the instability. For KEKB, $\phi_{e,y} = \omega_{e,y} \sigma_z / c \approx 2.8$ and $\phi_{e,x} = \omega_{e,x} \sigma_z / c \approx 1.1$.

The beam-beam system also has a potential to cause a two-stream instability because one beam oscillates in the electromagnetic field produced by the other beam with a certain frequency. The beam-beam force is expressed as

$$\frac{d^2 \mathbf{x}_\pm(z_\pm, s)}{ds^2} = -\frac{4\lambda_\mp r_e}{\gamma} \mathbf{F}_G[\mathbf{x}_\pm(z_\pm, s) - \mathbf{x}_\mp(z_\mp, -s)], \quad (4)$$

where the longitudinal coordinate is chosen so that beams with $z_\pm = 0$ collide at $s = 0$, namely $s = (z_\pm - z_\mp)/2$. There is a coherent frequency during the interaction between the two colliding beams given by

$$\omega_{\pm,y(x)} = \sqrt{\frac{2\lambda_\mp r_e c^2}{\gamma_\pm (\sigma_{\mp,x}^* + \sigma_{\mp,y}^*) \sigma_{\mp,y(x)}^*}}, \quad (5)$$

where $\gamma_\pm$ is the relativistic factor of the positron and/or the electron beam. We note that $\sigma_{\pm,x(y)}^*$, the beam size of the positron and/or the electron beam at the collision point, is typically much smaller than $\sigma_{x(y)}$ in Eq. (3), and that $\gamma \gg 1$.

The phase advance of the oscillation during a collision is expressed by

$$\phi_{\pm,y(x)} = \frac{\omega_{\pm,y(x)} \sigma_z}{c} = \sqrt{\frac{4\pi \xi_{y(x)} \sigma_z}{\beta_{y(x)}}} \equiv \sqrt{2D_{y(x)}}, \quad (6)$$

where ξ_y is the beam-beam tune shift parameter and we have assumed that the two beams have the same beam size. We call $D \equiv \phi_\pm^2/2$ the beam-beam disruption parameter. The vertical phase advance ϕ_y is approximately the order of unity for recent high luminosity colliders. The two-stream effect may be important under this condition. The horizontal phase advance $\phi_x = \phi_y \sqrt{\xi_x \beta_y / \xi_y \beta_x}$ is typically smaller than ϕ_y , but the horizontal effect may be important depending upon the tune as will be discussed later.

III. ONE-TWO-PARTICLE MODEL

We first study the phenomena using a small number of macroparticles; i.e., a one-two-particle model in which the electron and positron beams are represented by one and two macroparticles, respectively. The model is reliable approximation for considering the beam-beam interaction, since the phase advance, $\phi_\pm$, is less than 1 in most cases. We further make the approximation that the beam-electron cloud interaction can be described by a constant wake force. Although ϕ_e is comparable or larger than 1, and therefore the validity of the model is being stretched, we believe that the analysis remains reasonable. An analytic treatment becomes possible by this approximation.

We discuss vertical motion below. Motion of the two beams is characterized by a vector $\mathbf{Y}(s)$,

$$\mathbf{Y}(s) = \begin{pmatrix} y_1^+ \\ p_1^+ \\ y_2^+ \\ p_2^+ \\ y^- \\ p^- \end{pmatrix}, \quad (7)$$

where the suffix $\pm$ refers to the positron or electron macroparticle. We consider a revolution matrix to transfer $\mathbf{Y}(s^*)$ to $\mathbf{Y}(s^* + C)$, where s^* and C are the positions of the collision point and the circumference of a ring, respectively. The beam size (beta function) is temporarily assumed to be a constant during the collision. The synchrotron tune Q_s is assumed to be inverse of an integer ($Q_s = 1/n_s$). We postpone the study of the general synchrotron tune until later using a tracking simulation.

Since the beta function is assumed to be constant, the beam-beam force does not have a longitudinal component. The two macroparticles in the positron beam are assumed to have opposite synchrotron phases. The collision points of the two positron macroparticles and the electron macroparticle are given by $s^* + \Delta$, where

$$\Delta = \pm \frac{\sigma_z}{2} \sin(2\pi Q_s s/C). \quad (8)$$

The collision of the i th positron and the electron is represented by a matrix $B_i(\xi)$,

$$B_1(\xi) = \begin{pmatrix} I + b(2\xi) & 0 & -b(2\xi) \\ 0 & I & 0 \\ -b(\xi) & 0 & I + b(\xi) \end{pmatrix}, \quad (9)$$

$$B_2(\xi) = \begin{pmatrix} I & 0 & 0 \\ 0 & I + b(2\xi) & -b(2\xi) \\ 0 & -b(\xi) & I + b(\xi) \end{pmatrix}, \quad (10)$$

where

$$b(\xi) = \begin{pmatrix} 0 & 0 \\ -\pi\xi & 0 \end{pmatrix}, \quad (11)$$

and I is the 2×2 unit matrix.

The transfer matrix of collision at $s = \Delta$ is expressed by $B(\Delta) = D^{-1}(\Delta)BD(\Delta)$: that is, particles drift to $s = \pm\Delta$, collide, and return to the collision point. The matrix $D(\Delta)$ is expressed as

$$D(\Delta) = \begin{pmatrix} d(\Delta) & 0 & 0 \\ 0 & d(\Delta) & 0 \\ 0 & 0 & d(-\Delta) \end{pmatrix}, \quad (12)$$

where

$$d(\Delta) = \begin{pmatrix} 1 & \Delta \\ 0 & 0 \end{pmatrix}. \quad (13)$$

The transfer matrix of the collision event is expressed as

$$\mathbf{Y}_{\text{after}}(s^*) = T_{\text{BB}}(\Delta)\mathbf{Y}_{\text{before}}(s^*), \quad (14)$$

where T_{BB} has two ways of representations depending on the sign of Δ , i.e., depending on which macroparticle is at the bunch head or tail. When the first macroparticle stays at the head of the bunch ($\Delta > 0$) in a half synchrotron period, the matrix is given by

$$T_{\text{BB}}(\Delta) = D^{-1}(-\Delta)B_2D(-\Delta)D(\Delta)^{-1}B_1D(\Delta). \quad (15)$$

In the other half synchrotron period ($\Delta < 0$), it is given by

$$T_{\text{BB}}(\Delta) = D^{-1}(\Delta)B_1D(\Delta)D^{-1}(-\Delta)B_2D(-\Delta). \quad (16)$$

After each collision, the macroparticles are transferred along the arc section of the collider ring from the collision point of one turn ($s = s^*$) to the next turn ($s = s^* + C$). At the same time, the wake force describing the beam-electron cloud interaction affects the tail macroparticle depending on the betatron amplitude of the head macroparticle. Again, the transfer matrix from s^* to $s^* + C$ has two representations depending on the sign of Δ . The matrix (T_{arc}) for $\Delta > 0$ is

$$T_{\text{arc}} = \begin{pmatrix} T_2(\mu_1) & 0 & 0 \\ A(W, \mu_1) & T_2(\mu_2) & 0 \\ 0 & 0 & T_2(\mu_e) \end{pmatrix}, \quad (17)$$

while for $\Delta < 0$,

$$T_{\text{arc}} = \begin{pmatrix} T_2(\mu_1) & A(W, \mu_2) & 0 \\ 0 & T_2(\mu_2) & 0 \\ 0 & 0 & T_2(\mu_e) \end{pmatrix}, \quad (18)$$

where

$$T_2(\mu) = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix}. \quad (19)$$

The quantity $\mu_{+(-)} = 2\pi Q_{+(-)}$ is the bare betatron phase advance, and $\mu_{1,2}$ are the betatron phase advances including modulation characterized by the chromaticity Q'_+ ,

$$\begin{aligned} \mu_1 &= 2\pi \left(Q_+ + Q_s \chi \sin \frac{\omega_s}{c} s \right), \\ \mu_2 &= 2\pi \left(Q_+ - Q_s \chi \sin \frac{\omega_s}{c} s \right), \end{aligned} \quad (20)$$

where

$$\chi = \frac{2\pi Q'_+}{\alpha L} = \frac{Q'_+ \sigma_\delta}{Q_s}, \quad (21)$$

with α and L the momentum compaction factor and the circumference of the ring, respectively. The 2×2 matrix

$A(W, \mu)$ describes the transformation caused by the wake field,

$$A(W, \mu) = \frac{W}{2} \begin{pmatrix} \sin \mu & -\cos \mu \\ \cos \mu & \sin \mu \end{pmatrix}. \quad (22)$$

The revolution matrix including the transfer of the arc section and the beam-beam interaction is

$$T_{\text{rev}}(\Delta) = T_{\text{BB}}(\Delta)T_{\text{arc}}(W, \mu), \quad (23)$$

where Δ is given by Eq. (8). We then calculate the transfer matrix for one synchrotron period ($Q_s = 1/n_s$),

$$T_{\text{syn}} = \prod_{i=1}^{n_s} T_{\text{rev}}(\Delta_i). \quad (24)$$

Stability of the system can be discussed by eigenvalues (λ) of the 6×6 matrix T_{syn} . The matrix is not symplectic, although its determinant is unity. The eigenvalues are calculated numerically. When the absolute value of one (or more) of the eigenvalues becomes bigger than unity, the system becomes unstable. We discuss the stability using the imaginary part of eigentunes, which is represented by $\ln|\lambda|$. A positive value of $\ln|\lambda|$ means the system is unstable.

We first discuss vertical motion. Figure 1 shows the imaginary part of the eigentunes as functions of the

FIG. 1. Dependence of the imaginary part of the eigentunes on the betatron tune. The eigentunes are normalized by the synchrotron tune Q_s . Beam-beam parameter is chosen to be 0.05. (a) and (b) are for $W = 0$ and $W = 0.1$, respectively.

unperturbed betatron tune. For $W = 0$, nonzero values of the imaginary part occur only near the half integer tune as shown in the upper picture. For $W > 0$, the non-zero imaginary part occurs for at least one eigenmode for all tunes, i.e., the system is always unstable regardless of the tune.

Figure 2 shows the imaginary parts as functions of the strength of the wake field and beam-beam parameter. The behavior for the wake strength is simple but that for the beam-beam parameter is complex. One complication of the beam-beam kicks is that they depend on the longitudinal coordinates of the macroparticles. The complex dependence on the beam-beam kicks is similar to the complex dependence of the head-tail instability on the chromaticity.

Figure 3 shows the chromaticity dependence of the imaginary part of the eigentunes.

We now discuss horizontal effect. The horizontal phase advance related to the beam-beam disruption is less than the vertical one because $\phi_x \approx \phi_y \sqrt{\beta_y/\beta_x}$ in most colliders (assuming $\xi_x \approx \xi_y$), while $\beta_y \ll \beta_x$. However, at KEKB, we often adopt an operating point with a horizontal tune close to a half integer to benefit from dynamical beta effect. The horizontal effect may therefore be important even though ϕ_x is small. Figure 4 shows the imaginary part of the eigentunes of the horizontal matrix. The beam can be unstable when $W > 0$. This

FIG. 2. Dependence on the wake strength at $\xi = 0.05$ (a) and the beam-beam parameter at $W = 0.1$ (b).

FIG. 3. Chromaticity dependence. (a) $\xi = 0.03$; (b) $\xi = 0.05$.

means that the horizontal effect should be taken into consideration.

So far we have assumed that the synchrotron tune was inverse of an integer. We extended the model to a general synchrotron tune [7] to avoid potential unphysical resonance behavior by using the following trick. We write down the transfer matrix for one synchrotron period as

$$T_{\text{syn}} = T_{\text{rev}} \left(\frac{\sigma_z}{2} \right)^{1/2 Q_s} T_{\text{rev}} \left(-\frac{\sigma_z}{2} \right)^{1/2 Q_s}, \quad (25)$$

where Q_s is not necessarily an inverse of the integer. We treat the eigenvalue problem mathematically, i.e., in the eigensystem, a noninteger power of a matrix can be evaluated. The collision points are assumed to be at $\pm \sigma_z/2$ so that the transfer matrix is expressed by $1/2Q_s$ power of the revolution matrices. With this trick, we obtained results qualitatively consistent with the previous model. A more sophisticated model is seen in Ref. [2].

We tried a two-two particle model in which both beams are represented by two macroparticles. In this model we assumed the same synchrotron tunes for both beams. Similar results were obtained as for the one-two particle model. Further extensions are done by particle tracking simulation as described next.

FIG. 4. Dependence on the horizontal tune. (a) $W = 0$; (b) $W = 0.05$. Note that the $W = 0$ case has an expanded vertical scale.

IV. PARTICLE TRACKING SIMULATION USING A MULTIPARTICLE MODEL

We now proceed to a more realistic model, in which the beam-beam force is strongly nonlinear and the synchrotron tune is not the inverse of an integer. The two beams are allowed to have different beam-beam parameters and different synchrotron and betatron tunes. The electron cloud is represented by a distribution of electrons. The characteristic phase angle ϕ_e is larger than unity, and electrons are pinched by the beam force. We perform a particle tracking simulation to study beam stability under these general conditions.

We represent the beams as a sizable number of macroparticles (~ 1000) with a transverse Gaussian distribution of a fixed rms size characterized by $\sigma_{x(y)} = \sqrt{\beta_{x(y)}(s) \epsilon_{x(y)}}$ [4]. For easy visualization, we use a multiple air-bag model for the longitudinal distribution, in which the microbunches are distributed on concentric circles in the longitudinal phase space, characterized by the position z and the relative momentum deviation $\Delta p/p$. The interaction starts from collision between the pair of microbunches of the two bunches with the largest value of $z_+ + z_-$ (i.e., the head of the two bunches), and then continues for other microbunch pairs with progressively smaller $z_+ + z_-$. The collision point of a pair is $s_{\pm} = \pm(z_+ - z_-)/2$ from viewpoints of positron and electron beams. The coordinate should be

transferred to the collision point by a transformation $D(s_{\pm})$. The macroparticles are transferred around the ring using a linear transport matrix and applying a chromaticity kick.

The electron cloud is represented by a large number of macroparticle electrons ($\sim 10\,000$). The electron cloud is put at a fixed position in the positron ring.

Figure 5 shows the variation of the maximum vertical action $J_{y,\max}$ of macroparticles with and without beam-beam interaction. The electron cloud density ($\rho_e = 2 \times 10^{11} \text{ m}^{-3}$) used in the simulation is less than the threshold value ($\rho_{e,\text{th}} = 5 \times 10^{11} \text{ m}^{-3}$) expected in the absence of beam-beam interaction. We observe a remarkable difference between the cases with and without beam-beam interaction. The difference is due to the combined effect of beam-beam and beam-electron cloud interactions. There is no growth for pure beam-beam interaction without the electron cloud in the range of parameters studied.

Figure 6 shows the shape of the positron bunch projected onto the y - z plane of the macroparticles after 400 and 800 turns. We can see a head-tail motion for the positron bunch experiencing both the beam-beam and the beam-electron cloud interactions.

We next study the effects of chromaticity and synchrotron tune spread. For a regular head-tail instability, it is well known that chromaticity and synchrotron tune spread [8] affect its behavior. Figure 7 shows the dependence on chromaticity and synchrotron tune spread in our simulation. For the inclusion of tune spread effect, macroparticles are assumed to have a Gaussian distribution in the longitudinal phase space. These results indicate that the chromaticity or synchrotron tune spread work to suppress the combined instability. However these effects are limited. For example, these parameters do not work well at a larger beam-beam parameter.

FIG. 5. (Color) Combined effect of beam-beam and beam-electron cloud interactions. The two curves correspond to the variation of maximum $J_{y,\max}$ of the macroparticles with and without beam-beam interaction. The horizontal green line indicates $\sqrt{\epsilon_y}$.

We looked for signals of horizontal instability in the above simulation. For the horizontal tunes examined, $Q_x = 0.51, 0.515$, and 0.56 , however, a horizontal instability has not been seen in the simulations. The effect of horizontal wake force may not be as strong as expected in a linear theory.

The previous simulation must be extended in order to take into account the nonlinearity of the beam-beam interaction sufficiently. In the simulation, betatron phase space location for a given synchrotron phase space location of a macroparticle is unique. Actually since there are many particles with various betatron coordinates in a region of synchrotron phase space, the beam-beam force may smear the betatron motion. To estimate the nonlinearity correctly, a strong-strong beam-beam simulation, which treats interactions between many macroparticles, is required. Three dimensional beam-beam simulation is essential to study the present problem, since we are interested in a head-tail motion due to beam-beam and beam-cloud effects. However the three dimensional beam-beam simulation was problematic even without the electron cloud interaction. The beam was divided into longitudinal slices, and to get reliable results in the simulation, many longitudinal slices (~ 20 – 30) seemed required depending on bunch length and beam-beam

FIG. 6. (Color) y - z distribution of the positron bunch after 400 turns (a) and 800 turns (b). Different colors of the data points correspond to the bunch shapes with (red) and without (green) beam-beam interaction.

FIG. 7. (Color) Dependence on chromaticity and synchrotron tune spread. (a) depicts the evolution of J_y for $Q'_p = 5$ and $Q'_p = 0$, and (b) depicts those for $\Delta Q_s = 0$ and $\Delta Q_s = 0.005$. The horizontal green line indicates $\sqrt{\epsilon_y}$.

parameter [9,10]. It was not fully understood why many slices were necessary. Since the calculation time scales quadratically with the number of slices, a very long CPU time was required. To proceed efficiently, we need to study how to integrate the three dimensional beam-beam interaction.¹ A method is being developed to do this, and some preliminary results are shown in Ref. [11]. To check the reliability of the three dimensional calculation, however, more studies are needed, such as the study of synchrotron modes [2]. We will discuss the three dimensional beam-beam issue and its electron cloud related effects using a strong-strong beam-beam simulation elsewhere.

V. CONCLUSIONS AND DISCUSSIONS

We studied the combined phenomena of the beam-beam and beam-electron cloud effects using first a linear theory and then a simulation with Gaussian approximation. In the linear theory, a one-two-particle model was used to describe the electron and positron beams. The electron cloud effect was approximated by a constant wake field. We first discussed vertical effects, because

¹The Coulomb potential, which determines beam-beam force, is solved in the two dimensional space slice by slice, therefore this may be called a 2.5 dimensional simulation.

the disruption parameter of the vertical direction is larger than the horizontal one. The beam-beam system without the electron cloud effect was unstable at particular tune regions related to a synchrotron resonance, whereas the combined system was always unstable regardless of the tune.

The imaginary tune has a simple behavior, monotonously increasing with the strength of the electron cloud effect (or wake field), while its dependences on the beam-beam parameter and chromaticity exhibit rather complex behaviors. The beam-beam kicks, which depend on the longitudinal coordinate, seem to affect the beams in ways similar to the effect of chromaticity in a head-tail instability.

The simulation with Gaussian approximation was performed to study the phenomena under general conditions. Below the thresholds of both beam-beam and beam-cloud instabilities considered alone, an instability occurred due to their combined effect in the simulation. We studied effects for the chromaticity and synchrotron tune spread on this combined instability. The results are consistent with the results of the linear theory. The combined phenomena may be analogous in its characteristics to the regular head-tail effect.

We also studied the horizontal instability in the linear theory. The horizontal effects may be important depending on the choice of the horizontal tune, although this conclusion has not yet been born out in the Gaussian model.

The nonlinear effect is very strong in the beam-beam interaction. Landau damping is one important nonlinear effect that may affect this combined instability. The Gaussian model is insufficient to take into account the nonlinearity. A three dimensional beam-beam and beam-electron cloud simulation based on a strong-strong model will have to be implemented in order to determine whether or under what conditions Landau damping can suppress the instability.

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