Identification and correction of antisymmetric field components in combined function magnets

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This paper investigates the identification and correction of antisymmetric field components in combined function magnets. A measurement procedure is developed to identify small field components on the order of 10^{-3} in the plane of the reference path. Furthermore, a method is proposed to compensate them by surface correction coils on the poles of the magnet. As an example, the measurement and compensation of antisymmetric field components in a 90° bending magnet of the double sided microtron HDSM at the MAMI-C electron scattering facility is simulated with the help of the electromagnetic design code TOSCA.

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I. INTRODUCTION

Field components in the plane of the reference path lead to a coupling between the horizontal, vertical, and longitudinal phase spaces in combined function magnets [1,2]. They are typically on the order of 10^{-3} if the magnet is carefully designed and manufactured. To the author's knowledge, they were hitherto neglected in the identification and correction of field errors when hall probes were the only option for field measurement [3–5]. However, the construction of more advanced combined function magnets such as the 90° dipoles of the double sided microtron HDSM at MAMI-C (see Sec. IV) makes it necessary to compensate even those small field errors over an area greater than 6 m^2 .

In Sec. II, a procedure is proposed to experimentally determine antisymmetric field components. A method to compensate them by surface correction coils is given in Sec. III. Finally, the identification and compensation of antisymmetric field components in a 90° HDSM magnet is simulated with the help of TOSCA [6] in Sec. V.

If $\vec{j}(\vec{q})$ is the current density at location $\vec{q}$, the vector potential $\vec{A}$ is [7]

$$\vec{A}(\vec{q}) = \frac{1}{4\pi\epsilon_0 c^2} \int d\vec{q}' \frac{\vec{j}(\vec{q}')}{|\vec{q} - \vec{q}'|}. \quad (1)$$

The magnetization vector within the magnet is $\vec{M}$. It is equivalent to the following current density $\vec{j}_M$:

$$\vec{j}_M = \nabla \times \vec{M}. \quad (2)$$

Inserting $\vec{j}_M$ into Eq. (1) and integrating over the volume V and surface S of the magnet yields [7]

$$\begin{aligned} \vec{A}_M(\vec{q}) = & \frac{1}{4\pi\epsilon_0 c^2} \left(\int_V dV(\vec{q}') \frac{\nabla' \times \vec{M}(\vec{q}')}{|\vec{q} - \vec{q}'|} \right. \\ & \left. + \oint_S dS(\vec{q}') \frac{\vec{M}(\vec{q}') \times \vec{n}'(\vec{q}')}{|\vec{q} - \vec{q}'|} \right), \end{aligned} \quad (3)$$

where $\vec{n}'$ is the surface normal vector of S . In the following, it is assumed that the magnetization $\vec{M}$ is homogeneous in V , or that the gap of the magnet is much smaller than V . Then the volume integral in Eq. (3) can be considered as a constant. Assume that the magnetization $\vec{M}_e$ on the surface of the real magnet is somehow known. To bring it in accordance with the design calculation for $\vec{M}$, a correction current $\delta\vec{j}$ is needed:

$$\delta\vec{j} = \nabla \times (\vec{M} - \vec{M}_e). \quad (4)$$

In magnetostatics we have

$$\nabla \times \left(\vec{B} - \frac{1}{\epsilon_0 c^2} \vec{M} \right) = 0, \quad (5)$$

and thus

$$\delta\vec{j} = \epsilon_0 c^2 \nabla \times (\vec{B} - \vec{B}_e). \quad (6)$$

Therefore the magnetic field $\vec{B}_e$ at the surface of the poles of the magnet has to be determined experimentally. From a comparison of $\vec{B}_e$ with the design calculation for $\vec{B}$, surface correction coils can be constructed as shown in Eq. (6). For (anti)symmetrical field errors, the correction current $\delta\vec{j}$ is (anti)symmetrical with respect to the midplane. The corresponding field components are perpendicular to the midplane in the case of symmetry and are parallel to it in the case of antisymmetry.

II. MEASUREMENT OF ANTISYMMETRIC FIELD COMPONENTS

Assume that the reference path lies in the x - z plane, which is also the median plane of the magnet, and that the main field component is parallel to the y axis. It is not experimentally feasible to directly measure the B_x and B_z components of the field as a function of (x, z) in the presence of a B_y component that is approximately 10^3 times larger. Therefore, the procedure proposed by Wind [8] is not applicable. Harmonic coils [9] would be another option if the field errors are expected to be invariant with x or z , which is not assumed here.

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The following procedure circumvents a direct measurement of B_x and B_z . In the gap of the magnet, Maxwell's equations yield for the magnetostatic case:

$$\nabla \cdot \vec{B} = 0, \quad (7)$$

and

$$\nabla \times \vec{B} = 0. \quad (8)$$

Differentiation of Eq. (7) with respect to x and z gives

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_z}{\partial z \partial x} = -\frac{\partial^2 B_y}{\partial y \partial x}, \quad (9)$$

and

$$\frac{\partial^2 B_z}{\partial z^2} + \frac{\partial^2 B_x}{\partial z \partial x} = -\frac{\partial^2 B_y}{\partial y \partial z}, \quad (10)$$

respectively. From Eq. (8), it follows that

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}. \quad (11)$$

Inserting this relation into Eqs. (9) and (10) results in

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial y \partial x}, \quad (12)$$

and

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2} = -\frac{\partial^2 B_y}{\partial y \partial z}. \quad (13)$$

The right-hand sides of Eqs. (12) and (13) can be determined by a measurement of B_y below and above the median plane of the magnet. If $h_y/2$ is the distance from the median plane, then

$$\frac{\partial B_y}{\partial y}(x, 0, z) \approx \frac{1}{h_y} \left[B_y\left(x, \frac{h_y}{2}, z\right) - B_y\left(x, -\frac{h_y}{2}, z\right) \right]. \quad (14)$$

Thus if B_y is measured on a grid with spacing h_x and h_z , one can determine $\partial_{yx}B_y$ and $\partial_{yz}B_y$ with the help of Eq. (14) as follows:

$$\frac{\partial^2 B_y}{\partial y \partial x} \approx \frac{1}{h_y h_x} \left[B_y\left(x + \frac{h_x}{2}, \frac{h_y}{2}, z\right) - B_y\left(x + \frac{h_x}{2}, -\frac{h_y}{2}, z\right) + B_y\left(x - \frac{h_x}{2}, -\frac{h_y}{2}, z\right) - B_y\left(x - \frac{h_x}{2}, \frac{h_y}{2}, z\right) \right], \quad (15)$$

and

$$\frac{\partial^2 B_y}{\partial y \partial z} \approx \frac{1}{h_y h_z} \left[B_y\left(x, \frac{h_y}{2}, z + \frac{h_z}{2}\right) - B_y\left(x, -\frac{h_y}{2}, z + \frac{h_z}{2}\right) + B_y\left(x, -\frac{h_y}{2}, z - \frac{h_z}{2}\right) - B_y\left(x, \frac{h_y}{2}, z - \frac{h_z}{2}\right) \right]. \quad (16)$$

In mathematical terms, both Eqs. (12) and (13) are partial differential equations of the Dirichlet type. Assume that B_x and B_z are zero at the boundary of the measurement region, and that in the median plane $\partial_{yx}B_y$ and $\partial_{yz}B_y$ are known from measurement. Then each equation has a unique solution for B_x and B_z in the median plane. In general, both field components can be determined by numerical integrations of Eqs. (12) and (13). There exists an abundance of methods for this task [10]. Having solved both equations for B_x and B_z and additionally hav-

ing measured B_y in the median plane, all components of the magnetic field in the median plane of the magnet are determined.

III. SURFACE CORRECTION COILS

Now that $\vec{B}_e$ is known in the median plane of the real magnet, one can determine the magnetic field in the whole gap. In free space, $\vec{B}_e$ can be expressed as a gradient of a scalar potential Ψ [7]. In the vicinity of a point $(x_j, 0, z_i)$ on the measurement grid, Ψ can be expressed as follows:

$$\begin{aligned} \Psi = & a_{100}(x - x_j) + a_{010}y + a_{001}(z - z_i) + \frac{1}{2} a_{200}(x - x_j)^2 + \frac{1}{2} a_{020}(y)^2 + \frac{1}{2} a_{002}(z - z_i)^2 \\ & + a_{110}(x - x_j)y + a_{011}y(z - z_i) + a_{101}(x - x_j)(z - z_i) + \dots \end{aligned} \quad (17)$$

All coefficients are now known:

$$\begin{aligned} a_{100} &= B_x(x_j, 0, z_i), \\ a_{010} &= B_y(x_j, 0, z_i), \\ a_{001} &= B_z(x_j, 0, z_i), \end{aligned} \quad (18)$$

and

$$\begin{aligned}
 a_{200} &= \frac{\partial B_x}{\partial x}(x_j, 0, z_i) \approx \frac{1}{h_x} [B_x(x_j + h_x, 0, z_i) - B_x(x_j, 0, z_i)], \\
 a_{020} &= -a_{200} - a_{002}, \\
 a_{002} &= \frac{\partial B_z}{\partial z}(x_j, 0, z_i) \approx \frac{1}{h_z} [B_z(x_j, 0, z_i + h_z) - B_z(x_j, 0, z_i)],
 \end{aligned} \tag{19}$$

and

$$\begin{aligned}
 a_{101} &= \frac{\partial B_x}{\partial z}(x_j, 0, z_i) \approx \frac{1}{h_z} [B_x(x_j, 0, z_i + h_z) - B_x(x_j, 0, z_i)], \\
 a_{110} &= \frac{\partial B_y}{\partial x}(x_j, 0, z_i) \approx \frac{1}{h_x} [B_y(x_j + h_x, 0, z_i) - B_y(x_j, 0, z_i)], \\
 a_{011} &= \frac{\partial B_y}{\partial z}(x_j, 0, z_i) \approx \frac{1}{h_z} [B_y(x_j, 0, z_i + h_z) - B_y(x_j, 0, z_i)].
 \end{aligned} \tag{20}$$

Using this approach, Ψ and thus $\vec{B}_e$ can be determined at the poles of the magnet. Neglecting a possible curvature of the poles, the surface current density $\vec{\sigma}$ at the upper pole can be calculated from Eq. (6)

$$\begin{aligned}
 \sigma_x &= +\epsilon_0 c^2 (B_z - B_{z,e}), \\
 \sigma_z &= -\epsilon_0 c^2 (B_x - B_{x,e}),
 \end{aligned} \tag{21}$$

if the relative permeability of the magnet is assumed to be infinite. The current density at the lower pole has a reversed sign. If a function $I(x, z)$ is defined as

FIG. 1. (Color) Artist's view and pole profile of a 90° HDSM bending magnet.

$$I(x, z) = \int_0^x ds_x \sigma_z(s_x, z) - \int_0^z ds_z \sigma_x(0, s_z), \quad (22)$$

one gets

$$\frac{\partial I}{\partial x} = \sigma_z; \quad \frac{\partial I}{\partial z} = -\sigma_x. \quad (23)$$

Therefore the surface correction coils have to run along the equipotential lines of I with a current equal to the level distance ΔI .

FIG. 2. (Color) Simulated field errors B_x , B_z , and $B_y - B_{y,e}$ in the median plane of a 90° HDSM bending magnet with a slightly misaligned upper yoke.

FIG. 3. (Color) TOSCA calculation of B_x compared with the results of a simulated measurement of $\partial_{xy} B_y$ and subsequent integration of Eq. (12).

IV. MAMI-C

At the MAMI [11] electron scattering facility, three race-track microtrons are cascaded to provide a cw electron beam at 855 MeV. In a fourth stage, a double sided microtron HDSM is currently being built [12] to raise the beam energy to 1500 MeV in 43 cycles. A 250 ton 90° HDSM bending magnet [13] is shown in Fig. 1. Its field maximum for B_y is 1.5 T. To nullify the strong vertical defocusing forces of the fringe field for all beam energies between 855 and 1500 MeV, the magnets have an additional field gradient in the main field region [14]. Because of construction constraints on the sophisticated pole profile, it is expected that antisymmetric field components

FIG. 4. (Color) Surface correction coils for the upper pole of a 90° HDSM bending magnet.

FIG. 5. (Color) Field components B_x , B_z , and $B_y - B_{y,e}$ in the midplane of a 90° HDSM bending magnet with surface correction coils to compensate field errors due to a slightly misaligned upper yoke.

in the median plane of the order of 10^{-3} have to be compensated.

V. TOSCA SIMULATION

To simulate antisymmetric field components in the median plane of a 90° HDSM magnet, its upper yoke was not correctly aligned with respect to the lower yoke, but the maximum deviations of 0.3 mm were still within the manufacturing tolerance. The ensuing field errors were calculated with TOSCA and are shown in Fig. 2. From the TOSCA calculation, the main field component B_y in

TABLE I. Deviation of B_x and B_z from the TOSCA results B_{xT} and B_{zT} for various noise levels of B_y .

σ_{B_y} (Gauss)	0.0	0.5	1.0	2.0	4.0	8.0
$\sigma_{B_x - B_{xT}}$ (Gauss)	0.05	0.15	0.26	0.47	0.91	1.78
$\sigma_{B_z - B_{zT}}$ (Gauss)	0.03	0.17	0.22	0.44	0.86	1.71

the median plane and at $h_y/2 = 2$ cm above and below the median plane was extracted on a grid with spacing $h_x = 1$ cm and $h_z = 1$ cm. Then the field components B_x and B_z in the median plane were calculated according to Sec. II. In Fig. 3, the result for B_x is compared with the original data given by TOSCA. They agree within 5%. Finally, surface correction coils were calculated as described in Sec. III. On the lower pole, no correction coils were necessary in this example since the field errors were exclusively produced by the misaligned upper yoke. The surface correction coils for the upper pole are shown in Fig. 4. The magnetic field in the median plane was again calculated by TOSCA with the correction coils turned on. The result is shown in Fig. 5. In all cases, the field quality was improved by a factor of about 10.

To estimate the robustness of the proposed procedure, assume in the following that B_x is not a function of z and B_z is not a function of x . Then Eqs. (12) and (13) can be integrated with respect to x and z , respectively:

$$\delta B_x \approx -\frac{\delta x}{\delta y} \delta B_y, \quad (24)$$

and

$$\delta B_z \approx -\frac{\delta z}{\delta y} \delta B_y. \quad (25)$$

Therefore the error in determining B_x and B_z is proportional to the measurement error of B_y . To verify this relationship numerically, Gaussian noise was added to the TOSCA data for B_y , and then B_x and B_z were recalculated according to Eqs. (12) and (13). Table I shows the sigma width of the ensuing deviation from the original TOSCA result for B_x and B_z . The linear relationship as predicted by Eqs. (24) and (25) is well verified. For the used spacing $h_y = 4$ cm and $h_x = 1$, $h_z = 1$ cm, one would expect a slope of $1/4.0$. The simulation yields $1/4.4$, which is probably due to the simplifying assumption leading to Eqs. (24) and (25).

VI. CONCLUSION

In this paper, we demonstrated that it is possible to identify and compensate small antisymmetric field components on the order of 10^{-3} in combined function magnets. By measuring the main field components both in the median plane and below and above the median plane, one can derive all components of the magnetic field in the median plane. Thus the field in the whole gap of the magnet can be calculated from measurements. Surface correction coils

can be constructed from a comparison of the experimentally determined magnetic field at the poles with the field of the design calculation.

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