One-step tuning algorithm for π -mode cavities

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Cavities for π -mode operation are used in existing accelerators and are being built for many future systems. One of the steps in cavity construction is tuning individual cells within the cavity to ensure that the absolute values of relative peak on-axis electric field amplitudes in each cell are equal. This process may have multiple iterative steps: measuring the on-axis fields using bead-pull techniques, employing tuning methods to perturb the cells, making cell adjustments, and then repeating the process on the basis of the new results. This paper describes a one-step process for an N cell cavity that provides the necessary adjustment for each cell to achieve equal field amplitude absolute values from one bead-pull measurement. In addition, a method is shown for inferring relative cell fields while a cavity is operational in an accelerator complex.

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I. INTRODUCTION

Important elements of any particle accelerator are the rf structures providing beam acceleration. The usual choice made for these structures in synchrotrons or modern linacs is π -mode cavities. In order to achieve the best performance from these π -mode cavities it is important to have the absolute values of the on-axis electric field amplitudes in each cell as equal as possible. Ensuring that the fields are equal is a time-consuming process involving on-axis bead pulls to measure field distributions, determination of the sensitivity to various cell changes, taking action on specific cells, remeasuring the fields, taking action again, etc. Such an iterative procedure results in an acceptable relative field distribution cell-to-cell along the length of the cavity, but it requires much handling of the cavity. This paper describes a simplification to this procedure; a method that reduces the above process to one step. The initial bead-pull measurement is used to determine actions necessary to correct all cell frequencies such that the peak on-axis field distribution along the length of the cavity will be flat and the π -mode frequency will be that desired.

This one-step tuning procedure results in cavities with flat fields cell-to-cell. However, a remaining concern when cavities are put on-line and operated as part of an accelerator complex is whether cavity field distributions stay as measured. A method to infer possible changes to the field distributions while on-line is described in the following, using techniques developed for the one-step tuning.

II. ONE-STEP TUNING

Techniques developed in this paper for understanding specific details of π -mode cavities are based on coupled-circuit theory using an analog system consisting of coupled RLC resonant loops. Terminology and information developed in Ref. [1] are used extensively in the following discussion. Equation development [1] for relative fields in a π -mode cavity made it possible to determine methods for

evaluating fields in a cavity with errors and for the actions required to make fields as equal as possible cell-to-cell. Solving for relative fields using a stepwise method and determination of the π -mode frequency correction factor α were two significant steps making it possible to develop the theory for a one-step tuning algorithm that achieves flat fields in a π -mode cavity.

A. One-step field flattening from frequency corrections

The relative cell field amplitude expression for a full-cell terminated cavity operating in the π -mode given by Eq. (4) of Ref. [1] is simplified by eliminating terms whose effects are less than several percent of the main correction term. Using the absolute value for the cell field amplitude expression (taking into account cell frequency errors) gives

$$i_n(\text{full}) \approx 1 + n(n-1)\alpha - \frac{4}{k}[(n-1)\Delta_1 + \dots + 3\Delta_{n-3} + 2\Delta_{n-2} + \Delta_{n-1}]$$

on the basis of the first cell equation being $i_1 = 1$, where n is cell number from 1 to N , N is the total cell number, α is the important π -mode frequency correction factor defined later, k is the cavity coupling constant, Δ_n is the fractional frequency error in the n th cell given by $\Delta_n = df_n/f_0$, f_0 is the unperturbed frequency for each cell, and i_n represents the peak on-axis electric field amplitude in the n th cell. Because α can be written in terms of the frequency error df_n , the above equation can be written for each of the N cells in a form relating the field amplitude difference from unity to the frequency error df_n in a similar set of equations. The frequency correction factor is used basically for replacing k by $k(1 + \alpha)$ to obtain an accurate π -mode frequency value. Under the conditions of frequency errors $\alpha = 2(1 - k)(\sum_{n=1,N} df_n)/(f_0 N k)$ resulting in the set of equations:

$$\begin{aligned}
(i_2) - 1 &= 2 \frac{2(1-k)}{Nk} \left[\left(\sum_{n=1, N-1} \Delta_n \right) - \frac{4}{k} (\Delta_1) \right], \\
(i_3) - 1 &= 6 \frac{2(1-k)}{Nk} \left[\left(\sum_{n=1, N-1} \Delta_n \right) - \frac{4}{k} (2\Delta_1 + \Delta_2) \right], \\
(i_4) - 1 &= 12 \frac{2(1-k)}{Nk} \left[\left(\sum_{n=1, N-1} \Delta_n \right) - \frac{4}{k} (3\Delta_1 + 2\Delta_2 + \Delta_3) \right], \\
&\vdots \\
(i_N) - 1 &= N(N-1) \frac{2(1-k)}{Nk} \left\{ \left[\sum_{n=1, N-1} \Delta_n \right] - \frac{4}{k} [(n-1)\Delta_1 + \dots + 3\Delta_{N-3} + 2\Delta_{N-2} + \Delta_{N-1}] \right\}.
\end{aligned} \tag{1}$$

These equations written in matrix form with $c = 2(1-k)/Nk$ and $N_N = N(N-1)$ give

$$\begin{pmatrix} (i_2) - 1 \\ (i_3) - 1 \\ (i_4) - 1 \\ \vdots \\ (i_N) - 1 \end{pmatrix} = \begin{pmatrix} 2c - \frac{4}{k} & 2c & 2c & \cdots & 2c \\ 6c - \frac{8}{k} & 6c - \frac{4}{k} & 6c & \cdots & 6c \\ 12c - \frac{12}{k} & 12c - \frac{8}{k} & 12c - \frac{4}{k} & \cdots & 12c \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ N_N c - \frac{4(N-1)}{k} & N_N c - \frac{4(N-2)}{k} & N_N c - \frac{4(N-3)}{k} & \cdots & N_N c - \frac{4}{k} \end{pmatrix} \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_{N-1} \end{pmatrix}. \tag{2}$$

When on-axis field amplitudes are measured for an N cell cavity and normalized to the first being unity, the above equations can be used to determine frequency errors in the cells using the $(N-1)$ -by- $(N-1)$ inverted matrix. There are $N-1$ knowns (relative field amplitudes) to determine $N-1$ unknowns (cell frequency errors). The general form of the inverted matrix equation is given below for cell field amplitudes normalized such that the first cell field amplitude is unity ($i_1 = 1$):

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \Delta_{N-2} \\ \Delta_{N-1} \end{pmatrix} = \begin{pmatrix} -a & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & -b & b \\ 2a & -a & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & -b & b \\ -a & 2a & -a & 0 & 0 & \cdots & \cdots & \cdots & 0 & -b & b \\ 0 & -a & 2a & -a & 0 & \cdots & \cdots & \cdots & 0 & -b & b \\ 0 & 0 & -a & 2a & -a & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & -a & 2a & -a & 0 & -b & b \\ \vdots & \vdots & \vdots & \vdots & 0 & -a & 2a & -a & -b & b & b \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & -a & 2a & -g & b \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -a & d & -e \end{pmatrix} \begin{pmatrix} (i_2) - 1 \\ (i_3) - 1 \\ (i_4) - 1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ (i_{N-1}) - 1 \\ (i_N) - 1 \end{pmatrix}, \tag{3}$$

where

$$\begin{aligned}
a &= \frac{k}{4}, & b &= \frac{ck^2}{4[(N-1)ck - 2]}, \\
g &= \frac{k[Nck - 2]}{4[(N-1)ck - 2]}, & d &= \frac{k[(2N-3)ck - 4]}{4[(N-1)ck - 2]}, \\
e &= \frac{k[(N-2)ck - 2]}{4[(N-1)ck - 2]}.
\end{aligned} \quad \text{and}$$

$$IM_5 = \begin{pmatrix} -a & 0 & -b & b \\ 2a & -a & -b & b \\ -a & 2a & -g & b \\ 0 & -a & d & -e \end{pmatrix},$$

$$IM_6 = \begin{pmatrix} -a & 0 & 0 & -b & b \\ 2a & -a & 0 & -b & b \\ -a & 2a & -a & -b & b \\ 0 & -a & 2a & -g & b \\ 0 & 0 & -a & d & -e \end{pmatrix}.$$

The lower right-hand portion of the inverted matrix, (IM), is important as shown below for the 2×2 , 4×4 , and 5×5 inverted matrices used, respectively, for three-cell, five-cell, and six-cell cavity calculations:

$$IM_3 = \begin{pmatrix} -g & b \\ d & -e \end{pmatrix},$$

Use of this inverted matrix methodology was verified using data generated from LOOPER [2]. Accurate values for frequency errors were obtained for errors up to 1/2% (5 MHz errors in 1 GHz cavity) with cell field amplitudes

differing up to a factor of 2. Parameters for these verification calculations used k from 1% to 8%, quality factors down to 10 000, and cavity cell numbers to nine.

The method is illustrated using values for a five-cell cavity. Analysis of data from an on-axis bead pull determines the five peak on-axis fields in the cavity (E_1, E_2, E_3, E_4, E_5). An end cell, for instance $n = 1$, is used to normalize the fields used in the calculation ($E_1/E_1 = 1.0, E_2/E_1, E_3/E_1, E_4/E_1, E_5/E_1$). The four fields referenced to a unity value used for the $(i_n) - 1$ vector in Eq. (3) are then ($E_2/E_1 - 1.0, E_3/E_1 - 1.0, E_4/E_1 - 1.0, E_5/E_1 - 1.0$). Fractional frequency errors, $(\Delta_1, \Delta_2, \Delta_3, \Delta_4)$, in the first four cells are determined from the 4×4 matrix of Eq. (3). An error in the fifth cell of amount (Δ_5) shows up as $(-\Delta_5, -\Delta_5, -\Delta_5, -\Delta_5)$ with a $(-\Delta_5)$ value for all of the cells from one to four. Everything is relative, as illustrated in this last example. If the frequency for the π mode is δf low fractionally, the tuning action taken for each of the five cells to achieve a flat field distribution in the cavity at the desired frequency is $(\delta f - \Delta_1 + S, \delta f - \Delta_2 + S, \delta f - \Delta_3 + S, \delta f - \Delta_4 + S, \delta f + S)$ with $S = \sum_{n=1,4} \Delta_n/4$.

Data [3] from KfA, Germany, for a five-cell cavity ($k = 0.0185, f_0 = 500$ MHz) show relative field amplitudes from cells one to five of (1.0, 1.045, 1.0722, 1.0058, 1.0284) from which the above method gives frequency errors from cells one to five of (-152, -6.6, 169, -254, and 0 kHz) that need to be corrected to obtain flat field amplitudes. A check on the accuracy of the method and the assumption that there are no other errors contributing to field changes can be accomplished by using the method from the opposite end of the cavity. In this case, relative field amplitudes from cells five to one of (1.0, 0.9780, 1.0426, 1.016, 0.9724) yield frequency errors from cells five to one of (143, -108, 303, 131, 0 kHz). This set of errors converts to (-143, -12, 160, -251, and 0 kHz) for cells one to five, indicating that there is an additional small

effect probably associated with very small coupling constant errors between cells one and three (less than 0.005 fractional coupling error associated with differences between the two cases).

Analysis in Sec. IIIC of Ref. [1] showed that dimensional tolerances were a factor of 10 smaller for frequency requirements than for coupling constant requirements. Since frequency tolerances are much more stringent, it is assumed that relative field errors in a cavity are mainly related to frequency errors. In addition, using a large set of calculations on the basis of data generated with LOOPER, it was determined that frequency corrections compensate for coupling constant errors. The two cells affected by the coupling constant error are the two cells that need their frequencies adjusted. Relative field errors from coupling constant errors were reduced by at least a factor of 10 from this frequency tuning of the cells.

B. One-step field flattening from coupling corrections

Coupling constant errors are best determined (and possibly corrected) after initial assembly of cavity cells by making individual coupling constant measurements between cells when other cells are significantly detuned or shorted. If this process is inadequate to make all of the necessary corrections, then remaining errors can be accounted for by adjusting cell frequencies as described in Sec. IIA. Although errors in coupling constants can be ameliorated by this method, there may be a particular situation for which coupling constant corrections must be determined because it is known that the field errors depend on coupling constant errors only. Using Eq. (4) from Ref. [1], with only errors in coupling constants $[k_{n,n+1} = k(1 + \epsilon_{n,n+1})]$, where $\epsilon_{n,n+1}$ is the fractional coupling error, gives: $i_n(\text{full}) \approx 1 + n(n-1)\alpha - (\dots + 7\epsilon_{n-4,n-3} + 5\epsilon_{n-3,n-2} + 3\epsilon_{n-2,n-1} + \epsilon_{n-1,n})$ with $\alpha = (\sum_{n=1,N-1} \epsilon_{n,n+1})/N$ leading to the set of equations

$$\begin{aligned}
 (i_2) - 1 &= 2 \frac{1}{N} \left[\left(\sum_{n=1,N-1} \epsilon_{n,n+1} \right) - \epsilon_{1,2} \right], \\
 (i_3) - 1 &= 6 \frac{1}{N} \left[\left(\sum_{n=1,N-1} \epsilon_{n,n+1} \right) - (3\epsilon_{1,2} + \epsilon_{2,3}) \right], \\
 (i_4) - 1 &= 12 \frac{1}{N} \left[\left(\sum_{n=1,N-1} \epsilon_{n,n+1} \right) - (5\epsilon_{1,2} + 3\epsilon_{2,3} + \epsilon_{3,4}) \right], \\
 &\vdots \\
 (i_N) - 1 &= N(N-1) \frac{1}{N} \left[\left(\sum_{n=1,N-1} \epsilon_{n,n+1} \right) - (\dots + 7\epsilon_{N-4,N-3} + 5\epsilon_{N-3,N-2} + 3\epsilon_{N-2,N-1} + \epsilon_{N-1,N}) \right],
 \end{aligned}$$

of matrix form with $c = 1/N$ and $N_N = N(N-1)$,

$$\begin{pmatrix} (i_2) - 1 \\ (i_3) - 1 \\ (i_4) - 1 \\ \vdots \\ (i_N) - 1 \end{pmatrix} = \begin{pmatrix} 2c - 1 & 2c & 2c & \cdots & 2c \\ 6c - 3 & 6c - 1 & 6c & \cdots & 6c \\ 12c - 5 & 12c - 3 & 12c - 1 & \cdots & 12c \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N_{Nc} - (2N - 3) & N_{Nc} - (2N - 5) & N_{Nc} - (2N - 7) & \cdots & N_{Nc} - 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1,2} \\ \varepsilon_{2,3} \\ \varepsilon_{3,4} \\ \vdots \\ \varepsilon_{N-1,N} \end{pmatrix}. \quad (4)$$

When on-axis field amplitudes are measured for an N cell cavity and normalized to the first being unity, the above equations can be used to determine coupling constant errors in the cells using the $(N - 1)$ -by- $(N - 1)$ inverted matrix. There are $N - 1$ knowns (relative field amplitudes) to determine $N - 1$ unknowns (the $N - 1$ coupling constant errors). There are two general forms to the inverted matrix equation depending on whether the number of cells is odd or even as given below for cell fields normalized such that the first cell field is unity ($i_1 = 1$).

(i) Odd cell number with even numbered matrix,

$$\begin{pmatrix} \varepsilon_{1,2} \\ \varepsilon_{2,3} \\ \varepsilon_{3,4} \\ \vdots \\ \varepsilon_{N-2,N-1} \\ \varepsilon_{N-1,N} \end{pmatrix} = \begin{pmatrix} -a & b & -b & b & -b & \cdots & \cdots & \cdots & b & -b & b/2 \\ 3 & -1 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ -g & d & -a & b & -b & \cdots & \cdots & \cdots & b & -b & b/2 \\ 4 & -4 & 3 & -1 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ -g & g & -g & d & -a & \vdots & \vdots & \vdots & b & -b & b/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & g & -g & d & -a & b & -b & b/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & 4 & -4 & 3 & -1 & 0 & 0 \\ -g & g & -g & g & \cdots & \cdots & g & -g & d & -a & b/2 \\ 4 & -4 & 4 & -4 & \cdots & \cdots & \cdots & 4 & -4 & 3 & -1 \end{pmatrix} \begin{pmatrix} (i_2) - 1 \\ (i_3) - 1 \\ (i_4) - 1 \\ \vdots \\ (i_{N-1}) - 1 \\ (i_N) - 1 \end{pmatrix}, \quad (5)$$

where

$$a = \frac{(N + 3)c - 1}{(N - 1)c - 1}, \quad b = \frac{4c}{(N - 1)c - 1}, \quad g = \frac{4(Nc - 1)}{(N - 1)c - 1}, \quad d = \frac{(3N + 1)c - 3}{(N - 1)c - 1}.$$

(ii) Even cell number with odd numbered matrix,

$$\begin{pmatrix} \varepsilon_{1,2} \\ \varepsilon_{2,3} \\ \varepsilon_{3,4} \\ \vdots \\ \varepsilon_{N-2,N-1} \\ \varepsilon_{N-1,N} \end{pmatrix} = \begin{pmatrix} -a & -b & b & -b & b & \cdots & \cdots & \cdots & b & -b & b/2 \\ 3 & -1 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ -g & d & -a & -b & b & \cdots & \cdots & \cdots & b & -b & b/2 \\ 4 & -4 & 3 & -1 & 0 & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ -g & g & -g & d & -a & \vdots & \vdots & \vdots & b & -b & b/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 4 & -4 & 3 & -1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & g & -g & d & -a & -b & b/2 \\ 4 & -4 & 4 & -4 & \cdots & \cdots & 4 & -4 & 3 & -1 & 0 \\ -g & g & -g & g & \cdots & \cdots & \cdots & g & -g & d & -e \end{pmatrix} \begin{pmatrix} (i_2) - 1 \\ (i_3) - 1 \\ (i_4) - 1 \\ \vdots \\ (i_{N-1}) - 1 \\ (i_N) - 1 \end{pmatrix}, \quad (6)$$

where

$$a = \frac{(N - 4)c - 1}{Nc - 1}, \quad b = \frac{4c}{Nc - 1}, \quad g = \frac{4[(N - 1)c - 1]}{Nc - 1},$$

$$d = \frac{(3N - 4)c - 3}{Nc - 1}, \quad e = \frac{(N - 2)c - 1}{Nc - 1}.$$

The lower terms of the inverted matrices, (IM), are important as illustrated in Sec. II A for small matrices such as the 2×2 inverted matrix from Eq. (5) used for a three-cell cavity case, and the 3×3 inverted matrix from Eq. (6) used for a four-cell cavity case.

Use of this inverted matrix methodology was verified using data generated from LOOPER by completing calculations with various coupling constant errors.

A five-cell cavity is used to illustrate the method. Analysis of data from an on-axis bead pull determines the five peak on-axis field amplitudes in the cavity (E_1, E_2, E_3, E_4, E_5). An end cell, for instance $n = 1$, is used to normalize the field amplitudes used in the calculation ($E_1/E_1 = 1.0, E_2/E_1, E_3/E_1, E_4/E_1, E_5/E_1$). The four fields referenced to a unity value used for the $(i_n) - 1$ vector in Eq. (5) are then ($E_2/E_1 - 1.0, E_3/E_1 - 1.0, E_4/E_1 - 1.0, E_5/E_1 - 1.0$). Fractional coupling constant errors, ($\varepsilon_{1,2}, \varepsilon_{2,3}, \varepsilon_{3,4}, \varepsilon_{4,5}$), in the cavity are determined from the 4×4 matrix of Eq. (5).

Data from the KfA five-cell cavity with relative field amplitudes from cells one to five of (1.0, 1.045, 1.0722, 1.0058, 1.0284) give fractional coupling constant errors from cells one to five of (-0.19, 0.06, -0.11, -0.12) that would need to be corrected to obtain flat field amplitudes if indeed these were the errors. This case is used as an example only, because such large coupling errors are difficult to imagine and would be very hard to compensate for in a real cavity. The field errors are actually a result of frequency errors that are corrected as discussed in Sec. II A.

III. INFORMATION FROM MODE SPECTRA

A concern in operating π -mode cavities is the fact that relative on-axis field amplitudes are difficult to measure on-line. The assumption is that the relative field amplitudes are essentially those measured prior to locating the cavity on-line in the accelerator complex. Usually the cavity has to be taken off-line and opened to air in order for a

bead pull to be accomplished. However, results from the bead pull provide information on the relative field amplitudes for the conditions of the cavity off-line. It is possible to infer relative fields in an operating cavity by analyzing the measured mode spectra while on-line. Such a measurement of the mode spectra is not a difficult task, as demonstrated earlier [4].

Fields for all passband modes of a full-cell terminated cavity with end cells tuned to produce a flat field distribution for the π -mode have been investigated extensively. This investigation showed that field amplitudes for each mode under this end-cell tuning condition have relative field patterns given by $i_n \approx \sin[(2n - 1)\pi q/(2N)]$, where i_n is the field in the n th cell, $q = 1, 2, 3, \dots, N$ is the mode number ($q = 1$ for the 0-mode and $q = N$ for the π -mode), n is the cell number, and N is the number of cells in the cavity.

Using these field pattern solutions for the modes of a full-cell terminated cavity, and realizing that these patterns are symmetric about the cavity center, it is clear that cell frequency information from mode spectra can be obtained easily only for pairs of cells, i.e., the end cells as a pair, the next inner pair, and so on until all cells are included.

A frequency shift $dF(q)$ in a particular cavity mode frequency $F(q)$ is determined from the sum of each cell frequency error df_n times the normalized squares of the cell fields for each mode using the fact that $\sum_{n=1,N} \sin^2[(2n - 1)\pi q/(2N)] = N/2$ for all modes except for the π -mode ($q = N$) when the sum is equal to N . The following matrix form yields each mode frequency shift on the basis of actual frequency errors using $s(n, q) = \{\sin[(2n - 1)\pi q/(2N)]\}^2$. Units for df_n and $dF(q)$ are the same,

$$\begin{pmatrix} dF(1) \\ dF(2) \\ \vdots \\ dF(N) \end{pmatrix} = \begin{pmatrix} \frac{2s(1,1)}{N} & \frac{2s(2,1)}{N} & \dots & \frac{2s(N,1)}{N} \\ \frac{2s(1,2)}{N} & \frac{2s(2,2)}{N} & \dots & \frac{2s(N,2)}{N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{s(1,N)}{N} & \frac{s(2,N)}{N} & \dots & \frac{s(N,N)}{N} \end{pmatrix} \begin{pmatrix} df_1 \\ df_2 \\ \vdots \\ df_N \end{pmatrix}.$$

This matrix form was verified with extensive calculations using LOOPER for frequency errors up to 2 MHz in a 500 MHz cavity with coupling constants up to 8% and quality factors down to 10 000.

As stated above, when using the inverse matrix to determine cell frequency errors on the basis of the mode frequency passband, only one quadrant of a modified matrix is used because of the symmetry situation of the field patterns. (Complicated methods exist for inverting such symmetric matrices, but they are not used here.) In this case, pairs are combined to give the following matrix where the inverted matrix is not shown explicitly because of its complex multiple terms for each element. In the following representation, cell and mode numbering start at an end cell and the first mode, respectively, odd N has parameters $[N_{\text{mid}} = N/2, N_{\text{num}} = (N + 1)/2]$, and even N has parameters $(N_{\text{mid}} = N/4, N_{\text{num}} = N/2)$,

$$\begin{pmatrix} df_1 \\ df_2 \\ \vdots \\ df_{N_{\text{num}}} \end{pmatrix} = \begin{pmatrix} \frac{4s(1,1)}{N} & \frac{4s(2,1)}{N} & \dots & \frac{s(N_{\text{num}},1)}{N_{\text{mid}}} \\ \frac{4s(1,2)}{N} & \frac{4s(2,2)}{N} & \dots & \frac{s(N_{\text{num}},2)}{N_{\text{mid}}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{4s(1,N_{\text{num}})}{N} & \frac{4s(2,N_{\text{num}})}{N} & \dots & \frac{s(N_{\text{num}},N_{\text{num}})}{N_{\text{mid}}} \end{pmatrix}^{-1} \begin{pmatrix} dF(1) \\ dF(2) \\ \vdots \\ dF(N_{\text{num}}) \end{pmatrix}.$$

In this case, the result for df_1 applies to both cells 1 and N ; df_2 applies to both cells 2 and $N - 1$, etc. Having calculated frequency differences df_n in the cells, Eq. (2) in Sec. II A can now be used to determine relative field shifts for all cells.

Thus, an estimate of relative fields in an operating cavity can be determined.

As an aid to these calculations, Ref. [1] showed that the frequency difference between the last mode and its next nearest mode, $[F(N) - F(N - 1)]$, and between the next to last mode and the first, $[F(N - 1) - F(1)]$, can be used to determine both the cavity coupling constant and a global field difference between end cells and the rest of the cells in the cavity.

The lower part of the mode passband is not the only part that must be used to infer field changes. To use the upper part of the spectrum, the above matrix elements are transposed from the first quadrant to the last quadrant. This modification requires matrix element changes from the first element $4s(1, 1)/N$ becoming $s(N_{\text{num}}, N_{\text{num}})/N_{\text{mid}}$ to the last element $s(N_{\text{num}}, N_{\text{num}})/N_{\text{mid}}$ becoming $4s(N, N)/N$, with appropriate multipliers for the first and last columns of the matrix based on whether N is even or odd (whether there are cell pairs for the central part of the cavity). Data on mode spectra from KfA show that, relative to an errorless system (flat fields), mode frequency shifts for the last three modes starting from the middle mode are (15, -81, -39 kHz). These mode shifts lead to cell pair frequency errors from end cells to middle cells of (-74, -100, 153 kHz). In order to use this information with Eq. (2), frequency errors must be shifted so that $\Delta_5 = 0$, yielding frequency errors from cells one to five of (0, -26, 227, -26, 0 kHz). Normalizing these values to an f_0 of 500 MHz for the cavity yields a field pattern of (1.0, 1.015, 1.055, 1.013, 0.996) which is in fair agreement with the measured values of (1.0, 1.045, 1.072, 1.006, 1.028); fair agreement on the basis of knowing that cell pairs are not identically matched for this cavity as shown in Sec. II A.

IV. CONCLUSIONS

A simple and efficient tuning method was developed, incorporating a one-step process based on measuring rela-

tive on-axis fields in a multicell cavity operating in the π -mode. The method determines frequency tuning required for each cavity cell in order to obtain a flat field distribution throughout the cavity at the expected operating frequency.

In addition, a method for determining possible corrections to coupling constant errors was developed. This method is not as important as the frequency method since the latter is easier to accomplish and can compensate for any coupling errors that might exist in a cavity.

Also, measurement of the mode passband can be used to infer cavity field distributions for a π -mode cavity while operating in an accelerator environment, as long as it is possible to measure these mode frequencies and determine frequency changes that occurred since the spectra was last measured.

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