Erratum: Chromaticity of the lattice and beam stability in energy recovery linacs [Phys. Rev. ST Accel. Beams **15, 074401 (2012)]**

Vladimir N. Litvinenko

(Received 25 November 2014; published 31 March 2015)

DOI: [10.1103/PhysRevSTAB.18.039901](https://doi.org/10.1103/PhysRevSTAB.18.039901)

PACS numbers: 29.27.-a, 29.20.Ej, 99.10.Cd

The subject of emittance growth resulting from noncompensated chromaticity in single pass linac focusing systems was exhaustively studied in the era of linear colliders decades ago; see, for example, classical treatment in Refs. [1–4], and references therein. Effects described in these papers are fully applicable to energy recovery linacs with noncompensated chromaticity, e.g., a case presented in this paper. I suggest to use a strongly chromatic lattice to improve the beam’s stability in energy recovery linacs (ERLs). Recently, I realized, that, naturally, such lattices would exhibit a very high sensitivity to orbital errors—the effect described in details in Refs. [1–4]. Such errors could, in turn, significantly increase the beam’s emittance. Hence, this Erratum will address this shortcoming of this paper.

A simple case illustrating the importance of this shortcoming can be presented in a very simple form [5] using notations used in this paper. The orbit distortion caused by random field errors can be described as follows:

$$\delta x(s, \delta) = w(s, \delta) \cdot \int_0^s \frac{e\delta B(s_1)}{p_o c(1 + \delta)} \cdot \sin(\psi(s, \delta) - \psi(s_1, \delta)) \cdot w(s_1, \delta) ds_1 \quad (1)$$

wherein I used the definitions from Eq. (8) of this paper. The angle $\delta x'(s, \delta)$ can be obtained readily simply by differentiating (1). In a periodic lattice comprising N_{cell} identical cells, introducing an angular kicks θ_k would result in the following energy-dependent orbit at its exit:

$$\delta x(s, \delta) = w_N(s_N, \delta) \cdot \sum_{k=1}^{N_{\text{cell}}} \theta_k \cdot w(s_k, \delta) \cdot \sin(\psi(s_N, \delta) - \psi(s_k, \delta)). \quad (2)$$

For the case of uncorrelated errors, $\langle \theta_i \theta_k \rangle = \theta_i^2 \delta_{ik}$, located at the cells’ centers, with a Gaussian energy distribution (14) and expansion (10), one can show [5] that the projected normalized emittance at the exit of the lattice can be written as follows:

$$\begin{aligned} \varepsilon_N^2 &= \left(\varepsilon_{N0} + \gamma_o \beta \sum_{n=1}^{N_{\text{cell}}} A_n \theta_n^2 \right) \left(\varepsilon_{N0} + \gamma_o \beta \sum_{n=1}^{N_{\text{cell}}} B_n \theta_n^2 \right) + O\left(\left(v \sigma_\delta \sum_{n=1}^{N_{\text{cell}}} \theta_n^2 \right)^2 \right); \\ \beta &\equiv w^2; \quad E_n = \exp(-(2\pi n \cdot C \sigma_\delta)^2); \\ A_n &= \frac{1}{2} (1 - E_n) (1 - E_n \cos(2n\mu_o)); \\ B_n &= \frac{1}{2} (1 - E_n) (1 + E_n \cos(2n\mu_o)), \end{aligned} \quad (3)$$

where ε_{N0} is the beam’s initial normalized emittance, $\gamma_o = E_o/mc^2$ is the beam’s relativistic factor, μ_o is the phase-advance per cell, and C is the cell’s chromaticity. Again, for brevity, I discuss here only the main terms in (3) and neglect the small term $O(\dots)$ [5].

I note that for an achromatic lattice ($C = 0$) or for a monoenergetic beam ($\sigma_\delta = 0$) there is no growth in emittance. In contrast, for a lattice dominated by chromaticity with $E_n = \exp(-(2\pi n \cdot C \sigma_\delta)^2) \rightarrow 0$, which correspond to the “loss of memory,” we can estimate the normalized emittance as follows:

Published by the American Physical Society under the terms of the [Creative Commons Attribution 3.0 License](https://creativecommons.org/licenses/by/3.0/). Further distribution of this work must maintain attribution to the author(s) and the published articles title, journal citation, and DOI.

$$\Delta\epsilon_N = \epsilon_N - \epsilon_{No} \sim N_{\text{cell}} \frac{\gamma_o \beta}{2} \langle \theta^2 \rangle. \quad (4)$$

Thus, for a 5-GeV beam, $N_{\text{cell}} = 500$, and $\beta \sim 2m$, the rms angular kicks of 10^{-6} rad could engender a 10-mm mrad growth of normalized emittance. Such an increase would not acceptable for most ERL projects. Thus, unless a remedy is found to prevent the growth of emittance, a careful study will be required to determine whether the proposed Transverse Beam Break-Up instability-suppression scheme would be acceptable for a specific ERL project.

Furthermore, following modifying Eq. (34) in Ref. [3] for the case of the constant energy, one can arrive to the simple equation of the emittance delusion caused by a corrected orbit in periodic FODO lattice (see [1–3] for details on the orbit correction assumptions):

$$\Delta\epsilon_N \approx 32\pi \cdot \gamma_o \cdot N_{\text{cell}} \cdot C \frac{\langle \delta x_{\text{BPM}}^2 \rangle \sigma_\delta^2}{L_{\text{cell}}}, \quad (5)$$

where $\sqrt{\langle \delta x_{\text{BPM}}^2 \rangle}$ is an rms beam position monitor positioning error. For the above parameters and the beam with the energy spread of 0.03%, the chromaticity per cell ~ 0.25 , rms beam position monitor position errors ~ 0.2 mm, and cell length of 2 m, one should expect the emittance growth ~ 0.5 mm mrad.

Accordingly, the conclusion to this paper should be modified by adding the following:

The high chromaticity of the ERL's lattice, in combination with a sufficient energy spread that suppresses the response to the transverse kicks in the ERL's linac, also would result in a high sensitivity to random and correlated orbit errors. The latter, in turn, could engender a significant (at least for some applications) growth of emittance. Thus, the applicability of my proposed scheme to ERLs operating with extremely low emittance beams could be a challenging problem.

ACKNOWLEDGMENTS

The author thanks the reviewer of this Erratum for pointing out the key publications fully describing the emittance delusion in linear accelerators.

-
- [1] Ronald D. Ruth, Frontiers of particle beams, *Lect. Notes Phys.* **296**, 440 (1988).
 - [2] T. Raubenheimer, PhD thesis, Stanford University, SLAC-Report-387, 1991.
 - [3] T. Raubenheimer, Estimates of emittance dilution and stability in high-energy linear accelerators, *Phys. Rev. ST Accel. Beams* **3**, 121002 (2000).
 - [4] M. Syphers, Report No. FN-458, 1987.
 - [5] V.N. Litvinenko, Chromatic lattices, orbit errors and emittance growth, C-AD note, BNL, 2014.