Generic conditions for suppressing the coherent synchrotron radiation induced emittance growth in a two-dipole achromat

Yi Jiao,^{*} Xiaohao Cui, Xiyang Huang, and Gang Xu

Institute of High Energy Physics, CAS, Beijing 100049, People's Republic of China

(Received 2 December 2013; published 5 June 2014)

The effect of the coherent synchrotron radiation (CSR) becomes evident, and leads to increased beam energy spread and transverse emittance dilution, as both the emittance and bunch length of the electron beams are continuously pushed down in present and forthcoming high-brightness light sources and linear colliders. Suppressing this effect is important to preserve the expected machine performance. Methods of the R-matrix analysis and the Courant-Snyder formalism analysis have been proposed to evaluate and to suppress the emittance growth due to CSR in achromatic cells. In this paper a few important modifications are made on these two methods, which enable us to prove that these two methods are equivalent to each other. With the modified analysis, we obtain explicit and generic conditions of cancelling the CSR-driven emittance excitation in a single achromat consisting of two dipoles of arbitrary bending angles. In spite of the fact that the analysis constrains itself in a linear regime, based on the assumption that CSR-induced particle energy deviation is proportional to both θ and $\rho^{1/3}$, with θ being the bending angle and ρ the bending radius, it is demonstrated through ELEGANT simulations that the conditions derived from this analysis are still effective in suppressing the emittance growth when a more detailed one-dimensional CSR model is considered. In addition, it illustrates that the emittance growth can be reduced to a lower level with the proposed conditions than with the other two approaches, such as matching the beam envelope to the CSR kick and setting the cell-to-cell betatron phase advance to an appropriate value.

DOI: 10.1103/PhysRevSTAB.17.060701

PACS numbers: 29.20.Ej, 29.27.Eg, 41.60.Ap, 41.85.Gy

I. INTRODUCTION

Electron beams with low transverse normalized emittance (at the $\mu\text{m} \cdot \text{rad}$ or sub- $\mu\text{m} \cdot \text{rad}$ scale), short bunch length (at the subpicosecond scale), and high peak current (up to thousands of amperes) are generated or expected in high-brightness light sources (see, e.g., [1–5]) and linear colliders (see, e.g., [6]). In these machines as beams pass through bending magnets, the emission of the coherent synchrotron radiation (CSR) leads to beam quality degradation, by inducing increased beam energy spread and causing transverse emittance dilution. Suppressing this effect is necessary and important to preserve the expected machine performance that is evaluated without considering the CSR effect. This has stimulated extensive analytical, numerical, and experimental studies [7–25] on the CSR effect in the past few decades.

One important topic among these studies is to suppress the CSR-induced emittance growth. It has been shown that the CSR effect can be suppressed through shielding [23] or longitudinal pulse shaping [24]. Nevertheless, in this paper

we concentrate on the CSR suppression approaches of manipulation of beam optics. One approach [13] is to cancel the CSR kick through finely tuning the betatron phase advance between identical cells to an appropriate value and imposing certain symmetric conditions on the transport system (hereafter this approach will be denoted by the optical balance method). This method was recently expanded [25] with the Courant-Snyder (C-S) formalism, where the emittance growth is related to the beam C-S parameters and a single-kick approximation of the CSR effect is adopted. It was shown that the transverse emittance can be preserved after two identical double bend achromats (DBAs) with a π section between the DBAs. In this approach, however, the CSR kick is not canceled in a single achromatic cell. Thus, the CSR suppression efficiency relies highly on the identity of the optics of the adjacent achromatic cells as well as the phase advance of the quadrupole section between them.

Another approach [18] is to minimize the CSR-induced emittance growth by matching the beam envelope to the net CSR kick at the exit of a single achromatic cell (hereafter this approach will be denoted by CSR kick matching method). It has been shown that the CSR effect in a single achromatic cell can be analyzed by using the so-called R-matrix analysis. In this analysis, it was noted that for an electron bunch of Gaussian temporal distribution, the rms energy spread caused by CSR is [18,19]

^{*}jiaoyi@ihep.ac.cn

Published by the American Physical Society under the terms of the Creative Commons Attribution 3.0 License. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

$$\Delta E_{\text{rms}} = 0.2459 \frac{eQ\mu_0 c_0^2 L_b}{4\pi\sigma_z^{4/3} \rho^{2/3}}, \quad (1.1)$$

where Q is the bunch charge, μ_0 is the permeability of vacuum, c_0 is the speed of light, L_b is the particle bending path in a dipole, σ_z is the rms bunch length, and ρ is the bending radius. Based on this relation, the CSR effect in a dipole was linearized by assuming $\delta(\text{csr}) = \kappa L_b$, where $\delta(\text{csr})$ is the CSR-induced particle energy deviation and κ is the normalized CSR wake potential, and a first-order equation of the motion of a particle in a uniform field of a dipole magnet was given by

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho} [\delta_0 + \delta_{\text{csr}} + \kappa(s - s_0)], \quad (1.2)$$

where x is the horizontal displacement of an electron relative to the ideal trajectory, the prime represents the derivative with respect to longitudinal coordinate s , δ_0 is the initial momentum deviation at $s = 0$, $s = s_0$ indicates the entrance of the dipole, and δ_{csr} is the CSR-induced energy deviation in the upstream dipoles (at $s < s_0$). A 5×5 R -matrix derived from Eq. (1.2) was used to represent the motion specified by a vector $\mathbf{X}(s_0) = (x, x', \delta_0, \delta_{\text{csr}}, \kappa)^T$, and to evaluate the “CSR dispersion” ($x - \kappa$ correlation) function, which is found to behave differently from the momentum dispersion ($x - \delta$ correlation). Through matrix multiplication, the net CSR-induced coordinate deviations and emittance growth after passing through an achromat can be calculated. In addition, it was pointed out that the emittance growth will be minimized if the beam envelope matches the CSR kick, with the orientation of the beam envelope in (x, x') phase space measured by [20,26]

$$\tan 2\phi_{\text{phase}} = \frac{2\alpha_f}{\gamma_f - \beta_f}, \quad (1.3)$$

where α_f , β_f , and γ_f are the C-S parameters at the exit of the cell.

Then the question arises, whether there exist generic laws of eliminating rather than minimizing the CSR-induced emittance growth in a single achromatic cell. To this end, in Secs. II and III, several modifications are made on the R -matrix analysis and the C-S formalism analysis, such as (1) extending the scope of the R -matrix analysis to achromats with dipoles of different bending radii, based on a new assumption that $\delta(\text{csr}) \propto L_b/\rho^{2/3}$, (2) expressing the R -matrix for the quadrupole section between dipoles in terms of the betatron phase advance and C-S parameters, so as to make the analysis independent of concrete optics design, and (3) establishing a point-kick model of the CSR effect in a dipole, with which the analysis of the CSR-induced emittance growth can be reduced to a (x, x') two-dimensional (2D) plane. With the modified analysis, we obtain explicit conditions of cancelling the CSR-driven emittance excitation for a single achromat with two dipoles

FIG. 1. Schematic layout of a two-dipole achromat (a) and physical model for the two-dimensional point-kick analysis of the CSR effect in a two-dipole achromat (b). The points 1 and 2 indicate the centers of the first and the second dipole, respectively.

of arbitrary bending radii and bending angles (see Fig. 1 for the sketch),

$$\cos \varphi_{12} = -\text{sign}(\theta_2/\theta_1), \quad (1.4)$$

$$\rho_1 \theta_1^3 \cong \rho_2 \theta_2^3, \quad \text{or} \quad L_1 \theta_1^2 \cong L_2 \theta_2^2, \quad (1.5)$$

and

$$\frac{\alpha_1 - \alpha_2}{\beta_1} \cong -\frac{12}{L_1}, \quad (1.6)$$

where Eq. (1.4) ensures the achromatic condition, and Eqs. (1.5) and (1.6) together result in zero CSR kick in linear regime, which are justified as $\theta_1 \ll 1$ and $\theta_2 \ll 1$, φ_{12} is the horizontal phase advance between the centers of the two dipoles, θ_1 (θ_2), ρ_1 (ρ_2), and L_1 (L_2) are the bending angle, radius, and length of the first (second) dipole, and α_1 (α_2) and β_1 are the C-S parameters at the center of the first (second) dipole, respectively.

An equivalent but easier applied condition to Eqs. (1.4) and (1.6) is that the 2×2 transfer matrix for the beam line between the centers of the two dipoles, M_{c2c} , is of the form

$$M_{c2c} \cong \begin{pmatrix} -1/c & 0 \\ 12c/L_1 & -c \end{pmatrix}, \quad (1.7)$$

where $c = S_2/S_1$, $S_1 = \sin(\theta_1/2)$, and $S_2 = \sin(\theta_2/2)$. In the case with $\theta_1 \ll 1$ and $\theta_2 \ll 1$, $c \cong \theta_2/\theta_1$ gives a measure of the ratio between two bending angles.

In Sec. IV A, the proposed conditions are verified with the ELEGANT simulations [15] with various beam and optical parameters. Studies show that, in spite of the fact that these conditions are derived with linear analysis considering only the linear effect of the CSR wake in a dipole, they are still effective in suppressing the emittance growth when a more detailed one-dimensional (1D) physical model of the CSR wake is considered. In addition, a comparison between the proposed conditions and two other

approaches, the CSR kick matching method and the optical balance method, is made in Sec. IV B. It turns out that with the proposed conditions the emittance growth can be reduced to a lower level than with the other two approaches.

II. MODIFIED R-MATRIX ANALYSIS

A. *R*-matrices applicable to achromats with dipoles of different ρ

From Eq. (1.1), the CSR-induced rms energy spread ΔE_{rms} is proportional to both L_b and $\rho^{-2/3}$, or namely, $\Delta E_{\text{rms}} \propto \rho^{1/3}\theta$. It reveals through ELEGANT simulations that this relation applies well to the cases with θ ranging

from 1° to 12° and ρ ranging from 1 to 150 m, as shown in Fig. 2. Therefore the CSR effect can be linearized by assuming $\delta(\text{csr}) = kL_b/\rho^{2/3}$, where k depends only on the bunch charge Q and the bunch length σ_z , and is in unit of $m^{-1/3}$. The equation of the motion of a particle in a uniform field of a dipole magnet now becomes

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}[\delta_0 + \delta_{\text{csr}} + k(s - s_0)/\rho^{2/3}]. \quad (2.1)$$

Accordingly, the vector specifying the particle motion changes to be $\mathbf{X}(s_0) = (x, x', \delta_0, \delta_{\text{csr}}, k)^T$, and the 5×5 *R*-matrix for a sector bending magnet is given by

$$R_d = \begin{pmatrix} \cos \theta & \rho \sin \theta & \rho(1 - \cos \theta) & \rho(1 - \cos \theta) & \rho^{4/3}(\theta - \sin \theta) \\ -\sin \theta / \rho & \cos \theta & \sin \theta & \sin \theta & \rho^{1/3}(1 - \cos \theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho^{1/3}\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (2.2)$$

which is an extension of the 3×3 transfer matrix of the horizontal motion that includes the propagation of off-momentum rays, and where $R_d(1,4) = R_d(1,3)$ and $R_d(2,4) = R_d(2,3)$, since the relation between x and δ_0 is the same as for δ_{csr} in Eq. (2.1), and $R_d(1,4)$, $R_d(2,4)$, and $R_d(1,5)$, $R_d(2,5)$ are the CSR dispersive terms related to δ_{csr} and k , respectively. Thus, the coordinate deviations of a particle relative to the ideal path after passage through a sector bending magnet are given by

$$\begin{aligned} \Delta x &= \Delta x(\delta_0) + \Delta x(\text{csr}) \\ &= R_d(1,3)\delta_0 + R_d(1,4)\delta_{\text{csr}} + R_d(1,5)k, \\ \Delta x' &= \Delta x'(\delta_0) + \Delta x'(\text{csr}) \\ &= R_d(2,3)\delta_0 + R_d(2,4)\delta_{\text{csr}} + R_d(2,5)k. \end{aligned} \quad (2.3)$$

Because k is independent of the dipole parameters, the *R*-matrix in Eq. (2.2) can be used to investigate the CSR effect in achromats with dipoles of different ρ and θ . On the other hand, k varies with Q and σ_z . Therefore the

FIG. 2. Variation of the CSR-induced rms energy spread in a dipole with θ (left) and $\rho^{1/3}$ (right), respectively, obtained by ELEGANT simulations with $Q = 100$ pC and $\sigma_z = 30$ μm .

analysis with such a matrix is justified under the condition that the electron bunch does not significantly change either the bunch length or the longitudinal charge distribution.

The R -matrices for a quadrupole and a drift space can be obtained in a similar manner, whose dispersive terms [(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), and (2, 5) elements] are zero.

If one evaluates the CSR-induced emittance growth in an achromat by multiplying directly the R -matrices of the dipoles, quadrupoles, and drifts, the result will vary with a change in any of the element parameters. To avoid the dependency of the analysis on concrete optics design, the beam line between two dipoles (actually the beam line between the centers of the two dipoles) is treated as a whole and is formulated in terms of the betatron phase advance and C-S parameters. This, as will be shown below, makes it feasible to achieve generic and explicit CSR cancellation conditions for a two-dipole achromat.

B. R -matrix description of a two-dipole achromat

In practical designs of a transport system including several bending magnets, an achromat is usually made to avoid transverse emittance growth due to the momentum dispersive effect. As illustrated in Fig. 1(a), a two-dipole achromat can be generally modeled as (HD1, HD1, PbD, HD2, HD2), where HD1 and HD2 stand for half of the first and the second dipoles, respectively, and PbD represents the part between two dipoles, including drifts and quadrupoles.

The R -matrices for HD1 and HD2 take the same forms as R_d in Eq. (2.2), with ρ, θ replaced by $\rho_1, \theta_1/2$ and $\rho_2, \theta_2/2$, respectively. On the other hand, the beam line between the centers of the two dipoles, (HD1, PbD, HD2), is described with an R -matrix in terms of φ_{12} and the C-S parameters at the dipole centers,

$$R_{c2c} = \begin{pmatrix} r_{c2c,11} & r_{c2c,12} & r_{c2c,13} & r_{c2c,14} & r_{c2c,15} \\ r_{c2c,21} & r_{c2c,22} & r_{c2c,23} & r_{c2c,24} & r_{c2c,25} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & r_{c2c,45} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (2.4)$$

with [27]

$$\begin{aligned} r_{c2c,11} &= \sqrt{\beta_2/\beta_1} [\cos(\varphi_{12}) + \alpha_1 \sin(\varphi_{12})], \\ r_{c2c,12} &= \sqrt{\beta_1\beta_2} \sin(\varphi_{12}), \\ r_{c2c,21} &= \frac{(\alpha_1 - \alpha_2) \cos(\varphi_{12}) - (1 + \alpha_1\alpha_2) \sin(\varphi_{12})}{\sqrt{\beta_1\beta_2}}, \\ r_{c2c,22} &= \sqrt{\beta_1/\beta_2} [\cos(\varphi_{12}) - \alpha_2 \sin(\varphi_{12})]. \end{aligned} \quad (2.5)$$

In Eq. (2.4), since the CSR effect in two half-dipoles has been accounted for, the dispersive terms $r_{c2c,13} = r_{c2c,14}$,

$r_{c2c,23} = r_{c2c,24}$, $r_{c2c,15}$, and $r_{c2c,25}$ are nonzero, and $r_{c2c,45} = (\rho_1^{1/3}\theta_1 + \rho_2^{1/3}\theta_2)/2$, which can be easily justified. The R -matrix for PbD can be calculated by

$$R_{pbd} = R_{hd2}^{-1} R_{c2c} R_{hd1}^{-1}. \quad (2.6)$$

Note that the dispersive terms in R_{pbd} are zero, from which $r_{c2c,13} = r_{c2c,14}$, $r_{c2c,23} = r_{c2c,24}$, $r_{c2c,15}$, and $r_{c2c,25}$ can be derived as functions of $\beta_1, \beta_2, \alpha_1, \alpha_2, \varphi_{12}, \rho_1, \rho_2, \theta_1$ and θ_2 . They are omitted here due to lengthy expressions.

Then calculate the total R -matrix of the two-dipole section by

$$R_t = R_{hd2} R_{c2c} R_{hd1}. \quad (2.7)$$

For a two-dipole achromat $R_t(1,3) = R_t(2,3) = 0$ [$R_t(1,4)$ and $R_t(2,4)$ are also zero], from which one can obtain the *achromatic condition*,

$$\sin \varphi_{12} = 0, \quad \cos \varphi_{12} = -\sqrt{\frac{\beta_2}{\beta_1}} \sin \frac{\theta_2}{2} / \sin \frac{\theta_1}{2}. \quad (2.8)$$

It turns out that $|\cos \varphi_{12}| = 1$ and $(\beta_1/\beta_2)^{1/2} = |\sin(\theta_2/2)/\sin(\theta_1/2)|$, indicating that in a two-dipole achromat the phase advance between the centers of the two dipoles, φ_{12} , must be π (modulus 2π) for the case where two dipoles deflect beams in the same directions, or 2π (modulus 2π) for the case with two opposite-deflecting dipoles. It is worth noting that this achromatic condition has been obtained in a similar formula during the study of the Chasman-Green lattice for the third generation light sources (see, e.g., [28]).

Substituting the achromatic condition in Eq. (2.5), we obtain the 2×2 transfer matrix of the horizontal betatron motion for the beam line between the centers of the two dipoles,

$$M_{c2c} = \begin{pmatrix} -\frac{1}{c} & 0 \\ -\frac{(\alpha_1 - \alpha_2)}{\beta_1} c & -c \end{pmatrix}, \quad (2.9)$$

which indicates that for a two-dipole achromat with fixed dipole parameters, only $M_{c2c}(2,1)$ is variable. Next we will show that if a zero CSR-induced emittance growth is pursued, even this element is not arbitrary, but tends to $12c/L_1$.

C. CSR-cancellation conditions for a two-dipole achromat

Through matrix manipulations, we find that with the achromatic condition Eq. (2.8) or (2.9), the particle coordinate deviations relative to the ideal path after passage through a two-dipole achromat are proportional to k ,

$$\begin{aligned} \Delta x &= R_t(1,5)k = r_{15}k, \\ \Delta x' &= R_t(2,5)k = r_{25}k, \end{aligned} \quad (2.10)$$

with

$$\begin{aligned} r_{15} &= \left[\frac{C_2(2S_1 - C_1\theta_1)}{c} + S_2^2\theta_1 r + (\theta_2 - \sin\theta_2)r^{4/3} \right] \rho_1^{4/3} + \frac{cS_2(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1} \rho_1^{7/3} r, \\ r_{25} &= \left[S_2(2r^{1/3}S_2 + C_2\theta_1) - \frac{S_1(2S_1 - C_1\theta_1)}{r} \right] \rho_1^{1/3} + \frac{cC_2(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1} \rho_1^{4/3}, \end{aligned} \quad (2.11)$$

where $C_1 = \cos(\theta_1/2)$, $C_2 = \cos(\theta_2/2)$, and a factor $r \equiv \rho_2/\rho_1$ is defined to give a measure of the ratio between two bending radii.

The final geometric emittance in the presence of the CSR effect can be evaluated by

$$\begin{aligned} \varepsilon &= \sqrt{(\varepsilon_0\beta_f + \Delta x_{\text{rms}}^2)(\varepsilon_0\gamma_f + \Delta x_{\text{rms}}^2) - (\varepsilon_0\alpha_f - \Delta x_{\text{rms}}\Delta x'_{\text{rms}})^2} = \sqrt{\varepsilon_0^2 + \varepsilon_0 d\varepsilon}, \\ d\varepsilon &= \gamma_f \Delta x_{\text{rms}}^2 + 2\alpha_f \Delta x_{\text{rms}} \Delta x'_{\text{rms}} + \beta_f \Delta x_{\text{rms}}^2 = (\gamma_f r_{15}^2 + 2\alpha_f r_{15} r_{25} + \beta_f r_{25}^2) k_{\text{rms}}^2, \end{aligned} \quad (2.12)$$

where ε_0 is the unperturbed geometric emittance.

In most cases $d\varepsilon \ll \varepsilon_0$; therefore, the growth in geometric emittance and in normalized emittance due to CSR can be estimated by

$$\begin{aligned} \Delta\varepsilon &= \varepsilon - \varepsilon_0 \approx \frac{1}{2} d\varepsilon, \\ \Delta\varepsilon_n &= \varepsilon_n - \varepsilon_{n0} = \gamma\beta(\varepsilon - \varepsilon_0) \approx \frac{1}{2} \gamma\beta d\varepsilon, \end{aligned} \quad (2.13)$$

where γ is the Lorentz factor, and β is the particle velocity relative to the speed of light.

It is clear that $d\varepsilon = 0$ if $r_{15} = r_{25} = 0$, from which the CSR cancellation conditions can be derived, and are of the form

$$r^* = \text{sign}\left(\frac{\theta_2}{\theta_1}\right) \left(\frac{S_1(2S_1 - C_1\theta_1)}{S_2(2S_2 - C_2\theta_2)} \right)^{3/4}, \quad (2.14)$$

and

$$\left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right)^* = \frac{\theta_1 + r^{*1/3}\theta_2}{\rho_1(-2 + \theta_1 C_1/S_1)}, \quad (2.15)$$

where an asterisked quantity means the quantity leads to a zero emittance growth in a linear regime. To achieve a zero emittance growth, the above two conditions must be simultaneously satisfied.

For the case of particular interest with $\theta_1 \ll 1$ and $\theta_2 \ll 1$, Eqs. (2.14) and (2.15) reduce to Eqs. (1.5) and (1.6), and Eq. (2.9) reduces to Eq. (1.7), where only the first significant terms with respect to θ_1 and θ_2 are kept.

The obtained CSR cancellation conditions suggest a new way to suppress the CSR-induced emittance growth in a two-dipole achromat with dipoles of arbitrary ρ and θ . First, choose the dipole parameters to ensure condition (2.14) or (1.5), and then vary the quadrupole strengths (and the quadrupole and drift lengths if necessary) to tune $M_{c2c}(2, 1)$ to $M_{c2c}^*(2, 1) \cong 12c/L_1$.

Nevertheless, in an actual circumstance, due to various reasons (e.g., random errors, other optics constraints), $M_{c2c}(2, 1)$ may be close to, rather than at the exact optimal value. Therefore it is useful to investigate the scaling of the emittance growth when condition (2.14) is fulfilled while condition (2.15) is not satisfied. In such a case, Eq. (2.11) reduces to

$$\begin{aligned} r_{15}|_{r=r^*} &= S_2^2(\theta_1 + r^{*1/3}\theta_2)\rho_1^{4/3}r^* \left[1 - \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right) / \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right)^* \right], \\ r_{25}|_{r=r^*} &= S_2C_2(\theta_1 + r^{*1/3}\theta_2)\rho_1^{1/3} \left[1 - \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right) / \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right)^* \right], \end{aligned} \quad (2.16)$$

with the orientation of the CSR kick in (x, x') phase space (denoted by $\tan\phi_{\text{csr}}$) given by

$$\tan\phi_{\text{csr}}|_{r=r^*} = \frac{\Delta x'}{\Delta x}|_{r=r^*} = \frac{r_{25}}{r_{15}}|_{r=r^*} = \frac{\cot(\theta_2/2)}{\rho_2}. \quad (2.17)$$

By expressing β_f , α_f , and γ_f in terms of β_2 and α_2 , and combining Eqs. (2.8), (2.12), and (2.17), we obtain an explicit expression of $d\varepsilon$ in this case,

$$\begin{aligned} d\varepsilon|_{r=r^*} &= k_{\text{rms}}^2 r_{25}^2 \beta_2 / C_2^2|_{r=r^*} = k_{\text{rms}}^2 S_1^2(\theta_1 + r^{*1/3}\theta_2)^2 \rho_1^{2/3} \beta_1 \left[1 - \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right) / \left(\frac{\alpha_1 - \alpha_2}{\beta_1} \right)^* \right]^2, \\ &= k_{\text{rms}}^2 S_1^2(\theta_1 + r^{*1/3}\theta_2)^2 \rho_1^{2/3} \beta_1 [1 - M_{c2c}(2, 1)/M_{c2c}^*(2, 1)]^2. \end{aligned} \quad (2.18)$$

III. MODIFIED C-S FORMALISM ANALYSIS

A. Point-kick model of the CSR effect in a dipole

The emittance growth due to CSR in two identical DBAs was studied in Ref. [25] by using the C-S formalism analysis with a single-kick approximation of the CSR effect. In this approximation, the particle coordinate deviations due to CSR were written as $(\Delta x, \Delta x') = [\eta\delta(\text{csr}), \eta'\delta(\text{csr})]$, with η and η' being the momentum dispersion and its derivative. This resulted in explicit formulation of the emittance growth caused by CSR. Furthermore, since the analysis was reduced to an (x, x') 2D plane, the CSR suppression with an appropriate phase advance was able to be highlighted in this approach.

In the opinion of the authors, it, however, needs several modifications or specifications on the single-kick approximation to make the analysis self-consistent. In the following we will show that the CSR effect in a dipole can be equivalently formulated with a point kick, which is built based on the following considerations. First, the kick occurs at the center of a dipole; second, as shown in Sec. II A, the CSR kick contains two elements, related to δ_{csr} and k , respectively; third, the particle energy deviation is not constant but increases by $k\rho^{1/3}\theta$ after passage through a dipole; and finally, the kick should be such that the CSR-induced coordinate deviations at the end of the dipole should be the same as that from the more rigorous R-matrix analysis.

From Eqs. (2.2) and (2.3), the horizontal coordinate deviations of a particle with energy deviation of $\delta = (\delta_0 + \delta_{\text{csr}})$ after passage through a sector bending magnet are given by

$$X_e = \begin{pmatrix} x_e \\ x'_e \end{pmatrix} = \begin{pmatrix} \rho(1 - \cos \theta) \\ \sin \theta \end{pmatrix} \delta + \begin{pmatrix} \rho^{4/3}(\theta - \sin \theta) \\ \rho^{1/3}(1 - \cos \theta) \end{pmatrix} k. \quad (3.1)$$

Hereby we assume the CSR kick at the center of the dipole is in a similar form,

$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} D_k \\ D'_k \end{pmatrix} \delta + \begin{pmatrix} \zeta_k \\ \zeta'_k \end{pmatrix} k, \quad (3.2)$$

where D_k , D'_k , and ζ_k , ζ'_k represent the equivalent momentum dispersion and its derivative, the equivalent “CSR dispersion” and its derivative at the center of dipole, respectively; and we further assume that after the particle with a coordinate of X_k passes through half of the dipole, the coordinates take the same forms as in Eq. (3.1),

$$\begin{pmatrix} x_e \\ x'_e \end{pmatrix} = \begin{pmatrix} \cos \theta/2 & \rho \sin \theta/2 \\ -\frac{\sin \theta/2}{\rho} & \cos \theta/2 \end{pmatrix} \begin{pmatrix} x_k \\ x'_k \end{pmatrix}, \quad (3.3)$$

from which D_k , D'_k , ζ_k , and ζ'_k can be derived, and are of the form

$$\begin{aligned} D_k &= 0, \\ D'_k &= 2 \sin(\theta/2), \\ \zeta_k &= \rho^{4/3}[\theta \cos(\theta/2) - 2 \sin(\theta/2)], \\ \zeta'_k &= \rho^{1/3}\theta \sin(\theta/2). \end{aligned} \quad (3.4)$$

Substituting Eq. (3.4) in Eq. (3.2), we obtain the *point-kick model* of the CSR effect in a dipole,

$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} \rho^{4/3}k[\theta \cos(\theta/2) - 2 \sin(\theta/2)] \\ \sin(\theta/2)(2\delta + \rho^{1/3}\theta k) \end{pmatrix}. \quad (3.5)$$

B. CSR cancellation conditions derived with the CSR point-kick model

With the proposed CSR point-kick model, the analysis of the CSR effect in a two-dipole achromat can be greatly simplified. As illustrated in Fig. 1(b), at the center of the first dipole (point 1) the particle experiences a CSR kick, and then experiences another kick at the center of the second dipole (point 2) after a betatron transportation through the beam line between the centers of the two dipoles.

For simplicity, it is assumed that the particle starts from the entrance of the achromat with initial energy deviation of $\delta_0 = 0$ and with initial coordinates of $x_0 = x'_0 = 0$. The coordinates remain zero until the particle experiences the CSR kick at the center of the first dipole. The coordinates right before and after the CSR kick are given by

$$x_{1-} = 0, x'_{1-} = 0, \quad (3.6)$$

and

$$\begin{aligned} x_{1+} &= x_{1-} + x_{k,1} = k\rho_1^{4/3}(C_1\theta_1 - 2S_1), \\ x'_{1+} &= x'_{1-} + x'_{k,1} = k\rho_1^{1/3}S_1\theta_1. \end{aligned} \quad (3.7)$$

As shown in Sec. II B, the 2×2 transfer matrix for the beam line between the centers of the two dipoles is given by Eq. (2.9). Thus, the particle coordinates at the center of the second dipole, right before the second CSR kick, can be calculated by

$$\begin{pmatrix} x_{2-} \\ x'_{2-} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\beta_1} & 0 \\ -\frac{\alpha_1 - \alpha_2}{\beta_1} & -c \end{pmatrix} \begin{pmatrix} x_{1+} \\ x'_{1+} \end{pmatrix} = \begin{pmatrix} \frac{S_1}{S_2}(2S_1 - C_1\theta_1)\rho_1^{4/3}k \\ \left[\frac{(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1 S_1}\rho_1 - \theta_1\right]S_2\rho_1^{1/3}k \end{pmatrix}. \quad (3.8)$$

After the second CSR kick (now with $\delta = k\rho_1^{1/3}\theta_1$), the particle coordinates turn out to be

$$\begin{aligned} x_{2+} &= x_{2-} + x_{k,2} = \left[\frac{S_1}{S_2} (2S_1 - C_1\theta_1) + r^{4/3} (C_2\theta_2 - 2S_2) \right] \rho_1^{4/3} k = Ak, \\ x'_{2+} &= x'_{2-} + x'_{k,2} = \left[\frac{(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1 S_1} \rho_1 + (\theta_1 + r^{1/3}\theta_2) \right] S_2 \rho_1^{1/3} k = Bk. \end{aligned} \quad (3.9)$$

The final geometric emittance in the presence of the CSR effect is given by

$$\begin{aligned} \varepsilon &= \sqrt{(\varepsilon_0\beta_2 + x_{2+,rms}^2)(\varepsilon_0\gamma_2 + x_{2+,rms}'^2) - (\varepsilon_0\alpha_2 - x_{2+,rms}x_{2+,rms}')^2} = \sqrt{\varepsilon_0^2 + \varepsilon_0 d\varepsilon_1}, \\ d\varepsilon_1 &= \gamma_2 x_{2+,rms}^2 + 2\alpha_2 x_{2+,rms} x_{2+,rms}' + \beta_2 x_{2+,rms}'^2 = (\gamma_2 A^2 + 2\alpha_2 AB + \beta_2 B^2) k_{rms}^2. \end{aligned} \quad (3.10)$$

It is clear that $d\varepsilon_1 = 0$ if $A = B = 0$, from which the same solutions for r and $(\alpha_1 - \alpha_2)/\beta_1$, as in Eqs. (2.14) and (2.15), are obtained. In the case when only the condition (2.14) is fulfilled, the expression of $d\varepsilon_1$ is also the same as in Eq. (2.18). It indicates that with the proposed CSR point-kick model, the C-S formalism analysis is equivalent to the R -matrix analysis. The consequence is that to evaluate the CSR-induced emittance growth in an n -dipole achromat, one just needs to analyze the horizontal betatron motion with n -point kicks. Such a methodology, which we shall call 2D point-kick analysis, will greatly simplify the analysis of the CSR effect in an achromat consisting of three or even more dipoles.

IV. APPLICATIONS OF THE CSR CANCELLATION CONDITIONS

A. Verification of the proposed conditions

In the above sections, generic and explicit CSR cancellation conditions have been obtained for a two-dipole achromat with dipoles of arbitrary ρ and θ . However, these conditions are derived with the modified analysis based on the linearization of the effect of the CSR wake in a dipole, where the effects of the nonlinear components of the CSR wake in a dipole (denoted by n.c. CSR), the transient CSR at the edges of the dipole (denoted by tr. CSR), and the CSR wake in drift spaces following dipoles (denoted by d.s. CSR) are not considered. Therefore it is necessary to investigate the CSR suppression efficiency of the proposed conditions under the case where a more detailed 1D physical model of the CSR wake is included.

To this end, a two-dipole achromat with θ_1 of 6° , ρ_1 of 8 m, and θ_2 of 4° is considered. From Eqs. (1.4) to (1.7), the optimal parameters resulting in zero emittance growth in a linear regime can be determined, such as $c \cong 2/3$, $\varphi_{12} = \pi$, $r^* \cong 27/8$, $\rho_2^* \cong 27$ m, and $M_{c2c}^*(2, 1) \cong 30/\pi$. Such an achromat is designed with four families of quadrupoles located between the dipoles, whose optics can be varied flexibly. The emittance growth caused by CSR after

passage through the achromat is simulated with the ELEGANT program, where an electron bunch with typical parameters of initial normalized emittance of $2 \mu\text{m rad}$, mean energy of 1 GeV, energy spread of 0.05%, bunch charge of 500 pC, and bunch length of $30 \mu\text{m}$ is tracked.

First, only the CSR wake in dipoles is included in the tracking. In this case the variation of the growth in normalized emittance, $\Delta\varepsilon_n$, with $M_{c2c}(2, 1)$ (while fixing $r = r^*$) and r [while fixing $M_{c2c}(2, 1) = M_{c2c}^*(2, 1)$] is investigated. The results are presented in Fig. 3, with the C-S parameters at the entrance of the achromat, hereafter denoted by initial C-S parameters, (β_0, α_0) of (3 m, 8) and (3 m, 9), corresponding to β_1 of 0.1 m and 0.26 m, respectively. It shows that the found conditions r^* and $M_{c2c}^*(2, 1)$ result in a minimum $\Delta\varepsilon_n$ (or $\Delta\varepsilon_n$ close to the minimum with a few other sets of initial C-S parameters), and the variation of $\Delta\varepsilon_n$ with $M_{c2c}(2, 1)$ is in accordance with the analytical prediction from Eqs. (2.13) and (2.18) with $k_{rms} = 0.0012 \text{ m}^{-1/3}$ (dashed lines in Fig. 3), which indicates the validity of the modified analysis. Compared to $M_{c2c}(2, 1)$, the influence of r on the emittance growth is relatively small, especially with less than 50% deviation from the optimal value of r^* .

Note that the minimum $\Delta\varepsilon_n$ is not exactly zero, but on the scale of $0.001 \mu\text{m rad}$, which indicates that the effect of n.c. CSR becomes dominant at this point and distorts the regular beam distribution in transverse phase space. The effects of tr. CSR and d.s. CSR are also included in the tracking for the case with $r = r^*$, $M_{c2c}(2, 1) = M_{c2c}^*(2, 1)$, and $(\beta_0, \alpha_0) = (3 \text{ m}, 8)$. The results are listed in Table I. In the presence of these effects, the relative emittance growth, $\Delta\varepsilon_n/\varepsilon_{n0}$, can increase to about 0.6%. However, compared with the rapid increase in $\Delta\varepsilon_n/\varepsilon_{n0}$ as $M_{c2c}(2, 1)$ and r deviate from the optimal values (see Fig. 3), these effects are very weak relative to the linear effect of the CSR wake in dipoles, at least in our particular case. It appears that in the presence of these effects, the emittance growth due to CSR can still be well controlled with the proposed conditions.

FIG. 3. Variation of the emittance growth $\Delta\epsilon_n$ due to CSR with r (in units of r^*) and $M_{c2c}(2,1)$ [in units of $M_{c2c}^*(2,1)$] for the cases with initial C-S parameters (β_0, α_0) of (3 m, 8) and (3 m, 9) in a two-dipole achromat, obtained by ELEGANT simulations. The dashed lines are the analytical prediction from Eqs. (2.13) and (2.18) with $k_{\text{rms}} = 0.0012\text{ m}^{-1/3}$ and with a shift of the minimum $\Delta\epsilon_n$.

B. Comparison between the proposed conditions and two other CSR suppression approaches

In the following we will compare the proposed conditions with two other approaches, such as the CSR kick matching method and the optical balance method. For the convenience of comparison, a DBA with symmetric optics is considered, where $L_1 = L_2 = L_B$, $\theta_1 = \theta_2 = \theta$, $\beta_2 = \beta_1$, $\alpha_2 = -\alpha_1$, $\alpha_f = -\alpha_0$, and $\beta_f = \beta_0$.

In such a DBA, one of the CSR cancellation conditions, Eq. (2.14), is naturally satisfied; and the other condition, Eq. (2.15), reduces to

$$\left(\frac{\alpha_1}{\beta_1}\right)^* = \frac{\theta}{\rho[-2 + \theta \cot(\theta/2)]}. \quad (4.1)$$

By expressing β_1 and α_1 in terms of β_0 and α_0 , the above equation can be written as

TABLE I. The emittance growth in presence of the effects of n.c. CSR, tr. CSR, and d.s. CSR in a two-dipole achromat with $r = r^*$, $M_{c2c}(2,1) = M_{c2c}^*(2,1)$ and $(\beta_0, \alpha_0) = (3\text{ m}, 8)$, obtained by ELEGANT simulations.

CSR effects	Initial normalized emittance ϵ_{n0} (mm.rad)	Final normalized emittance ϵ_{nf} (mm.rad)	Relative emittance growth $\Delta\epsilon_n/\epsilon_{n0}$
n.c. CSR	2	2.0030	1.5×10^{-3}
n.c. CSR + tr.CSR	2	2.0056	2.8×10^{-3}
n.c. CSR + tr.CSR + d.s.CSR	2	2.0119	5.95×10^{-3}

$$\begin{aligned} &\beta_0^2 \sin \theta + \beta_0 \rho \alpha_0 [2 \cos \theta - \theta \cot(\theta/2)] \\ &+ \rho^2 (1 + \alpha_0^2) (\theta - \sin \theta) = 0. \end{aligned} \quad (4.2)$$

From Eq. (4.2), it is found that two solutions (denoted by $\beta_{0,1}$ and $\beta_{0,2}$) are possible as $\alpha_0 > 2\sqrt{6}$, and are of the form

$$\begin{aligned} \beta_{0,1} &\cong \frac{L_B}{12} \left(5\alpha_0 - \sqrt{\alpha_0^2 - 24} \right), \\ \beta_{0,2} &\cong \frac{L_B}{12} \left(5\alpha_0 + \sqrt{\alpha_0^2 - 24} \right), \end{aligned} \quad (4.3)$$

where only the first significant terms with respect to θ are kept.

On the other hand, in the case that the CSR-induced coordinate deviations are not zero, the CSR kick matching method minimizes the emittance growth by matching the beam envelope [with $\tan 2\phi_{\text{phase}}$ identified by Eq. (1.3)] to the CSR kick [with $\tan \phi_{\text{csr}}$ identified by Eq. (2.17)]. It requires

$$\tan 2\phi_{\text{phase}} = \tan 2\phi_{\text{csr}} = \frac{2 \tan \phi_{\text{csr}}}{1 - \tan^2 \phi_{\text{csr}}}, \quad (4.4)$$

which can also be written in terms of β_0 and α_0 ,

$$\begin{aligned} &\beta_0^2 \rho \cot(\theta/2) - \beta_0 \alpha_0 [\rho^2 - \cot^2(\theta/2)] \\ &- \rho \cot(\theta/2) (1 + \alpha_0^2) = 0. \end{aligned} \quad (4.5)$$

TABLE II. Parameters of the electron beam and of the DBA used in the ELEGANT simulations.

Parameter	Value	Unit
Bunch charge	500	pC
Beam energy	1000	MeV
Energy spread	0.05	%
Beam normalized emittance	2	$\mu\text{m} \cdot \text{rad}$
Bunch length	30	μm
Dipole bending radius	7	m
Dipole bending angle	3	degrees

Solving Eq. (4.5), we obtain

$$\beta_{0,\text{match}} = \frac{\alpha_0}{2\rho} [\rho^2 \tan(\theta/2) - \cot(\theta/2)] + \frac{\sqrt{2\rho^2(2 + \alpha_0^2) + \alpha_0^2[\cot^2(\theta/2) + \rho^4 \tan^2(\theta/2)]}}{2\rho}. \quad (4.6)$$

With aid of the Eqs. (4.3) and (4.6), the proposed conditions and the CSR kick matching method can be compared conveniently in the (β_0, α_0) space. To do so, a DBA with three families of quadrupoles symmetrically located between the dipoles is designed, whose optics are varied in a large range, with β_0 from 0.1 to 3 m and α_0 from 0.1 to 15. The main parameters of the beam and of the DBA are listed in Table II. The dependency of the relative emittance growth $\Delta\epsilon_n/\epsilon_{n0}$ on the initial C-S parameters is investigated with ELEGANT simulations. For each set of (β_0, α_0) , the initial electron beam distribution in phase space is generated accordingly to match the optics. The results with and without considering the effect of d.s. CSR, as well as the solutions $\beta_{0,1}$, $\beta_{0,2}$, and $\beta_{0,\text{match}}$ are presented in Fig. 4. In the case where d.s. CSR is not accounted for, very low emittance growth (with $\Delta\epsilon_n/\epsilon_{n0}$ on the scale of 10^{-4}) appears in the area neighboring the solutions predicted by the proposed conditions ($\beta_{0,1}$ and $\beta_{0,2}$, especially $\beta_{0,1}$). The situation changes slightly when d.s. CSR is considered. The available minimum $\Delta\epsilon_n/\epsilon_{n0}$ increases by about an order of magnitude, and occurs in the area between $\beta_{0,1}$ and $\beta_{0,2}$ while closer to $\beta_{0,2}$. This change is mainly due to the fact that the solution $\beta_{0,2}$ is near $\beta_{0,\text{match}}$ in the (β_0, α_0) space, with beam envelope in a similar orientation to the CSR kick, which partially counterbalances the emittance growth caused by a stronger CSR kick in the presence of the effect of d.s. CSR. It indicates that the CSR kick matching method is really helpful in CSR-suppression. Nevertheless, it can be clearly seen in both cases that, the proposed conditions ($\beta_{0,1}$ and $\beta_{0,2}$) promise lower emittance growth and have higher tolerance to the mismatch of the beam optics, compared to the CSR kick matching method ($\beta_{0,\text{match}}$).

FIG. 4. Variation of the relative emittance growth $\Delta\epsilon_n/\epsilon_{n0}$ after passage through a DBA with β_0 and α_0 , obtained by ELEGANT simulations, with (lower plot) and without (upper plot) considering the effect of d.s. CSR. The white solid, white dotted, and black solid lines represent the solutions $\beta_{0,1}$, $\beta_{0,2}$, and $\beta_{0,\text{match}}$, respectively.

Different from the proposed conditions and the CSR kick matching method, the optical balance method cancels the CSR kick over several identical achromatic cells by tuning the phase advance of the quadrupole section between adjacent cells to an appropriate value (e.g., with a π section between two DBAs [25]). In order to compare the proposed conditions with the optical balance method, a transport system consisting of two identical DBAs is considered, with the same parameters of the beam and of the DBA as in Table II. The variation of the emittance growth with the phase advance of the quadrupole section between two DBAs (denoted by $\Delta\mu$) is investigated with ELEGANT simulations. In the simulations the effects of n.c. CSR, tr. CSR, and d.s. CSR are included, and three sets of (β_0, α_0) of (1.261 m, 10), (0.8 m, 10), and (20 m, 0.1), which are exactly on, close to, and far away from the $\beta_{0,1}$ solution, are considered. The results are presented in Fig. 5. The effect of the optical balance method can be seen that $\Delta\epsilon_n/\epsilon_{n0}$ is reduced as $|\Delta\mu - \pi|$ decreases from 40° towards 0, by a maximum factor of 4. By contrast, the order of magnitude of $\Delta\epsilon_n/\epsilon_{n0}$ is essentially determined by the distance of (β_0, α_0) from the $\beta_{0,1}$ solution curve, ranging from 10^{-3} to 10^{-1} .

FIG. 5. Variation of the relative emittance growth $\Delta\epsilon_n/\epsilon_{n0}$ after passage through two identical DBAs with the phase advance of the quadrupole section between two DBAs, $\Delta\mu$, obtained by ELEGANT simulations.

V. DISCUSSION AND CONCLUSIONS

By adopting a few important modifications on the R -matrix analysis and the C-S formalism analysis, we have derived the generic conditions for eliminating the CSR kick in a linear regime applicable to a two-dipole achromat with dipoles of arbitrary bending radii and bending angles. These conditions suggest a new way to suppress the CSR-induced emittance growth in a two-dipole achromat. First, choose the dipole parameters to ensure $L_1\theta_1^2 \cong L_2\theta_2^2$, and second, vary the quadrupole strengths to make $M_{c2c}(2, 1) \cong 12/L_1 * \sin(\theta_2/2)/\sin(\theta_1/2)$. Numerical simulations illustrate that the emittance growth can still be well controlled under the case where a more detailed 1D physical model of the CSR wake is considered. In addition, it shows that the CSR-induced emittance growth can be kept to a lower level with the proposed conditions than with two other approaches, such as the CSR kick matching method and the optical balance method.

With the modified analysis, we illustrate the equivalence between the C-S formalism and the R -matrix analysis, and show that the CSR-induced emittance growth in a n -dipole achromat can be evaluated with the analysis of only the motion of particle in an (x, x') 2D plane with n -point kicks, which can be, to a large extent, counted separately. This methodology, which we shall call 2D point-kick analysis, greatly simplifies the analysis and results in explicit formulations, and can be applied to an achromat consisting of three or more dipoles.

At last it is worth mentioning that constant bunch charge and bunch length are assumed in the analysis. Thus, the results presented in this paper are more appropriate to the transport system with a small momentum compaction (R_{56}) than to that with a large R_{56} , e.g., a specified functional bunch compressor. Further study is being carried out to cover the cases where the bunch length has a large variation.

ACKNOWLEDGMENTS

The authors thank J. Q. Wang and S. H. Wang of IHEP for their encouragement and support on this study, and also A. Chao of SLAC for helpful discussions. This work was supported by the National Natural Science Foundation of China (No. U1332108).

- [1] P. Emma *et al.*, *Nat. Photonics* **4**, 641 (2010).
- [2] W. Ackermann *et al.*, *Nat. Photonics* **1**, 336 (2007).
- [3] M. Altarelli, *Nucl. Instrum. Methods Phys. Res., Sect. B* **269**, 2845 (2011).
- [4] G. P. Williams, *Rev. Sci. Instrum.* **73**, 1461 (2002).
- [5] Cornell Energy Recovery Linac: Project Definition Design Report (PDDR), edited by G. S. Hoffstaetter, S. Gruner, and M. Tigner, <http://www.classe.cornell.edu/Research/ERL/PDDR.html> (2013).
- [6] International Linear Collider, Technical Design Report, 2013, <http://www.linearcollider.org/ILC/Publications/Technical-Design-Report>.
- [7] T. Nakazato, M. Oyamada, N. Niimura, S. Urasawa, O. Konno, A. Kagaya, R. Kato, T. Kamiyama, Y. Torizuka, T. Nanba, Y. Kondo, Y. Shibata, K. Ishi, T. Ohsaka, and M. Ikezawa, *Phys. Rev. Lett.* **63**, 2433 (1989).
- [8] Ya. S. Derbenev, J. Rossbach, E. L. Saldin, and V. D. Shiltsev, Deutsches Elektronen-Synchrotron, Report No. TESLA-FEL 95-05, 1995.
- [9] E. L. Saldin, E. A. Schneidmiller, and M. V. Yurkov, *Nucl. Instrum. Methods Phys. Res., Sect. A* **398**, 373 (1997).
- [10] Ya. S. Derbenev and V. D. Shiltsev, Stanford Linear Accelerator Center, Report No. SLAC-PUB-7181, 1996.
- [11] P. Emma and R. Brinkmann, Stanford Linear Accelerator Center, Report No. SLAC-PUB-7554, 1997.
- [12] M. Dohlus and T. Limberg, *Nucl. Instrum. Methods Phys. Res., Sect. A* **393**, 494 (1997).
- [13] D. Douglas and T. Jefferson, National Accelerator Laboratory, Technical Note, JLAB-TN-98-012 (1998).
- [14] H. Braun, F. Chautard, R. Corsini, T. O. Raubenheimer, and P. Tenenbaum, *Phys. Rev. Lett.* **84**, 658 (2000).
- [15] M. Borland, *Phys. Rev. ST Accel. Beams* **4**, 070701 (2001).
- [16] S. Heifets, S. Krinsky, and G. Stupakov, *Phys. Rev. ST Accel. Beams* **5**, 064401 (2002).
- [17] Z. Huang and K. J. Kim, *Phys. Rev. ST Accel. Beams* **5**, 074401 (2002).
- [18] R. Hajima, *Nucl. Instrum. Methods Phys. Res., Sect. A* **528**, 335 (2004).
- [19] G. Bassi, T. Agoh, M. Dohlus, L. Giannessi, R. Hajima, A. Kabel, T. Limberg, and M. Quattromini, *Nucl. Instrum. Methods Phys. Res., Sect. A* **557**, 189 (2006).
- [20] M. Shimada, M. Okazaki, K. Harada, and O. Tsukuba, in *Proceedings of ERL07, Daresbury, UK, 2007*, p. 108.
- [21] D. Sagan, G. Hoffstaetter, C. Mayes, and U. Sae-Ueng, *Phys. Rev. ST Accel. Beams* **12**, 040703 (2009).

- [22] C. Mayes and G. Hoffstaetter, [Phys. Rev. ST Accel. Beams](#) **12**, 024401 (2009).
- [23] V. Yakimenko, M. Fedurin, V. Litvinenko, A. Fedotov, D. Kayran, and P. Muggli, [Phys. Rev. Lett.](#) **109**, 164802 (2012).
- [24] C. Mitchell, J. Qiang, and P. Emma, [Phys. Rev. ST Accel. Beams](#) **16**, 060703 (2013).
- [25] S. Di Mitri, M. Cornacchia, and S. Spampinati, [Phys. Rev. Lett.](#) **110**, 014801 (2013).
- [26] H. Wiedemann, *Particle Accelerator Physics I* (Springer-Verlag, Berlin, 1999), p. 152.
- [27] E. D. Courant and H. S. Snyder, [Ann. Phys. \(N.Y.\)](#) **3**, 1 (1958).
- [28] A. Jackson, *Part. Accel.*, **22**, 111 (1987).