Microbunching instability in storage rings: Link between phase-space structure and terahertz coherent synchrotron radiation radio-frequency spectra

E. Roussel, C. Evain,* C. Sz waj, and S. Bielawski

Laboratoire de Physique des Lasers, Atomes et Molécules, UMR CNRS 8523,

Centre d'Études et de Recherches Lasers et Applications, FR CNRS 2416,

Université des Sciences et Technologies de Lille, F-59655 Villeneuve d'Ascq Cedex, France

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In storage rings, when the number of electrons in a bunch exceeds a threshold value, microstructures appear spontaneously in the bunch. Experimental observations of these microstructures and their evolution are indirect to obtain. In practice, most information is obtained from the terahertz coherent synchrotron radiation (THz CSR), since microstructures in the millimeter range radiate THz CSR. Here we show, thanks to a simple analytical formula associated to numerical simulations, that a frequency component in the THz CSR signal permits one to obtain information about the structure of the electron bunch longitudinal phase space during the microbunching instability. The proposed formula permits one also to link experimental information together, in particular the bunch length and the THz CSR spectrum.

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I. INTRODUCTION

Contrary to the synchrotron radiation emitted by free-electron lasers, the light emitted by electron bunches circulating in a storage ring is usually incoherent longitudinally, since there is no phase relation between emission by different electrons in a bunch. However, in special situations, emission can become coherent in the THz domain, and thus highly more intense. It happens when the electron bunch is shorter than the THz wavelength (typically, with a bunch length of few millimeters) [1] or when the longitudinal charge density has been modified by an external laser [2–4]. Beside these controlled situations, terahertz coherent synchrotron radiation (THz CSR) is also emitted spontaneously in storage rings when the number of electrons exceeds a threshold value, with the apparition of the so-called microbunching instability (e.g., [5–9]).

This instability is characterized by the presence of microstructures in the bunch, typically in the millimeters scale, thus associated with THz frequencies. It is usually a limitation for storage-ring operations; mainly because of the intense noise in the THz signal associated to an erratic behavior of the microstructures. However, when the THz signal fluctuations remain small, for example near the instability threshold, this instability can also be used as an intense THz source with a good signal-to-noise ratio [10].

It is known, from numerical simulations [11] and stability analysis [8,12], that the microstructures appear due to the interaction of the electrons with their own electric field (wakefield), in particular the so-called CSR wakefield emitted in bending magnets. However, little is known about the electron bunch characteristics during this instability, in particular how the electrons composing the bunch are distributed. The dynamics occurs essentially in a 2D phase space (longitudinal position z and associated energy E), where direct experimental information is very difficult to obtain. Inversely, the THz CSR emitted by one electron bunch is relatively easy to get, but is an indirect measurement of the 2D longitudinal phase space. Indeed, two projections are needed to get the value of the coherent THz signal P_{THz} from a 2D longitudinal electron distribution $f(z, E)$ (see Sec. II).

Here we show that the presence, in this coherent THz signal, of a frequency component in the radio-frequency (rf) domain gives precise information on the 2D phase-space structure of the electron bunch during the microbunching instability. This frequency, which we call f_m (m for microstructures), has been observed in several storage rings (BESSY-II [13], MLS [14], SOLEIL [15], ANKA [16], and DIAMOND [17]). It is known to be a convenient diagnostic of the presence of microstructures (even small) in the bunch, and can be used for example to observe precisely the microbunching instability threshold [15].

We show here that this frequency f_m is directly linked to the angle between the branches forming the patterns of the electron distribution. The value of this angle, obtained with a simple formula, is compared to the phase-space structure obtained by the numerical integration of the Vlasov-Fokker-Planck (VFP) equation using the CSR wakefield.

*clement.evain@phlam.univ-lille1.fr

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To test the validity of this work, two storage rings with very different parameters are considered, the Synchrotron SOLEIL and the facility UVSOR-II.

Moreover, thanks to a simple formula derived from the previous one, it is possible to link, with a rather good accuracy, information about the bunch length, the THz CSR spectrum and f_m . This can be used for example to estimate the value of f_m in the case when this value is not known (from experiment or simulation).

II. EQUATIONS AND PARAMETERS

Numerical results are obtained by integrating the Vlasov-Fokker-Planck (VFP) equation, with the shielded CSR wakefield [Eqs. (1) and (2)] [11] (using the opposite sign for the definition of p compared to Ref. [11]). It is well known that this model (or the associated one using macro-particles) permits one to retrieve important characteristics of experimental THz CSR signal, in particular the modulation at f_m [15] and at lower frequencies (oscillation or burst behavior) [11,12].

The VFP equation describes the spatiotemporal evolution of $f(q, p, \tau)$, the phase-space density in the frame of the synchronous electron (the electron circulating along the circular orbit of radius R , with R the bending magnet radius). q, p, τ are dimensionless variables, associated to $z, E,$ and t respectively ($\omega_s = 2\pi f_s$, $q = \frac{z}{\sigma_z}$, $p = \frac{E-E_0}{\sigma_E}$, $\tau = \omega_s \times t$, with f_s the synchrotron frequency, E_0 the ring nominal energy, σ_z and σ_E the rms width, without collective effects, in position and in energy respectively). Without collective effects (i.e., without wakefields), the phase-space density is Gaussian in the two dimensions, and is rotating at the synchrotron frequency f_s in phase space (due to an interplay between energy gain by rf cavities, energy losses by synchrotron radiation, and a path length depending on the electron energy). ϵ is the ratio between the damping time τ_d (due to synchrotron radiation) and ω_s ($\epsilon = \frac{1}{\tau_d \omega_s}$):

$$\begin{aligned} \frac{\partial f(q, p, \tau)}{\partial \tau} - p \frac{\partial f}{\partial q} + \frac{\partial f}{\partial p} (q - I_c E_{wf}) \\ = 2\epsilon \frac{\partial}{\partial p} \left(p f + \frac{\partial f}{\partial p} \right). \end{aligned} \quad (1)$$

The microstructures come from the interaction of the electrons with their own electric field E_{wf} , which is the convolution product of the longitudinal charge density $\rho(q)$ [where $\rho(q, \tau) = \int_{-\infty}^{+\infty} f(q, p, \tau) dp$] with the electric field of one electron. In this work, only the simplified model of the radiation of an electron circulating in an circular orbit (of radius R) between two infinitely conductive parallel plates (separated by an height $2 \times h$) is taken into account [Eq. (2)]. I_c is the normalized current, proportional to N , the number of electrons in the bunch ($I_c = \frac{Ne}{T_0 \sigma_E \omega_s T_0}$ with e the electron charge and T_0 the revolution period of the

reference particle). $\gamma = E_0/mc^2$, m the electron mass, c the velocity of light, and ϵ_0 is the vacuum permittivity. The expressions of the function w and G_2 are given by Eqs. (3.12) and (5.22b) of Ref. [18]. Numerical estimation of [Eq. (2)] is done following a method similar to the one used in [19], and details are given in the Appendix:

$$\begin{aligned} E_{wf}(q) = \frac{4}{3} \frac{\gamma^4 T_0}{4\pi\epsilon_0 R^2} \int_{-\infty}^{+\infty} d\zeta \rho(q-\zeta) w \left(\frac{3\gamma^3}{2R} \sigma_z \zeta \right) \\ - \frac{T_0}{8\pi\epsilon_0 h^2} \int_{-\infty}^{+\infty} d\zeta \rho(q-\zeta) G_2 \left(\frac{\sigma_z}{2R(h/R)^{3/2}} \zeta \right). \end{aligned} \quad (2)$$

The recorded coherent THz signal $P_{\text{CSR}}(\tau)$ corresponds to the CSR power emitted in the detector bandwidth (ν_0, ν_f) [cf. Eqs. (3)–(5)]. $P_{1e}(\nu)$ is the power emitted by one electron at the frequency ν , and is equal to the real part of the impedance of one electron [20]:

$$P_{\text{CSR}}(\tau) = \int_{\nu_0}^{\nu_f} P_{\text{CSR}}(\nu, \tau) d\nu, \quad (3)$$

$$P_{\text{CSR}}(\nu, \tau) = P_{1e}(\nu) N^2 |\tilde{\rho}(\nu, \tau)|^2, \quad (4)$$

$$\tilde{\rho}(\nu, \tau) = \int_{-\infty}^{+\infty} \rho(q, \tau) e^{-i2\pi\nu q} dq. \quad (5)$$

To test the validity of our work over a large range of parameters (in particular the frequency $f_m = 10$ kHz for SOLEIL and $f_m = 650$ kHz for UVSOR, see Sec. III), these equations have been integrated with two sets of very different parameters, given in Table I. The UVSOR case is studied in normal alpha mode, whereas the SOLEIL case is studied in a low-alpha configuration (with alpha, the momentum compaction factor, linking the variation of the electron position q to its energy p [20]). For

TABLE I. Parameters of SOLEIL and UVSOR II storage rings used in the numerical simulations.

Parameter	Unit	SOLEIL	UVSOR II
Nominal energy E_0	MeV	2750	600
Relative energy spread σ_E		10^{-3}	3.4×10^{-4}
Bunch length σ_z	mm	1.4	23.4
Damping time τ_d	ms	3.3	19
Synchrotron frequency f_s	kHz	1.47	19.4
Vacuum chamber height $2 \times h$	mm	25	38
Bending radius R	m	5.36	2.2
Current I	mA	0.3	40

SOLEIL, experimental THz signals are published for this configuration [15].

III. ORIGIN OF THE RF FREQUENCY IN THE THz COHERENT SYNCHROTRON RADIATION

The modulation of the THz signal in the rf domain is due to the spatiotemporal evolution of the microstructures in the 2D longitudinal phase space. The presence of microstructures in the phase space [Figs. 1(a) and 1(b)] induces a modulation in the longitudinal charge distribution $\rho(z, t)$ [Figs. 1(c) and 1(d)]. With time, this modulation drifts along the longitudinal coordinates because the phase space

FIG. 1 (color online). From microstructures in the electron-bunch longitudinal phase space $f(z, p, t)$ to modulation of the THz CSR power $P_{CSR}(t)$. Numerical results with (a), (c), (e), (g), and (i) SOLEIL parameters and (b), (d), (f), (h), and (j) UVSOR II parameters. (a) and (b) Electron bunch phase space $f(z, p, 0)$ at the time $t = 0$. (c) and (d) Associated longitudinal charge density $\rho(z, 0)$. (e) and (f) Time evolution of $\rho(z, t)$. (g) and (h) Associated THz power $P_{CSR}(t)$. (i) and (j) Norm of the Fourier transform of $P_{CSR}(t)$: (black line) calculated from all the 2D phase-space distribution, (red line) calculated from the 2D distribution, where $-0.5 < p < 0$ (distribution between the red dashed lines in (b)). In (h), the red line is shifted vertically for figure clarity.

is rotating [Figs. 1(e) and 1(f)]. This behavior, coupled to the fact that the microstructures continuously appear in the bottom of the phase space, induces a modulation in the THz CSR signal [Figs. 1(g) and 1(h)]. As we can see on the Fourier transform of $P_{CSR}(t)$ [Figs. 1(i) and 1(j)], this modulation is typically in the radio-frequency range, about 10 KHz for SOLEIL and about 650 kHz for UVSOR-II. For the UVSOR case, we observe that the peak at the second harmonic of f_m is very dominant compared to the one at the fundamental frequency. The peak at the fundamental frequency can be nevertheless observed clearly if we record the CSR from only one part of the phase space [Figs. 1(b), 1(h), and 1(j), red curve].

IV. LINK BETWEEN ANGLE IN THE PHASE SPACE AND THE FREQUENCY MODULATION f_m OF THE THz SIGNAL

Our goal here is to link the frequency f_m of the THz CSR signal to the angle between branches forming the patterns in the 2D phase space. Our hypothesis is that the period T_m ($T_m = 1/f_m$) corresponds to the time needed to rotate by the angle φ , the angle between two branches. Since the angular velocity is $\omega_s = 2\pi f_s$, this implies the following relation between the angle φ and f_m :

$$\varphi = \frac{2\pi f_s}{f_m}. \quad (6)$$

Using this relation with the parameters of Table I and the value of f_m obtained with numerical simulations [$f_m \approx 10$ kHz for SOLEIL and $f_m \approx 650$ kHz for UVSOR II, cf. Figs. 1(i) and 1(j)], we find $\varphi \approx 53^\circ$ for SOLEIL and

FIG. 2 (color online). Angle φ calculated from the Eq. (6) (in red) superposed to the numerical phase space (in grey scale). (a) and (b) SOLEIL case and (c) and (d) UVSOR II case. (a) and (c) Cartesian (q, p) coordinates and (b) and (d) polar (r, θ) coordinates (same parameters as Fig. 1).

$\varphi \simeq 10^\circ$ for UVSOR II. As shown in Figs. 2(a) and 2(c), these angles are in good agreement with the angles between the branches of the 2D phase space density obtained by the numerical integration of the Vlasov-Fokker-Planck equation. This can also be observed in polar coordinates r, θ (with $q = r \times \cos \theta$, $p = r \times \sin \theta$) [Figs. 2(b) and 2(d)]. We predict that the relation (6) is all the more accurate since the phase-space structure keeps a similar shape during the rotation, so that the angle φ does not change significantly with time. This implies that the relation (6) is more accurate near the instability threshold than far above the threshold, when the phase-space distortions are important (see for example Fig. 7 in [12]). It implies also that this relation is more accurate for systems with a low ϵ (i.e., a high value of the product damping time by synchrotron frequency); thus more accurate for UVSOR-II ($\epsilon = 4.3 \times 10^{-4}$) than for SOLEIL in low-alpha configuration ($\epsilon = 3.3 \times 10^{-2}$).

V. LINK BETWEEN BUNCH LENGTH, THZ CSR SPECTRUM, AND f_m

It is also possible to estimate the angle φ with another formula, using the bunch length σ_z and the THz CSR average spectrum $\langle P_{\text{CSR}}(\nu, \tau) \rangle$ (the brackets denote averaging over the time τ):

$$\varphi \simeq \lambda_{\text{CSR peak}} / \sigma_z, \quad (7)$$

with $\lambda_{\text{CSR peak}}$, the wavelength associated to the peak in $\langle P_{\text{CSR}}(\nu, \tau) \rangle$ (cf. Fig. 4). This equation describes that at a distance σ_z from the center of the phase space, the microstructure wavelength is about $\lambda_{\text{CSR peak}}$ (cf. Fig. 3).

The two expressions of φ permit one to link the frequency f_m with σ_z and $\lambda_{\text{CSR peak}}$:

FIG. 3. Schematic representation of an electron-bunch phase space with a modulation located at a distance σ_q from the bunch center. The distance λ is associated to the projection on the horizontal axis (in unit of meters) of the arclength corresponding to one modulation period of the phase space. The distance λ is generally associated to a coherent emission, and thus can be measured experimentally.

FIG. 4. Average THz CSR spectrum $\langle P_{\text{CSR}}(\nu, \tau) \rangle$ in the case of (a) SOLEIL and (b) UVSOR-II. Vertical lines indicate $\lambda_{\text{CSR peak}}$ (same parameters as in Fig. 1).

$$f_m \lambda_{\text{CSR peak}} \simeq 2\pi f_s \sigma_z. \quad (8)$$

In the case where only one of the parameters of Eq. (8) is not known, it is then possible to estimate it. Indeed, all the parameters in this equation can be obtained experimentally, but with very different techniques. σ_z can be measured with a resolution of several picoseconds with a streak camera or from the fluctuations of the incoherent radiation [21]. $\langle P_{\text{CSR}}(\nu, \tau) \rangle$ can be obtained experimentally with a Michelson interferometer coupled to a bolometer, at the condition that the fluctuations of the THz signal remain sufficiently small. The temporal THz CSR signal permits one to know f_m , provided the resolution of the THz detector is good enough.

As an example, we compare the value of f_m given by this equation, with the value obtained from numerical simulations. Numerical THz spectra $\langle P_{\text{CSR}}(\nu, \tau) \rangle$ of SOLEIL and of UVSOR-II are presented in Fig. 4. For SOLEIL, $\nu_{\text{CSR peak}} = 1/\lambda_{\text{CSR peak}} \simeq 6 \text{ cm}^{-1}$, and for UVSOR $\nu_{\text{CSR peak}} \simeq 1.5 \text{ cm}^{-1}$. For the value of σ_z , we use the value given by the analytical formula (values in Table I), and thus without taking into account collective effects. Results are given in Table II for the case of SOLEIL and UVSOR-II. In both cases, the agreement between the value of f_m obtained

TABLE II. Value of the modulation frequency f_m of the THz signal obtained from numerical simulations and with Eq. (8).

	f_m from simulations	f_m from Eq. (8)	Ratio
SOLEIL	10 kHz	8 kHz	1.25
UVSOR II	650 kHz	428 kHz	1.5

with numerical simulations and with Eq. (8) is equal or better to a factor 1.5.

VI. CONCLUSION

Precise information on the structure of an electron-bunch longitudinal phase space during the microbunching instability can be obtained from a radio-frequency component in the coherent terahertz signal emitted during this instability. More precisely, the angle between branches forming the pattern can be deduced using a simple analytical formula. We propose also an estimation of this angle using the bunch length and the THz CSR spectrum, which permit one to deduce possible unknown experimental parameters.

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APPENDIX: NUMERICAL METHOD FOR THE COMPUTATION OF THE SHIELDED CSR WAKEFIELD

We consider the shielded CSR wakefield E_{wf} generated by an electron moving on a circular orbit in the midplane of two infinitely conductive parallel plates that are separated by a distance $2 \times h$. The wakefield is composed of two contributions: the free-space term (w) and the parallel plates term (G_2).

Concerning the free-space contribution, we have to compute the convolution of $w(\frac{3\gamma^3}{2R}\sigma_z q)$ and the electron density $\rho(q)$. For that, we use an integration by parts. First, we compute the fast Fourier transform (FFT) of $v(\mu)$ [the antiderivative of $w(\mu)$, with $\mu = \frac{3\gamma^3}{2R}\sigma_z q$] with a very fine mesh [typically 100 times finer than the mesh for $\rho(q)$] and multiply it by ik . The function $v(\mu)$ is given by

$$v(\mu) = \frac{9}{16} \left\{ -\frac{2}{\mu} + \frac{1}{\mu\lambda} [(\lambda + \mu)^{1/3} + (\lambda + \mu)^{-1/3}] + \frac{2}{\lambda} [(\lambda + \mu)^{2/3} - (\lambda + \mu)^{-2/3}] \right\} \quad (A1)$$

with $\lambda = \sqrt{\mu^2 + 1}$. But because it is near zero, the function $v(\mu)$ varies very quickly (at a scale much shorter than one mesh step), a special process before the FFT computation is needed to increase the order of the computation method. We evaluate the exact value of the integration of $v(\mu)$ near zero within a δq step size of the fine mesh. After this preprocess, we perform the FFT of the array v_i as follows:

$$v_0 = \frac{1}{\delta q} \int_0^{\delta q/2} v\left(\frac{3\gamma^3}{2R}\sigma_z q\right) dq, \quad (A2)$$

$$v_i = v\left(\frac{3\gamma^3}{2R}\sigma_z i\delta q\right) \quad \text{for } i > 0. \quad (A3)$$

Concerning the parallel plates contribution, we have to compute the convolution of $G_2(\frac{\sigma_z}{2R(h/R)^{3/2}} q)$ and the density $\rho(q)$. For that, we compute the FFT of $G_2(x)$ (with $x = \frac{\sigma_z}{2R(h/R)^{3/2}} q$) using a very fine mesh. The function G_2 is given by

$$G_2(x) = 2 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \left[\frac{4Y_k^4(3 - Y_k^4)}{(1 + Y_k^4)^3} \right], \quad (A4)$$

where Y_k is a root of the equation

$$\frac{x}{k^{3/2}} = \frac{Y_k^4 - 3}{6Y_k}. \quad (A5)$$

To avoid border effects for long electron bunches, in the simulations, we sum k up to 150.

After computing the FFT of the two terms of the wakefield, we compute the FFT of $\rho(q)$, multiply it by the previous calculated arrays, and then compute the inverse FFT.

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