BNL alternating gradient synchrotron with four helical magnets to minimize the losses of the polarized proton beam

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The principle of using multiple partial helical magnets to preserve the polarization of the proton beam during its acceleration was applied successfully to the alternating gradient synchrotron (AGS) which currently operates with two partial helical magnets. In this paper we further explore this idea by using four partial helical magnets placed symmetrically in the AGS ring. This provides many advantages over the present setup of the AGS, which uses two partial helical magnets. First, the symmetric placement of the four helical magnets and their relatively lower field of operation allows for better control of the AGS optics with reduced values of the beta functions especially near beam injection and allows both the vertical and horizontal tunes to be placed within the “spin tune gap,” therefore eliminating the horizontal and vertical intrinsic spin resonances of the AGS during the acceleration cycle. Second, it provides a wider spin tune gap. Third, the vertical spin direction during beam injection and extraction is closer to vertical. Although the spin tune gap, which is created with four partial helices, can also be created with a single or two partial helices, the high field strength of a single helical magnet which is required to generate such a spin tune gap makes the use of the single helical magnet impractical, and that of the two helical magnets rather difficult. In this paper we will provide results on the spin tune and on the optics of the AGS with four partial helical magnets, and compare them with those from the present setup of the AGS that uses two partial helical magnets. Although in this paper we specifically discuss the effect of the four partial helices on the AGS, this method which can eliminate simultaneously the vertical and horizontal intrinsic spin resonances is a general method and can be applied to any medium energy synchrotron which operates in similar energy range like the AGS and provides the required space to accommodate the four helices. In addition, the four partial helix solution is an optimum solution because it eliminates all the spin resonances for any synchrotron which operates in the same energy range as the AGS.

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I. INTRODUCTION

The preservation of the polarization of a polarized proton beam during its acceleration in a synchrotron is not a trivial task since the polarized beam encounters many imperfection and intrinsic resonances [1,2] which may depolarize the beam. To overcome the spin resonances, an idea was suggested [1] of inserting two partial helical magnets (helices) in the alternating gradient synchrotron (AGS). These helices excite strong artificial imperfection-type resonances which minimizes the polarization loss due to the weak imperfection resonances of the synchrotron, and also, under certain conditions, allows the elimination of the vertical intrinsic spin resonances. This idea of using multiple helices to preserve the polarization of a polarized beam was successfully applied to the AGS [2] by inserting into the AGS ring two partial helices. Figure 1 shows a schematic diagram of the RHIC accelerator complex and the location of the two helices on the AGS ring. Although high field helices

are beneficial for the preservation of the beam polarization during its acceleration, the high field of the helices generates local orbit bumps, distorts the betatron ($\beta_{x,y}$) and dispersion ($\eta_{x,y}$) functions, and also introduces linear beam coupling. These adverse effects can be mitigated by the introduction of compensation quadrupoles placed at the vicinity of the helices and by generating local orbit bumps at the location of the helices [3]. In this paper we further explore the idea of using multiple helices in a synchrotron [1] by introducing four helices, symmetrically placed around the AGS ring, and we present results from calculations, which help compare with the two helices setup [4,5].

Although the use of four partial helices appears to be a mere application and extension of the use of two, we show that the basis of this idea of using four partial helices is independent from that of using two partial helices, and can be applied to any medium energy synchrotron like the AGS which has available space for the placement of the partial helices and has superperiodicity which is a multiple of two. In addition, the four partial helix solution is an optimum solution for any synchrotron which operates in the same energy range as AGS because it eliminates all the spin resonances (imperfection, and intrinsic) and offers a beam optics which is much easier to control from injection to

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FIG. 1. Schematic diagram of the RHIC accelerator complex with the location of the A20 and E20 helices of the AGS.

extraction energies. Unlike the four helices which eliminate the horizontal and vertical spin resonances, the two helices eliminate only the vertical resonances and the horizontal spin resonances are eliminated by the implementation of the “jump quads” method which currently is very tedious. Currently, the operation of the AGS with two partial helices provides proton beam polarization at extraction energy in the range of 65% to 70% when the polarization at injection is 80%. This experimentally measured transmission of the beam polarization is in good agreement with simulations. The proposed insertion of four helices in the AGS will allow 100% transmission of the injected beam polarization because of the elimination of all spin resonances as we show in this paper. In addition, beam optics calculations show that the insertion of four helices in the AGS generates beam optics with low beta functions which allow 100% transmission of the beam current. In the rest of this paper the word helix will be referring to a partial helix.

II. THE BASIS OF THE IDEA OF USING FOUR HELICES IN A MEDIUM ENERGY SYNCHROTRON

The purpose of this section is to prove that the use of the four helix idea is not a mere extension of the two helix idea but has its own logical basis which develops to a general method to suppress all spin resonances if applied to any synchrotrons which operate in the same energy range as the AGS. To start with, we consider a synchrotron which

operates in the same energy range like AGS that has stable spin direction along the vertical and accelerates polarized protons. To minimize the polarization loss of the beam during its acceleration, we can install a single helix at a location of the synchrotron’s ring as shown in Fig. 2(a). The function of this single helix is twofold, first to overcome the imperfection spin resonances which occur at $G\gamma = \text{integer}$, and second to eliminate the vertical and horizontal intrinsic resonances by placing the fractional part of the horizontal and vertical betatron tunes in the “spin tune gap.” The spin tune gap is explained below in a separate section. Since a realistic helix contains multipoles, it is apparent that the strength of the single helix which generates the desired spin tune gap is too strong for the synchrotron to support a stable circulating beam at injection energies. To overcome this problem, we can introduce multiple helices which should provide the same spin tune gap as the single helix, but the strength of the helices should be low enough for the optics of the synchrotron to maintain a stable beam at injection energies. In addition, the placement of the helices should provide a spin tune gap large enough to allow for the placement of the fractional part of the vertical and horizontal betatron tunes thus eliminating both the vertical and horizontal intrinsic spin resonances. To achieve this, one can easily notice that the arrangement of the helices in Fig. 2(b) is equivalent to the arrangement of the helices in Fig. 2(a) because it provides the same spin tune at a reduced strength for

FIG. 2. Each picture shows partial helix arrangement in a synchrotron. The equal signs between the pictures indicate that the helix arrangement shown in each picture provides equivalent spin tune. The arrangement of the helices in the last picture (d) provides an optimum beam optics for the synchrotron.

each helix. But this arrangement of the helices is not possible since the straight section is not long enough to accommodate two helices, with strength $(\delta/2, \delta/2)$ each. In this discussion, we assume that the helices with strength $(\delta/2, -\delta/2)$ both located diametrically opposite to the single helix shown in Figs. 2(a) and 2(b) do not have any effect on either the spin or the beam optics. To overcome the space limitation problem of two helices in the same straight section of the synchrotron, we search for available straight sections in the synchrotron ring that can accommodate four helices as shown in Fig. 2(c). This four helix arrangement shown in Fig. 2(c) has identical spin tune as the single helix in Fig. 2(a) but with reduced strength of the helices. Note the spin rotation angle of one of the helices has negative rotation angle $-\delta/2$. Having found a solution for the spin tune with reduced strength of the helices, we are looking for a beam optics solution which will provide a stable circulating beam at injection energies and also fractional betatron tunes which can be placed in the “spin tune gap.” A reasonable choice is to place the helices in locations such that each helix belongs to one superperiod of the synchrotron. Such a placement is shown in Fig. 2(d). In particular, when this arrangement of the helices is applied to the AGS it reduces its superperiodicity of the AGS from 12-fold to fourfold. This arrangement is an optimum one because it eliminates all the imperfection spin resonances and provides a beam optics which allows the placement of the horizontal and vertical fractional betatron tunes in the spin tune gap, thus eliminating all intrinsic spin resonances from injection to extraction.

III. SPIN CONSIDERATION DURING THE ACCELERATION

A. Theoretical background

The function of the helices is twofold: first, to introduce artificial spin resonances which are stronger than the imperfection resonances, and occur when the spin tune ν_{sp} of the synchrotron satisfies the condition $\nu_{sp} = n$ and, second, to generate a spin tune ν_{sp} during the acceleration cycle such that the condition for the intrinsic resonances,

$$\nu_{sp} = n \pm Q_{x,y} \quad (1)$$

is never satisfied during the acceleration cycle. In the expression (1) above, n is an integer, and $Q_{x,y}$ is the horizontal/vertical betatron tune. In this section we briefly describe the spinor algebra [6] method we use to derive an expression of the spin tune ν_{sp} as a function of $G\gamma$, for a proton circulating in a synchrotron which employs four helices. From the calculated expression of the spin tune, it becomes obvious that, by setting properly the values of the betatron tunes $Q_{x,y}$ of the AGS, the condition (1) is never satisfied during the acceleration cycle, thus the horizontal and vertical intrinsic spin resonances are eliminated. The spin tune ν_{sp} is derived from the expression

$$\cos(\pi\nu_{sp}) = \frac{1}{2} \text{Tr}\{M(G\gamma)\}, \quad (2)$$

where the symbol $M(G\gamma)$ represents the 2×2 spin rotation matrix of the AGS which, in the case where the four helices are arranged as shown in Fig. 3, is given by the expression

$$M(G\gamma) = M_3(-\delta)M_2(G\gamma\theta_4)M_3(\delta)M_2(G\gamma\theta_3)M_3(\delta) \times M_2(G\gamma\theta_2)M_3(\delta)M_2(G\gamma\theta_1). \quad (3)$$

In the expression (3), $M_2(G\gamma\theta_i)$ $\{i = 1, 2, 3, 4\}$ corresponds to the spin rotation matrix of the AGS main dipoles which are placed in the arcs between the helices, as shown in Fig. 3, and is given by the equation

$$M_2(G\gamma\theta_i) = \{I \cos(G\gamma\theta_i/2) - i\sigma_2 \sin(G\gamma\theta_i/2)\}, \quad (4)$$

where $M_3(\delta)$ corresponds to the spin rotation matrix of each of the helices and is given by

$$M_3(\delta) = \{I \cos(\delta/2) - i\sigma_3 \sin(\delta/2)\}. \quad (5)$$

The angle θ_i $\{i = 1, 2, 3, 4\}$, appearing in Eqs. (3) and (4), corresponds to the azimuthal angle shown in Fig. 3, and the symbol δ appearing in Eq. (5) corresponds to the spin rotation angle about the spin rotation axis of each helix. Note in Fig. 3 the spin rotation angle of one of the helices is $-\delta$, and the spin rotation angle of the other three is $+\delta$. The spin rotation axis of each helix is along the beam axis [7]. The symbols $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices

FIG. 3. Schematic diagram of the AGS ring with four helices symmetrically placed around the ring. Each pair of helices(1,3) and helices(2,4) are separated by an angle π . The spin rotation angles about the stable spin direction of each of the helices is $+\delta$, except for the helix4 which provides a spin rotation angle $-\delta$.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and correspond along the radial, longitudinal, and vertical directions, respectively. In the expression (2) above, $\text{Tr}\{M(G\gamma)\}$ is the trace of the spin rotation matrix $M(G\gamma)$ given by the expression (3). The stable spin direction $\vec{b}$ about which the spin rotates by an angle ϕ is given by the equation

$$\vec{b} = \left[\frac{i}{2 \sin(\phi/2)} \right] \text{Tr}(\sigma M) \quad (6)$$

and the $\sin(\phi/2)$ can be calculated from the expression

$$\cos\left(\frac{\phi}{2}\right) = \frac{1}{2} \text{Tr}[M(G\gamma)] \quad (7)$$

with the symbol ϕ in Eqs. (6) and (7) being the spin rotation angle about the stable spin direction $\vec{b}$.

B. Calculation of spin tune using four helical magnets in a synchrotron

The knowledge of the spin tune, as defined by the Eq. (2) in the previous section, identifies the location of the intrinsic spin resonances ($\nu_{\text{sp}} = n \pm Q_{x,y}$) which occur during the acceleration. A computer program calculates the fractional part of the spin tune ν_{sp} , by making use of Eq. (2) above. The spin tune depends on $G\gamma$, the strength δ of the helical magnets, and is independent of the azimuthal angle along the ring. For the case of four helices placed as shown in Fig. 3, the fractional spin tune ν_{sp} is calculated as a function of $G\gamma$, for three different strengths of the helices, namely, 1.9, 2.1, and 2.5 T, which correspond to a spin rotation angle of 14.5° , 19.0° , and 25.5° about the spin rotation axis of each of the helices. The calculated spin tune for the three different magnetic field settings of the helices is plotted as a

FIG. 4. The fractional part of the spin tune of AGS with four helices. Each of the three curves corresponds to a given strength for each set of the four helices. The region of the spin tune gap which corresponds to a strength of the helices of $B = 2.1$ T is highlighted in pink.

function of $G\gamma$ in Fig. 4. The pink shaded area on Fig. 4 is the spin tune gap which corresponds to the excitation of 2.1 T of each of the four helices. Note that the spin tune gap becomes wider as the magnetic field of each of the four helices increases. If the value of the fractional part of the vertical betatron tune Q_y is within the upper or lower part of the spin tune gap shown in Fig. 4, the condition $\nu_{\text{sp}} = n \pm Q_y$ will never be satisfied, therefore the vertical intrinsic spin resonances are eliminated. If we consider the fractional parts of the spin tune $\nu_{\text{sp},f}$ and the fractional part of the betatron tune $Q_{y,f}$, it is easy to show that the condition $\nu_{\text{sp}} = n \pm Q_y$ can be written as two conditions, $\nu_{\text{sp},f} = Q_{y,f}$ and $(1 - \nu_{\text{sp},f}) = Q_{y,f}$. Therefore, in the rest of this paper we will plot the functions $\nu_{\text{sp},f}$ and $(1 - \nu_{\text{sp},f})$ as a function of $G\gamma$, in a single plot, and compare them with the value of the fractional tune $Q_{y,f}$.

C. Comparison of the spin tune generated by four helical magnets with the spin tune generated by one or two helical magnets in AGS

In Fig. 5, the thick red trace corresponds to the fractional part of the spin tune in AGS with four helices arranged along the ring as shown in Fig. 3, and the thinner blue trace is the fractional part of the spin tune when the AGS employs only a single helix. Although the spin tune generated by the four helices is identical to that generated by a single helix, the single helix requires a field of 3.0 T, as compared to the 2.1 T field required by each of the four helices. A MAD [7] model of the AGS which employs a single helix of 3.0 T generates an unstable beam at injection energies; therefore the use of a single 3.0 T helix is not practical. However, the MAD model of the AGS with four helices of 2.1 T each generates a stable beam, as we show in a later section. The comparison of the fractional spin tune

FIG. 5. The fractional part of the spin tune generated by four helices (thick red curve) and by a single helix (thinner blue curve). Note that the single helix requires a strength of 3 T to generate the same spin tune as that of four helices which require a field of 1.9 T each.

generated by four helices with that of two helices [2] as we presently run the AGS is shown in Fig. 6. The red solid line in Fig. 6 is the fractional spin tune as generated by the four helix arrangement shown in Fig. 3, and the green solid line is the fractional spin tune generated by the present arrangement of the two helices as they are installed and operate in AGS [2]. Since the spin tune shown in Fig. 6 is periodic as a function of $G\gamma$, we only plot one period of the fractional spin tune. Figure 6 demonstrates that the spin tune gap generated by the four helices is wider than that generated by the two helices. As we mentioned earlier, although the strong magnetic field of the helices generates a larger spin tune gap which is desired to eliminate the intrinsic spin

FIG. 6. The fractional part of the spin tune generated by four helices (red curve) each at a field of 2.1 T, and by two helices (green curve) of 2.1 and 1.5 T, respectively, as they are presently installed in AGS.

resonances, it also generates challenges in setting up the beam optics of the accelerated beam. Reference [3] describes in detail how we coped with the challenges which emerged during the setup of the beam optics of the AGS as it operates with two helices. The main advantages of the four helix arrangement over the present two helix one are, first, the simpler beam optics of the AGS at injection energies, and this is due to the symmetric arrangement of the four helices, as it is described in the next section, and second the flexibility which allows the placement of the fractional part of the horizontal and vertical betatron tunes in the spin tune gap. A fair comparison of the spin tunes generated by the four and two helical magnet placement would be to replace the second helix of the AGS which presently rotates the spin by 10.6° with a helix which rotates the spin by the same angle of 19° as the first helix. Such a hypothetical comparison is shown in Fig. 7 with the black trace corresponding to the fractional part of the spin tune generated by the two helices of equal strength, and the red trace corresponding to the spin tune generated by four helices as placed in Fig. 3. Although the hypothetical increase in the strength of the second helix is very beneficial because it increases of the spin tune gap at the location of $G\gamma \sim 6$ as it occurs at the location of a strong spin resonance, this arrangement of the two helices having equal strength of 2.1 T each, as it appears in Fig. 1, cannot provide a stable beam optics at injection energies. This is verified by calculations which are based on the MAD modeling of the AGS equipped with two helices having equal strength of 2.1 T each. In contrast, the symmetric placement of the four helices makes the beam optics of the AGS a very easy problem to solve, and most important, it allows the placement of the horizontal and vertical betatron tunes within the spin tune gap. Another issue is the stable spin direction at the injection and extraction points of the AGS. Specifically, since there are no spin rotators in the

FIG. 7. The fractional part of the spin tune generated by four helices (red curve) each at a field of 2.1 T, and by two helices (black curve) of 2.1 T each.

TABLE I. The values of the directional cosines of the stable spin direction with the vertical, at the injection point (2nd column) and extraction point (3rd column), for the cases of two helices (2nd and 3rd rows) and the case of four helices (4th row).

	Dir_cos S_z at injection SS_L20 $G\gamma = 4.5$	Dir_cos S_z at extraction SS_H10 $G\gamma = 45.5$
Two helices A20 and E20	0.9979	-0.9763
Two equal helices A20 and E20	1.0000	-0.9635
Four helices	1.0000	-0.9828

AGS injection-beam-transport line to align the stable spin direction to that at the injection point of AGS, and similarly, because there are no spin rotators at the AGS extraction-beam-transport line to align the stable spin direction along the stable spin direction at the injection point of RHIC, it is preferable that the stable spin direction at the injection and extraction points of AGS be along the vertical. Table I, contains the values of the spin directional cosines with the vertical at the injection point (2nd column) and the extraction point (3rd column) of AGS, for three different placements of helical magnets (1st column) in the AGS ring. Table I shows also that the four helices provide stable spin directions at the injection and extraction points of AGS which are closer to the vertical. The calculations of the stable spin direction at the injection and extraction points of the AGS which employs helical magnets are based on Eqs. (6) and (7) which appear in an earlier section.

IV. BEAM OPTICS CONSIDERATION OF THE AGS WITH HELICAL MAGNETS

The insertion of helical magnets into a medium energy synchrotron poses the challenging task of coping with the

FIG. 8. The beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions over three superperiods of the bare AGS. The horizontal and vertical tune quadrupoles (one horizontal and one vertical in each superperiod) are set to zero. The corresponding betatron tunes appear in the 2nd row of Table II.

TABLE II. The horizontal and vertical betatron tunes (Q_x, Q_y) for three different setups of the AGS, as they appear in the 1st column.

	Q_x	Q_y
Bare_AGS	8.7127	8.7675
Bare_AGS4 (cold helices)	9.1864	8.9884
Non-Bare-AGS4 (cold helices)	8.9785	8.9874

effect of the high field of the helices on the beam optics of the circulating beam. This effect is especially large at injection energy when the beam rigidity is relatively low and the beam size is large. The beam optics design for the AGS with two helices [2] is described in Ref. [3], and the design requires insertion of thin compensation quadrupoles [8] to maintain low beta function in the ring and also to satisfy the constraint that the vertical tune Q_y is near integer to be placed within the spin tune gap. In the following subsection we describe the beam optics of the circulating beam at injection energy, when the AGS employs four helical magnets as shown in Fig. 3.

A. Beam optics of bare AGS and AGS with four helical magnets

The AGS consists of 240 combined function magnets which form a synchrotron of superperiodicity $P = 12$. The horizontal and vertical betatron tunes can be adjusted with 12 pairs of focusing/defocusing quadrupoles, with each pair located at specified straight sections of every superperiod of the AGS. Figure 8 shows the beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions of the closed orbit's circulating beam, over three superperiods of the bare AGS, where the tune quadrupoles and the helical magnets are set to zero excitation. The corresponding tunes for a bare AGS at injection energy are shown in columns 2 and 3 of the 2nd row in Table II. The

FIG. 9. The beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions over three superperiods of the AGS with four helices. A single helix of strength 1.9 T is placed at the middle of the three superperiods of the AGS. The corresponding betatron tunes appear in the 3rd row of Table II. The horizontal and vertical tune quadrupoles of the AGS are not excited.

TABLE III. The strength in m^{-2} (4th row) and the location of the compensation quadrupoles (3rd row) relative to the location of the helical magnet for each of the three superperiod sections of the AGS.

1st superperiod		2nd superperiod			3rd superperiod	
CQ1	CQ2	CQ3	Helix	CQ4	CQ5	CQ6
SS_03	SS_07	SS_09	SS_10	SS_11	SS_13	SS_17
0.0405	-0.3031	0.3201		-0.4707	0.1594	0.0031

insertion of four helices symmetrically placed around the AGS ring, as shown in Fig. 3, reduces the superperiodicity of the AGS to four. Figure 9 shows the beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions of the closed orbit's circulating beam, over three superperiods of the bare AGS, with a helical magnet of 2.1 T strength, placed at the center of each of the three superperiods. This placement of the four helices, each having a strength of 2.1 T, provides a spin tune shown in Fig. 6 by the red curve. The corresponding betatron tunes Q_x , and Q_y , for a bare AGS which includes four helical magnets, at injection energy are shown in the 3rd row of Table II in the columns 2 and 3, respectively. In the beam optics calculations we assume that the focusing properties of each of the four helical magnets [9] are identical and the optics of each helical magnet is represented as a first order transfer matrix, which was calculated by raytracing through the three-dimensional field map [9] of the helices. In order to match the beam optical functions ($\alpha_{x,y}$, $\beta_{x,y}$) and the dispersion functions ($\eta_{x,y}$) of each section of the AGS consisting of three superperiods each, with a helical magnet in the middle, we had to vary and make identical the beam

parameters ($\alpha_{x,y}$, $\beta_{x,y}$) and the dispersion functions ($\eta_{x,y}$) at the entrance and exit of each of the three superperiod sections. The results from the beam optics calculations appear in Fig. 9, which shows the beta functions and the dispersion functions of one quarter of the AGS's ring. The comparison of the beta and dispersion functions as they appear in Figs. 8 and 9 indicates that the helices affect the 12-fold superperiodicity of the bare AGS and introduce a beta wave with a maximum value $\beta_x \sim 50$ m and $\beta_y \sim 30$ m. Similarly, the horizontal dispersion reaches the value of $\eta_x \sim -4$ m as shown in Fig. 9. Our experience with the AGS running with two helical magnets [3] guarantees that such a lattice as shown in Fig. 9 can lead to an easily controlled circulating beam from the injection to the acceleration.

B. Beam optics of the AGS with four helical magnets and compensation quadrupoles

In order to improve the beam optics of the AGS running with four helical magnets and provide better beam and tune control during the acceleration cycle, we introduce in the lattice of the AGS running with four helical magnets, compensation quadrupoles, which compensate for the effect of the helices on the beam optics. In this study, in each of the four “three superperiod” sections, we placed compensation quadrupoles in the straight sections of the AGS. The location of each of these quadrupoles appears in the 3rd row of Table III. Figure 10 shows the beta and dispersion functions which correspond to the strength of the compensation quadrupoles appearing in the 4th row of Table III. From this figure we conclude that the compensation quadrupoles help reduce the beta and dispersion functions, and also set the values of the horizontal and vertical betatron tunes, as they appear in the 4th row of Table II. By setting the values of the betatron tunes within the spin tune gap, the intrinsic resonance condition $\nu_{\text{sp}} = n \pm Q_{x,y}$ is never satisfied during the acceleration cycle; therefore the horizontal and vertical intrinsic spin resonances are eliminated.

V. SUMMARY

In this section we summarize the most important conclusions of the paper and we make a detailed comparison of the performance of the four helices with the performance of the two helices, to convince the reader that

FIG. 10. The beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions over three superperiods of the AGS with four helices and the placement of the compensation quadrupoles as shown in Table III. In each three superperiod section of the AGS a helix of strength 1.9 T is placed at the middle of the three superperiods of the AGS. The strengths of the horizontal and vertical tune quadrupoles and those of the compensation quadrupoles are set to match the optical and dispersion function among each of the four sections of the AGS. The corresponding betatron tunes for this beam optics appear in the 4th row of Table II.

the four helices provides the optimum solution to a synchrotron regarding the beam optics and the preservation of the polarization. We conclude that the four helices installed in a synchrotron provide 100% transmission of beam current and polarization. We start this section by clarifying the terminology we use in this paper, like spin tune gap and “spin resonances,” and we discuss their relation.

A. The spin tune gap

For a synchrotron without helices, the spin tune ν_{sp} is defined by the equation $\nu_{\text{sp}} = G\gamma$, and the spin tune can take any value of $G\gamma$ corresponding to the energy range the synchrotron can accelerate the beam. The spin tune ν_{sp} for the AGS with no helices can take any value between 4.5 and 45.5 corresponding to the values of $G\gamma$ from beam injection to beam extraction, respectively. For a synchrotron with helical magnets, the spin tune as calculated by Eq. (2) may take only specific values. For example if the values of $G\gamma$ are between two consecutive integers say 40 and 41 ($40 \leq G\gamma \leq 41$) and if the spin tune ν_{sp} as calculated by Eq. (2) can only take values in the range of 40.2 and 40.8 only, ($40.2 \leq \nu_{\text{sp}} \leq 40.8$), then the values of the spin tune ν_{sp} in the ranges between (40.0, 40.2) and (40.8, 41.0) are not allowed. The spin tune gap is defined in this paper, as the set of values in the ranges of $G\gamma$ (40.0, 40.2) and (40.8, 41.0). An illustration of the spin tune gap is shown by the pink rectangular box in Figs. 4 and 5 where we plot the values of the fractional spin tune ν_{fsp} as a function of $G\gamma$. This spin tune gap corresponds to the spin tune plotted by the red curve of Fig. 4.

B. The horizontal and vertical intrinsic spin resonances

In this subsection we state the condition for a horizontal and vertical intrinsic spin resonance to occur, and we show how these spin resonances can be eliminated. The condition for a horizontal or a vertical spin resonance to occur is $\nu_{\text{sp}} = n \pm Q_x$ (horizontal) and $\nu_{\text{sp}} = n \pm Q_y$ (vertical). The symbols in these equations have been defined in an earlier section, and as it is explained in this paper, if the fractional part of the betatron tune $Q_{x,y}$ is located within the spin tune gap the condition $\nu_{\text{sp}} = n \pm Q_{x,y}$ for a horizontal or vertical intrinsic to occur is never satisfied ($\nu_{\text{sp}} \neq n \pm Q_{x,y}$); therefore the intrinsic resonances are eliminated.

C. The imperfection spin resonances of a synchrotron with two or four helices

The imperfection spin resonances occur during the acceleration of a polarized proton beam when the condition $G\gamma = n$ is satisfied. The insertion of partial helices in a synchrotron ring is equivalent to the introduction of strong artificial spin resonances which occur when the energy

of the beam corresponds to $G\gamma = n$, therefore when a polarized beam crosses an imperfection resonance a total spin flip occurs according to the Froissart-Stora formula $\frac{P_f}{P_i} = 2 \exp(-\frac{\pi|\epsilon_r|^2}{2\alpha}) - 1.0$, where $\alpha = \frac{G\dot{\gamma}}{\omega_{\text{rev}}}$. In the equations above, the symbols P_i and P_f are the polarization of the beam before and after the resonance, ϵ_r is the strength of the imperfection spin resonance, $\dot{\gamma}$ is the acceleration rate, and ω_{rev} is the revolution frequency of the beam bunch in the synchrotron. For a reasonably large value of the resonance strength (ϵ_r), the first term of the Froissart-Stora formula is much smaller than 1.0 [$2 \exp(-\frac{\pi|\epsilon_r|^2}{2\alpha}) \ll 1.0$]; as a result, complete spin flip occurs ($P_f \approx -P_i$) when the beam crosses the resonance during acceleration. In conclusion, the imperfection spin resonances of a synchrotron are eliminated when one or more partial helices are placed in a synchrotron ring.

D. The vertical intrinsic spin resonances of a synchrotron with two or four helices

As we mentioned earlier, by placing the vertical betatron tune within the spin tune gap the vertical intrinsic resonances are eliminated because the condition $\nu_{\text{sp}} = n \pm Q_y$ is never satisfied. In the case of two partial helices, the elimination of the vertical intrinsic resonances requires that the fractional vertical betatron tune be raised to values $Q_y > 0.960$ to be placed within the spin tune gap (see green curve in Fig. 6). This is not possible to be achieved at near injection energies ($G\gamma = 4.5$ to ~ 7.0) because the cold helix which operates at a constant field of 2.1 T strongly affects the relatively low rigidity beam at injection; thus, by raising the fractional vertical betatron tune to values $Q_y > 0.96$ the beam optics becomes unstable. This instability of the beam optics has been observed in the calculations by modeling the beam optics of the AGS using the MAD computer code, and also experimentally. Indeed Fig. 11 shows a theoretical plot of the beta ($\beta_{x,y}$) and dispersion functions ($\eta_{x,y}$) at near injection energy of $G\gamma = 5.0$ along the whole AGS ring when it operates with two helices, as modeled by the MAD computer code. The corresponding betatron tunes for the beam optics of Fig. 11 are $Q_x = 8.694$ and $Q_y = 8.935$. From Fig. 11 we conclude that when the AGS operates with two helices, the beam optics provides large beta functions $\beta_y \sim 100$ m as compared with the nominal beta functions of the AGS $\beta_{x,y} \sim 23$ m (see Fig. 8) and the fractional part of the vertical betatron tune is too low ($Q_y = 0.935$) to be placed within the spin tune gap which requires values of betatron tune $Q_y > 0.960$. The adverse effect of the two helix operation is also apparent from the dispersion functions as shown by the green curve in Fig. 11. Namely, the horizontal dispersion function fluctuates greatly along the AGS ring as compared with the nominal variation of the horizontal dispersion function which is plotted in

FIG. 11. The beta ($\beta_{x,y}$) and eta ($\eta_{x,y}$) functions along the whole ring of the AGS operating with two helices at beam energy corresponding to $G\gamma = 5.0$. The locations of the cold and warm helices along the ring are indicated by arrows. The values of the horizontal and vertical betatron tunes are also shown on the plot. Note the large value of the $\beta_y \sim 100$ m function as compared to the nominal one of $\beta_y \sim 23$ m shown in Fig. 8. Similarly, the horizontal dispersion function η_y shows large variation along the ring as compared to that of the nominal AGS shown in Fig. 8. The appearance of the vertical dispersion function indicates beam coupling which is introduced by the 2.1 T cold helix.

Fig. 8. In addition, the 2.1 T strength of the cold helix introduces beam coupling which is apparent by the vertical dispersion shown as the green dashed line in Fig. 11. The experimental proof which shows that the vertical

FIG. 12. The spin tune and the experimentally measured fractional part of the vertical betatron runs plotted as a function of $G\gamma$ at near injection energies. Note that in these polarization runs, the fractional vertical betatron tune is near or crosses the spin tune line. This indicates the nonelimination of the vertical spin intrinsic resonances at near injection energies.

intrinsic resonances cannot be eliminated at near injection energies when the AGS is operating with two helices is shown in Fig. 12 where we plot the spin tune and the vertical fractional betatron tune for various polarized protons runs, as a function of $G\gamma$. It is clear from Fig. 12 that the fractional betatron tune is not within the spin tune gap at near injection energies. This inability of placing the vertical betatron tune within the gap at near injection energies is the result of some loss of beam polarization. In the case of four partial helices, it is very easy to place the vertical betatron tune within the spin tune gap not only because the spin tune gap is wider than that generated by the two helical magnets (compare red and green curve of Fig. 6), but mainly because of the AGS fourfold superperiodicity which makes the control of the beam optics of the AGS an easy task. To recapitulate, although the spin tune gap generated by the two and four helices is not much wider than that of the two helices, the beam optics of the synchrotron with four helices is such that it allows the placement of the vertical betatron tune in the spin tune gap in the whole acceleration range, as opposed to the two helix case where this betatron spin tune placement in the spin tune gap is not possible at injection energies thus some polarization loss occurs. A comparison of the beam optics generated by four helices to the beam optics generated by two helices is provided by Figs. 10 and 11, respectively, where we observe the smaller beta ($\beta_{x,y}$ and dispersion ($\eta_{x,y}$) functions shown in Fig. 10 as compared to those in Fig. 11. From the comparison of Figs. 10 and 11, we also observe smaller variation of the horizontal dispersion function and the near absence of beam coupling introduced by the four helices.

E. Horizontal intrinsic spin resonances using two or four helices

The beam optics of the AGS with two helices does not allow the placement of the horizontal betatron tune within the spin tune gap and this is verified theoretically by modeling the AGS with two helical magnets [3] and also experimentally. To correct for this deficiency of the two helices which do not allow for the elimination of the horizontal intrinsic resonances, and minimize the polarization loss during the beam acceleration, we employ the jump quads [10]. In the case of the AGS with four helices, the beam optics allows the placement of both the horizontal betatron tune as well as the vertical betatron tune within the spin tune gap, thus eliminating both the horizontal and vertical intrinsic spin resonances. No jump quads are needed in a synchrotron with four helices. Beam optics calculations using the MAD computer code show that the simultaneous elimination of the horizontal and vertical intrinsic spin resonances may also occur when we use the same four helix arrangement, but with lower strength of the helices of 1.9 T instead of 2.1 T. This is expected because of the lower field of the helices.

VI. FURTHER STUDIES, INVESTIGATIONS, AND SUGGESTIONS ON THE AGS RUNNING WITH FOUR HELICAL MAGNETS

In the design of the beam optics of the AGS with two helical magnets [3], we designed a local orbit bump at the location of the helical magnet, to displace the beam horizontally on the outside of the AGS ring. Such a displacement of the beam at the location of each helical magnet is required because the beam is deflected on the left when it is inside the helix; therefore, by displacing the beam on the outside of the ring or equivalently on the right of the helical magnet before the beam enters the magnet, we prevent the beam from hitting the magnet's vacuum pipe. From our experience with the AGS running with two helical magnets, we realized that the use of horizontal corrector magnets at the entrance and exit of the helical magnets would have provided us with better control of the beam. We therefore plan to install such horizontal correctors if new helical magnets are built. Similarly, the introduction of compensation quadrupoles at the entrance and exit of the helical magnets will provide additional flexibility in the beam optics of the AGS. In this study of the AGS with the four helical magnets, we considered the AGS as comprised of four identical sections, of three superperiods each, with each section having the helical magnet in the middle. As a future study we suggest considering the AGS with four helical magnets as being comprised of two identical sections, with each section containing six superperiods and two helical magnets.

VII. CONCLUSION

Calculations show that the insertion of four helices in the AGS provides a significantly improved beam optics and 100% transmission of beam current and polarization, as compared to the two helices. The four helices can eliminate

both the horizontal and vertical intrinsic spin resonances during the acceleration of polarized protons and there is no need of using the jump quads. This solution of four helices is a general solution which can be applied to any synchrotron which operates in the same energy range as the AGS. Also, the elimination of all the spin resonances makes the four helix solution an optimum solution. The effect of the helices on the optics of a synchrotron depends not only on the type of the helices but also the optics of the synchrotron. As a result, although the four helix solution can be applied to any synchrotron, the optics of each synchrotron which employs four helices must be given its own consideration.

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