Two-chicane compressed harmonic generation of soft x rays

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Seeding an electron bunch prior to compression simultaneously shifts the laser modulation to shorter wavelengths while decreasing the required modulation amplitude. The final x-ray wavelength is then tunable by controlling the compression factor with the rf phase. In this paper we describe a two-chicane scheme that allows for large modulation amplitudes, extending the method to photocathode beams with significant uncorrelated energy spreads. The downside of such compressed seeding is the need to maintain bunching across an extended accelerator region. We present analytical estimates and computer simulations to study tolerances for a sample lattice. We also note that transportation of the fine compressed modulation structure is helped by error self-correction in the second chicane, an effect that may be of more general interest.

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I. INTRODUCTION

A free-electron laser (FEL) [1] can theoretically produce fully coherent x rays, a promising tool for the fields of physics, chemistry, and biology. Current x-ray FELs in use or under construction rely on self-amplified spontaneous emission (SASE) [2,3]. SASE FELs can reach the hard x-ray region, but are saddled by long saturation lengths and poor longitudinal coherence. In contrast, FELs “seeded” by optical or UV lasers promise full coherence and shorter FEL lengths. One leading seeded FEL scheme is high gain harmonic generation (HGHG) [4,5]. However, single-stage HGHG requires high laser power (expensive and potentially problematic for the FEL process), and is limited to wavelengths around 20 nm [6]. Multiple-stage HGHG can reach shorter wavelengths, but is more technically challenging. A recent and promising seeding scheme is echo enabled harmonic generation, which manipulates hidden structure in phase space to produce high harmonic seeding [7,8].

Seeding the electron beam prior to bunch compression, which we will call compressed harmonic generation (CHG), is an alternative approach [9–11]. In this paper we discuss a variation on CHG in which dispersion from the compression stage smears out the longitudinal modulation, but the structure remains imprinted in phase space and can be revived later [10,12].

The scheme uses two dispersive sections, the first to compress the beam and the second to bunch the modulation. Starting from the electron gun, an accelerator section brings the beam to energy E_a , while adding a linear chirp, h . A laser then modulates the beam energy by $A_L \cos(k_L z)$. A dispersive section, $R_{56}^{(a)}$, simultaneously compresses the bunch length by a factor of $\alpha = 1/(1 + R_{56}^{(a)}h)$, while also

strongly overbunching the laser modulation. Another accelerator section flattens the beam with a second chirp, $-\alpha h$, while increasing the energy to $E_b \equiv gE_a$. Finally, a second dispersive section with effective opposite sign, $R_{56}^{(b)} \approx -R_{56}^{(a)}g/\alpha \pm g/(A_L\alpha^2k_L)$, unwinds the overbunched laser modulation, recovering a maximally bunched beam at wave vector αk_L . As an added benefit, the second dispersive region also reverses second order effects from the preceding accelerator section. The full multistep process is summarized in Fig. 1.

In this paper we give an analytical description of the two-chicane CHG scheme, and extend the simulations of [12] to include higher energies and smaller β functions (desirable for sending the beam through a radiator). The major drawback of compressed seeding is the need to preserve fine phase space structure across long accelerator regions. In the final section we consider the technical requirements for effective transport of this phase space modulation.

II. MOTIVATION

A. Advantages of compressed seeding

Seeding before compression offers several advantages. First, the bunch compressor reduces the final modulation wavelength by a factor of α . Second, the rf phase controls α , so changing the phase tunes the final wavelength. Third, the combination of chirp and bunch compressor amplifies the laser modulation by the compression factor, reducing the required laser power by α^2 . CHG also requires relatively few components, even for the two-chicane variation; the first chicane doubles as the bunch compressor (required to increase current for all FELs), so we need only one additional modulator and chicane, the same as for

FIG. 1. Diagram of CHG scheme. (a) The first accelerator section raises the particle energy to E_a and introduces a linear chirp, h . After modulating with a laser, the first dispersive section, BC1, compresses the beam and overbunches the modulation. A second rf section accelerates the particles to E_b , and cancels the chirp of the first section. The final dispersive section, BC2, unwinds the overbunching. (b) Operating BC1 in overcompression rotates the electron beam head to tail, allowing the use of a chicane for both BC1 and BC2. In both cases it is possible to add a third accelerator section to reach a final energy of E_f .

single-stage HGHG. Finally, the low power requirement allows seeding with short-wavelength lasers.

B. Advantages of two-chicane CHG

The simplest compressed seeding schemes use a single dispersive section to both compress the beam and bunch the laser modulation [9]. For simultaneous compression and bunching, the chirp slope must match the modulation strength; i.e., the same dispersive section, of strength R_{56} , both compresses the chirp (moving electrons on the order of the mm-scale bunch length) and maximizes the bunching (by compressing the sub- μm -scale modulation). This matching condition implies a laser modulation approximately 1000 times weaker than the chirp, limiting the modulation amplitude to a few tens of eV. Such a small amplitude is sufficient for the small energy spread of a thermionic gun (as used in [9]), but is far below the incoherent energy spread in high current, photocathode beams.

To extend CHG to photocathode beams, we have considered the addition of a second dispersive section to allow for larger modulation strengths [12]. The additional dispersive region, $R_{56}^{(b)}$, separates the compression, carried out by the first dispersive section, $R_{56}^{(a)}$, and the bunching condition, determined by the modified sum, $R_{56}^{(T)} = R_{56}^{(a)} + \frac{\alpha}{g} R_{56}^{(b)}$. As a side benefit, the second dispersive region corrects upstream errors, reversing second order effects in the first chicane and subsequent accelerator sections. We note that two-chicane compressed seeding was first suggested in [10]. A modified version of the scheme described in [12] was proposed recently [13,14].

The downside of compressed seeding is the need to preserve the modulation throughout the ensuing accelerator sections. A second dispersive region can help reverse smearing from certain effects (e.g., emittance), but also requires strong dispersion with very tight tolerances.

The advantages of CHG must be weighed against these technical constraints.

III. ANALYTICAL MODEL

A. Bunching factor

The seeding can be quantified by the bunching amplitude at the end of BC2,

$$b_f(k) = \int dz_f \int dp_f e^{ikz_f} \Psi_f(z_f, p_f), \quad (1)$$

with final longitudinal particle position z_f , normalized energy p_f , and final distribution $\Psi_f(z_f, p_f)$. [For a particle of energy, E , we define $p \equiv (E - E_b)/E_b$.] To evaluate Eq. (1) analytically, we assume an initially uniform longitudinal distribution within the electron beam. We also assume an initial Gaussian energy distribution, $\Psi_i(p_i) \propto \exp[-p_i^2/2\sigma_p^2]$, with relative energy spread, $\sigma_p \equiv \sigma_{E0}/E_a$, defined in terms of the absolute energy spread, σ_{E0} , and energy prior to the first bunch compressor, E_a .

We describe the CHG process as follows: (1) We chirp the beam and modulate with a laser. (2) The first dispersive section (abbreviated BC1) compresses the beam. (3) We accelerate again and add a second chirp. (4) A second dispersive section (BC2) unwinds the overbunching,

$$\begin{aligned} z_1 &= z_i, & p_1 &= p_i + A_L \sin(k_L z_i) + h z_i, \\ z_2 &= z_1 + R_{56}^{(a)} p_1, & p_2 &= p_1, \\ z_3 &= z_2, & p_3 &= (p_2 - \alpha h z_2)/g, \\ z_f &= z_3 + R_{56}^{(b)} p_3, & p_f &= p_3. \end{aligned} \quad (2)$$

If we transform to the initial coordinates, $dz_f dp_f \rightarrow dz_i dp_i/g$, we can evaluate Eq. (1) by integrating over the simple uncorrelated initial distribution, $\Psi_i(p_i)$. Solving for z_f in terms of z_i , we find

FIG. 2. In the first step (upper left) we add a chirp and laser modulation. We show only one modulation wavelength, so the chirp effect is small. After the first chicane, the modulation is overbunched (zoom, upper right). A second chirp reverses the first chirp (zoom, middle left, shows reversed slope in phase space). After more acceleration, the second chicane revives bunching (middle right). Note the compression of the wavelength by $|\alpha| = 10$, and the increase in modulation amplitude by $|\alpha|/g = 2.5$. We find strong bunching past the 5th harmonic (bottom).

$$z_f = z' + R_{56}^{(T)}[p_i + A_L \sin(k_L \alpha z')], \quad (3)$$

with definitions $z' \equiv z_i/\alpha$ and $R_{56}^{(T)} \equiv R_{56}^{(a)} + R_{56}^{(b)}\alpha/g$. We note that Eq. (3) has the same form as from a single modulation, but at compressed wave vector αk_L , and with effective dispersive section, $R_{56}^{(T)}$, and relative laser

modulation amplitude, A_L/σ_p . It is possible to set $R_{56}^{(T)} \ll R_{56}^{(a)}, R_{56}^{(b)}$, so that a small effective $R_{56}^{(T)}$ bunches the short-wavelength modulation despite the strong dispersion of the bunch compressors. Consequently, the two terms in $R_{56}^{(T)}$ must have opposite signs, requiring either opposite sign dispersive sections (e.g. one chicane, one 2-bend dogleg), or negative α (overcompression) (Fig. 1). Chicanes have relatively favorable geometric properties compared to doglegs, so for this paper we choose negative α and use chicanes for both dispersive regions.

Having changed $dz_f dp_f \rightarrow dz_i dp_i/g$, and $\Psi_f \rightarrow \Psi_i$, we integrate Eq. (1) to find bunching at the harmonics as in HGHG [4]

$$b_f(m\alpha k_L) \propto e^{-[(m\alpha k_L R_{56}^{(T)} \sigma_p)^2/2]} J_m(m\alpha k_L R_{56}^{(T)} A_L), \quad (4)$$

with maxima at $\alpha k_L R_{56}^{(T)} A_L \approx \pm 1$. To avoid suppression by the energy spread, we require $|m\alpha k_L R_{56}^{(T)} \sigma_p| < 1$, giving significant bunching at the m th harmonic when $A_L > m\sigma_p$.

We can now confirm the advantages of CHG. First, we produce radiation at the harmonics of the compressed wavelength $\lambda_L/\alpha \ll \lambda_L$. Second, the rf phase can shift $|\alpha|$, tuning the final wavelength. Third, the precompression energy spread, σ_p , rather than the larger postcompression energy spread, $|\alpha|\sigma_p$, determines the required

TABLE I. Parameter list for both 1D (MATLAB) and 3D (ELEGANT) simulations. We choose L-band rf frequencies because the larger diameter structures will mitigate the effect of wakefield fluctuations. We have not listed bunch charge because without wakefield effects the simulations are charge independent.

Laser wavelength (λ_L)	157 nm
Laser power (P_L)	6 MW
Laser modulation amplitude (A_L)	5×10^{-5}
Uncorrelated rms E -spread before BC1 (σ_p)	1×10^{-5}
Normalized transverse emittance (ϵ_N)	$0.4 \mu\text{m}$
Initial energy spread (σ_{E0})	2.5 keV
Initial bunch length (σ_z)	$400 \mu\text{m}$
Electron energy (E_a, E_b, E_f)	0.25, 1.0, 3.5 GeV
BC1, BC2 dispersion ($R_{56}^{(a)}, R_{56}^{(b)}$)	50 mm, 20 mm
Compression factor ($ \alpha $)	10
Linac rf frequency	1.3, 3.9 GHz

laser modulation amplitude, A_L . A 1D simulation with parameters from Table I illustrates the process (Fig. 2).

B. Error self-correction from second dispersive region

The primary appeal of employing two dispersive regions is the separation of the bunching and compression requirements. However, the second dispersive region provides a second advantage by reversing longitudinal smearing. In our simple model [Eq. (2)], we assumed a longitudinally frozen beam outside of the dispersion regions BC1 and BC2. In reality, longitudinal deviations, such as second order transport elements, will smear out the phase space structure even outside of dispersive regions. The second chirp and dispersive region help to cancel such effects.

We model the self-correction in three steps. (1) The j th particle acquires a longitudinal error, Δz_j , due to second order effects following BC1. (2) The second chirp imparts a relative energy modulation of $\Delta E_j = -\frac{\alpha}{g} h \Delta z_j$. (3) BC2 produces a longitudinal shift, resulting in the final longitudinal error

$$\Delta z_j^{(F)} = \Delta z_j - R_{56}^{(b)} \frac{\alpha}{g} h \Delta z_j \approx \Delta z_j / \alpha, \quad (5)$$

where we have assumed $R_{56}^{(a)}, R_{56}^{(b)} \gg R_{56}^{(T)}$ so that we can approximate $R_{56}^{(b)} \approx -g R_{56}^{(a)} / \alpha$ in the second step. We find that the second chirp and dispersive regions effectively recompress the phase space structure, reducing smearing between the two chirps by the compression factor, $|\alpha|$. (Smearing from within the chirps and second chicane is suppressed more weakly, due to the decreased effective values of h and $R_{56}^{(b)}$.) This recompression of the fine density modulation is essential to CHG at short wavelengths. We consider a specific case of error self-correction in Sec. VB.

IV. SIMULATIONS

A. Simulation inputs

To test the practicality of a CHG scheme, we have constructed a sample accelerator lattice (Fig. 3) using MAD [15]. We emphasize that we have not fully optimized the lattice, but use it merely as a demonstration to study tolerances. We then simulate the CHG process by particle tracking in ELEGANT [16]. The simulations include incoherent synchrotron radiation (ISR) and first and second order transport elements for both longitudinal and transverse motion. We have not simulated coherent effects (e.g. coherent synchrotron radiation in the chicanes) due to computational constraints. To simulate the modulation, we use a 6 MW laser with 2 mm focal width, interacting with the beam over a 12 period, 50 cm long undulator. The undulator has sufficient bandwidth to seed the entire beam even with a strong 2% chirp. We assume a 157 nm modulation wavelength (corresponding to an F₂ laser), so that with compression of $|\alpha| \approx 10$, we find 3 nm radiation at the 5th harmonic of the density modulation (50th harmonic of the initial λ_L). By modulating at low energy (250 MeV), the moderate laser power still produces a relative modulation of $A_L = 5 \times 10^{-5}$. A partial list of parameters used in the simulations can be found in Table I and simulation results are given in Figs. 4–6.

We divide both the first and second linac sections into two parts, with the first half providing acceleration (phase “on-crest”) and the second providing chirp (phase at “zero crossing”). We assume accelerating gradients of approximately 25 MV/m for each linac section. A third-harmonic cavity linearizes the phase space prior to the first bunch compressor, providing a flat central region (Fig. 4). We do not require a third-harmonic cavity in the second stage, because after compression, the bunch length ($\sigma_z/|\alpha| \sim 40 \mu\text{m}$) is short compared to the rf wavelength ($\lambda_{\text{rf}} \sim 20 \text{ cm}$). We then find that the deviation due to rf curvature,

FIG. 3. Courant-Snyder parameters for 3D ELEGANT simulations showing β functions (solid and dotted lines) and dispersion (η_x , dash-dotted line). The weak second chicane is designed to minimize ISR effects. Following the second chicane, a third linac section increases the electron energy while simultaneously decreasing the β function, enhancing the radiation power.

FIG. 4. Particle phase space for ELEGANT simulation of parameters in Table I. The two horizontal stripes of higher density are signatures of a sinusoidal modulation. A third-harmonic accelerating cavity produces the flat central portion of the beam.

FIG. 5. A zoom of Fig. 4 shows the compressed modulation bunched to optimize higher harmonics. The vertical stripes (“standing up” the modulation) in phase space produce sharp density spikes that drive harmonic generation. The T_{566} component of the transfer matrix causes slight scalloping.

FIG. 6. Bunching factor at two wavelengths with 0.15% bandwidth from ELEGANT [16] simulation (parameters in Table I). Bunching is lower than for the 1D case because of 2nd order effects (e.g. emittance and T_{566}), ISR and finite laser radius.

$1 - \cos(k_{\text{tr}}\sigma_z/\alpha)$, is negligible compared to the final energy spread ($|\alpha|A_L \sim 5 \times 10^{-4}$).

The compression and bunching of the modulation is evident in a zoom of the phase space (Fig. 5). “Standing up” the modulation produces sharp density spikes with high harmonic bunching. To calculate the bunching factor for bandwidth $\Delta\lambda/\lambda$ at a position z_0 , we sum the particle phases, $b(k) = \sum_j \exp(ikz_j)$, for all particles in the region $z_0 - \lambda/2\Delta\lambda < z_j < z_0 + \lambda/2\Delta\lambda$. For bunching at the fifth harmonic of the compressed modulation, we set $k = 5|k_L|$. The result is given in Fig. 6.

B. Radiation

With high levels of density modulation from the seeding process, we can pass the electron bunch directly into a radiator. Assuming a 12% bunching factor and 30 m β function at the end of the lattice, feeding a 3.5 GeV, 1 kA beam through a 2.5 m undulator with $K = 3.5$ and 4 cm period produces nearly 10 MW of power. The beam could also feed a radiator directly following the second chicane, where the lower energy and larger β function are partially compensated by the higher bunching factor (above 15%) and shorter accelerator.

Alternatively, the final premodulated 3.5 GeV beam could drive an FEL. For the parameters of Table I with kA beam current, we find a Pierce parameter of $\rho \approx 10^{-3}$ [17]. The strong initial bunching requires only a few gain lengths of undulators to reach saturation, where Ming Xie scaling estimates nearly 5 GW of power [18].

V. FEASIBILITY

The drawback to compressed seeding is the need to preserve the phase space modulation across long accelerator regions. In particular, CHG is most sensitive to errors originating between the dispersive sections; errors prior to the first chicane are largely canceled by the second chicane. In the following section we study tolerances by estimating the effects of incoherent synchrotron radiation (ISR), dispersion errors, longitudinal smearing from 3D effects, and changes to the bunching condition due to jitter in the second chirp strength. Finally, we consider how modulating near the beginning of the accelerator may affect the microbunching instability.

A. ISR induced energy spread

Incoherent energy spread, $\Delta p^{(\text{ISR})}$, from ISR between the two dispersive sections could destroy the fine phase space structure. To preserve high harmonics, we must limit the longitudinal broadening, $\Delta z^{(\sigma_p)}$, from the total energy spread to less than $\lambda_L/2\pi m|\alpha| \sim 0.5$ nm. We can estimate

$$\Delta z^{(\sigma_p)} = R_{56}^{(a)}\sigma_p + R_{56}^{(b)}\frac{\alpha}{g}\sigma_p + R_{56}^{(b)}\Delta p^{(\text{ISR})}, \quad (6)$$

where σ_p is the uncorrelated relative energy spread at BC1. The sum of the first two terms is small by design, so for $R_{56}^{(b)} = 20$ mm, we find $\Delta p^{(\text{ISR})} \sim 10^{-8}$ for each bend, achievable with weak dipoles in BC2 at 1 GeV (Fig. 3). Figure 6 shows bunching including ISR effects.

B. Second order lattice effects

The laser modulation must survive transport through two strong chicanes, and approximately 60 m of accelerator. Smearing from second order effects (e.g. emittance and curvature from energy modulation) could broaden the fine 3 nm structure. An electron with coordinates $X = [x, x', y, y', z, p]$ acquires a longitudinal deviation of $\Delta z = \tilde{X}T_{5ij}X$, with T_{5ij} the second order transport coefficient from the modulator to the beginning of the second chirp. As described in Sec. III B, the second chirp helps to cancel such effects. The chirp imparts a relative energy modulation of $-\frac{\alpha}{g}h\Delta z$, reducing the longitudinal shift following the second chicane by a factor of $|\alpha|$. Though the recompression is weaker for errors from further down the accelerator, for our lattice (Fig. 3), the transverse components (T_{5ij}) are dominated by the first chicane, where we increase the beta function after the small beam radius of the modulator. As a result, we expect the error self-correction effect to be significant for our sample lattice.

To estimate the size of the second order terms, we tracked a longitudinal delta slice (vanishing bunch length and energy spread, but finite emittance) from the modulation to the end of the accelerator. We observe that the acceleration following the first chicane broadens the slice, but also imparts a chirp. The second chicane then recompresses the slice, decreasing the smearing effect by approximately the compression factor, $|\alpha|$ (Fig. 7).

C. rf phase and wakefield stability

The rf phase errors, wakefields, and rf curvature alter the chirp between dispersive sections and degrade the final bunching. An error in the second, canceling chirp shifts the unwinding process and leaves the modulation either under or overbunched. (The first chirp is less problematic because the two dispersive sections have canceling effects.)

To estimate the sensitivity to a linear chirp error, we repeat the earlier analysis, with the addition of an error, ε , in the final chirp. Step three becomes

$$z_3 = z_2, \quad p_3 = (p_2 - \alpha h z_2 - \varepsilon z_2)/g. \quad (7)$$

Again solving for z_f in terms of z_i , we find

$$z_f = z' + (R_{56}^{(T)} - \delta_R) \left[p_i + A_L \sin\left(\frac{\alpha k_L}{1 - \delta_k} z'\right) \right], \quad (8)$$

with definitions $\delta_k \equiv R_{56}^{(b)}\varepsilon/g$, $\delta_R \equiv R_{56}^{(a)}R_{56}^{(b)}\varepsilon/g$, and $z' \equiv z_i(1 - \delta_k)/\alpha$. The chirp error has two effects: the wave vector shifts by the factor $1 - \delta_k$ and the $R_{56}^{(T)}$ required to unwind the overcompression shifts by δ_R . While the shift in wave vector is small, the second condition implies a tight constraint on the phase stability of the second chirp; if the phase shifts, the effective $R_{56}^{(T)}$ either overbunches or underbunches the modulation. To maintain maximal bunching we need chirp error $|\varepsilon| \ll gR_{56}^{(T)}/R_{56}^{(a)}R_{56}^{(b)} \approx 0.2/\text{m}$, requiring control of the second chirp to better than 0.1%. The dot-dashed curve in Fig. 8 shows a $\pm 0.005^\circ$ phase error reduces bunching to 10%.

Longitudinal wakefields produce an energy modulation that is proportional to the derivative of the beam density profile. For a Gaussian bunch, we approximate the wakefield effect as an additional linear chirp near the center of

FIG. 7. To demonstrate the emittance cancellation effect, we track a longitudinal delta slice (zero length) following the laser modulation. Emittance effects increase the slice length before the second chicane, BC2 (top left, x vs z), but the off-crest accelerating section introduces a chirp to the beam (bottom left, energy vs z). The second chicane then recompresses the bunch by the compression factor, $|\alpha|$ (center). Following the final accelerator section, L3, the slice starts to spread out again (right). The bunch head is to the left.

FIG. 8. Wide bandwidth bunching for two simulations with a 0.01° phase shift (solid and dotted blue lines). An FEL would pick out a narrow $\Delta\lambda/\lambda = 0.15\%$ bandwidth, leaving lower bunching (narrow peak, red dot-dashed line). To calculate the bunching factor at a position z_0 and bandwidth $\Delta\lambda/\lambda$, we sum the phases of all particles in the region $z_0 - \lambda/2\Delta\lambda < z < z_0 + \lambda/2\Delta\lambda$. The high baseline in the wide bandwidth curves is due to the relatively low number of particles per bunching calculation when $\lambda/\Delta\lambda$ is small. A bunched sine wave makes a sawtooth that may either be right leaning, as in Fig. 5, or left leaning, depending on the sign of the effective dispersion, $R_{56}^{(T)}$. In our simulation, $R_{56}^{(T)}$ changes as a function of longitudinal beam position, resulting in the double peak seen in the wide bandwidth bunching.

the beam. (There will be nonlinear effects at the edges, but we are primarily interested in the central region where CHG is most effective). With charge fluctuations of $\sim 1\%$, we can only ignore the wakefield if it contributes less than 10% of the chirp. To minimize the wakes, we assume L -band superconducting structures with large geometric apertures.

D. Dispersion leakage

The most serious technical limitations come from dispersion leakage in the first chicane, which can smear the fine structure over the ensuing accelerator sections. For example, jitter in the dipole magnet strengths and quadrupole terms in the magnets will both broaden the modulation. We study dispersion leakage by tracking a longitudinal delta slice of finite emittance, but vanishing bunch length and energy spread (as we did for second order effects). We then adjust the field strength and quadrupole terms of a bend magnet in the first chicane, and observe the effect on the final bunch length. For the bend magnets in the middle of the chicane, we find the quadrupole jitter must have geometric strengths below $K = 10^{-5}$ (corresponding to fields of 0.1 G/m) to preserve 3 nm bunching. Likewise, jitter in the dipole strengths must be kept below 0.01%. Both effects are primarily second order, so they decrease with shorter accelerating sections (i.e. higher accelerating gradients), which can be achieved with higher

frequency accelerating structures. We note that the laser chirp method of Ref. [13] may be less sensitive to dispersion errors due to the short distance between chicanes. However, higher frequency structures and laser chirps may both be highly susceptible to phase errors, as discussed in the previous section. Lowering the x emittance (e.g. in 20 pC operation [19]) may also improve the tolerances.

In a CHG scheme, the tolerances for the accelerator are comparable to those of an undulator in an FEL; in both cases, preservation of the fine bunching is essential. For this reason, we also track a longitudinal delta slice while shifting the position of individual dipoles and quadrupoles. We find longitudinal errors of below 1 mm in the bend magnets and individual quadrupole offsets below 300 μm have negligible smearing effect. Finally, we note that all tolerances are looser if we place the radiator directly following BC2.

E. Microbunching

The microbunching instability (MBI) can degrade linac and FEL performance [20]. Incoherent energy spread damps the MBI effect by the exponential factor $e^{-k^2 R_{56}^2 \sigma_p^2}$, suppressing radiation for $k R_{56} \sigma_p > 1$. For the parameters of Table I, we expect suppression of MBI effects for $\lambda < 3 \mu\text{m}$. If long wavelength MBI is still problematic, a laser heater could push the incoherent energy spread as high as 10 keV, damping radiation out to wavelengths of 12 μm [21,22].

While laser heaters have proved effective at suppressing MBI [23], the larger initial energy spread would require higher seed laser power and enhance distortions from T_{566} . Given the similarity between the laser heater and the first stage of CHG, it is interesting to consider the extent to which the CHG process naturally suppresses MBI without additional heating.

We model the MBI in two steps; first, shot noise in the electron beam produces broadband space charge forces that modulate the beam energy. Second, a dispersive region converts the energy modulation into amplified density modulation. To evaluate the MBI amplitude, we rewrite the first line of Eq. (4) with the addition of a shot noise driven energy term, $A_{\text{sn}} \sin(k_{\text{sn}} z_i)$, with the understanding that the wave vector k_{sn} may take on any value, and the amplitude, A_{sn} , is generally small. As in Sec. III A, our strategy is to use the normalized initial coordinates, $z'_i \equiv z_i/\alpha$, to express the coordinates following the first chicane,

$$z_f = z'_i + R_{56} p_i + R_{56} [A_L \sin(\alpha k_L z'_i) + A_{\text{sn}} \sin(\alpha k_{\text{sn}} z'_i)]. \quad (9)$$

If we again assume a uniform longitudinal distribution with Gaussian energy spread, $\Psi_i(p_i)$, we find the bunching factor [from Eq. (1)] due to MBI:

FIG. 9. Double-horn energy distribution, left, from a sinusoidal modulation. Approximating a sawtooth with two frequencies gives the more uniform energy distribution at right.

$$\langle b(k) \rangle \propto e^{-(k^2 R_{56}^2 \sigma_p^2 / 2)} \alpha k_{\text{sn}} R_{56} A_{\text{sn}} \sum_n J_n(k R_{56} A_L) \times \int dz'_i e^{-ikz'_i} e^{i\alpha k_L z'_i} \left(\frac{e^{i\alpha k_{\text{sn}} z'_i} - e^{-i\alpha k_{\text{sn}} z'_i}}{2} \right), \quad (10)$$

where we have used the Jacobi-Anger expansion on the laser modulation term, and we have assumed $k_{\text{sn}} R_{56} A_{\text{sn}} \ll 1$ to linearize the shot noise term. We expect significant bunching at frequencies that satisfy $k = n\alpha k_L \pm \alpha k_{\text{sn}}$. The initial, uncorrelated, Gaussian energy spread again drives the exponential prefactor, while the smeared laser modulation provides additional suppression by way of the Bessel function factor.

If we consider only long wavelength MBI ($n = 0$, so that $k = k_{\text{sn}}$), we reproduce Eq. (11) from Ref. [22]; the laser modulation suppresses the MBI by an additional factor of $J_0(k R_{56} A_L)$, so we now expect moderate damping for $k R_{56} A_L \gtrsim 1.5$. Compared to the exponential prefactor, the laser suppression extends to longer wavelengths by the ratio A_L / σ_p , though the slower decay of the Bessel function weakens the effect. For the parameters of Table I, we find additional suppression at wavelengths as long as $10 \mu\text{m}$ even without the use of a laser heater.

When $n \neq 0$, there may be additional long wavelength MBI driven by frequency beating between the shot noise and seed laser. For example, with $n = -1$, we have long wavelength $k = -k_L + k_{\text{sn}}$ for $k_L \sim k_{\text{sn}} \gg k$. Further studies are needed to fully characterize the microbunching instability in CHG.

It is worth noting that the weak Bessel suppression stems from the double-horn energy distribution of a sinusoidal modulation [22]. A perfectly flat energy distribution (a longitudinal sawtooth) would maximize both the microbunching suppression and high harmonic bunching. Even a two-frequency laser modulation approximating a sawtooth would substantially improve the suppression over the single frequency case analyzed above (Fig. 9).

VI. CONCLUSION

Seeding an electron beam prior to the compression stage provides short-wavelength bunching with relatively low seed laser power. In this paper, we have studied a scheme for two-stage CHG, which extends the method to photocathode beams, and aids in the transport of the fine phase space structure. Through 3D simulations, we have demonstrated that modulation of a beam with $0.4 \mu\text{m}$ emittance can survive an extended accelerator region and still provide bunching at higher harmonics. We have also considered an effect by which smearing errors may be suppressed. However, strict stability requirements on the rf phase (to suppress variation in the chirp) and tight limits on dispersion leakage from the chicanes (to prevent smearing of the fine phase space structure) may make practical implementation of CHG schemes challenging.

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