Nonlinear interaction of charged particles with strong laser pulses in a magnetic undulator

H. K. Avetissian* and Kh. V. Sedrakian

Centre of Strong Fields Physics, Yerevan State University, 1 A. Manukian, Yerevan 0025, Armenia

(Received 17 June 2010; published 4 August 2010)

Laser acceleration due to the nonlinear-threshold phenomena of charged particle “reflection” and capture by slowed wave in a magnetic undulator is considered. The obtained numerical results prove the particle reflection and capture phenomena in the field of actual laser pulses with temporal and space profiles which lead to the particles acceleration. In contrast to the reflection regime where particle acceleration takes place already at the constant undulator step, in the capture regime it is necessary to increase adiabatically the undulator step along the laser pulse propagation direction by the certain self-consistent variation law corresponding to acceleration rate.

DOI: 10.1103/PhysRevSTAB.13.081301

PACS numbers: 41.75.Jv, 41.75.Ht

I. INTRODUCTION

During the past three decades diverse schemes have been proposed for acceleration of charged particles by powerful laser fields [1]. Meanwhile the experimental testing of laser acceleration is a very new field, made practical only in the past few years by the commercialization of chirped pulse amplified lasers [2]. The recent advance in laser technology and availability of superintense femto-second laser pulses of relativistic intensities [3,4] open real opportunities for realization of laser-plasma accelerators of superhigh energies [5–11]. The considered schemes involve mainly the laser-plasma-wake field and laser-beat-wave mechanisms of acceleration in underdense plasma. In the relativistic laser-plasma interaction process, quasimonoenergetic relativistic electron beams in laser-driven plasma accelerators have been generated [12–19]. Regarding the state of the art of laser accelerator schemes, one should refer to the papers [20–25].

The laser acceleration problem, in principle, reduces to realization of the inverse stimulated effect of energy absorption from the external laser beam by free particles in the processes with the possible large coherent length, such as induced Cherenkov, Compton, undulator, synchrotron processes, and channeling radiation [1]. In these processes, effective coherent energy exchange between the high brightness electron beams and superpower laser beams of relativistic intensities (both particles acceleration and x-ray lasers schemes) has been shown. In particular, the nonlinear-threshold phenomena of particles “reflection” and capture by a plane electromagnetic (e.m.) wave in induced Cherenkov, Compton, and undulator processes have been revealed, which open principally new opportunities for laser acceleration and nonlinear schemes of x-ray lasers with the strong pump optical lasers [1]. In this connection becomes important the implementation of these phenomena for actual nonplane laser pulses of a few cycle

durations with the certain time-space forms of the envelopes. The latter has been made in Ref. [26] for the induced Cherenkov process in the gaseous media, where the mentioned nonlinear-threshold phenomena have been justified for laser pulses of short enough durations and nonplane transversal-space envelopes. However, it is more important to investigate the induced Compton and undulator processes with actual time-space profiles of current laser pulses of relativistic intensities, since in these vacuum processes there is not a restriction on the field intensity taking place for induced Cherenkov interaction in dielectric-gaseous media. Thus, one can apply laser fields of arbitrary large intensities in vacuum and achieve considerable acceleration of charged particles.

In the present paper, the nonlinear dynamics of charged particles interaction with the strong laser pulses of actual time-space profiles in a magnetic undulator is investigated to clear up the influence of finite duration and transverse width of actual laser pulses on the phenomena reflection and capture towards the implementation of the particles acceleration.

II. NONLINEAR DYNAMICS OF A CHARGED PARTICLE IN THE FIELD OF STRONG LASER PULSE IN A MAGNETIC UNDULATOR

Consider the nonlinear dynamics of a charged particle at the interaction with a strong laser pulse in a magnetic undulator. Let a particle enter into a magnetic undulator with circularly polarized magnetic field

$$\mathbf{H}_u(x) = \left\{ 0, -H_u \cos \frac{2\pi}{l} x, H_u \sin \frac{2\pi}{l} x \right\} \quad (1)$$

(l is the space period or step of an undulator) along the axis of which propagates a laser pulse of finite time-space (transversal) envelope. We will assume that the laser pulse with the carrier frequency ω has a circular polarization and electric field strength envelope $E_0(y, z, t - \frac{x}{c})$:

*avetissian@ysu.am

$$\mathbf{E}(x, t) = \left\{ 0, E_0 \left(y, z, t - \frac{x}{c} \right) \sin \omega \left(t - \frac{x}{c} \right), E_0 \left(y, z, t - \frac{x}{c} \right) \times \cos \omega \left(t - \frac{x}{c} \right) \right\}. \quad (2)$$

For numerical investigation of the full dynamics of a charged particle and its acceleration in the field of the laser pulse, Eqs. (2), it is convenient to represent the particle's equations of motion in the form of dimensionless quantities:

$$\frac{d\mathbf{\Pi}}{dt'} = \xi_E(t', \mathbf{r}') + \frac{1}{\sqrt{1 + \mathbf{\Pi}^2}} [\mathbf{\Pi} \times \xi_H(t', \mathbf{r}')] + \frac{1}{\sqrt{1 + \mathbf{\Pi}^2}} \frac{2\pi c}{\omega l} [\mathbf{\Pi} \times \xi_u(x')], \quad (3)$$

$$\frac{d\mathbf{r}'}{dt'} = \frac{\mathbf{\Pi}}{\sqrt{1 + \mathbf{\Pi}^2}}, \quad (4)$$

where the dimensionless momentum $\mathbf{\Pi}$, time t' , and coordinate $\mathbf{r}'$ are introduced:

$$\mathbf{\Pi} = \frac{\mathbf{p}}{mc}, \quad t' = \omega t, \quad \mathbf{r}' = \frac{\omega}{c} \mathbf{r},$$

and the dimensionless parameters

$$\xi_E = \frac{e\mathbf{E}}{mc\omega}, \quad \xi_H = \frac{e\mathbf{H}}{mc\omega}$$

characterize the electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) field strengths of the nonplane wave pulse. The magnetic undulator is characterized by the relativistic parameter:

$$\xi_u = \frac{e\mathbf{H}_u l}{2\pi mc^2}. \quad (5)$$

It is well known that, for the large magnitudes of the undulator field strength H_u and space period l when $\xi_u > 1$, such an undulator is called a wiggler.

The particle energy change is given by the equation

$$\frac{d\gamma}{dt'} = \frac{\mathbf{\Pi} \xi_E(t', \mathbf{r}')}{\sqrt{1 + \mathbf{\Pi}^2}},$$

where $\gamma = \sqrt{1 + \mathbf{\Pi}^2}$ is the Lorentz factor. For the plane monochromatic wave, the exact analytical solution of Eqs. (3) and (4) is well known [1]. Thus, for a particle with an initial velocity $v_0 = v_{0x}$, taking into account the relation between the electric and magnetic field strengths of a transverse e.m. wave,

$$\xi_H(t', \mathbf{r}') = [\hat{\mathbf{x}} \xi_E(t', \mathbf{r}')] \quad (\hat{\mathbf{x}} \text{ is the unit vector along the pulse propagation direction}),$$

one can obtain the integral of motion in the induced undulator process:

$$\gamma(t') - \frac{1}{n_{\text{eff}}} \Pi_x(t') = \text{const}; \quad n_{\text{eff}} = 1 + \frac{\lambda}{l}, \quad (6)$$

and the particle energy in the field of a strong plane

monochromatic wave in the magnetic undulator is given as

$$\gamma = \frac{\gamma_0}{n_{\text{eff}}^2 - 1} \left\{ n_{\text{eff}}^2 \left(1 - \frac{v_0}{cn_{\text{eff}}} \right) \mp \left[\left(1 - n_{\text{eff}} \frac{v_0}{c} \right)^2 - \frac{n_{\text{eff}}^2 - 1}{\gamma_0^2} [\xi_0^2 + \xi_u^2 - 2\xi_0 \xi_u \cos(t' - n_{\text{eff}} x')] \right]^{1/2} \right\}. \quad (7)$$

Here γ_0 , ξ_0 are the normalized energy of the particle and amplitude of the plane monochromatic wave, respectively, ξ_{u0} is the normalized amplitude of the undulator field, and $n_{\text{eff}} = 1 + \lambda/l$ is the refractive index of the formed slowed interference wave in the induced nonlinear undulator process, which causes the threshold phenomenon of particle reflection.

As follows from the formula (7), there is a critical value of the field ξ_{cr} (corresponding to nonlinear resonance in the undulator) above which ($\xi_{\text{eff}} > \xi_{\text{cr}}$) the root becomes complex and then changes its sign after the reflection, which leads to particle acceleration or deceleration depending on its initial velocity in respect to the resonant one [1].

III. NUMERICAL TREATMENT FOR LASER PULSES OF SHORT DURATION AND NONPLANE CONFIGURATION

To justify the mentioned peculiarity in the dynamics of the induced nonlinear undulator process with the acceleration effect for short laser pulses, we present the results of numerical integration of Eqs. (3) and (4) for the plane laser pulse of finite duration, neglecting at first the transversal-space dependence of actual strong laser beams. The simulations have been made for a plane e.m. wave pulse of the following form:

$$\xi(\tau) = \xi_0 \frac{\cos \tau}{\cosh(\frac{\tau}{\tau_0})}, \quad (8)$$

where $\tau = x' - t'$ is the wave coordinate, τ_0 is the pulse duration, and ξ_0 is the maximal value of the pulse intensity parameter.

Since the existence of the critical point is conditioned by the nonlinear resonance of a particle with a transverse e.m. wave in the undulator, for quantitative exploration of the acceleration effect we will consider the parameters' values in this range—initially near the nonlinear resonance. For the parameters' values initially far from resonance, the required critical fields increase too, corresponding to ultrarelativistic intensities of laser pulses. In these cases the acceleration effect is correspondingly larger as well, however for high relativistic intensities some impeding factors arise connected with the essential decreasing of the Rayleigh length for strongly focused laser beams, especially in the case of continuous acceleration with variable undulator step, etc. In optimal cases of soft relativistic intensities for moderately focused laser beams of considering form and undulator parameters, the mean initial

FIG. 1. Reflection of a particle from a plane laser pulse of finite duration. The energy versus the position x is plotted when the laser intensity is above the critical point.

energy of particles in the nonlinear resonance corresponds to $\mathcal{E}_0 = 40$ MeV. Hence, the particle initial energy should be in this range by magnitude, i.e., we take $\mathcal{E}_0 = 38$ MeV and the initial velocity is directed along the wave propagation direction. Figure 1 illustrates the evolution of the particle energy: the energy versus the position x is plotted for a Nd:YAG laser ($\hbar\omega \approx 1.17$ eV) with $\xi_0 = 0.5$ and pulse duration $\tau_0 = 50T$ (where $T = 2\pi/\omega$ is the laser-wave period). For the undulator field intensity parameter is $\xi_u = 0.1$ with the constant step $l = 2$ cm. For these values of the main parameters the effective field intensity (peak value) is above the critical point and, as we see from this figure, the particle energy is changed abruptly corresponding to the reflection from the slowed effective wave pulse formed in the undulator. Figure 2 illustrates the evolution

FIG. 2. The evolution of the energies of particles at the reflection from the plane laser pulse with the various initial phases.

of the particle energies with different initial phases—the positions x in respect to laser pulse center, with the same initial energies.

The analytical treatment of the entire dynamics of a particle—law of motion—corresponding to the mentioned phenomenon is impossible even for a pure plane wave. Consequently, we will treat this problem and then particle acceleration effect by actual laser pulses via numerical integration of equations of the particle law of motion $\mathbf{r}' = \mathbf{r}'(t')$, i.e., Eq. (3) with Eq. (4). For the numerical integration of the set of Eqs. (3) and (4), we will choose a definite form for a laser pulse. The latter is modeled by a special form for the focused laser pulses of circular polarization (for a linear one, see [27]), the electric and magnetic fields of which are given by the formulas

$$\begin{aligned}
 E_x &= E_0 g(\tau) \frac{\lambda w_0^2 e^{-r_\perp^2/w^2(x)}}{\pi w^4(x)} \left\{ \left[-2 \frac{xy}{x_R} - \left(1 - \frac{x^2}{x_R^2} \right) z \right] \cos\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) + \left[\left(1 - \frac{x^2}{x_R^2} \right) y - 2 \frac{xz}{x_R} \right] \sin\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) \right\}, \\
 E_y &= E_0 g(\tau) \frac{w_0^2 e^{-r_\perp^2/w^2(x)}}{w^2(x)} \left\{ \cos\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) + \frac{x}{x_R} \sin\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) \right\}, \\
 E_z &= E_0 g(\tau) \frac{w_0^2 e^{-r_\perp^2/w^2(x)}}{w^2(x)} \left\{ -\frac{x}{x_R} \cos\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) + \sin\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) \right\}, \\
 B_z &= E_0 g(\tau) \frac{\lambda w_0^2 e^{-r_\perp^2/w^2(x)}}{\pi w^4(x)} \left\{ \left[-2 \frac{xz}{x_R} + \left(1 - \frac{x^2}{x_R^2} \right) y \right] \cos\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) + \left[\left(1 - \frac{x^2}{x_R^2} \right) z + 2 \frac{xy}{x_R} \right] \sin\left(\tau + \frac{r_\perp^2 x}{x_R w^2(x)} \right) \right\}, \\
 B_y &= -E_z; \quad B_z = E_y,
 \end{aligned} \tag{9}$$

where $r_\perp^2 = y^2 + z^2$, $w(x) = w_0 \sqrt{1 + x^2/x_R^2}$, $x_R = \pi w_0^2/\lambda$ is the Rayleigh length, and $g(\tau) = 1/\cosh(\tau/\tau_0)$.

To illustrate the peculiarity of nonlinear interaction of a charged particle with a strong e.m. wave in a magnetic undulator with the acceleration effect based on the reflection phenomenon, we will present the graphics of numerical solution for the particle energy in the field and after the reflection.

Figure 3 manifests the particle reflection phenomenon for actual strong laser pulses of nonplane form: with the transverse size (space width) of the focused pulse waist $w_0 = 500\lambda$ (where $\lambda = 2\pi c/\omega$ is the laser wavelength). The particles move in this case along the undulator axis x ($y_0 = z_0 = 0$). Figure 4 illustrates the case of interaction when the particles initially situated at the different distances from the undulator axis: $z_0 = w_0/4$ (due to

FIG. 3. Reflection of a particle from a nonplane laser pulse with the transverse size of the focused pulse waist $w_0 = 500\lambda$, at $\xi_0 = 0.5$, $\tau_0 = 50T$, and $\xi_u = 0.1$, for step $l = 2$ cm. The particles move in this case along the undulator axis x ($y_0 = z_0 = 0$).

axial symmetry it is assumed $y_0 = 0$ without loss of generality).

As we see, in considering the scheme of laser acceleration by the nonlinear-threshold mechanism ($\xi_{\text{eff}} > \xi_{\text{cr}}$), when the particles reflection phenomenon from the real nonplane laser pulse (slowed effective wave pulse in the undulator) occurs, the acceleration rate neither depends on the field magnitudes (only should be above threshold field) nor on the interaction length, as it takes place for a plane wave pulse [1].

To find out the influence of energy spread of a real particle bunch in the acceleration process due to reflection phenomenon, in Fig. 5 the initial and final distribution functions over the particles' energies after the interaction are plotted. Here we consider a particle bunch which is described by the Gaussian distribution function:

FIG. 5. The initial (dashed line) and final (solid line) energy distribution functions for a particle bunch. The interaction parameters are the same as in Fig. 3.

$$f_0(\mathcal{E}) = \frac{1}{\sqrt{2\pi}\Delta} e^{-[(\mathcal{E}-\mathcal{E}_0)^2/2\Delta^2]}, \quad (10)$$

where Δ is the energy spread in the bunch. The latter is taken to be $\Delta = 0.5$ MeV at the mean energy of the bunch $\mathcal{E}_0 = 40$ MeV. As we see from this figure, a considerable part of the particles in the bunch accelerated due to the reflection phenomenon.

For efficient acceleration, one needs to realize multiple reflections from the effective slowed wave pulse in the undulator, which can be achieved by continuously changing the undulator step along the laser pulse propagation direction. In Fig. 6 the case of a variable undulator step at the certain variation law along the undulator axis is illustrated for the maximal acceleration effect (continuous reflection) at $\xi_0 = 0.2$, $\tau_0 = 150T$, $w_0 = 6000\lambda$, and the initial value for an undulator parameter is chosen to be $\xi_u = 0.5$ at the initial step $l_0 = 10$ cm. The self-consistent law for an undulator step change is plotted in Fig. 7.

FIG. 4. The same dependence as in Fig. 3 when the particles initially situated at the different distances from the undulator axis: $z_0 = w_0/4$ ($y_0 = 0$).

FIG. 6. Particle continuous reflection and acceleration at the variable undulator step along the axis is illustrated at $\xi_0 = 0.2$, $\tau_0 = 150T$, $w_0 = 6000\lambda$, and $\xi_u = 0.5$, for initial step $l_0 = 10$ cm.

FIG. 7. The self-consistent law $l(x)$ of the undulator step change corresponding to maximal acceleration.

To generalize the acceleration picture for real particle bunches with the energy spreads in the regime of continuous acceleration by nonplane laser pulses due to the reflection phenomenon in a magnetic undulator of variable step, in Fig. 8 the acceleration effect—final energy distribution of particles after the interaction—is plotted for the initial distribution function (10). The parameters are chosen as in Fig. 6. As seen from this figure, the final energy distribution function of the particles is peaked around 12 GeV with the relative energy spread $\Delta_f/\mathcal{E} \sim 10^{-2}$, which is comparable with initial relative energy spread.

To reveal the influence of space effects connected with the finite-nonplane configurations of actual laser pulses, we present direct comparison of pure plane wave and a real nonplane laser pulse under the same condition, which is illustrated in Fig. 9, where the final energy dependence on the initial transversal momenta of the particles is plotted. As seen from Fig. 9, the particles with the large initial transversal momenta are not accelerated in the field of the real laser pulse because of finite transversal-space sizes of focused laser beams, in contrast to a plane wave pulse.

FIG. 8. The final energy distribution function for continuous reflection and acceleration at the variable undulator step. The parameters are chosen as in Fig. 6.

FIG. 9. The final energy versus initial transversal momentum $p_{\perp}$ for the same interaction parameters as in Fig. 6. The solid line corresponds to a plane wave, while the dashed line corresponds to a focused nonplane laser beam.

IV. CONCLUSION

We have presented a theoretical and numerical treatment of the charged particle nonlinear dynamics in the field of strong (under the relativistic intensities) laser pulses of finite duration and certain space form of the envelope in the strong magnetic undulator. The acceleration effect due to the nonlinear-threshold phenomena of particles reflection and capture by actual laser pulses of nonplane configuration, at the constant and variable (continuous reflection-acceleration) undulator steps, is considered. These phenomena, which have previously been revealed for a pure plane e.m. wave, are developed for current strong laser pulses of short duration and finite transversal-space configurations. The obtained numerical results prove the particle reflection and capture phenomena in the field of nonplane short laser pulses, which lead to the particles efficient acceleration.

ACKNOWLEDGMENTS

This work was supported by International Science and Technology Center (ISTC) Project No. A-1307.

- [1] H. K. Avetissian, *Relativistic Nonlinear Electrodynamics* (Springer, New York, 2006).
- [2] M. D. Perry *et al.*, *Opt. Lett.* **24**, 160 (1999); Th. Stohlker *et al.*, *Phys. Rev. Lett.* **86**, 983 (2001).
- [3] C. N. Danson *et al.*, *Nucl. Fusion* **44**, S239 (2004).
- [4] G. A. Mourou, T. Tajima, and S. V. Bilanov, *Rev. Mod. Phys.* **78**, 309 (2006).
- [5] R. Wagner, S.-Y. Chen, A. Maksimchuk, and D. Umstadter, *Phys. Rev. Lett.* **78**, 3125 (1997).
- [6] T. Tajima and J. M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979).
- [7] E. Esarey *et al.*, *IEEE Trans. Plasma Sci.* **24**, 252 (1996); *IEEE J. Quantum Electron.* **33**, 1879 (1997).

- [8] Th. Stohlker *et al.*, *Phys. Rev. Lett.* **86**, 983 (2001).
- [9] M. Drescher *et al.*, *Sci. Exp.* **291**, 1923 (2001).
- [10] A. Pukhov and J. Meyer-ter-Vehn, *Appl. Phys. B* **74**, 355 (2002).
- [11] S. V. Tochitsky *et al.*, *Phys. Rev. Lett.* **92**, 095004 (2004).
- [12] H. Schwoerer *et al.*, *Phys. Rev. Lett.* **86**, 2317 (2001).
- [13] S. P. D. Mangles *et al.*, *Nature (London)* **431**, 535 (2004).
- [14] C. G. R. Geddes *et al.*, *Nature (London)* **431**, 538 (2004).
- [15] J. Faure *et al.*, *Nature (London)* **431**, 541 (2004).
- [16] S. Fritzler *et al.*, *Phys. Rev. Lett.* **92**, 165006 (2004).
- [17] E. Miura *et al.*, *Appl. Phys. Lett.* **86**, 251501 (2005).
- [18] B. Hidding *et al.*, *Phys. Rev. Lett.* **96**, 105004 (2006).
- [19] M. I. K. Santala *et al.*, *Phys. Rev. Lett.* **86**, 1227 (2001); *Science* **298**, 1596 (2002).
- [20] K. Nakajima *et al.*, *Phys. Rev. Lett.* **74**, 4428 (1995).
- [21] P. Mora and B. Quesnel, *Phys. Rev. Lett.* **80**, 1351 (1998).
- [22] K. T. McDonald, *Phys. Rev. Lett.* **80**, 1350 (1998).
- [23] K. Nakajima, *Laser Part. Beams* **18**, 519 (2000).
- [24] P. X. Wang *et al.*, *Appl. Phys. Lett.* **78**, 2253 (2001).
- [25] Ch. Varin, M. Piche, and M. A. Porras, *Phys. Rev. E* **71**, 026603 (2005).
- [26] H. K. Avetissian, S. S. Israelyan, and Kh. V. Sedrakian, *Phys. Rev. ST Accel. Beams* **10**, 071301 (2007).
- [27] K. T. McDonald, [arXiv:physics/0003056v2](https://arxiv.org/abs/physics/0003056v2).