Energy loss due to eddy current in linear transformer driver cores

A. A. Kim,¹ M. G. Mazarakis,² V. I. Manylov,¹ V. A. Vizir,¹ and W. A. Stygar²

¹*Institute of High Current Electronics, Russian Academy of Sciences, Tomsk 634055, Russia*

²*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

(Received 5 January 2010; published 2 July 2010)

In linear transformer drivers [Phys. Rev. ST Accel. Beams **12**, 050402 (2009); Phys. Rev. ST Accel. Beams **12**, 050401 (2009)] as well as any other linear induction accelerator cavities, ferromagnetic cores are used to prevent the current from flowing along the induction cavity walls which are in parallel with the load. But if the core is made of conductive material, the applied voltage pulse generates the eddy current in the core itself which heats the core and therefore also reduces the overall linear transformer driver (LTD) efficiency. The energy loss due to generation of the eddy current in the cores depends on the specific resistivity of the core material, the design of the core, as well as on the distribution of the eddy current in the core tape during the remagnetizing process. In this paper we investigate how the eddy current is distributed in a core tape with an arbitrary shape hysteresis loop. Our model is based on the textbook knowledge related to the eddy current generation in ferromagnetics with rectangular hysteresis loop, and in usual conductors. For the reader's convenience, we reproduce some most important details of this knowledge in our paper. The model predicts that the same core would behave differently depending on how fast the applied voltage pulse is: in the high frequency limit, the equivalent resistance of the core reduces during the pulse whereas in the low frequency limit it is constant. An important inference is that the energy loss due to the eddy current generation can be reduced by increasing the cross section of the core over the minimum value which is required to avoid its saturation. The conclusions of the model are confirmed with experimental observations presented at the end of the paper.

DOI: 10.1103/PhysRevSTAB.13.070401

PACS numbers: 84.70.+p, 84.60.Ve

I. INTRODUCTION

In linear transformer drivers, the cavity walls represent the single-turn winding around the core and the core itself is made of ET3425 anisotropic iron tape [1]. When the pulsed voltage is applied to the single-turn winding that surrounds the core (actually the cavity walls), it generates the eddy current in the iron tape of the core, which together with inductance of the core determine the total current flowing along the cavity walls.

The energy loss due to generation of the eddy current in the linear transformer driver (LTD) core is modeled by using the resistance R_C , which simulates the core effect in the LTD circuit [2,3]. The value of this resistance depends on the specific resistivity of the core material, the design of the core, as well as on the distribution of the eddy current in the iron tape during the remagnetizing process.

A rigorous theory describing the physics of the eddy current generation exists only for ferromagnetic material with rectangular hysteresis loop [4–6]. In such ferromagnetics, the eddy current is evenly distributed at the surface layer of the iron tape; the thickness of this layer constantly increases during the remagnetizing process from zero to $\delta/2$ when the core completely saturates. Because of this continuous increase of the thickness of the current carrying layer, the resistance R_C of the core with rectangular hysteresis loop continuously decreases during the length of the applied voltage pulse.

The hysteresis loop of the ET3425 iron is not rectangular, and to define the resistance R_C and the corresponding energy loss in the LTD core it is necessary to precisely specify how the eddy current distributes in the tape with an arbitrary shape hysteresis loop. The present paper is devoted to this problem. In Sec. II we outline the logic and some useful conclusions of the theory describing the eddy current generation in ferromagnetic materials with rectangular hysteresis loop. In Sec. III we demonstrate how this theory can be applied to ferromagnetic materials with arbitrary hysteresis loop. In Sec. IV we present experimental results supporting the approach taken in Sec. III.

II. EDDY CURRENT IN THE CORE WITH RECTANGULAR HYSTERESIS LOOP

In presenting the theory of eddy current generation in the core with rectangular hysteresis loop (shown in Fig. 1), we follow Ref. [6], where the core considered is composed of numerous identical isolated ferromagnetic sheets (turns of the tape) (see Fig. 2). Each sheet is δ thick (in the x direction), a wide (in the z direction), and ℓ long (in the y direction), and it is assumed that

$$\delta \ll a \ll \ell. \quad (1)$$

Initially the induction in the core is $B_0 = -B_S$; the applied voltage pulse generates in the winding (here cavity wall) the current $I(t)$ which flows around the y axis. This

FIG. 1. Rectangular hysteresis loop: B_S is the induction at saturation, H_C the coercive force.

FIG. 2. Ferromagnetic core consisting of numerous δ -thick sheets.

current subsequently generates inside the winding, where the core is located, the uniform magnetic field (all equations are in SI units)

$$H(t) = \frac{I(t)}{\ell}, \quad (2)$$

which shifts the induction of the core into the $+B_S$ direction. Until the magnetic field H reaches the value H_C , it penetrates easily into the core and the induction remains at $-B_S$. When the value of H reaches H_C , the magnetic induction on the surface of the tape (at $x = \pm\delta/2$, see Fig. 3) jumps from $-B_S$ to $+B_S$, and stays at $+B_S$ for the rest of the pulse. At further increase of H , the tape continues to remagnetize, this remagnetizing process happens by the expansion of the ξ -thick layers with induction $+B_S$ from the tape surface into the bulk of the material. Because the thickness of the tape δ is assumed to be much less than its width, there are two plane layers expanding with the same velocity towards the center from both sides of the tape. The remagnetizing process finishes and the core saturates when the expanding layers, inside all the tape sheets composing the core, reach the middle plane of the tapes at $x = 0$. Then the magnetic induction everywhere throughout the core becomes equal to $+B_S$.

Because of assumption (1), the edge effects can be neglected and the problem becomes one-dimensional, where all the variables depend on x only. Since the current in the winding flows around the y axis, the magnetic field $H(x, t)$ and the induction $B(x, t)$ are directed along the y axis, and the electric field $E(x, t)$ along the z axis. The field distribution is found by solving the Maxwell equations, which inside the tape take the form

FIG. 3. Field distribution in the δ -thick iron tape with rectangular hysteresis loop at some fixed time t , when $H(t) > H_C$.

$$\frac{\partial E(x, t)}{\partial x} = \frac{\partial B(x, t)}{\partial t}, \quad \frac{\partial H(x, t)}{\partial x} = \frac{1}{\rho} E(x, t), \quad (3)$$

where ρ is specific resistivity of the tape material.

Figure 3(a) shows the distribution of the induction inside the tape at fixed time t , when $H(t) > H_C$. As appears from this figure, the variation of the induction during the time t , averaged over the tape cross section, is

$$\Delta B(t) = B(t) - B_0 = \Delta B \frac{2\xi(t)}{\delta}, \quad (4)$$

where $B(t)$ is the induction which is averaged over the cross section of the tape, $\Delta B = B_S - B_0$ ($B_0 = -B_S$ for the tape with rectangular hysteresis loop considered here), $\xi(t)$ the thickness of the saturated surface layer where the induction is $+B_S$.

From Eq. (3) follows that the electric field in the sheet has the following x dependence:

$$E(x, t) = \begin{cases} 0, & \text{at } 0 \leq x < (\frac{\delta}{2} - \xi(t)) \\ \Delta B \frac{d\xi}{dt}, & \text{at } x \geq (\frac{\delta}{2} - \xi(t)) \end{cases} \quad (5)$$

and because of symmetry

$$E(x) = -E(-x). \quad (6)$$

Equations (5) and (6) mean that the eddy current with density $j(x, t) = E(x, t)/\rho$ is closed and evenly distributed inside the ξ -thick saturated surface layer of the tape.

By using Eq. (5) and the initial condition $H(t = 0) = H_C$, the magnetic field H is determined from Eq. (3) as

follows:

$$H(x, t) = H_C + \begin{cases} 0, & \text{at } 0 \leq x < (\frac{\delta}{2} - \xi(t)) \\ \frac{\Delta B}{\rho} (x - \frac{\delta}{2} + \xi) \frac{d\xi}{dt}, & \text{at } (\frac{\delta}{2} - \xi(t)) \leq x \leq \frac{\delta}{2}. \end{cases} \quad (7)$$

The distribution of the electric and magnetic fields inside the tape at time t is shown in Figs. 3(b) and 3(c).

From Eq. (7), the magnetic field on the surface of the tape at $x = \delta/2$ is

$$H(t) = H_C + \frac{\Delta B}{\rho} \xi \frac{d\xi}{dt}. \quad (8)$$

Now by using Eq. (4) to exclude $\xi(t)$ from Eq. (8), we can get

$$H(t) = H_C + \frac{\delta^2}{4\rho\Delta B} \Delta B(t) \frac{dB(t)}{dt}. \quad (9)$$

If the cross section of the core is S , and the voltage applied to its single-turn winding is $U(t)$, then the variation of the averaged induction in the core is

$$\Delta B(t) = \frac{1}{S} \int_0^t U(t) dt, \quad (10)$$

and the magnetic field on the surface of the tape becomes equal to

$$H(t) = H_C + \frac{\delta^2}{4\rho\Delta B} \frac{U(t)}{S^2} \int_0^t U(t) dt. \quad (11)$$

Equation (2) relates this field with the current $I(t)$ in the winding allowing one to obtain the volt-ampere characteristics of the core:

$$I(t) = \ell H_C + \ell \frac{\delta^2}{4\rho\Delta B} \frac{U(t)}{S^2} \int_0^t U(t) dt. \quad (12)$$

Here the first term on the right depends on the coercive force; it is responsible for the energy loss due to the hysteresis effect (see below). The second term is the effect of the eddy current generation in the conductive material of the core which defines the resistance of the core $R_C(t)$:

$$R_C(t) = 4 \frac{\rho\Delta B}{\ell\delta^2} \frac{S^2}{\int_0^t U(t) dt} = 4 \frac{\rho S}{\ell\delta^2} \frac{\Delta B}{\Delta B(t)}. \quad (13)$$

Equation (13) indicates that for any single-polarity voltage pulse the resistance $R_C(t)$ decreases with time. This occurs because during the remagnetization of the core with rectangular hysteresis loop the eddy current penetrates deeper and deeper into the tape during the whole length of the pulse. If the remagnetizing process of the entire core finishes at the end of the τ -long voltage pulse, then $\Delta B(\tau) = \Delta B$, and the resistance $R_C(\tau)$ becomes equal to

$$R_C(\tau) = 4 \frac{\rho S}{\ell\delta^2}. \quad (14)$$

By using Eqs. (10) and (12), the energy loss in the core during the length τ of the voltage pulse $U(t)$ can be expressed in the form

$$\begin{aligned} E &= \int_0^\tau U(t) I(t) dt \\ &= S\ell H_C \Delta B(\tau) + \ell \int_0^\tau \frac{\delta^2}{4\rho\Delta B S^2} U(t)^2 \left(\int_0^t U(t') dt' \right) dt. \end{aligned} \quad (15)$$

Here the first term on the right is the loss due to hysteresis effect, E_H , whereas the second term is the loss E_{EC} due to generation of the eddy current:

$$E_{EC} = \ell \int_0^\tau \frac{\delta^2}{4\rho\Delta B S^2} U(t)^2 \left(\int_0^t U(t') dt' \right) dt. \quad (16)$$

Equation (16) indicates that the loss E_{EC} depends on the shape of the voltage pulse $U(t)$. For example, at the same length of the pulse τ , the same induction swing ΔB , and the same core, the loss E_{EC} for a rectangular pulse is $\frac{\pi^2}{8} \sim 1.23$ times less than that for the sinusoidal pulse $U(t) = U_0 \sin(2\pi \frac{t}{T})$, $0 \leq t \leq \tau = T/2$. The relation between the E_{EC} and the $U(t)$ appears because the resistance R_C depends on the thickness $\xi(t)$ of the current carrying layer, which expands with the velocity

$$\frac{d\xi}{dt} = \frac{\delta}{2\Delta B} \frac{dB}{dt} = \frac{\delta}{2\Delta B S} U(t), \quad (17)$$

depending on $U(t)$.

When the total energy loss during the pulse length τ is important and not so much the actual resistance of the core at any given time during the pulse, it is convenient for simplicity of energy loss calculations to introduce the equivalent constant resistance R_{CEQ} that gives the same energy losses as Eq. (16). This resistance is given by

$$R_{CEQ} = \frac{\int_0^\tau U(t)^2 dt}{E_{EC}(\tau)}. \quad (18)$$

For any symmetric, around its center $\tau/2$, τ -long voltage pulse [such as rectangular, sinusoidal, etc., which satisfies the condition $U(\frac{\tau}{2} + t) = U(\frac{\tau}{2} - t)$], the resistance R_{CEQ} equals

$$R_{CEQ} = 8 \frac{\rho S}{\ell\delta^2} \frac{\Delta B}{\Delta B(\tau)}. \quad (19)$$

If the voltage pulse completely remagnetizes the core during the pulse length τ , i.e. $\Delta B(\tau) = \Delta B$, then

$$R_{CEQ} = 8 \frac{\rho S}{\ell\delta^2}. \quad (20)$$

This was first shown for a square pulse in [7]. From Eq. (13), the resistance $R_C(t)$ reaches this value of R_{CEQ} at the time t^* given by

$$\Delta B(t^*) = \frac{1}{2}\Delta B. \quad (21)$$

For a symmetric voltage pulse of length τ this happens at $t^* = \tau/2$.

III. EDDY CURRENT IN THE CORE WITH ARBITRARY HYSTERESIS LOOP

The generation of the eddy current in ferromagnetic materials with arbitrary hysteresis loop can be interpreted by utilizing the well-known “skin effect” analysis. If, for example, some homogeneous conductor with constant permeability μ fills the region $x > 0$, and the field on its surface (at $x = 0$) is perpendicular to the x axis and varies in time like

$$E(t) = E_0 \sin \omega t, \quad (22)$$

then in the bulk of the conductor at $x > 0$ this field equals

$$E(x, t) = E_0 e^{-px} \sin(\omega t - px), \quad (23)$$

where

$$x_S = \frac{1}{p} = \sqrt{\frac{2\rho}{\mu\omega}} \quad (24)$$

is the “skin depth.” So, between the field on the surface and in the bulk of the conductor there exists a phase shift, and the amplitude of the field exponentially decreases with x , i.e., the eddy current $j(x, t) = E(x, t)/\rho$ concentrates mainly in the x_S -thick surface layer.

In ferromagnetics the permeability is not a constant, but if we consider the *unsaturated* core only, its hysteresis loop can be characterized by some effective constant permeability μ_E . Then depending on how fast the field varies in time, i.e., on ω , for the δ -thick tape of the core, we distinguish two extreme limits: (i) high frequency limit: $x_S \ll \frac{\delta}{2}$; (ii) low frequency limit: $x_S \gg \frac{\delta}{2}$.

In the high frequency limit

$$\omega \gg \frac{8\rho}{\mu_E \delta^2}, \quad (25)$$

the skin depth x_S is much less than $\delta/2$, and therefore the remagnetizing process occurs in a similar way as described above for the ferromagnetics with rectangular hysteresis loop. Indeed, the rectangular loop means $\mu(H_C) = \infty$, in terms of the skin depth this corresponds to $x_S = 0$. Since $x_S \propto (\mu\omega)^{-1/2}$, as far as the skin depth is concerned, infinite permeability $\mu(H_C) = \infty$ at finite frequency ω is equivalent to infinite frequency $\omega = \infty$ at finite permeability μ_E . Therefore, for ferromagnetic materials with finite permeability, an applied voltage pulse of infinite high frequency causes the material to behave as if it had a rectangular hysteresis loop. In the high frequency limit, at the beginning of the pulse all the current flows in the x_S -thick surface layer of the tape. The tape begins to saturate from the surface, where the magnetic field is the

highest. The electric field and the eddy current density in the two layers that are closest to both surfaces of the tape become uniform following the magnetic saturation of those very thin layers. Later in the pulse these saturated layers expand to the center of the tape, as do the ξ -thick layers in Fig. 3. The only difference compared to Fig. 3 is that the shape of these layers at $x = \pm(\frac{\delta}{2} - \xi)$ is not a steplike function but the x_S -thick exponent. According to Eq. (17), the velocity of the expansion, $\frac{d\xi}{dt}$, depends on the shape of the voltage pulse, but at any shape the region of the tape with homogeneous current flow becomes thicker and thicker. The resistance $R_C(t)$ decreases with time, and the tape (and the whole core) completely gets remagnetized when the thickness of each layer reaches $\delta/2$. If both ω and μ_E are finite, the skin depth increases and the resistance decreases, but not to a larger extent, as it will be shown below.

For practical purposes, in the high frequency limit, and for a symmetric voltage pulse saturating the entire core, the energy loss due to generation of the eddy current E_{EC} can be estimated by using the constant resistance R_{CEQ} given by Eq. (20). This resistance R_{CEQ} should be equal to the ratio of the voltage to the current in the single-turn winding at the middle of the voltage pulse.

The loss E_{EC} can be reduced by increasing the core cross section S over the minimum value $S_{\min}$ given by Eq. (10) at $\Delta B(\tau) = \Delta B$. At $S > S_{\min}$, the core does not completely remagnetize during the pulse length, and in the high frequency limit the eddy current does not completely penetrate the entire thickness of the tape. From Eq. (19), for the symmetric voltage pulse, the equivalent resistance R_{CEQ} increases directly proportional to the ratio of the core cross sections square

$$R_{CEQ} = 8 \frac{\rho S}{\ell \delta^2} \left(\frac{\Delta B}{\Delta B(\tau)} \right) = 8 \frac{\rho S_{\min}}{\ell \delta^2} \left(\frac{S}{S_{\min}} \right)^2. \quad (26)$$

In the low frequency limit

$$\omega \ll \frac{8\rho}{\mu_E \delta^2}, \quad (27)$$

the phase shift and the reduction of the field amplitude in the center of the tape at $x = 0$ are negligible compared to the phase and the amplitude on its surface. This means that the electric field and the eddy current density distribution inside the tape is almost homogeneous from the very beginning of the pulse. The tape remagnetizes in a similar manner as in the high frequency limit—beginning (somewhere on the front of the current pulse) from the surface of the tape where the magnetic field is the highest. But unlike in the high frequency limit, now the saturation of the external parts does not change the current distribution throughout the cross section of the tape which is already almost homogeneous. At homogeneous current flow the resistance R_C is constant; its value is given by Eq. (14). This is because at the end of the remagnetizing

process in a core with a rectangular hysteresis loop, the current flow becomes also homogeneous throughout the entire cross section of the tape. Note that this resistance R_C characterizing the energy loss in the low frequency limit is 2 times less than the resistance R_{CEQ} [see Eq. (20)] characterizing the energy loss in the high frequency limit. Note also that the difference between the two limits in terms of the resistance R_C is only a factor of 2, which is not that much when the wide range of frequency variation from zero to infinite is taken into account.

In the low frequency limit, the loss E_{EC} can be also reduced by increasing the core cross section S over the minimum value $S_{\min}$ given by Eq. (10) at $\Delta B(\tau) = \Delta B$. In the high frequency case R_C was proportional to the square of the cross section ratio. Here in the low frequency limit the resistance R_C is directly proportional to the cross section ratio

$$R_C = 4 \frac{\rho S}{\ell \delta^2} = 4 \frac{\rho S_{\min}}{\ell \delta^2} \left(\frac{S}{S_{\min}} \right). \quad (28)$$

IV. EXPERIMENTAL RESULTS

Below we present the results obtained with the cores made of ET3425 iron tape. The specific resistivity of this iron is $\rho = 5 \times 10^{-7}$ Ohm m, the induction at saturation $B_S = 2$ T, it is reached at $H_S = 30$ kA/m, and the coercive force is $H_C = 0.03$ kA/m. Since $H_S \gg H_C$, at pulsed remagnetization of such cores the energy loss E_H due to the hysteresis effect is negligible compared to the loss E_{EC} due to generation of the eddy current. This means that in Eq. (12), the first term on the right is much less than the second term. Then the current in the real core winding is the inductive current plus the impact from the eddy current in the core. Moreover, as shown below, the impact from the eddy current predominates over the inductive current.

Figure 4 shows the applied voltage and the current flowing in the single-turn winding of the ring-shaped core, where the voltage pulse rises in ~ 15 ns to ~ 3.2 kV and afterwards is almost constant during ~ 70 ns, whereas the current increases almost linearly in time and reaches ~ 1.5 kA at the time the voltage falls down. The core consists of two rings, each ring is wound with a $50\text{-}\mu$ -thick, 18-mm-wide iron tape sandwiched in a $10\text{-}\mu$ -thick, 20-mm-wide Mylar film insulation. The inner diameter of the core is 1.2 cm, the outer diameter 2 cm, the length of the center line of the core $\ell \sim 5$ cm, and the cross section of the iron $S \sim 1$ cm². The initial induction in the core was $B_0 = -B_R = -1.2$ T; the length of the voltage pulse is limited by the core saturation.

With a rectangular voltage pulse, the inductive current

$$I_\mu(t) = \frac{1}{L_\mu} \int_0^t U(t) dt \sim \frac{U_0 t}{L_\mu} \quad (29)$$

risks linearly in time, as the current in Fig. 4. Assuming the relative permeability of the ET3425 iron is $\mu_{ER} \sim 600$

FIG. 4. (Color) Fast voltage pulse and the current in the single-turn winding of a core made of ET3425 iron tape with thickness $\delta = 50$ μ m.

($\mu_{ER} = \mu_E/\mu_0$, $\mu_0 = 1.257 \times 10^{-6}$ V s/A m—permeability of the free space), the inductance of the winding is quite high and equal to $L_\mu \sim 2.2$ μ H. Hence, with $U_0 \sim 3.2$ kV the inductive current in $t \sim 70$ ns reaches only $I_\mu \sim 100$ A, which is an order of magnitude less than the total winding current observed in the experiment.

The traces in Fig. 4 can be explained by the eddy current generation in the core operating in the high frequency limit. Indeed, the voltage rise time in Fig. 4 corresponds to the frequency $\omega \sim 10^8$ s⁻¹, which is much more than $\frac{8\rho}{\mu_E \delta^2} \sim 2 \times 10^6$ s⁻¹. In this case the condition (25) is well satisfied, and the core should behave as if its hysteresis loop were rectangular. Then, as predicted, the resistance $R_C(t)$ should decrease, and the current $I(t)$ should increase with time. The top of the voltage pulse in Fig. 4 is almost flat; the voltage begins to decrease when the core starts saturating. Then the ratio $U(t)/I(t)$ should reach the value R_{CEQ} close to the middle of the flat part of the pulse. From Eq. (20), for the core studied, this value is $R_{CEQ} \sim 3.2$ Ohm. In Fig. 4 the voltage when $U(t)/I(t) = 3.2$ Ohm is marked with a black cross, which actually is located approximately in the middle of the flat part of the pulse.

Figure 5 shows the voltage and the current in the ten-turn winding of the other core where the voltage rise time is around 5 μ s. This core is wound using the same type of iron tape and Mylar film insulation. The inner diameter of the core is ~ 1.75 m, the outer ~ 2 m, the length of the center line $\ell \sim 6$ m, and the cross section of the iron $S \sim 16$ cm². The initial inductance is $B_0 = -B_R = -1.2$ T; similar to Fig. 4, the voltage pulse here is almost rectangular and its length is limited by saturation of the core. However, unlike in Fig. 4, before the core begins to saturate the current in Fig. 5 seems to be proportional to the voltage rather than linearly increasing with time (this means that

FIG. 5. (Color) Slow voltage pulse and the current in the ten-turn winding of the core made of ET3425 iron tape of thickness $\delta = 50 \mu\text{m}$.

here the inductive current $I\mu$ is also negligible). The ratio between the voltage and current in the $n = 10$ turn winding during the first $\sim 20 \mu\text{s}$ is

$$R_{10} = \frac{U(t)}{I(t)} \sim 18 \text{ Ohm}, \quad (30)$$

which for single-turn winding gives the equivalent resistance

$$R_1 = \frac{R_{10}}{n^2} \sim 0.18 \text{ Ohm}. \quad (31)$$

This result is in good agreement with the eddy current generation in a core operating in the low frequency limit. The rise time of the voltage in Fig. 5 corresponds to the frequency $\omega \sim 3 \times 10^5 \text{ s}^{-1}$, which is much less than $\frac{8\rho}{\mu_E \delta^2} \sim 2 \times 10^6 \text{ s}^{-1}$, and the condition (27) is satisfied. The resistance of the core should be constant and equal to $R_C \sim 0.21 \text{ Ohm}$, as it is predicted for this core by Eq. (14).

In the fast LTD stages, the output voltage rises in $\sim 50\text{--}100 \text{ ns}$, and hence the corresponding frequency is $\omega \sim (1.5\text{--}3) \times 10^7 \text{ s}^{-1}$. At such frequency, and $\delta = 50\text{--}80 \mu\text{m}$, the LTD cores operate in the high frequency range, and their resistance should decrease during the pulse. As the output LTD pulse is almost sinusoidal in the matched mode, the equivalent resistance of the core can be determined as a ratio between the voltage and current close to the time of peak voltage. According to [2], for a core with $S = 56 \text{ cm}^2$, $\delta = 80 \mu\text{m}$, and $\ell = 6 \text{ m}$, this ratio is $R_C \sim 0.56 \text{ Ohm}$ which is close to $R_{CEQ} = 0.58 \text{ Ohm}$ estimated from Eq. (20). According to the information received from Fred Bayol [8], who is the lead scientist of the fast LTD development project at ITHPP, Thegra, France, for a core with $S = 64 \text{ cm}^2$, $\delta = 50 \mu\text{m}$, and $\ell = 6 \text{ m}$, that ratio is $\sim 1.5 \text{ Ohm}$ compared to $R_{CEQ} = 1.7 \text{ Ohm}$ estimated from Eq. (20).

According to [9], for the LTD16-8 core (the prototype of this stage is described in [10]) with $S = 52 \text{ cm}^2$, $\delta = 50 \mu\text{m}$, and $\ell = 2 \text{ m}$, the ratio of the voltage to the current close to the middle of the voltage pulse is $\sim 7.8 \text{ Ohm}$ compared to $R_{CEQ} = 4.16 \text{ Ohm}$ from Eq. (20). However, since the minimum cross section for this stage is $S_{\min} \sim 27 \text{ cm}^2$, with an actual $S = 52 \text{ cm}^2$ core the output voltage pulse does not remagnetize it completely and therefore the value of R_{CEQ} must be estimated from Eq. (26), which gives a very close to the experiment value of $R_{CEQ} = 8 \text{ Ohms}$.

V. CONCLUSIONS

We have demonstrated analytically and experimentally that we can analyze the remagnetization of the LTD cores with finite permeability μ_E by considering two distinct categories; the high and low frequency limits. These two limits strongly differ by the current density distribution inside the core tape and by the effective resistance of the core.

In the high frequency limit, the core with finite permeability behaves as if its hysteresis loop were rectangular. Initially the eddy current concentrates in the skin layer in the vicinity of the surface of the tape. As the surface side of this layer saturates, the current expands deeper and deeper to the center of the tape. Therefore the resistance R_C decreases with time; the speed of this reduction depends on the shape of the applied voltage pulse. If this pulse is close to symmetrical, the energy loss at complete remagnetizing of the core can be found by modeling the core as a constant resistance R_{CEQ} given by Eq. (20). The cores of the $\sim 100\text{-ns}$ time scale LTDs operate in the high frequency limit.

In the low frequency limit, just from the beginning of the pulse the eddy current distributes inside the tape almost homogeneously, with no phase shift. Therefore the resistance R_C is constant in time, and does not depend on the shape of the voltage pulse. This resistance is given by Eq. (14), and it is 2 times less than the resistance R_{CEQ} , which determines the energy loss in the high frequency limit. The cores of the $\sim 1\text{-}\mu\text{s}$ time scale LTDs operate in the low frequency limit.

The energy loss due to generation of the eddy current can be reduced by increasing the cross section of the core, S , above the minimum value $S_{\min}$, which is needed to avoid saturation before the end of the voltage pulse. At $S > S_{\min}$, the loss of energy in the high frequency limit decreases inversely proportional to $(S/S_{\min})^2$; in the low frequency limit it decreases inversely proportional to $S/S_{\min}$.

ACKNOWLEDGMENTS

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the U.S. Department of Energy under Contract No. DE-AC04-94-AL85000.

- [1] http://www.amet.ru/stan720_1.html (in Russian).
- [2] A. A. Kim, M. G. Mazarakis, V. A. Sinebryukhov, B. M. Kovalchuk, V. A. Visir, S. N. Volkov, F. Bayol, A. N. Bastrikov, V. G. Durakov, S. V. Frolov, V. M. Alexeenko, D. H. McDaniel, W. E. Fowler, K. LeChien, C. Olson, W. A. Stygar, K. W. Struve, J. Porter, and R. M. Gilgenbach, *Phys. Rev. ST Accel. Beams* **12**, 050402 (2009).
- [3] Michael G. Mazarakis, William E. Fowler, Alexander A. Kim, Vadim A. Sinebryukhov, Sonrisa T. Rogowski, Robin A. Sharpe, Dillon H. McDaniel, Craig L. Olson, John L. Porter, Kenneth W. Struve, William A. Stygar, and Joseph R. Woodworth, *Phys. Rev. ST Accel. Beams* **12**, 050401 (2009).
- [4] M. A. Rosenblat, *Automatika i Telemekhanika* XIX **1**, 75 (1958) (in Russian).
- [5] A. Goral, *IEEE Trans. Magn.* **1**, 84 (1965).
- [6] L. A. Meerovich, I. M. Vatin, E. F. Zaitsev, and V. M. Kandykin, *Magnetic Pulsed Generators* (Sovetskoje Radio Publishing House, Moscow, 1968), in Russian.
- [7] V. A. Vizir, A. D. Maksimenko, V. I. Manylov, and G. V. Smorudov, in *Proceedings of the 13th Symposium on High Current Electronics*, edited by B. Kovalchuk and G. Remnev (Institute of High Current Electronics SD RAS, Tomsk, Russia, 2004), p. 198.
- [8] F. Bayol (private communication).
- [9] F. Bayol (private communication).
- [10] A. A. Kim, V. A. Sinebryukhov, B. M. Kovalchuk, A. N. Bastrikov, V. G. Durakov, S. N. Volkov, S. V. Frolov, V. M. Alexeenko, F. Bayol, C. Drouilly, F. Cubaynes, L. Veron, M. Toury, C. Vermare, and R. Nicolas, in *Proceedings of the 16th IEEE International Pulsed Power Conference*, edited by E. Schamuloglu and F. Peterkin (IEEE, Piscataway, NJ, 2007), p. 148.