Laser-assisted H^- charge exchange injection in magnetic fields

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The use of stripping foils for charge exchange injection can cause a number of operational problems in high intensity hadron accelerators. A recently proposed three-step method of laser-assisted injection is capable of overcoming these problems. This paper presents advances in the physical model of laser-assisted charge exchange injection of H^- beams and covers a wide field of atomic physics. The model allows the calculation of the evolution of an H^0 beam taking into account spontaneous emission, field ionization, and external electromagnetic fields. Some new data on the hydrogen atom related to the problem are calculated. The numerical calculations in the model use realistic descriptions of laser field and injection beam. Generally, the model can be used for design and optimization of a laser-assisted injection cell within an accelerator lattice. Example calculations of laser-assisted injection for an intermediate experiment at SNS in Oak Ridge and for the PS2 accelerator at CERN are presented. Two different schemes, distinctively characterized by various magnetic fields at the excitation point, are discussed. It was shown that the emittance growth of an injected beam can be drastically decreased by moving the excitation point into a strong magnetic field.

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1. INTRODUCTION

At present, it is considered that using thin carbon foils for H^- charge exchange injection will cause serious operational problems in high intensity machines. Numerical estimations and current experimental experience in the SNS accelerator in Oak Ridge show overheating of the carbon foils, leading to shorter lifetimes and rapid failure. This situation complicates facility maintenance. It is also believed that carbon foils will not survive multi-MW operation. This factor is considered to be one of the main obstacles to ramping up the power for SNS and for other hadron machines. Under normal operating conditions, the foil causes uncontrolled beam losses through the mechanisms of nuclear scattering and Lorentz stripping of H^{0+} excited states created when the beam passes through the foil. For this reason, the SNS team is developing foils made of beam resistant materials [1] and exploring an alternative foil-less injection scheme based on laser-assisted charge exchange injection. This paper is devoted to the second injection technique, which we call laser stripping (LS) for simplicity.

The three-step LS scheme has been considered in the work of Yamane [2], and then developed [3] and demonstrated experimentally [4]. Paper [3] contains the basic physical model of the three-step LS, and it allows the estimation of key parameters necessary for successful beam stripping. The estimations have been used for a proof-of-principle experiment [4] that proved the feasibility of LS and the consistency of the theory. This paper presents a continuation of the development of the theory that includes a wider range of physical phenomena necessary for describing a real experiment and for determining

in detail the LS efficiency and other injection parameters. The physics of the three-step LS scheme focuses mainly on the resonant excitation of the H^0 beam by a laser, with the goal being the determination of the LS efficiency. Paper [3] takes a quantum mechanical approach to the excitation by considering the Schrödinger equation and a two-level unperturbed atom. This approach is valid for the calculation of the H^0 excitation efficiency by a linearly polarized field without external electric and magnetic fields. A very important insight of paper [3] was the recognition that adiabatic rapid passage of a laser beam with angular spread could provide efficient resonant excitation of an H^0 beam with angular and energy spread.

The first section of the paper presents a quantum mechanical model for calculating the evolution of the H^0 atom in a superposition of laser and external electric and magnetic fields and taking into account the phenomena of spontaneous emission and field ionization. A useful prediction of the model is the probability of ionization of H^0 atoms in the beam as a function of time. The model uses some new data on the hydrogen atom in electric fields that has been calculated within this work. The numerical calculations use a realistic description of the laser field and the H^0 beam. The proposed model can be used for design and optimization of LS experiments. The calculations can be performed for an entire injection cell within the accelerator lattice.

In the case of the three-step LS scheme, the magnetic field from the first and second stripping magnets has a negative effect. The field perturbs the H^0 atoms via the Stark effect and affects the resonant excitation. The model can calculate the impact of the magnetic field on the total ionization efficiency of LS. The model also computes the

loss of LS efficiency due to spontaneous emission. The model can also be used to optimize the dependence of the losses on the distance between the interaction point and the second stripping magnet. Generally, the design of LS involves the self-consistent optimization of laser beam optics, H^0 beam optics, and magnetic field. Injection requirements defining the optimization are conditioned by the LS efficiency and emittance growth. The latter unavoidably results from the Lorentz stripping of H^- and H^0 beams by the magnetic field. As a rule the injection stripping efficiency is required to be 96%–99%. However, the injection emittance requirements can vary widely, depending on the requirements of the facility. In any case the optimization will provide maximum stripping efficiency with minimum laser power, thus playing a valuable role in the design of LS. Some results of calculations for an intermediate experiment [5] at SNS in Oak Ridge are presented in this paper.

Although the magnetic field causes emittance growth in the three-step scheme, there is an alternative stripping scheme in which it can have a positive effect. The three-step scheme considers Lorentz stripping of an excited beam in a fringe field of the second stripping magnet, which leads to significant emittance growth of the injected beam. The alternative scheme of LS proposes excitation of the H^0 beam by a laser in a strong magnetic field. It was first considered by Yamane in Ref. [6] and then estimated in Ref. [7]. This approach shows only a small emittance growth of the injected beam, in contrast with the three-step scheme. The last section of the paper pays particular attention to this case, presenting and discussing the results of an investigation for the PS2 accelerator characterized by small emittance injection into recycler.

II. PHYSICAL MODEL

Lorentz stripping of the first electron in a high energy H^- beam by a magnetic field is a relatively simple method. The process can be completely calculated using the lifetime of H^- as a function of the electric field in the rest frame of the particle [8]. The calculation was outlined in [3] and we are not concentrating on it in this paper, except to say that it provides an H^0 beam with parameters that can be used as initial conditions for the problem considered here. This problem can be formulated in the following way: it is required to find the probability of ionization $p(t)$ as a function of time for an H^0 atom traveling through the superposition of external static electric, static magnetic, and laser fields. The ionization probability functions, calculated independently for every particle of the beam, completely define the injection process. The problem defined in this way presents a universal approach for the calculation of any LS scheme. There are two quantum mechanical approaches for solving the problem: the Schrödinger equation and the density matrix. Let us now consider each case

in more detail. We use atomic units throughout this paper unless otherwise specified.

A. Schrödinger equation

It is convenient to solve the problem in the rest frame of the H^0 particle. The evolution of the wave function $\Psi(\vec{r}, t)$ of the electron in the hydrogen atom is driven by electromagnetic fields that can be transformed from the lab frame to the particle rest frame using Lorentz transformations. The probability of ionization of the H^0 atom can be calculated by the following expression:

$$p(t) = 1 - \int_{V_{in}} \Psi^*(\vec{r}, t) \Psi(\vec{r}, t) d^3r. \quad (1)$$

In our case, we integrate the probability over a region V_{in} that will be defined and explained below. The wave function $\Psi(\vec{r}, t)$ can be found by solving the Schrödinger equation:

$$i \frac{\partial \Psi}{\partial t} = [\hat{H}_{E,B} + \hat{V}(t)] \Psi. \quad (2)$$

Here $\hat{H}_{E,B}$ is the Hamiltonian of the hydrogen atom in the static electric and magnetic fields, and $\hat{V}(t)$ is the interaction between the electron and the laser field. The magnetic field in the rest frame of the particle can be neglected because it has much less effect than the electric field. We will also assume that the external static field has a slow dependence on time, in contrast with the laser field. This slow time dependence is caused by the passage of the H^0 atom through the system of magnets with nonuniform field. The equation cannot be solved analytically in the general case without simplifying assumptions. First of all, it is necessary to perform a rotation from the inertial frame $\vec{r}$ of the particle to the frame $\vec{r}'$ in which the static electric field is directed along the z' axes. Equation (2) is then transformed into the following:

$$i \left(\frac{\partial \Psi}{\partial t} + \vec{\nabla} \Psi \cdot [\mathbf{M} \mathbf{M}^{-1} \vec{r}] \right) = [\hat{H}_E + \hat{V}(t)] \Psi, \quad (3)$$

where $\hat{H}_E = \hat{H}_0 - e z E_z$ is the new Hamiltonian of the hydrogen atom in the electric field directed along the z axis, $\hat{H}_0$ is the Hamiltonian of the unperturbed atom, $\hat{V}(t) = -e \vec{r} \vec{E}_l(t)$ is the interaction between electron and laser field $\vec{E}_l(t)$ in the new frame, and $\mathbf{M}$ is a 3D rotation transformation matrix from the inertial frame $\vec{r}$ to the noninertial frame $\vec{r}'$: $\vec{r}' = \mathbf{M} \vec{r}$. $\mathbf{M}$ is not unique in 3D space, and the choice of the matrix affects the solution of (3) but does not affect the solution of (2). The probability (1) will be the same for both solutions. The authors used the following rotation matrix because of its simplicity:

$$M(\vec{n}_E) = \begin{pmatrix} 1 - \frac{n_x^2}{n_z+1} & -\frac{n_x n_y}{n_z+1} & -n_x \\ -\frac{n_x n_y}{n_z+1} & 1 - \frac{n_y^2}{n_z+1} & -n_y \\ n_x & n_y & n_z \end{pmatrix}. \quad (4)$$

The components of electric field direction n_x , n_y , and n_z are functions of time depending on the external fields affecting the H⁰ atom. The initial condition for Eq. (2) or Eq. (3) is given by the ground state of the hydrogen atom at the initial instant of time: $\Psi(\vec{r}, 0) = \psi_0(\vec{r})$.

To solve Eq. (3) we apply a well-known method for solving linear differential equations (see, for example, [9], Sec. 40) and present the solution in the form of an eigenfunction expansion:

$$\Psi(\vec{r}, t) = \sum_{n=1}^N c_n(t) \psi_n(\vec{r}). \quad (5)$$

The solution (5), strictly speaking, must be supplemented by an expansion over a complete set of functions of the continuum spectrum. However, we neglect the continuum spectrum within the framework of the paper. Analyzing Eq. (3) and marking out the time-dependent terms it is reasonable to use eigenfunctions of the hydrogen atom in the external static field as the system of functions for (5). The eigenfunction problem can be obtained by taking $\hat{V}(t) = 0$, $\dot{M} = 0$ and leaving the Hamiltonian $\hat{H}_E$ in (3), which incorporates the Stark effect. The Stark effect problem deals with the wave functions of quasistationary states that, strictly speaking, belong to the class of functions of the continuum energy spectrum. The functions cannot be normalized to 1 over an infinite volume and it is necessary to use a limited region within the system [9]. For a quantum system that can disintegrate, it is reasonable to consider the region within the potential well confined by its edges. For a hydrogen atom in an electric field E , the region can be defined by the maximum of the potential barrier $U = -e/r + Ez$ on the path from the center of the atom. The region is calculated to be the half space: $-\sqrt{e/E} < z$. The assumptions listed here allow us to describe the quantum mechanical evolution using a finite number N of quasistationary states. The model can break down at very strong fields when energy levels are widely broadened and interfere with each other, thus forming a continuum spectrum. This case requires a separate investigation.

In order to solve the Stark effect problem, it is necessary to find eigenfunctions $\psi_n(\vec{r})$ of the Schrödinger equation with complex eigenvalues E_n : $\hat{H}_E \psi_n(\vec{r}) = E_n \psi_n(\vec{r})$, where $E_n = E_{0n} - i\Gamma_n/2$ specifies the energy levels E_{0n} and decay widths Γ_n . The problem was solved using the original definition of values E_n as roots of the equation $B_{in}(E_{0n} - i\Gamma_n/2) = 0$, where B_{in} is the amplitude of the incoming wave [9]. The problem was solved numerically by the method described in Ref. [10]. The eigenfunctions

were normalized within the system and were found to be orthogonal to a high precision.

Continuing with the general method, we substitute (5) into (3) recalling that $\hat{H}_E \psi_n(\vec{r}) = (E_{0n} - i\Gamma_n/2) \psi_n(\vec{r})$. Then after multiplying both sides of the equation on the left by ψ_m^* and integrating within the system V_{in} defined by $-\infty < x, y < \infty$, $-\sqrt{e/E} < z < \infty$, we obtain

$$i\dot{c}_m = \left(E_{0m} - \frac{i\Gamma_m}{2}\right)c_m + \sum_{n=1}^N c_n [\vec{E}_l(t) V_{mn} - iG_{mn}], \quad (6)$$

where

$$m = 1, \dots, N \quad V_{mn} = \int_{V_{in}} \psi_m^* \vec{r} \psi_n d^3r$$

$$G_{mn} = \int_{V_{in}} \psi_m^* \vec{\nabla} \psi_n [\hat{M}^{-1} \vec{r}] d^3r.$$

Equation (6) is a system of N differential equations for the complex amplitudes $c_n(t)$ appearing in the wave function (5). The initial condition of the equation is defined by the probability of 1 of the atom to be in the ground state at the initial instant of time: $c_n(t_0) = \delta_{1n}$. It must be remembered that values E_{0m} , Γ_n , V_{mn} , and G_{mn} are functions of the external electric field that are both external drivers of the equation and time dependent. Solving Eq. (6) and substituting (5) into (1) we obtain

$$p(t) = 1 - \sum_{n=1}^N |c_n(t)|^2. \quad (7)$$

We shall now use the rotating wave approximation (RWA) for solving Eqs. (6). The coefficients $c_n(t)$ contain rapidly oscillating factors $e^{-i \int E_{0n}(t) dt}$ related to the time dependence of the quasistationary states. The laser field, due to its rapidly oscillated nature, can be represented in the following form:

$$\vec{E}_l(t) = \vec{E}_{l0}(t) \cos[\varphi(t)] = \vec{E}_{l0}(t) \frac{e^{i\varphi(t)} + e^{-i\varphi(t)}}{2}. \quad (8)$$

After a change of variable $c_n(t) = a_n(t) e^{-i \int E_{0n}(t) dt}$ and using (8), Eq. (6) becomes

$$\dot{a}_m(t) = \sum_{n=1}^N a_n(t) S_{mn}(t),$$

where

$$S_{mn} = \frac{\vec{E}_{l0}(t)}{2i} (Q_{mn} + Q_{nm}^*) V_{mn} - G_{mn} R_{mn} - \frac{\delta_{nm} \Gamma_m}{2} \quad (9)$$

$$Q_{mn} = e^{i[\varphi(t) - \int (E_{0n} - E_{0m}) dt]} \quad R_{mn} = e^{i \int (E_{0m} - E_{0n}) dt}.$$

The change of variable does not change (7), so that $c_n(t)$ can be simply changed to $a_n(t)$. Following the RWA, the terms Q_{mn} or R_{mn} with rapidly oscillated phases are canceled, leaving only terms with slowly oscillating phases.

For Q_{mn} the phase oscillation is small when the laser frequency is closer to the transition frequency between the n and m levels $\dot{\varphi}(t) \approx E_{n0} - E_{m0}$. For R_{mn} terms the oscillations are small for levels with approximately the same energies $E_{n0} \approx E_{m0}$. Thus, the laser field provides resonant excitation of higher states, and the rotation of the external electric field redistributes states with very close energy levels. If the laser frequency is adjusted for excitation of hydrogen level with principal quantum number n , that is $\dot{\varphi}(t) \approx (1 - n^{-2})/2$, then, according to the number of parabolic Stark states, we must include $N = 1 + n^2$ states into Eqs. (9), not considering an exotic cases.

The Schrödinger equation approach cannot take into account the phenomenon of spontaneous emission. Nevertheless, the method can be applied when the evolution time is much less than the typical time for spontaneous emission. For example, an intermediate LS experiment at the SNS will use a 1 GeV hydrogen beam and excitation to the 3rd level along a 1 mm distance. The time of interaction is approximately 10^{-12} sec, which is much less than the typical time for spontaneous decay $\tau_{3p \rightarrow 1s} = 6.1 \times 10^{-9}$ sec. The effect of spontaneous emission can be omitted in this case and the Schrödinger equation can be applied.

B. Density matrix

The density matrix approach in quantum mechanics takes into account the phenomenon of spontaneous emission. This phenomenon contributes significantly to the loss of LS efficiency when considering the three-step LS scheme with a long distance between excitation point and the second stripping magnet.

A calculation of the density matrix elements for the wave function (5) using the density matrix operator $\hat{\rho} = |\Psi\rangle\langle\Psi|$ gives $\rho_{nm}(t) = c_n(t)c_m^*(t)$. The probability of ionization (7) is defined by diagonal elements:

$$p(t) = 1 - \sum_{n=1}^N \rho_{nn}(t). \quad (10)$$

The evolution of the density operator for the hydrogen atom obeys the optical Master equation [11]:

$$i \frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}] - \frac{1}{2} i \{\hat{\Gamma}, \hat{\rho}\} + D\{\hat{\rho}\}, \quad (11)$$

where the curly brackets denote an anticommutator, and $\hat{\Gamma}$ is the autoionization operator with matrix elements $\Gamma_{nm} = \delta_{nm}\Gamma_n$. The operator $D\{\hat{\rho}\}$ is responsible for spontaneous emission. The equation for density matrix elements can be derived using (11), but it is very easy to derive it in the following way (temporarily omitting $D\{\hat{\rho}\}$). Substituting (9) into $\dot{\rho}_{nm}(t) = \dot{a}_n(t)a_m^*(t) + a_n(t)\dot{a}_m^*(t)$ and bearing in mind that $a_n(t)a_m^*(t) = \rho_{nm}(t)$, we obtain

$$\dot{\rho}_{nm}(t) = \sum_{j=1}^N \rho_{jm}(t)S_{nj}(t) + \rho_{nj}(t)S_{mj}^*(t) + D_{nm}[\rho]. \quad (12)$$

Here $n, m = 1, \dots, N$. The initial condition requires that $\rho_{nm}(t_0) = \delta_{n1}\delta_{m1}$. It is necessary to be careful when choosing the form of the dissipation term added to the right of (12) [12]. For the hydrogen atom evolving in an external field and satisfying the requirements of a non-degenerate system, the dissipation term can be written in terms of parabolic basis as follows:

For diagonal elements,

$$D_{nn}[\rho] = \sum_{j=n+1}^N \gamma_{jn}\rho_{jj} - \rho_{nn} \sum_{j=1}^{n-1} \gamma_{nj}, \quad (13)$$

and for off-diagonal elements,

$$D_{nm}[\rho] = \frac{\rho_{nm}}{2} \left(\sum_{j=1}^{m-1} \gamma_{mj} + \sum_{j=1}^{n-1} \gamma_{nj} \right),$$

where $\gamma_{nm} = \frac{4}{3}\alpha^3(E_{0n} - E_{0m})^3|V_{mn}|^2$ are radiative transition rates and α is the fine structure constant. The form of (13) implies that $E_{0n} > E_{0m}$ at $n > m$. It must be remembered that (13) is not valid for calculation of the hydrogen atom in a parabolic basis without an external electric field. The Master equation (12), in contrast with the Schrödinger equation, must take into account the resonant levels and the levels below them. Hence, $N = 1^2 + 2^2 + \dots + n^2 = n(1+n)(1+2n)/6$.

C. Diagonal density matrix

The equations (12) and (13) are more general than the Schrödinger equation in that they can be applied for the calculation of any scheme of LS. However, when considering the three-step LS scheme it is easier for the calculation of the 3rd step to use a simplified version of (12) and (13). After excitation at the 2nd step, a hydrogen atom travels to the strong field of the 2nd stripping magnet without further interaction with the laser field: $\vec{E}_{l0}(t) = 0$. When omitting the rotation of the field in the particle rest frame $\dot{\mathbf{M}} = 0$, the evolution of the diagonal density matrix elements is separated from the evolution of the off-diagonal elements. As a result the number of equations is reduced from N^2 to N and, after simplification, we obtain

$$\dot{\rho}_{nn} = \sum_{j=n+1}^N \gamma_{jn}\rho_{jj} - \rho_{nn} \left(\Gamma_n + \sum_{j=1}^{n-1} \gamma_{nj} \right). \quad (14)$$

In this way, it is more convenient for calculation of the three-step LS scheme to apply Eq. (9) for calculation of the 2nd step and then Eq. (14) for calculation of the 3rd step, obtaining the same result for $p(t)$ as in the case of application of (12) and (13) continuously for both steps.

D. Laser field

Most laser beams can be well approximated by the elliptical Gaussian mode in the laboratory frame [13]. The expression for the electric field of the mode can be

written in SI units as follows:

$$E(\vec{r}, t) = \frac{2\sqrt{cP\mu_0 w_x w_y}}{\sqrt{\pi h_x h_y}} e^{ik(ct-z) - (x^2/h_x) - (y^2/h_y)} \quad (15)$$

$$h_{x,y} = w_{x,y}^2 - \frac{2i(z - f_{x,y})}{k}.$$

The laser beam presented here propagates in the positive direction along the z axis. Here $E(V/m)$ is the absolute value of the electric field, $P(W)$ is the power of the beam; $c(m/s)$ is the speed of light, and $k = 2\pi/\lambda(m^{-1})$, where $\lambda(m)$ is the wavelength. Polarization of the beam can be well approximated by assuming linear polarization of the electric field directed along the x or the y axis: $\vec{E} = E\vec{e}_{x,y}$. The magnetic field can be written as $\vec{B} = (E/c)\vec{e}_{-y,x}(T)$. Other types of polarization can be obtained by superposition of (15).

The laser beam (15), without considering its orientation, is given by four geometric parameters: the waists of the laser beam w_x , w_y and their positions f_x , f_y on the z axis. The parameters, generally speaking, define the beam sizes and beam divergences along the x and y axes at the interaction point z_0 of the laser beam with the H⁰ beam. The parameters directly affect the LS efficiency and are considered to be important variables in the problem of optimization of LS.

For the practical applications of LS in accelerator physics it is necessary to provide injection of multipicosecond beams. This leads to the necessity of using a pulsed laser beam with approximately the same duration of micropulse. The picosecond laser pulse cannot be considered as an ultrashort laser pulse, and the power P in (15) can be described by a Gaussian function of time characterized by its peak power and rms duration.

III. MAGNETIC FIELD IMPACT IN THE THREE-STEP LS SCHEME

Consider the three-step LS scheme and the excitation of the hydrogen atom in the second step. The magnetic field from the first and the second stripping magnets reaches the excitation region and decreases the LS efficiency due to the Stark effect in the rest frame of the particles and the detuning of the resonance excitation. This effect requires moving the magnets as far as possible from the excitation point. On the other hand, the longer distance between the excitation point and the second magnet will lead to bigger losses of efficiency due to spontaneous emission. The optimal design of magnets and magnetic field can be calculated using the method described above.

The small magnetic field considered in this case section can be characterized by negligibly small lifetimes Γ_i at the excitation point. As a starting point, we estimate the magnetic field effect on the excitation efficiency. First, we determine the polarization of the laser field in the laboratory frame that provides the most favorable conditions for

excitation. Static electric fields and laser fields can have relative orientation α between 0 and 90 degrees in the particle rest frame. According to (9), the laser field excites only levels in the parabolic basis with transition dipole moments not orthogonal to the laser field: $\vec{E}_{l0} \cdot V(\{0, 0, 0\} \rightarrow \{n_1, n_2, m\}) \neq 0$. Integrating the φ dependences for x , y , and z components of the transition dipole moments in parabolic coordinates with the static electric field along the z axis, we obtain $V_x \sim \delta_{1,|m|}$, $V_y \sim \delta_{1,|m|}$, and $V_z \sim \delta_{0,m}$ [14]. Thus, the parallel polarization in the particle rest frame excites parabolic states with $m = 0$ while the perpendicular polarization excites parabolic states with $m = \pm 1$. Analyzing the formula for energy perturbation of the hydrogen atom $\Delta E_{\text{pert}} = \frac{3}{2}n(n_1 - n_2)F$, we notice the following regularity for the states that can be excited from the ground state (Table I). The sublevels are vertically arranged in order of increasing $\Delta E_{\text{pert}}/n$.

It is seen from Table I that the perpendicular polarization leads to excitation of Stark levels with less energy splitting and therefore better excitation efficiency. For particles moving in the laboratory frame with momentum $\vec{p}$ in magnetic field $\vec{B}$, we can formulate the following rule for choosing the polarization of the laser field $\vec{E}_l$: All three vectors must lie in the same plane, or

$$\vec{E}_l \cdot [\vec{B} \times \vec{p}] \approx 0. \quad (16)$$

It should be noted that the laser must be adjusted for resonant excitation of the unperturbed atom for most efficient excitation in weak magnetic fields. It can be seen from Table II that the 2nd state of the hydrogen atom is unique because the polarization (16) leads to excitation of a single state $\{0, 0, \pm 1\}$ in a strong magnetic field without the Stark effect splitting and with the same transition dipole momentum as the unperturbed atom.

Estimation of the effect of the magnetic field on excitation efficiency cannot be done analytically for the general case because there are too many parameters for the H⁰

TABLE I. Structure of exited Stark sublevels with parabolic quantum number $\{n_1, n_2, |m|\}$ for the first five principal levels. The marked states are excited by a perpendicularly polarized laser.

$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
				400
			300	301
		200	201	310
	100	101	210	211
000	001	110	111	220
	010	011	120	121
		020	021	130
			030	031
				040

TABLE II. Laser and ion beam parameters.

Parameter	Value
Ion beam	
Energy	1 GeV
rms energy spread	0.5×10^{-3}
Microbunch duration, FWHM	30 ps
β_x	25 m
β_y	0.75 m
α_x	0
α_y	2.0
x dispersion	0 m
x-dispersion derivative	2.6
y dispersion	0 m
y-dispersion derivative	0
$\varepsilon_{x,y}$ norm. rms emittance	0.2π mm mrad
Laser beam	
Wavelength	355 nm
Incidence angle	39.33 deg
Peak power	2.2 MW
Micropulse duration, FWHM	55 ps
x-rms size, r_x	1.69 mm
y-rms size, r_y	0.77 mm
x divergence, α_x	0.36 mrad
y divergence, α_y	0.24 mrad

beam and laser beam. In this section we present the results of estimations for the LS intermediate experiment at the SNS [5]. Parameters of the H^0 beam and laser beam at the injection point are listed in Table II. Horizontal and vertical parameters are marked by x and y subscripts correspondingly. The H^0 beam propagates along the z axis and the laser beam lies in the x - z plane. Head-on interaction between the two beams corresponds an incident angle of zero.

The parameters r_x , r_y , α_x , α_y , and microbunch duration presented in Table I are optimized parameters corresponding to maximum efficiency of excitation. The parameters express the vertical and horizontal rms sizes and angular divergences of the beam at the interaction point. They also define the geometry of the elliptical Gaussian beam (15) and can be directly expressed via w_x , w_y , f_x , and f_y by the following formulas:

$$r_{x,y} = w_{x,y} \sqrt{1 + \left(\frac{f_{x,y} \lambda}{\pi w_{x,y}^2} \right)^2}, \quad \alpha_{x,y} = - \frac{f_{x,y} \lambda^2}{\pi^2 w_{x,y}^2 r_{x,y}}. \quad (17)$$

The laser beam parameters are those of an existing laser measured at the SNS [15]. The parameters of the ion beam are calculated and are in agreement with those expected in the experiment [5,13]. Figure 1 presents the dependence of the efficiency of excitation of the 3rd level as a function of magnetic field $f_1(B)$ at the interaction region. Here we must distinguish *excitation region* from *excitation point*

FIG. 1. Excitation efficiency as a function of magnetic field B for two types of polarization.

because it is simple to create exactly $B = 0$ at some point $z = z_0$ using two stripping dipole magnets of opposite polarities. However, it is much more difficult to provide $B = 0$ in the whole region of interaction.

The dependence in Fig. 1 can be used directly for the preliminary design of the magnetic field in the following way: The spontaneous emission losses for a beam traveling the distance L_2 between the excitation point and the second stripping magnet can be approximately estimated as $f_2(L_2) = 1 - \exp[-L_2/(\nu\tau\gamma)]$. Here ν , τ , and γ are beam velocity, lifetime of upper level, and relativistic factor correspondingly. The requirement for total losses to be less than k : $f_1(0) - f_1(B) + 1 - \exp[-L_2/(\nu\tau\gamma)] < k$ defines tolerances for interrelated parameters L_2 and B in the design of the magnetic field. It should be noted that the inequality is just a simplified example that must be supplemented at least by seven principal parameters of magnetic field configuration when considering the three-step LS scheme: namely, the magnetic field strengths and derivatives at the stripping points of the first and the second stripping magnets; the distances between each magnet and the excitation point; and the magnetic field at the interaction region. These can be used to define figures of merit to provide guidance for the design of optimal magnetic field and the magnets.

In this way we have shown the complexity of realistic design of LS that can be calculated and optimized using the model presented in this paper.

IV. LS SCHEME IN STRONG MAGNETIC FIELDS

Here we consider another scheme of LS in which the hydrogen beam is excited in a strong magnetic field accompanied by simultaneous Lorentz ionization [6,7]. The strong magnetic field can be characterized by the following

criterion: $\Gamma_i \approx \Omega_R$, where Γ_i is the width of the i th upper level of the hydrogen atom and $\Omega_R = \mu_i E_{l0}$ is the Rabi frequency of the level. In this case the lifetime of the excited atom $1/\Gamma_i$ will be comparable with excitation time. The scheme results in small emittance growth of the stripped particles compared to the three-step LS scheme. In this section we will consider the particular case of the PS2 accelerator [16]. The PS2 requires the emittance growth of injected particles to be below $\sim 1-2\pi$ mm mrad. The design parameters of the beam at the injection point are listed in Table III [16,17]. The normalized emittances were assigned in accordance with the SNS experience.

First of all, we estimate the emittance growth when applying the three-step LS scheme. The beam is supposed to have 100% excitation of the n th upper level. A laser field

TABLE III. Parameters of the beam for the PS2.

Parameter	Value
Energy	4 GeV
rms energy spread	0.5×10^{-3}
Microbunch duration, (rms)	15 ps
β_x	30.0 m
β_y	7.0 m
$\alpha_{x,y}$	0
x, y dispersions	0 m
x, y -dispersion derivatives	0
$\varepsilon_{x,y}$ norm. rms emittance	0.25π mm mrad

with polarization directed at an angle α to a small external electric field in the particle rest frame leads to the following distribution among populations of parabolic sublevels:

$$\rho_{k,k} = 3 \frac{\delta_{m,0}(n_1 - n_2)^2 \cos^2(\alpha) + \delta_{|m|,1}(n_1 + 1)(n_2 + 1) \sin^2(\alpha)}{n(n^2 - 1)}. \quad (18)$$

where $n = n_1 + n_2 + |m| + 1$ is the principal quantum number and k denotes a parabolic state with quantum numbers $\{n_1, n_2, m\}$. Formula (18) was obtained by expanding the wave function of the upper spherical np state directed at an angle α to the z axis in parabolic states. The angle $\alpha = \pi/2$ corresponds to the type of polarization of Eq. (16).

The beam travels a distance of 25 cm from the excitation point to the dipole fringe field of the second stripping magnet. The fringe field for a 50 mm gap can be described by formula [18] for the purpose of estimation. Solving Eq. (14) with initial conditions (18) for a single atom yields a probability of ionization (10) with maximum derivative in fringe field region. The rms angular spread α_{rms} of ionized particles can be calculated via integrations with the help of the ionization distribution $p'(z)$. The normalized rms emittance growth of stripped particles can be estimated as $\varepsilon_{\text{norm}} = (\beta\gamma)\alpha_{\text{rms}}^2\beta_{x,y}$, where $(\beta\gamma) = 5.17$ is the relativistic factor and $\beta_{x,y}$ are the beta functions, taken to be 22.5 m [17]. Figure 2 plots the emittance growth of the stripped particles as a function of the peak magnetic field of the dipole magnet.

The results of Fig. 2 show that it is necessary to choose the 3rd level in order to obtain satisfactory emittance growth. However, estimations show that the peak powers required for 98% excitation of the 2nd and the 3rd levels are 3 and 75 MW correspondingly. This difference is caused by significantly different transition dipole moments and incidence angles between the laser beam and the H⁰ beam. It should be noted that the 2nd and the 3rd levels can be excited by 1064 and 532 nm wavelengths, correspondingly.

The dependences in Fig. 2 will be changed when transferring an excitation point into a strong magnetic field. The

Stark effect will strongly split energy levels from each other and it is most optimal in this case to excite some individual parabolic state. The equations (9) will be transformed into a system of two differential equations coupling the ground state and the excited state amplitudes. It can be shown that the twice degenerate upper states $\{n_1, n_2, \pm 1\}$ can be considered as a single state with transition dipole moment $|V_{mn}| = \sqrt{2}|V'_{mn}|$, where $|V'_{mn}|$ is the transition dipole moment of $\{n_1, n_2, +1\}$ or $\{n_1, n_2, -1\}$ state. Figure 3 presents the dipole transition moments from the

FIG. 2. Emittance growth as a function of peak magnetic field B_{max} for different excited states $n = 2, 3$, and 4 of a hydrogen atom and different polarizations of the laser.

FIG. 3. (Color) Transition dipole moments from the ground state to different parabolic states as a function of electric field in the particle rest frame. The dashed lines represent the moments calculated using second order perturbation theory [18]. The upper scale presents the dependence on transverse magnetic field in the laboratory frame for a 4 GeV beam.

ground state to different parabolic states as a function of electric field in the particle rest frame. It allows the estimation of the peak power of the laser beam when choosing the state for excitation in strong fields.

FIG. 4. Emittance growth (solid line) and stripping efficiency (dashed line) as functions of uniform transverse magnetic field B for different parabolic states. The four parabolic states on the left belong to the 3rd principal state and have the following sequence from left to right: $\{0, 2, 0\}$, $\{0, 1, \pm 1\}$, $\{1, 0, \pm 1\}$, $\{2, 0, 0\}$. The three parabolic states on the right belong to the 2nd principal state and have the following sequence from left to right: $\{0, 1, 0\}$, $\{0, 0, \pm 1\}$, $\{1, 0, 0\}$.

TABLE IV. The optimal parameters of the laser beam required for 98% of the stripping efficiency.

Levels with principal quantum number $n = 2$	
Wavelength	1064 nm
Incidence angle	47.40 deg
Peak power	3.0 MW $\{0, 0, \pm 1\}$ 5.4 MW $\{1, 0, 0\}$ 6.5 MW $\{0, 1, 0\}$
Micropulse duration, rms	24 ps
x -rms size, r_x	1.3 mm
y -rms size, r_y	1.5 mm
x divergence, α_x	2.6 mrad
y divergence, α_y	0.0 mrad
Levels with principal quantum number $n = 3$	
Wavelength	532 nm
Incidence angle	90.79 deg
Peak power	150 MW $\{2, 0, 0\}$ 185 MW $\{1, 0, \pm 1\}$ 165 MW $\{0, 1, \pm 1\}$ 190 MW $\{0, 2, 0\}$
Micropulse duration, rms	21 ps
x -rms size, r_x	2.2 mm
y -rms size, r_y	1.5 mm
x divergence, α_x	1.0 mrad
y divergence, α_y	0.7 mrad

Numerically calculated curves agree very well with the second order perturbation theory [19]. The 2nd level with parabolic quantum number $\{0, 0, \pm 1\}$ is definitely the best for high efficiency excitation with a low power laser. At high fields, the level behaves like the $2p$ state of an unperturbed hydrogen atom with transition dipole moment $128\sqrt{2}/243$ a.u. negligibly decreasing with the field.

Figure 4 shows the emittance growth and stripping efficiency as a function of uniform transverse magnetic field for different parabolic states. The optimal parameters of the laser beam and the peak powers required for 98% of stripping efficiency are listed in Table IV.

The emittance growth for the second level gives satisfactory results in this case. Moreover, the scheme of stripping in a strong field avoids spontaneous decay losses. The minima of emittance growth approximately satisfy the condition of excitation: $\Gamma_i \approx \Omega_R$. The stripping efficiency falls because of the extremely small lifetime of the upper level, which leads to a small upper level population and complicates its excitation according to the equation. Figure 4 and the parameters of Table IV suggest that it is most optimal to use level $\{0, 0, \pm 1\}$ for excitation in the 1.9 T magnetic field.

V. CONCLUSIONS

We have presented a quantum mechanical model to calculate laser-assisted injection in the presence of small

and strong magnetic fields. Using the model, it is possible to optimize magnetic field design and the parameters of the laser beam and the particle beam. As a result, it is possible to increase the power efficiency of the laser and to significantly decrease the required laser power. It is shown that for polarization of the laser beam lying in the same plane as the magnetic field and direction of the H^0 beam is most optimal for any scheme of laser stripping. It is shown that excitation of the $\{0, 0, \pm 1\}$ parabolic state of the hydrogen atom is most optimal and can be ruled out only when the beam energy is insufficient to provide Lorentz stripping of the state in the magnetic field. It is shown that the laser-stripping scheme of exciting the H^0 beam in a strong magnetic field leads to much smaller emittance growth of injected beam in contrast with the three-step scheme.

For laser stripping in the strong magnetic field, the spectral broadening, Γ_i , of the energy levels begins to be comparable with the laser frequency spread in the particle rest frame. This factor can significantly affect the excitation of the atom and its evolution. For this reason, in the future we have to develop a quantum mechanical model of the hydrogen atom taking into account the continuum spectrum of the electron and the broadening of its energy levels.

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