Erratum: Pion linac as an energy-tagged ν source **[Phys. Rev. ST Accel. Beams **11**, 124701 (2008)]**

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We have found that the Lorentz transformation in Eq. (19) is incorrectly implemented. On correcting this error, it is necessary to change several of the subsequent equations in Sec. IID, as well as some of the text in that subsection. The basic point and conclusions of the paper are unchanged and there are only minor changes in the numerical details in Table II and in Figs. 2 and 4.

For clarity in showing what is changed, we present here a revised version of the entire Sec. IID. This version is to be substituted for the original in its entirety.

D. Neutrino energy spectrum and angular divergence

The neutrino energies will range over a wide spectrum, but for any specific time from injection of the pion bunch and angle of divergence from the beam axis of a neutrino scattering event in the detector, only one specific neutrino energy is possible.

The momentum and energy, E_0 , of a neutrino emitted in the pion rest frame is

$$E_0 = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV.} \quad (18)$$

For a decay where the neutrino is emitted at a polar angle θ_R (in the rest frame) with respect to the pion beam axis, the momentum, polar angle θ_L , and energy E_ν in the lab frame are given by

$$\mathbf{p} = E_0[\gamma(\beta + \cos\theta_R), \sin\theta_R], \quad (19)$$

$$\tan\theta_L = \sin\theta_R/[\gamma(\beta + \cos\theta_R)], \quad (20)$$

$$E_\nu = E_0\sqrt{\sin^2\theta_R + \gamma^2(\beta + \cos\theta_R)^2} = \gamma(1 + \beta\cos\theta_R)E_0, \quad (21)$$

where we show only the longitudinal and transverse momentum components, ignoring the azimuthal angle since the distribution in that angle is uniform.

From Eq. (21), neutrinos emitted exactly backward from the beam direction will have lab energies and momenta of $\gamma(1 - \beta)E_0$, i.e., approximately zero for the most relativistic pions. Those emitted precisely forward will have lab energies

TABLE II. Extractions of the decay point x_{dec} (in m), neutrino laboratory angle θ_L (in mrad), and E_ν (in MeV) for a scattering event taking place at various scattering times Δt in the detector at three values of off-axis radial position ρ_{evt} . This case is for an event occurring in the detector at $x_{\text{evt}} = 40.0$ m from the accelerator injection point. We have assumed here that $T_\pi = 350$ MeV at injection into PILAC, as in Table I.

Δt (nsec)	ρ_{evt}	$G = 10 \text{ MeV/m}$			$G = 50 \text{ MeV/m}$		
		x_{dec}	θ_L	E_ν	x_{dec}	θ_L	E_ν
0.250	0.3	1.81	7.9	213	2.15	7.9	252
	0.6	1.72	15.7	213	2.02	15.8	249
	0.9	1.58	23.4	212	1.82	23.6	245
0.500	0.3	3.77	8.3	222	5.58	8.7	327
	0.6	3.67	16.5	221	5.35	17.3	322
	0.9	3.50	24.7	221	4.99	25.7	314
0.750	0.3	5.89	8.9	231	11.9	10.7	463
	0.6	5.77	17.5	231	11.3	20.9	450
	0.9	5.58	26.1	230	10.5	30.6	433

FIG. 2. The neutrino energy, E_ν (in MeV), as a function of laboratory polar angle θ_L from decay in the acceleration section at a given time from bunch injection (or position in the accelerator) for three pion kinetic energies at the time of decay. Each pion energy corresponds to a specific location for the decay in the accelerator section.

and momenta of $\gamma(1 + \beta)E_0$. For neutrinos emitted transverse (in the pion rest frame) to the beam direction, the lab energy and momentum will be γE_0 .

From Eq. (20), the singularity at $\cos\theta_R = -\beta$ corresponds to a laboratory angle of 90° . Neutrinos at this angle come from a very backward decay and have a low energy of E_0/γ . Those that come from even more backward decays have yet lower energies. The maximum laboratory angle is 180° . That is, θ_L as a function of θ_R rises monotonically from 0 to 180° , and does so more steeply in the backward θ_R direction.

Neutrinos are emitted between the two extreme energies along a curve in E_ν - θ_L space. To make such a plot, one needs to find the value of $\cos\theta_R$ corresponding to a given laboratory angle, θ_L . From the inverse Lorentz transformation, from the laboratory to the pion rest frame, and with some algebra,

$$\cos\theta_R = \cos\left[\tan^{-1}\left(\frac{\sin\theta_L}{\gamma(\cos\theta_L - \beta)}\right)\right] = \frac{\cos\theta_L - \beta}{1 - \beta\cos\theta_L}. \quad (22)$$

Having found the value of $\cos\theta_R$ for a given (or derived) θ_L , the neutrino energy E_ν is then given by Eq. (21), or more directly, $E_\nu = (1 - \beta\cos\theta_L)^{-1}E_0/\gamma$. Figure 2 shows curves of the neutrino energy as a function of θ_L for three different

FIG. 4. Relation between the reacting neutrino's energy, E_ν , and the time difference between the time of bunch injection and the scattering event time in the detector for a particular event position, here at 40 meters from the start of the pion accelerator and at an off-axis distance of 1 meter. It shows the differences in E_ν and time difference Δt for three different pion decay positions. The kinetic energy of pions at injection, T_π , has been taken to be 350 MeV and the accelerating gradient is 10 MeV/m. Thus, in this example, if the absolute time of this scattering event can be determined relative to the injection time with an accuracy of better than 500 ps, then the energy of the incident neutrino that initiated the event will be determined to an accuracy of better than ≈ 15 MeV.

(total) energies of the pion at the time of its decay. Note that, for the higher pion energies, the neutrino energy E_ν is more forward peaked and, of course, is larger there. Taking into account that neutrino cross sections are proportional to E_ν^2 at low E_ν , this suggests that, for economy, the neutrino detector could well be long and narrow, subtending a small solid angle along the beam direction.

E. Other corrections

Minor changes of wording in Sec. II E are also needed to reflect the fact that the maximum neutrino laboratory angle is 180° rather than 45°) and the correction of Eq. (23). These changes are as follows.

1. In the middle of the second paragraph, drop the phrase “apart from the almost-everywhere two-valuedness of this energy shown in Fig. 2.”
2. At the end of the second paragraph, replace the last sentence with the following: “However, low-energy neutrino cross sections vary as E_ν^2 , so most of the neutrino events will come at forward angles less than $\approx 30^\circ$.”
3. The caption for Table I should read: “The calculations shown here are for $T_\pi = 350$ MeV at injection into PILAC.”
4. After Eq. (29), drop the phrase “(there are two values).”
5. Equation (30) should now read

$$\cos\theta_R = \frac{\cos\theta_L - \beta_{\text{dec}}}{1 - \beta_{\text{dec}} \cos\theta_L}. \quad (30)$$

6. After Eq. (30), drop the sentence beginning “The scattering event is overwhelmingly most likely to ...”.
- Table I in Sec. II E is unchanged from the published version, but Table II and Fig. 4 have small changes in the calculated neutrino energies, E_ν . Section III, “DISCUSSION,” is unchanged, except that we also now thank James Jenkins for a useful conversation.