Pion linac as an energy-tagged ν source

T. Goldman* and Richard R. Silbar†

Theoretical Division, MS-B283, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 9 July 2008; published 16 December 2008)

The energy spectrum and flux of neutrinos from a linear pion accelerator are calculated analytically under the assumption of a uniform accelerating gradient. The energy of a neutrino from this source reacting in a detector can be determined from timing and event position information.

DOI: 10.1103/PhysRevSTAB.11.124701

PACS numbers: 29.20.Ej, 14.60.Lm, 25.30.Pt, 13.15.+g

I. INTRODUCTION

There has been considerable analysis [1] of the value of a high energy muon storage ring for the purpose of producing well-defined, high energy beams of neutrinos. This approach takes advantage of the relatively long muon lifetime to allow for collection at modest energies and acceleration to very high energies, where the decay neutrinos form an intense, highly collimated beam. The proposals for such systems are quite expensive since the (generally hadronically) produced muons must be captured, cooled, and accelerated, all on a microsecond time scale.

Some time ago, Los Alamos proposed [2] a pion post-accelerator (PILAC) to be appended to the then extant meson factory, LAMPF. The proposal took advantage of the high pion flux produced at modest energies, to collimate and inject into a high gradient linac. Phase rotation was introduced to reduce the energy spread of the beam considerably. The main practical difficulty with this relatively modest cost proposal was the short pion lifetime—26 nanoseconds. This required a very high accelerating gradient in order to achieve a usable flux at the exit of the acceleration section.

The decay of a positively charged pion is almost exclusively into antimuons and muon neutrinos (or muons and muon antineutrinos for negatively charged pions) with a purity at the part in the ten thousand level. Thus, a PILAC also provides an intense, collimated beam of neutrinos with an energy spectrum ranging from negligible to initially (almost) half the pion input energy and finally to (almost) half the output energy value. We show here that the neutrino spectrum, dN_ν/dE_ν , can be adjusted to be almost flat in energy, if sufficiently high accelerating gradients for the pions can be achieved. The necessary values are below those already achieved [3] using superconducting rf cavities [4].

However, there is another advantage to such a neutrino source that is not dependent upon whether or not high gradients can be achieved. Currently, it is very difficult to determine the energy of the neutrino involved in a particu-

lar scattering event in the detector. Usually, a complicated series of Monte Carlo simulations of the detector and its response defines a statistical determination only. In the scheme we are discussing, however, it is possible to obtain the incident neutrino energy *directly, event by event*.

We analyze here the theoretically simple case of a uniform accelerating gradient analytically. A more realistic case with drift sections is left for the accelerator physicists who would actually design and build such a system.

II. CALCULATIONS

Although we ultimately want to obtain the neutrino energy spectrum as a function of distance from the injection point of the accelerator, to follow the decay rate, it turns out to be more convenient to first find the pion energy and decay rate as functions of time. Throughout, we will use units where $\hbar = c = 1$.

A. Time dependence

We begin with pions injected at energy E_I and $\gamma_I \equiv E_I/m_\pi$, where m_π is the pion mass. For simplicity, we assume a spatially constant, uniform accelerating gradient, G , even though the actual acceleration structure will consist of short, high acceleration regions interspersed with drift regions. Refinement to include this detail should only change our smooth curves to segments of alternating flat and higher slope regions but with essentially the same average behavior. For convenience, we scale G by the pion mass so that $g \equiv G/m_\pi$ has the dimensions of an inverse length,

$$gm_\pi = dE_\pi/dx. \quad (1)$$

The energy of a pion as a function of distance along the accelerator is therefore

$$E_\pi(x) = E_I + gm_\pi x. \quad (2)$$

However, to follow the decay process, we will need the time rate of change of pion energy, viz.,

$$dE_\pi/dt = \frac{dE_\pi}{dx} \frac{dx}{dt} = gm_\pi \beta(t), \quad (3)$$

*tgoldman@lanl.gov

†silbar@lanl.gov

where $\beta(t)$ is the pion velocity in units of c . Since $\gamma(t) = E_\pi/m_\pi$, we can rewrite Eq. (3) as

$$d\gamma/dt = g\beta \quad (4)$$

and since $\gamma = 1/\sqrt{1-\beta^2}$,

$$\frac{d\gamma}{dt} = \frac{\beta}{[1-\beta^2]^{3/2}} \frac{d\beta}{dt}. \quad (5)$$

Equation (4) thus becomes

$$\frac{1}{[1-\beta^2]^{3/2}} \frac{d\beta}{dt} = g. \quad (6)$$

This is easily integrated to obtain

$$\frac{\beta(t)}{[1-\beta^2(t)]^{1/2}} = gt + \frac{\beta(0)}{[1-\beta^2(0)]^{1/2}}, \quad (7)$$

or equivalently,

$$\beta(t)\gamma(t) = gt + \beta(0)\gamma(0) \equiv gt + \alpha_0. \quad (8)$$

This allows us to identify

$$\beta(t) = \frac{gt + \alpha_0}{\sqrt{1 + (gt + \alpha_0)^2}}, \quad (9)$$

$$\gamma(t) = \sqrt{1 + (gt + \alpha_0)^2}. \quad (10)$$

Bear in mind that α_0 is implicitly a function of the injected pion energy. Equation (8) is just the well-known result that a constant accelerating energy gradient causes the momentum to increase linearly with time, i.e., $p(t) = m_\pi gt + p_I$, where $p_I = m_\pi \alpha_0$ is the pion momentum at injection into the accelerator.

B. Decay

With $\gamma(t)$ in hand, we can solve for the pion decay function in the rest frame of the accelerator, viz.,

$$\frac{dN_\pi}{dt} = -\frac{\Gamma}{\gamma(t)} N_\pi(t) = -\frac{\Gamma}{\sqrt{1 + (gt + \alpha_0)^2}} N_\pi(t), \quad (11)$$

where Γ is the decay rate of the pion in its own rest frame. Integration of this equation yields

$$N_\pi(t) = N_\pi(0) \left[\frac{(gt + \alpha_0) + \sqrt{1 + (gt + \alpha_0)^2}}{\alpha_0 + \sqrt{1 + \alpha_0^2}} \right]^{-\Gamma/g}. \quad (12)$$

Since the number of neutrinos is just equal to the number of pions that have decayed, $N_\nu(t) = N_\pi(0) - N_\pi(t)$, it follows immediately that

$$N_\nu(t) = N_\pi(0) \left(1 - \left[\frac{\alpha_0 + \sqrt{1 + \alpha_0^2}}{(gt + \alpha_0) + \sqrt{1 + (gt + \alpha_0)^2}} \right]^{\Gamma/g} \right). \quad (13)$$

C. Distance dependence

We next convert this time dependence into spatial dependence, as a function of distance from the input of the linear accelerator. We can integrate $\beta = dx/dt$ given by Eq. (9) to obtain

$$x(t) = \int_0^t dt' \beta(t') = \frac{\sqrt{1 + (gt + \alpha_0)^2} - \sqrt{1 + \alpha_0^2}}{g}, \quad (14)$$

where we chose $x(0) = 0$. Equation (14) can easily be inverted to obtain

$$t(x) = \frac{[(gx + \sqrt{1 + \alpha_0^2})^2 - 1]^{1/2} - \alpha_0}{g}, \quad (15)$$

and this may be substituted into Eq. (13) to obtain $N_\nu(x)$. Our result for the neutrino flux at a distance x is, therefore,

$$N_\nu(x) = N_\pi(0) \left[1 - \left(\frac{\alpha_0 + \sqrt{1 + \alpha_0^2}}{gx + \sqrt{1 + \alpha_0^2} + [(gx + \sqrt{1 + \alpha_0^2})^2 - 1]^{1/2}} \right)^{\Gamma/g} \right]. \quad (16)$$

Noting that $m_\pi \sqrt{1 + \alpha_0^2} = E_I$, we convert this back to the original variables,

$$N_\nu(x) = N_\pi(0) \left[1 - \left(\frac{p_I + E_I}{gm_\pi x + E_I + \sqrt{[gm_\pi x + E_I]^2 - m_\pi^2}} \right)^{\Gamma/g} \right] = N_\pi(0) \left[1 - \left(\frac{E_I + p_I}{E_x + p_x} \right)^{m\Gamma/G} \right], \quad (17)$$

where E_x and p_x are the energy and momentum of the pion after traveling a distance x through the accelerator.

Figure 1 shows the behavior of $N_\nu(x)$ for two different accelerating gradients and for two different initial pion

kinetic energies (in the laboratory). As expected, the x dependence becomes more and more linear as the injection energy increases. Also, the higher the gradient, the flatter the curves become. The reason for this is that a bigger

FIG. 1. Dependence of $N_\nu(x)$, normalized by the number of input pions, $N_\pi(0)$, as a function of distance along the pion accelerator, x , for two pion kinetic energies spanning the production peak at injection into the accelerator. The solid curves are for a gradient of 10 MeV/m, while the dashed curves are for 50 MeV/m.

gradient means higher-energy pions, which have longer lifetimes (in the laboratory), whence fewer decays into neutrinos during acceleration.

The accelerator designer may therefore want to optimize between the number of neutrinos showing up in the detector (normalized by the number of pions captured in the accelerator) and their ultimate energies. One might also want to consider the possibility of making a pion *decelerator*, if it turns out that lower energy neutrinos have desirable properties for some experiments.

D. Neutrino energy spectrum and angular divergence

The neutrino energies will range over a wide spectrum, but for any specific time from injection of the pion bunch and angle of divergence from the beam axis of a neutrino scattering event in the detector, only two specific neutrino energies are possible. (The neutrino energy E_ν is unique at the laboratory maximum divergence angle of 45° with respect to the beam axis from the decay point, as will be shown below.)

The momentum and energy, E_0 , of a neutrino emitted in the pion rest frame is

$$E_0 = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV}. \quad (18)$$

For a decay where the neutrino is emitted at a polar angle θ_R (in the rest frame) with respect to the pion beam axis, the momentum, polar angle θ_L , and energy E_ν in the lab frame are given by

$$\mathbf{p} = E_0[\gamma(1 + \beta \cos\theta_R), \sin\theta_R], \quad (19)$$

$$\tan\theta_L = \sin\theta_R/[\gamma(1 + \beta \cos\theta_R)], \quad (20)$$

$$E_\nu = E_0\sqrt{\sin^2\theta_R + \gamma^2(1 + \beta \cos\theta_R)^2}, \quad (21)$$

where we show only the longitudinal and transverse momentum components, ignoring the azimuthal angle since the distribution in that angle is uniform.

Neutrinos emitted exactly backward from the beam direction will have lab energies and momenta of $\gamma(1 - \beta)E_0$, i.e., approximately zero for the most relativistic pions, while those emitted precisely forward will have lab energies and momenta of $\gamma(1 + \beta)E_0$. For neutrinos emitted transverse (in the pion rest frame) to the beam direction, the lab energy and momentum will be uniquely $\sqrt{\gamma^2 + 1}E_0$. The maximal divergence angle in the lab frame is always 45° , determined by $\partial \tan\theta_L / \partial \theta_R = 0$, or

$$\cos(\theta_R) = -\beta \quad \text{at } E_\nu = (\sqrt{2}/\gamma)E_0. \quad (22)$$

Neutrinos are emitted between the two extreme energies along a curve in E_ν - θ_L space with two values of energy E_ν for each value of θ_L , as shown in Fig. 2.

By squaring Eq. (20) and solving a quadratic equation, one finds

$$\cos\theta_R = \frac{-\beta\gamma^2\tan^2\theta_L \pm \sqrt{1 - \tan^2\theta_L}}{1 + \beta^2\gamma^2\tan^2\theta_L}. \quad (23)$$

(This has the expected values when θ_L takes values of 0° and 45° .) This formula then allows the neutrino energy E_ν to be determined directly from θ_L , once that has been determined.

FIG. 2. The neutrino energy, E_ν (in MeV), as a function of laboratory polar angle θ_L from decay in the acceleration section at a given time from bunch injection (or position in the accelerator) for three pion kinetic energies at the time of decay. Each pion energy corresponds to a specific location for the decay in the accelerator section. There is a twofold degeneracy of E_ν for each value of θ_L , except at the maximum neutrino angle in the lab frame, 45° , which occurs at a low neutrino energy.

Pions which decay in a drift volume after leaving the accelerator will have the same character as any other decay-in-flight source. Thus, the gradient, if large enough, could provide approximately uniform contributions from every segment of the accelerator. However, since the neutrino energy spectrum, dN_ν/dE_ν , is flat from any specific pion energy, it never rises at high energies.

E. Determining E_ν from event timing

Figure 3 depicts an example of what we have in mind, a pion accelerator which is 10 m long [5]. This is followed by an air-filled drift space (i.e., a decay volume) of 14 m, a 10 m long beam dump, such as earth or iron, and the neutrino detector, presumed, for cost effectiveness, to be a long cylinder aligned with the beam axis [6]. The drift space could also provide space for experiments using the pions which emerge from the accelerator. It also might contain bending magnets to deflect charged particles from the beam axis [7].

Neutrinos all travel at velocity c , which is discernibly larger than the velocity of the pions (up to ≈ 1 GeV pion energy). Thus the time of scattering and the position of the event in the detector are uniquely correlated (up to resolution limits) with the location and hence energy of the pion decay that produced that neutrino and so also the decay angle and energy of the neutrino emitted in the decay, apart from the almost-everywhere two-valuedness of this energy shown in Fig. 2. However, low-energy neutrino cross sections vary as E_ν^2 , and thus the probability for neutrino scattering at the lower energy in this regime is much smaller than that at the higher energy.

We discuss first the case for those higher-energy neutrinos emitted at 0° along the beam direction. Figure 3 shows

how the change in pion velocity along the accelerator, followed by speed-of-light propagation of the neutrino from the decay point, determines the neutrino energy. Note that this figure is for an *unrealistic* injection energy of $T_\pi = 100$ keV, simply to show the principle involved [8]. The idea is to measure the difference between the time when the neutrino scattering event occurs and that for a lightlike signal that begins at the time the pions are injected into the pion accelerator. The neutrino energy E_ν in this on-axis case is given by

$$E_{\nu,\text{on-axis}} = \gamma(t_{\text{dec}})[1 + \beta(t_{\text{dec}})]E_0, \quad (24)$$

where $t_{\text{dec}} = t(x_{\text{dec}})$ is given by Eq. (15) and E_0 by Eq. (18). The value of x_{dec} needed for t_{dec} is obtained by solving the more general expression for the event time, Eq. (25), given below.

Note also that some pions will decay after leaving the accelerator. In such cases, as in the dot-dashed curve in Fig. 3, the timing for the event will be later than the case shown when $x_{\text{decay}} = 10$ m, but E_ν will be the same, i.e., 93.0 MeV in this example.

For *more realistic* injection energies, e.g., $T_\pi = 350$ MeV, the curves in the pion accelerator section are much, much flatter than those shown in Fig. 3. The time differences are quite small but technically feasible. For an injection energy of $T_\pi = 350$ MeV, Table I shows, for two choices of accelerating gradient G , the time differences, in nanoseconds, and the associated energies of the higher-energy neutrino in each case. Note that, to distinguish the two highest energies shown for $G = 50$ MeV/m, one must have timing resolution of about 50 picoseconds.

A similar but more complicated relation occurs for radially off-axis points, where the decay angle must be included in the correlation. (See Fig. 2.) The procedure for determining E_ν from the time the event occurs (relative to the time of injection of the pions into the accelerator) is as follows.

If the position of the event in the detector is given by $(x_{\text{evt}}, \rho_{\text{evt}})$, where x_{evt} is measured from the injection point and ρ_{evt} is the transverse distance from the beam axis, the time of the event is

FIG. 3. Relation between decay time after pion bunch injection and time of scattering event in detector for on-axis ν 's of the highest energy. This is for an *unrealistic* injection energy of $T_\pi = 100$ keV, so that the curvatures before decay are highly exaggerated.

TABLE I. For $T_\pi = 350$ MeV at injection into PILAC. The E_ν here are for the higher of the two neutrino energies shown in Fig. 2.

x_{decay} (m)	$G = 10$ MeV/m		$G = 50$ MeV/m	
	Δt (nsec)	E_ν (MeV)	Δt (nsec)	E_ν (MeV)
0.0	0.000	205	0.000	205
2.0	0.275	214	0.236	249
4.0	0.528	223	0.401	292
6.0	0.762	231	0.522	335
8.0	0.978	240	0.616	379
10.0	1.179	249	0.691	422

$$t_{\text{evt}}(x_{\text{dec}}, x_{\text{evt}}, \rho_{\text{evt}}) = t_{\text{dec}}(x_{\text{dec}}) + t_{\nu}(x_{\text{evt}} - x_{\text{dec}}, \rho_{\text{evt}}), \quad (25)$$

where $t_{\text{dec}}(x)$ is given by Eq. (15) and $t_{\nu}(x, \rho) = \sqrt{x^2 + \rho^2}/c$. Equation (25) can be solved numerically for x_{dec} . The implicit dependence in t_{dec} on the gradient, G , and the kinetic energy T_{π} of the injected pions should be kept in mind.

Given the extracted value for x_{dec} , we can infer that the neutrino was emitted at a laboratory angle of

$$\theta_L = \tan^{-1}[\rho_{\text{evt}}/(x_{\text{evt}} - x_{\text{dec}})]. \quad (26)$$

The pion energy at the time it decays is also determined by x_{dec} . From Eqs. (10) and (15),

$$\gamma_{\text{dec}} = \sqrt{1 + [gt(x_{\text{dec}}) + \alpha_0]^2}, \quad (27)$$

$$\beta_{\text{dec}} = \sqrt{\gamma_{\text{dec}}^2 - 1}/\gamma_{\text{dec}}, \quad (28)$$

$$E_{\pi} = m_{\pi}\gamma_{\text{dec}}. \quad (29)$$

From the values of θ_L , β_{dec} , and γ_{dec} the cosine of the decay angle in the pion rest frame (there are two values) is then found by substituting these values into Eq. (23):

$$\cos\theta_R = \frac{-\beta_{\text{dec}}\gamma_{\text{dec}}^2 \tan^2\theta_L \pm \sqrt{1 - \tan^2\theta_L}}{1 + \beta_{\text{dec}}^2\gamma_{\text{dec}}^2 \tan^2\theta_L}. \quad (30)$$

The neutrino energy E_{ν} is then determined by substituting this value of $\cos\theta_R$ into Eq. (21). The scattering event is overwhelmingly most likely to have occurred for a neutrino with the larger value of E_{ν} , i.e., the positive solution given by the plus sign before the root in Eq. (30).

TABLE II. Extractions of the decay point x_{dec} (in m), neutrino laboratory angle θ_L (in mrad), and E_{ν} (in MeV) for a scattering event taking place at various scattering times Δt in the detector at three values of off-axis radial position ρ_{evt} . This case is for an event occurring in the detector at $x_{\text{evt}} = 40.0$ m from the accelerator injection point. We have assumed here that $T_{\pi} = 350$ MeV at injection into PILAC, as in Table I.

Δt (nsec)	ρ_{evt}	$G = 10$ MeV/m			$G = 50$ MeV/m		
		x_{dec}	θ_L	E_{ν}	x_{dec}	θ_L	E_{ν}
0.250	0.3	1.81	7.9	213	2.15	7.9	252
	0.6	1.72	15.7	212	2.02	15.8	249
	0.9	1.58	23.4	211	1.82	23.6	243
0.500	0.3	3.77	8.3	222	5.58	8.7	326
	0.6	3.67	16.5	222	5.35	17.3	320
	0.9	3.50	24.7	221	4.99	25.7	310
0.750	0.3	5.89	8.9	231	11.9	10.7	460
	0.6	5.77	17.5	230	11.3	20.9	443
	0.9	5.58	26.1	228	10.5	30.6	418

FIG. 4. Relation between the reacting neutrino's energy, E_{ν} , and the time difference between the time of bunch injection and the scattering event time in the detector for a particular event position, here at 40 m from the start of the pion accelerator and at an off-axis distance of 1 m. It shows the differences in E_{ν} and time difference Δt for three different pion decay positions. The kinetic energy of pions at injection has been taken to be 350 MeV and the accelerating gradient is 10 MeV/m. (We have assumed here the higher neutrino energy at this laboratory angle.) Thus, in this example, if the absolute time of this scattering event can be determined relative to the injection time with an accuracy of better than ~ 500 ps, then the energy of the incident neutrino that initiated the event will be determined to an accuracy of better than ~ 15 MeV.

Table II shows the correlations between measured scattering times Δt and off-axis event positions ρ_{evt} and the extracted values for the decay point x_{dec} , laboratory neutrino angle θ_L , and E_{ν} , for the same two accelerating gradients and injection energy used in Table I. Because of the small solid angle subtended by the detector, in this example, the time differences at off-axis points are not much different from those shown in Table I. Thus transverse spatial resolution becomes a significant contributor to determination of the neutrino energy, implicitly demanding a fine-grained detector.

Figure 4 illustrates the correlation for an off-axis detection of a neutrino between the time of the event and the pion decay point and angle of decay (as seen in the lab frame), whence the energy of the reacting neutrino.

III. DISCUSSION

Note that, as shown by Eq. (17), the important scale for the acceleration gradient is set by the lifetime of the pion in units of pion mass, viz.,

$$G = m_{\pi}\Gamma_{\pi}/c = 17.9 \text{ MeV/m}. \quad (31)$$

Gradients exceeding this value are now routinely available, which makes possible very tightly focused neutrino beams and unusual neutrino energy spectral distributions.

The higher the gradient can be made, the more pions survive to the exit energy and the neutrino spectrum becomes more and more weighted to higher energies. The higher-energy component of the neutrino beam also becomes more focused, projecting a smaller image on the target.

The pion particle flux expected from the design in Ref. [2] was 10^9 s^{-1} at the target end of the accelerator. With the gradient given above, the accelerator would be on the order of 5 lifetimes in length. Thus, while the neutrino flux at the highest energy available would be on the order of 10^9 s^{-1} , the total flux over the entire spectrum would be on the order of 10^{11} s^{-1} (since nearly every injected pion will decay into a neutrino). This certainly presents a challenge to experimenters to design detectors with a useful event rate, as the flux is not as large as produced by simple beam-target-decay volume sources.

An additional order of magnitude in flux should be available if the PILAC system were to be mounted at the Spallation Neutron Source at Oak Ridge National Laboratory.

ACKNOWLEDGMENTS

We thank C. Morris for a valuable conversation. This work was carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396.

-
- [1] S. Geer, Phys. Rev. D **57**, 6989 (1998). Subsequent development of the neutrino factory idea includes work, for example, by A. Bandyopadhyay *et al.*, arXiv:0710.4947 and by C. H. Albright *et al.*, arXiv/physics/0411123. See also references therein.
 - [2] Barbara Blind, Report No. LA-UR-04-3797, provides a useful, brief synopsis. Earlier published work includes H. A. Thiessen and D. Hywel White, Z. Phys. C **56**, S285 (1992); H. A. Thiessen, Nucl. Phys. A **547**, 339 (1992); B. Blind, Particle Accelerator Conference, 1991, pp. 899–901; S. Nath, G. Swain, R. Garnett, and T. P. Wangler, OSTI 6541038, 1990.
 - [3] See, e.g., B. Aune *et al.*, Phys. Rev. ST Accel. Beams **3**, 092001 (2000); W. D. Moeller, The Performance of the 1.3 GHz Superconducting RF Cavities in the First Module of the TESLA Test Facility Line, XIX International Linear Accelerator Conference, Chicago, Illinois, USA, 1998; M. Ono *et al.*, in Proceedings of the 8th RF Superconducting Workshop, Padova, 1997.
 - [4] In the examples given in the tables below, we will use two values of gradients, 10 and 50 MeV/m. The latter value is rather speculative, but might be achievable in the future. In this regard, for a FODO linac rather than the continuous gradient design we assume here (for simplicity of presentation), the average gradient will be less than that of a given rf cavity. Also, the gradient achievable could depend upon on aperture required to match the emittance of the pion beam being accelerated. Our only intention, in showing the 50 MeV/m case, is to indicate the effects of achieving high gradients. As noted in the Introduction, even at lower gradients, the design has a significant value in affording the determination of the energy of the incident neutrino event by event.
 - [5] The PILAC design [2] was for a rather longer accelerator, 102.9 m, but this was to get the highest possible energies for the pions. Also, the accelerating gradient in that design was only 12.5 MeV/m, befitting the technology available at that time.
 - [6] Any realistic detector proposal could only be made after a realistic accelerator design has been developed. Our hope is that a highly instrumented device, long but with a relatively small transverse size, might be feasible.
 - [7] Without such a bend, muons from pion decays might otherwise penetrate the beam dump and give background events in the neutrino detector which follows. The background from neutrons in the beam direction should be small. Note that the injection “snake” severely limits the pions in the linac in both momentum and charge, so opposite-sign pions and muons should be totally negligible.
 - [8] Details dependent on realistic beam dispersion and timing errors will blur these lines.