Resistive-wall impedance of two-layer tube

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In contrast to electron machines, beam particles travel in most proton or heavy ion circular accelerators with nonrelativistic speed. The resistive-wall impedance of the cylindrical two-layer tube with finite thickness is studied in this paper for any material in the nonrelativistic case. The nonrelativistic correction for a special case of metallic-type layers is given afterward. Finally, we derive the general expression of the resistive-wall impedance for the multilayer tubes.

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I. INTRODUCTION

Most modern proton or heavy ion accelerators are run or designed in nonrelativistic or moderate relativistic regions with high beam power. The impedance of vacuum chamber plays an important role for the stability of the circulating beams. As we all know, the wakefield generated by a beam of velocity v smaller than the speed of light c is different from that of the relativistic beam, and hence the impedance. Therefore the study of impedance and wakefield for the nonrelativistic beam is an important issue.

The resistive-wall impedance of a vacuum chamber for $v < c$ has been derived for a single layer tube with infinite [1,2] and finite wall thickness [3,4]. In Refs. [5–7], Ivanyan studied the longitudinal and transverse impedance for the two-layer tubes, and emphasized particularly on the relativistic beam impedance. Zotter and Kheifets studied the impedances and wakes of uniform charged beams in multilayer pipes [8].

In this paper, we will apply the field matching method to derive the general expression of the longitudinal and transverse impedances of a cylindrical two-layer tube. Then a special case of metallic-type layers is considered. The corresponding nonrelativistic corrections will also be derived. Finally, we will give a general expression for the multilayer tube with a finite thickness and arbitrary material.

II. LONGITUDINAL IMPEDANCE OF TWO-LAYER TUBE

The definition of impedance and wake function for the ultrarelativistic beam has been introduced in [8,9]. For the beam with an arbitrary velocity, we will use the generalized concept of wake and impedance. In the following, we will first derive the longitudinal impedance per unit length for the two-layer tube, and then the general result for the arbitrary multilayer will be given.

Let us consider a beam of line charge density $\lambda_b e^{iks-i\omega t}$, where ω is angular frequency of beam and $k = \omega/v$, moving with a velocity v along the axis of the beam pipe with an offset a_1 . By using the definition of $z = (s - vt)$, every quantity V is dependent on s and t as $V = \tilde{V} e^{ikz}$. The beam can be represented by a ring beam of radius a_1 , whose charge density can be obtained from Eq. (1.7) in Ref. [9]:

$$\rho_0(\vec{r}, z; t) = \frac{\lambda_b}{2\pi a_1} \delta(r - a_1) e^{ikz}, \quad (1)$$

and the corresponding axial current density

$$\vec{j}_0(\vec{r}, z; t) = \frac{\lambda_b v}{2\pi a_1} \delta(r - a_1) e^{ikz} \hat{s}. \quad (2)$$

Here we have adopted a cylindrical coordinate system (r, φ, s) , where r , φ , and s are radial, azimuthal, and longitudinal coordinates, respectively.

The beam pipe can be divided into four regions as sketched in Fig. 1:

FIG. 1. Side view of the beam pipe.

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$$\begin{cases} \text{(I) vacuum} & a_1 < r < a_2, \\ \text{(II) 1st layer} & a_2 < r < a_3, \\ \text{(III) 2nd layer} & a_3 < r < a_4, \\ \text{(IV) vacuum} & r > a_4, \end{cases}$$

where a_i ($i = 1, 2, 3$, and 4) are the radii of different layers.

Similar to that in Ref. [1], we use the wave equation of the scalar potential ϕ and magnetic vector potential $\vec{A}$ as the starting point of our calculation:

$$-\Delta \vec{A} + \frac{\mu_r \varepsilon_r}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} + \mu_0 \mu_r \sigma \frac{\partial \vec{A}}{\partial t} = \mu_0 \mu_r (\vec{j} - \sigma \vec{E}), \quad (3)$$

$$-\Delta \phi + \frac{\mu_r \varepsilon_r}{c^2} \frac{\partial^2 \phi}{\partial t^2} + \mu_0 \mu_r \sigma \frac{\partial \phi}{\partial t} = \frac{\rho}{\varepsilon_0 \varepsilon_r}, \quad (4)$$

where ε_r is the relative dielectric constant, μ_r the relative magnetic permeability, σ the conductivity for arbitrary medium, $\vec{j}$ the current density, ρ the charge density, and E the electric field. ε_0 and μ_0 are the dielectric constant and magnetic permeability of vacuum, respectively.

The relation of the electric and magnetic potentials is given by the Lorentz condition

$$\left(\vec{\nabla} \cdot \vec{A} + \frac{\mu_r \varepsilon_r}{c^2} \frac{\partial \phi}{\partial t} + \mu_0 \mu_r \sigma \phi \right) = 0. \quad (5)$$

Let us consider that $\vec{j} = 0$ and $\sigma = 0$ in the vacuum or insulated material, and Ohm's law $\vec{j} = \sigma \vec{E}$ exists in a medium with $\sigma \neq 0$. Additionally, ρ is zero in the region outside the beam. Then the right-hand sides of Eqs. (3) and (4) are all zeros except at $r = a_1$. Because of the symmetry of the structure, the equation above can be simplified to

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{A}_s}{\partial r} \right) - (k^2 - \mu_r \varepsilon_r k_0^2 - i\omega \mu_0 \mu_r \sigma) \tilde{A}_s &= 0, \\ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{\phi}}{\partial r} \right) - (k^2 - \mu_r \varepsilon_r k_0^2 - i\omega \mu_0 \mu_r \sigma) \tilde{\phi} &= 0, \\ ik \tilde{A}_s - i\omega \frac{\mu_r \varepsilon_r}{c^2} \tilde{\phi} + \mu_0 \mu_r \sigma \tilde{\phi} &= 0. \end{aligned} \quad (6)$$

By introducing

$$\lambda^2 = k^2 - \mu_r \varepsilon_r k_0^2 - i\omega \mu_0 \mu_r \sigma, \quad (7)$$

and the complex dielectric constant

$$\varepsilon' = \varepsilon_r \varepsilon_0 + i\sigma/\omega, \quad (8)$$

the general solution of the wave equations is given by

$$\tilde{A}_s = p I_0(\lambda r) + q K_0(\lambda r), \quad \tilde{\phi} = \frac{k}{\omega \mu \varepsilon'} \tilde{A}_s. \quad (9)$$

Here p and q are unknown coefficients to be solved. This gives the nonvanishing magnetic and electric field

$$B_\varphi = [-p \lambda I_1(\lambda r) + q \lambda K_1(\lambda r)] e^{ikz},$$

$$E_s = i\omega \frac{\lambda^2}{\lambda^2 - k^2} [p I_0(\lambda r) + q K_0(\lambda r)] e^{ikz}, \quad (10)$$

$$E_r = \omega \frac{\lambda k}{\lambda^2 - k^2} [p I_1(\lambda r) - q K_1(\lambda r)] e^{ikz},$$

where I_0 and I_1 are the modified Bessel functions of the first kind, and K_0 and K_1 are the modified Bessel functions of the second kind.

According to the wakefield expressions above, the tangential field can be written in the form of matrix

$$\begin{pmatrix} \tilde{H}_\varphi \\ \tilde{E}_s \end{pmatrix}_i = F(\lambda_i r) \begin{pmatrix} p \\ q \end{pmatrix}_i, \quad (11)$$

where i is the index of each layer of the chamber,

$$F(\lambda_i r) = \begin{pmatrix} -b_i I_1(\lambda_i r) & b_i K_1(\lambda_i r) \\ c_i I_0(\lambda_i r) & c_i K_0(\lambda_i r) \end{pmatrix}, \quad (12)$$

and

$$b_i = \lambda_i / \mu_i, \quad c_i = i\omega \frac{\lambda_i^2}{\lambda_i^2 - k^2}. \quad (13)$$

In order to obtain the unknown coefficients, we use the matching conditions of tangential field at the transition boundaries

$$\begin{pmatrix} \tilde{H}_\varphi \\ \tilde{E}_s \end{pmatrix}_i = \begin{pmatrix} \tilde{H}_\varphi \\ \tilde{E}_s \end{pmatrix}_{i+1}, \quad (14)$$

i.e.,

$$F(\lambda_i a_{i+1}) \begin{pmatrix} p \\ q \end{pmatrix}_i = F(\lambda_{i+1} a_{i+1}) \begin{pmatrix} p \\ q \end{pmatrix}_{i+1}. \quad (15)$$

Then we can derive that

$$\begin{pmatrix} p \\ q \end{pmatrix}_1 = \prod_{i=1}^3 F^{-1}(\lambda_i a_{i+1}) F(\lambda_{i+1} a_{i+1}) \begin{pmatrix} p \\ q \end{pmatrix}_4, \quad (16)$$

where

$$F^{-1}(\lambda_i r) = -\frac{\lambda_i r}{b_i c_i} \begin{pmatrix} c_i K_0(\lambda_i r) & -b_i K_1(\lambda_i r) \\ -c_i I_0(\lambda_i r) & -b_i I_1(\lambda_i r) \end{pmatrix}. \quad (17)$$

As the Bessel functions I_0 and I_1 diverge with increasing r , we have $p_4 = 0$. Additionally, the parameter q_1 is determined by the source boundary. In the beam region, we have $\lambda = k_r = \omega/(\gamma\beta c)$, and the wave equation for the vector potential $\tilde{A}_s$ satisfies

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{A}_s}{\partial r} \right) - k_r^2 \tilde{A}_s = -\mu_0 \tilde{j}_0. \quad (18)$$

In order to solve the equation, we first derive the solution for beam source in the free space. Concerning that $\tilde{A}_s$ is continuous at $r = a_1$, the solution can be written as

$$\tilde{A}_s = \tilde{A}_{s0} \begin{cases} I_0(k_r r) K_0(k_r a_1), & r \leq a_1 \\ I_0(k_r a_1) K_0(k_r r), & r \geq a_1, \end{cases} \quad (19)$$

where $\tilde{A}_{s0}$ is a coefficient to be determined. Notice that $I_0(k_r r)$ diverges with increasing r and $K_0(k_r r)$ diverges as r goes to zero. By integrating Eq. (18) over r from $r = a_1 - \delta$ to $r = a_1 + \delta$, where the parameter δ is infinitesimal, the left-hand side (lhs) of the integration turns to be $2\pi \tilde{A}_{s0} k_r a_1 [-I_0(k_r a_1) K_1(k_r a_1) - I_1(k_r a_1) K_0(k_r a_1)]$, and the right-hand side (rhs) to be $-\mu_0 \lambda_b v$. With the property of Bessel functions [10], the parenthesis on the lhs is $1/k_r a_1$, and thus we can obtain

$$\tilde{A}_{s0} = \frac{\mu_0 \lambda_b v}{2\pi}. \quad (20)$$

So in the region of $r \geq a_1$, we have

$$q_1 = \frac{\mu_0 \lambda_b v}{2\pi} I_0(k_r a_1), \quad \tilde{A}_s = \frac{\mu_0 \lambda_b v}{2\pi} I_0(k_r a_1) K_0(k_r r). \quad (21)$$

It should be mentioned that the expression of $\tilde{A}_s$ above is only applicable for the beam source in free space. When we impose the beam pipe boundary, a term proportional to $I_0(k_r r)$ should be added, as shown in Eq. (9).

So the only unknown coefficients are p_1 and p_4 . Then we can obtain the ratio of p_1 and q_1 (see Appendix A), which is

$$\frac{p_1}{q_1} = \frac{\kappa M K_1(k_r a_2) - K_0(k_r a_2)}{\kappa M I_1(k_r a_2) + I_0(k_r a_2)} \quad (22)$$

with

$$\kappa = -\frac{b_1 c_2}{c_1 b_2}, \quad (23)$$

$$M = \frac{(S_{2,3} S_{3,1} + S_{2,4} S_{3,3}) K_1(k_r a_4) - (S_{2,3} S_{3,2} + S_{2,4} S_{3,4}) K_0(k_r a_4)}{(S_{2,1} S_{3,1} + S_{2,2} S_{3,3}) K_1(k_r a_4) - (S_{2,1} S_{3,2} + S_{2,2} S_{3,4}) K_0(k_r a_4)}, \quad (24)$$

and

$$\begin{aligned} S_{i,1} &= c_i b_{i+1} [I_1(\lambda_i a_i) K_0(\lambda_i a_{i+1}) + K_1(\lambda_i a_i) I_0(\lambda_i a_{i+1})], \\ S_{i,2} &= b_i c_{i+1} [I_1(\lambda_i a_i) K_1(\lambda_i a_{i+1}) - K_1(\lambda_i a_i) I_1(\lambda_i a_{i+1})], \\ S_{i,3} &= c_i b_{i+1} [I_0(\lambda_i a_i) K_0(\lambda_i a_{i+1}) - K_0(\lambda_i a_i) I_0(\lambda_i a_{i+1})], \\ S_{i,4} &= b_i c_{i+1} [I_0(\lambda_i a_i) K_1(\lambda_i a_{i+1}) + K_0(\lambda_i a_i) I_1(\lambda_i a_{i+1})]. \end{aligned} \quad (25)$$

Here, $i = 2$ and 3 . The vector potential is

$$\begin{aligned} \tilde{A}_s &= \frac{\mu_0 \lambda_b v}{2\pi} I_0(k_r a_1) \left[K_0(k_r r) + I_0(k_r r) \right. \\ &\quad \times \left. \frac{\kappa M K_1(k_r a_2) - K_0(k_r a_2)}{\kappa M I_1(k_r a_2) + I_0(k_r a_2)} \right]. \end{aligned} \quad (26)$$

Following Eq. (2.64) in [9] and $J_0 = \lambda_b v$, the longitudinal impedance per unit length becomes

$$\begin{aligned} Z_{\parallel}(\omega) &= \frac{i Z_0 c k_r^2}{2\pi \omega} I_0(k_r a_1) \left[K_0(k_r r) + I_0(k_r r) \right. \\ &\quad \times \left. \frac{\kappa M K_1(k_r a_2) - K_0(k_r a_2)}{\kappa M I_1(k_r a_2) + I_0(k_r a_2)} \right], \end{aligned} \quad (27)$$

where $Z_0 = \mu_0 c \approx 377 \Omega$ is the impedance of free space.

Until now, we have obtained the exact expression of the longitudinal impedance of a two-layer tube. It has a similar form as that obtained by Zimmermann and Oide for single layer beam pipe with infinite thickness. As we will see later, the longitudinal impedance of any multilayer tubes

can be written in the form of Eq. (27), except for the different expression of M .

It is easy to show that, for the point beam source moving on-axis of the beam pipe, all the corresponding results can be obtained by removing the factor of $I_0(k_r a_1)$.

III. TRANSVERSE IMPEDANCE OF TWO-LAYER TUBE

In the previous section we obtained the longitudinal impedance of monopole mode with the method of field matching. The transverse impedance can be obtained using the Panofsky-Wenzel theorem [11], i.e., one has to obtain the longitudinal impedance of the dipole mode first.

Considering a beam source with dipole line density of $\lambda_b d e^{ikz}$, the equivalent charge and current density can be expressed as

$$\rho_1(\vec{r}, z; t) = \frac{\lambda_b d}{\pi a_1^2} \delta(r - a_1) \cos \varphi e^{ikz}, \quad (28)$$

$$\vec{j}_1(\vec{r}, z; t) = \beta c \rho_1 \hat{s}. \quad (29)$$

Here we suppose $d = 2a_1$ is the distance of the charges in the dipole.

First, we derive the dipole electromagnetic field generated by the beam source described above in free space. For the dipole moment, the wave equation for the vector potential is [see Eq. (67) in Ref. [1]]

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{A}_s}{\partial r} \right) - \left(\frac{1}{r^2} + k_r^2 \right) \tilde{A}_s = -\mu_0 \tilde{j}_s. \quad (30)$$

Done with the same analyses as the longitudinal case, the solution of $\tilde{A}_s$ can be written as

$$\tilde{A}_s = \tilde{A}_{s1} \cos \varphi \begin{cases} I_1(k_r r) K_1(k_r a_1), & r \leq a_1 \\ I_1(k_r a_1) K_1(k_r r), & r \geq a_1, \end{cases} \quad (31)$$

where $\tilde{A}_{s1}$ is an unknown coefficient. Integrating Eq. (30) over r from $r = a_1 - \delta$ to $r = a_1 + \delta$, we can find the lhs of the equation turns to

$$\text{lhs} = \tilde{A}_{s1} k_r a_1 (I_1(k_r a_1) K_1'(k_r a_1) - I_1'(k_r a_1) K_1(k_r a_1)) \cos \varphi, \quad (32)$$

and the rhs is

$$\text{rhs} = -\frac{\mu_0}{\pi a_1} \lambda_b dv \cos \varphi. \quad (33)$$

So we can obtain the coefficient $\tilde{A}_{s1}$, which is

$$\tilde{A}_{s1} = \frac{\mu_0}{\pi a_1} \lambda_b dv, \quad (34)$$

and the vector potential in the region $r \geq a_1$ is

$$\tilde{A}_s = \frac{\mu_0}{\pi a_1} \lambda_b dv \cos \varphi I_1(k_r a_1) K_1(k_r r). \quad (35)$$

Here the properties of Bessel functions have also been used. Similar to the longitudinal case, the solutions correspond to the beam source in free space, and the solution with beam pipe boundary should be added by a term of $I_1(k_r r)$.

We obtain the expressions for the field components E_s , E_φ , B_φ , and B_s by repeating the calculation process in Ref. [1], except that there is a factor of 1/2 missing in the expressions A_r and A_φ of Eq. (82) in Ref. [1] and the argument of Bessel functions is replaced by $\lambda_i r$. The electric and magnetic field strengths can be written as

$$\begin{aligned} B_\varphi &= \left(\frac{ik}{2} [p_+(I_2 - I_0) + q_+(K_2 - K_0)] \right. \\ &\quad \left. - \frac{\lambda}{2} [p_s(I_0 + I_2) - q_s(K_0 + K_2)] \right) \cos \varphi e^{ikz}, \\ B_s &= \lambda(p_+ I_1 - q_+ K_1) \sin \varphi e^{ikz}, \\ E_\varphi &= \left[\frac{i\omega}{2} [p_+(I_2 + I_0) + q_+(K_2 + K_0)] \right. \\ &\quad \left. + \frac{1}{r} \frac{k\omega}{k^2 - \lambda^2} (p_s I_1 + q_s K_1) \right] \sin \varphi e^{ikz}, \\ E_s &= -\frac{i\omega\lambda^2}{k^2 - \lambda^2} (p_s I_1 + q_s K_1) \cos \varphi e^{ikz}. \end{aligned} \quad (36)$$

Here we use a gauge transformation combining with the Lorentz condition. The gauge function is taken to be $\chi = [(p_+ + p_-)I_1(\lambda r) + (q_+ + q_-)K_1(\lambda r)]/\lambda e^{ikz} \cos \varphi$, and the following restrictions have been used:

$$\begin{aligned} p_+ &= -p_-, & q_+ &= -q_-, \\ p_s &= \frac{k^2 - \lambda^2}{k\omega} p_0, & q_s &= \frac{k^2 - \lambda^2}{k\omega} q_0. \end{aligned} \quad (37)$$

So the unknown coefficients in the wakefield expressions are determined by the following matching conditions:

$$G(\lambda_i a_{i+1}) \begin{pmatrix} p_+ \\ p_s \\ q_+ \\ q_s \end{pmatrix}_i = G(\lambda_{i+1} a_{i+1}) \begin{pmatrix} p_+ \\ p_s \\ q_+ \\ q_s \end{pmatrix}_{i+1}, \quad (38)$$

where $G(\lambda r)$ is given by

$$G(\lambda r) = \begin{pmatrix} -\frac{ik}{\mu\lambda r} I_1 & -\frac{\lambda}{\mu} I_1' & \frac{ik}{\mu\lambda r} K_1 & -\frac{\lambda}{\mu} K_1' \\ \frac{\lambda}{\mu} I_1 & 0 & -\frac{\lambda}{\mu} K_1 & 0 \\ i\omega I_1' & \frac{1}{r} \frac{k\omega}{k^2 - \lambda^2} I_1 & -i\omega K_1' & \frac{1}{r} \frac{k\omega}{k^2 - \lambda^2} K_1 \\ 0 & -\frac{i\omega\lambda^2}{k^2 - \lambda^2} I_1 & 0 & -\frac{i\omega\lambda^2}{k^2 - \lambda^2} K_1 \end{pmatrix}, \quad (39)$$

and I' and K' denote the derivation of the modified Bessel functions, which have the argument of λr .

According to the source boundary presented at the beginning of this section, we have

$$\begin{aligned} q_{s1} &= \frac{\mu_0}{\pi a_1} \lambda_b dv I_1(k_r a_1), & q_{01} &= \frac{c^2 k}{\omega} q_{s1}, \\ q_{+1} &= 0, & q_{-1} &= 0. \end{aligned} \quad (40)$$

The convergence of Bessel function $I(\lambda r)$ gives $p_{04} = p_{+4} = 0$. Applying the conditions above to Eq. (38), we have

$$\begin{pmatrix} p_+ \\ p_s \\ 0 \\ q_s \end{pmatrix}_1 = \prod_{i=1}^3 G^{-1}(\lambda_i a_{i+1}) G(\lambda_{i+1} a_{i+1}) \begin{pmatrix} 0 \\ 0 \\ q_+ \\ q_s \end{pmatrix}_4, \quad (41)$$

where G^{-1} is the inverse matrix of G , and

$$G^{-1}(\lambda r) = \begin{pmatrix} 0 & -\mu r K_1' & -\frac{i\lambda r}{\omega} K_1 & -\frac{k}{\lambda\omega} K_1 \\ -\mu r K_1 & -\frac{i\mu k}{\lambda^2} K_1 & 0 & -\frac{i(k^2 - \lambda^2)r}{\lambda\omega} K_1' \\ 0 & -\mu r I_1' & -\frac{i\lambda r}{\omega} I_1 & -\frac{k}{\lambda\omega} I_1 \\ \mu r I_1 & \frac{i\mu k}{\lambda^2} I_1 & 0 & \frac{i(k^2 - \lambda^2)r}{\lambda\omega} I_1' \end{pmatrix}. \quad (42)$$

In deriving the inverse matrix of G , we have used the property of Bessel functions [10]:

$$I_p(x)K_{p+1}(x) + I_{p+1}(x)K_p(x) = \frac{1}{x}. \quad (43)$$

For simplicity, we express the 4×4 matrices of G and G^{-1} in the 2×2 block form as

$$G(\lambda r) = \begin{pmatrix} A(\lambda r) & B(\lambda r) \\ C(\lambda r) & D(\lambda r) \end{pmatrix}, \quad (44)$$

$$G^{-1}(\lambda r) = \begin{pmatrix} \xi B^*(\lambda r) & \zeta D^*(\lambda r) \\ \xi A^*(\lambda r) & \zeta C^*(\lambda r) \end{pmatrix},$$

where

$$A(\lambda r) = \begin{pmatrix} -\frac{ikI_1}{\mu\lambda r} & -\frac{\lambda I_1'}{\mu} \\ \frac{\lambda I_1}{\mu} & 0 \end{pmatrix},$$

$$B(\lambda r) = \begin{pmatrix} \frac{ikK_1}{\mu\lambda r} & -\frac{\lambda K_1'}{\mu} \\ -\frac{\lambda K_1}{\mu} & 0 \end{pmatrix}, \quad (45)$$

$$C(\lambda r) = \begin{pmatrix} i\omega I_1' & \frac{1}{r} \frac{k\omega I_1}{k^2 - \lambda^2} \\ 0 & -\frac{i\omega\lambda^2 I_1}{k^2 - \lambda^2} \end{pmatrix},$$

$$D(\lambda r) = \begin{pmatrix} -i\omega K_1' & \frac{1}{r} \frac{k\omega K_1}{k^2 - \lambda^2} \\ 0 & -\frac{i\omega\lambda^2 K_1}{k^2 - \lambda^2} \end{pmatrix},$$

and $A^* = \det(A)A^{-1}$, $B^* = \det(B)B^{-1}$, $C^* = \det(C)C^{-1}$, $D^* = \det(D)D^{-1}$, $\xi = -\mu^2 r/\lambda$, $\zeta = (k^2 - \lambda^2)r/(\omega^2 \lambda)$.

Then Eq. (41) turns to be

$$\begin{pmatrix} p_+ \\ p_s \\ 0 \\ q_s \end{pmatrix}_1 = \begin{pmatrix} \xi_1 B^*(k_r a_2) & \zeta_1 D^*(k_r a_2) \\ \xi_1 A^*(k_r a_2) & \zeta_1 C^*(k_r a_2) \end{pmatrix} \begin{pmatrix} L_{11}^{(2)} & L_{12}^{(2)} \\ L_{21}^{(2)} & L_{22}^{(2)} \end{pmatrix} \\ \times \begin{pmatrix} L_{11}^{(3)} & L_{12}^{(3)} \\ L_{21}^{(3)} & L_{22}^{(3)} \end{pmatrix} \begin{pmatrix} A(k_r a_4) & B(k_r a_4) \\ C(k_r a_4) & D(k_r a_4) \end{pmatrix} \\ \times \begin{pmatrix} 0 \\ 0 \\ q_+ \\ q_s \end{pmatrix}_4, \quad (46)$$

where matrices (L_{ij}) 's are defined by Eq. (B4). The solution will be (see Appendix B)

$$\begin{pmatrix} p_+ \\ p_s \end{pmatrix}_1 = [\xi_1 B^*(k_r a_2)N_1 + \zeta_1 D^*(k_r a_2)N_2] \\ \times [\xi_1 A^*(k_r a_2)N_1 + \zeta_1 C^*(k_r a_2)N_2]^{-1} \begin{pmatrix} 0 \\ q_s \end{pmatrix}_1, \quad (47)$$

and

$$N_1 = \begin{pmatrix} L_{11}^{(2)} & L_{12}^{(2)} \\ L_{21}^{(2)} & L_{22}^{(2)} \end{pmatrix} \begin{pmatrix} L_{11}^{(3)} & L_{12}^{(3)} \\ L_{21}^{(3)} & L_{22}^{(3)} \end{pmatrix} \begin{pmatrix} B(k_r a_4) \\ D(k_r a_4) \end{pmatrix},$$

$$N_2 = \begin{pmatrix} L_{21}^{(2)} & L_{22}^{(2)} \\ L_{11}^{(2)} & L_{12}^{(2)} \end{pmatrix} \begin{pmatrix} L_{11}^{(3)} & L_{12}^{(3)} \\ L_{21}^{(3)} & L_{22}^{(3)} \end{pmatrix} \begin{pmatrix} B(k_r a_4) \\ D(k_r a_4) \end{pmatrix}. \quad (48)$$

By expanding the matrices in Eq. (47), we can easily get the ratio of p_{s1}/q_{s1} , which will be a very complex expression. According to Eq. (36), one can obtain the longitudinal impedance of dipole mode as

$$Z_{\parallel}^{(1)}(\omega) = -\frac{E_s}{d\lambda_b v r \cos \varphi e^{ikz}} \\ = \frac{q_{s1}}{d\lambda_b v r} \frac{ic^2 k_r^2}{\omega} \left(\frac{p_{s1}}{q_{s1}} I_1(k_r r) + K_1(k_r r) \right). \quad (49)$$

The transverse impedance can be derived from the Panofsky-Wenzel theorem.

Similar to the longitudinal treatments, we can get the results for the point beam source on-axis by expanding the factor of $I_1(k_r a_1)$ as $\frac{k_r a_1}{2} (1 + \frac{k_r^2 a_1^2}{8} + \dots)$.

IV. SPECIAL CASE OF METALLIC LAYERS

For metallic layers of a resistive-wall impedance, it is reasonable to take the conception that the beam aperture is much larger than the skin depth $\delta_{2,3}$, which means $|\lambda_2 r| \gg 1$ and $|\lambda_3 r| \gg 1$. So we can assume that

$$\lambda_2 \approx \lambda_{20} \approx \frac{1 - i \operatorname{sgn}(\omega)}{\delta_2}, \quad (50)$$

$$\lambda_3 \approx \lambda_{30} \approx \frac{1 - i \operatorname{sgn}(\omega)}{\delta_3},$$

and the Bessel functions can be approximated by [10]

$$K_p(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x} \quad (p \geq 0, x \rightarrow \infty),$$

$$I_p(x) \sim \frac{1}{\sqrt{2\pi x}} e^x \quad (p \geq 0, x \rightarrow \infty). \quad (51)$$

First, we take the above approximations to the longitudinal impedance expressed as Eq. (27), and the parameters κ and M can be simplified as

$$\kappa = -\frac{\lambda_2}{k_r \mu_0 \epsilon_2' c^2}, \quad (52)$$

and

$$M = -\frac{\varepsilon_0 \lambda_3 [\alpha \tanh(x_1) + \tanh(x_2)] K_1(k_r a_4) + \varepsilon'_3 k_r [\alpha \tanh(x_1) \tanh(x_2) + 1] K_0(k_r a_4)}{\varepsilon_0 \lambda_3 [\alpha + \tanh(x_1) \tanh(x_2)] K_1(k_r a_4) + \varepsilon'_3 k_r [\alpha \tanh(x_2) + \tanh(x_1)] K_0(k_r a_4)}, \quad (53)$$

where $\alpha = (\lambda_2 \varepsilon'_3) / (\varepsilon'_2 \lambda_3) \approx (\delta_2 \mu_2) / (\delta_3 \mu_3)$, $x_1 = \lambda_2(a_3 - a_2)$, and $x_2 = \lambda_3(a_4 - a_3)$.

For $k_r r \ll 1$, we simplify $Z_{\parallel}(\omega)$ to the first order of $\delta_{\text{skin}} \omega / c$ and the second order of k_r / k_0 , keeping the term $\delta_{\text{skin}} k_r^2$, the longitudinal impedance is then

$$Z_{\parallel}(\omega) = \frac{Z_0}{2\pi c} \left[-\frac{ic^2 k_r^2}{\omega} \ln\left(\frac{r}{a_2}\right) - \frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \right. \\ \left. \times \left(1 - \frac{k_r^2(2a_2^2 - r^2 - a_1^2)}{4}\right) M \right]. \quad (54)$$

The first term in the parentheses of the rhs of Eq. (54) is the space charge impedance, which is the same as that in Ref. [1]. The second one is supposed to be the resistive-wall impedance of the two-layer tube, i.e.,

$$Z_{\parallel, rw}(\omega) = \frac{Z_0}{2\pi c} \left[-\frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \right. \\ \left. \times \left(1 - \frac{k_r^2(2a_2^2 - r^2 - a_1^2)}{4}\right) M \right]. \quad (55)$$

For the ultrarelativistic case with $v = c$ and $k_r \rightarrow 0$, the above expression turns to be

$$Z_{\parallel, rw}(\omega) = \frac{Z_0}{2\pi c} \left[\frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \right. \\ \left. \times \frac{\alpha \tanh(x_1) + \tanh(x_2)}{\alpha + \tanh(x_1) \tanh(x_2)} \right], \quad (56)$$

which is consistent with the result obtained in Ref. [5] if we assume that the first term of Eq. (19) in Ref. [5] is much smaller than the second one, which is always correct for metal layers. For the single layer tube with infinite thickness, i.e., $\lambda_2 = \lambda_3$, $\varepsilon'_2 = \varepsilon'_3$, and $a_4 \rightarrow \infty$, Eq. (55) is reduced to Eq. (57) in Ref. [1] if we have $a_1 \rightarrow 0$ for the on-axis beam source. It also demonstrates that, for the same zeroth moment of the beam, the impedance is independent of the detailed shape of the beam distribution in the ultrarelativistic limit [9], but this is no longer true for the nonrelativistic or moderate relativistic beam.

Correspondingly, in order to obtain the transverse impedance, we first derive the longitudinal impedance of the dipole mode, which can be obtained by substituting Eqs. (50) and (51) into Eq. (49). The result is much more complicated than that of the monopole mode. Through further expanding the expression to the first order of $\delta_{\text{skin}} \omega / c$ and the second order of k_r / k_0 , and for the nonrelativistic case we always have $\delta_{\text{skin}} < k_r^2$, the longitudinal electric field becomes

$$E_s = \frac{ic^2 k_r^2 q_s}{\omega} \left(\frac{(r^2 - a_2^2)}{k_r a_2^2 r} + \delta_2 \frac{1 + i \text{sgn}(\omega)}{2} \frac{\omega^2 \mu_2 r}{2a_2^3 c^2 k_r^3 \mu_0} \right. \\ \left. \times \left(4 - k_r^2 a_2^2 + \frac{k_r^2 r^2}{2}\right) \frac{1 + \alpha \tanh(x_1) \tanh(x_2)}{\alpha \tanh(x_2) + \tanh(x_1)} \right) \\ \times \cos \varphi e^{ikz}. \quad (57)$$

The longitudinal impedance of the dipole mode is then

$$Z_{\parallel}^{(1)}(\omega) = -i \frac{Z_0}{c} \left(\frac{c^2 k_r^2 (r^2 - a_2^2)}{2\pi a_2^2 \omega r^2} + \delta_2 \frac{1 + i \text{sgn}(\omega)}{2} \right. \\ \left. \times \frac{\omega \mu_2}{4\pi a_2^3 \mu_0} \left(4 - k_r^2 a_2^2 + \frac{1}{2} k_r^2 r^2 + \frac{1}{2} k_r^2 a_1^2\right) \right. \\ \left. \times \frac{1 + \alpha \tanh(x_1) \tanh(x_2)}{\alpha \tanh(x_2) + \tanh(x_1)} \right). \quad (58)$$

The first term in Eq. (58) is independent of skin depth and equals the case of the one-layer tube [see Eq. (121) in Ref. [1]] with $b = a_2$ and $a_1 \rightarrow 0$, which is defined as the space charge impedance. The second term is the longitudinal resistive-wall impedance of the dipole mode.

Then the transverse impedance for the resistive wall can be obtained according to the Panofsky-Wenzel theorem:

$$Z_{\perp, rw}(\omega) = \frac{Z_{\parallel, rw}^{(1)}(\omega)}{k} \\ \approx \frac{1}{\sqrt{\omega^2/c^2 + k_r^2}} \frac{1 - i \text{sgn}(\omega)}{4\pi a_2^3 \delta_2 \sigma_2} \\ \times \left(4 - k_r^2 a_2^2 + \frac{1}{2} k_r^2 r^2 + \frac{1}{2} k_r^2 a_1^2\right) \\ \times \frac{1 + \alpha \tanh(x_1) \tanh(x_2)}{\alpha \tanh(x_2) + \tanh(x_1)}, \quad (59)$$

where α , x_1 , and x_2 have the same definitions as those in Eq. (53). For the case of the single layer tube with infinite thickness, the fraction part containing x_1 and x_2 is reduced to 1, and Eq. (59) becomes

$$Z_{\perp, rw}(\omega) = \frac{Z_{\parallel, rw}^{(1)}(\omega)}{k} \\ \approx \frac{1}{\sqrt{\omega^2/c^2 + k_r^2}} \frac{1 - i \text{sgn}(\omega)}{4\pi a_2^3 \delta_2 \sigma_2} \\ \times \left(4 - k_r^2 a_2^2 + \frac{1}{2} k_r^2 r^2 + \frac{1}{2} k_r^2 a_1^2\right), \quad (60)$$

which is consistent with Eq. (124) in Ref. [1] when $a_1 \rightarrow 0$.

The transverse impedance for the relativistic case can be obtained using the similar method if we consider $k_r^2 < \delta_{\text{skin}}$.

V. IMPEDANCE OF MULTILAYER TUBE

According to the impedance calculation of two-layer tube, we can generalize the expression to describe the longitudinal and transverse impedances for a multilayer tube. For the longitudinal impedance, the unknown parameter p_1 can be derived from the equation below as described in Eq. (16):

$$\begin{pmatrix} p \\ q \end{pmatrix}_1 = \prod_{i=1}^{n-1} F^{-1}(\lambda_i a_{i+1}) F(\lambda_{i+1} a_{i+1}) \begin{pmatrix} p \\ q \end{pmatrix}_n, \quad (61)$$

where F and F^{-1} are given by Eqs. (12) and (17), respectively, and $n > 1$ is the number of regions in the structure including the vacuum area.

The longitudinal monopole impedance can then be expressed as a function of M (see Appendix A):

$$Z_{\parallel}(\omega) = \frac{iZ_0 c k_r^2}{2\pi\omega} I_0(k_r a_1) \left[K_0(k_r r) + I_0(k_r r) \times \frac{\kappa M K_1(k_r a_2) - K_0(k_r a_2)}{\kappa M I_1(k_r a_2) + I_0(k_r a_2)} \right], \quad (62)$$

where κ and M are defined as

$$M = \frac{\begin{pmatrix} 0 & 1 \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} c_i b_{i+1} \cosh[\lambda_i(a_{i+1} - a_i)] & -b_i c_{i+1} \sinh[\lambda_i(a_{i+1} - a_i)] \\ -c_i b_{i+1} \sinh[\lambda_i(a_{i+1} - a_i)] & b_i c_{i+1} \cosh[\lambda_i(a_{i+1} - a_i)] \end{pmatrix} \begin{pmatrix} K_1(\lambda a) \\ -K_0(\lambda a) \end{pmatrix}_n}{\begin{pmatrix} 1 & 0 \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} c_i b_{i+1} \cosh[\lambda_i(a_{i+1} - a_i)] & -b_i c_{i+1} \sinh[\lambda_i(a_{i+1} - a_i)] \\ -c_i b_{i+1} \sinh[\lambda_i(a_{i+1} - a_i)] & b_i c_{i+1} \cosh[\lambda_i(a_{i+1} - a_i)] \end{pmatrix} \begin{pmatrix} K_1(\lambda a) \\ -K_0(\lambda a) \end{pmatrix}_n}. \quad (66)$$

The first term of Eq. (65) is the space charge impedance, and the second one the resistive-wall impedance for metallic multilayer tubes. Thus, the longitudinal impedance of arbitrary multilayers can always be split into two parts: the space charge impedance and the resistive-wall impedance. Theoretically, we can extend the result to any material of layers, as long as the first layer is metallic, which relates to κ .

The transverse impedance of the multilayer beam pipe can be obtained in the similar way. The corresponding longitudinal impedance of the dipole mode is (see Appendix B)

$$Z_{\parallel}^{(1)}(\omega) = \frac{ic^2 k_r^3 \mu_0}{2\pi r \omega} \left(1 + \frac{k_r^2 a_1^2}{8} \right) \begin{pmatrix} p_s \\ q_s \end{pmatrix} \left(\frac{p_s}{q_s} I_1(k_r r) + K_1(k_r r) \right), \quad (67)$$

where the ratio of p_s/q_s can be obtained from the expression

$$\kappa = -\frac{b_1 c_2}{c_1 b_2},$$

$$M = \frac{\begin{pmatrix} S_{1,3} & S_{1,4} \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda a) \\ -K_0(\lambda a) \end{pmatrix}_n}{\begin{pmatrix} S_{1,1} & S_{1,2} \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda a) \\ -K_0(\lambda a) \end{pmatrix}_n}, \quad (63)$$

with S 's for $i > 1$ given by Eq. (25), and for $i = 1$,

$$\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{1,3} & S_{1,4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (64)$$

According to the expressions we get above, we can tell that the impedance is mainly determined by the parameters of the first layer except the parameter M , which is dependent on every layer.

Correspondingly, for the case of metallic multilayers, the longitudinal impedance can be simplified to

$$Z_{\parallel}(\omega) = \frac{Z_0}{2\pi c} \left[-\frac{ic^2 k_r^2}{\omega} \ln\left(\frac{r}{a_2}\right) + \frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \times \left(1 - \frac{k_r^2(2a_2^2 - r^2 - a_1^2)}{4} \right) M \right], \quad (65)$$

with

$$\begin{pmatrix} p_+ \\ p_s \end{pmatrix}_1 = [\xi_1 B^*(k_r a_2) N_1 + \zeta_1 D^*(k_r a_2) N_2] \times [\xi_1 A^*(k_r a_2) N_1 + \zeta_1 C^*(k_r a_2) N_2]^{-1} \begin{pmatrix} 0 \\ q_s \end{pmatrix}_1, \quad (68)$$

with

$$N_1 = \begin{pmatrix} L_{11}^{(1)} & L_{12}^{(1)} \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix},$$

$$N_2 = \begin{pmatrix} L_{21}^{(1)} & L_{22}^{(1)} \end{pmatrix} \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix}. \quad (69)$$

Comparing with the results in Ref. [8] obtained by Zotter and Kheifets, who also studied the impedance of multilayer pipes, we find that Eqs. (100)–(101) in Ref. [8] can be written as

$$Z_{\parallel}(\omega) = \frac{iZ_0 c}{\pi \omega r^2} \left[[1 - 2I_1(k_r r)K_1(k_r r)] + 2I_1^2(k_r r) \right. \\ \left. \times \frac{Y_1 K_1(k_r a_2) - K_0(k_r a_2)}{Y_1 I_1(k_r a_2) + I_0(k_r a_2)} \right], \quad (70)$$

if the transverse size a of a uniform beam equals r . Like that in Ref. [8], Y_1 in Eq. (70) is determined as

$$Y_{i-1} = -X_i \frac{R_i + Y_i U_i}{R'_i + Y_i U'_i}, \quad Y_{n-1} = X_n \frac{K_0(\lambda_n a_{n-1})}{K_1(\lambda_n a_{n-1})}, \\ X_i = \frac{\lambda_i \varepsilon'_{i-1}}{\varepsilon'_i \lambda_{i-1}}, \quad (71)$$

with

$$R_i(\lambda_i r) = K_0(\lambda_i a_i)I_0(\lambda_i r) - I_0(\lambda_i a_i)K_0(\lambda_i r), \\ U_i(\lambda_i r) = -K_1(\lambda_i a_i)I_0(\lambda_i r) - I_1(\lambda_i a_i)K_0(\lambda_i r), \quad (72)$$

and R'_i, U'_i are the derivatives with respect to the argument. If Y_1 is replaced with κM , which is used here, we can get the following expression of the longitudinal impedance of the multilayer pipes when we expand Eq. (70) up to first order in $\delta_{\text{skin}} \omega/c$ and second order in $k_r c/\omega$:

TABLE I. Main parameters of three proton rings.

	CSNS	J-PARC	SNS
γ	1.1	1.4	2.1
a_2 (cm)	11	12.5	8
$d_1 = a_3 - a_2$ (nm)	10	15	100
$d_2 = a_4 - a_3$ (mm)	5	5	5

$$Z_{\parallel}(\omega) = \frac{Z_0}{2\pi c} \left[-\frac{ic^2 k_r^2}{\omega} \ln\left(\frac{r}{a_2}\right) + \frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \right. \\ \left. \times \left(1 - \frac{k_r^2 (2a_2^2 - r^2)}{4} \right) M \right]. \quad (73)$$

Equation (73) looks much like our expression of Eq. (65), but missing a term $-a_1^2$ on the numerator in the parentheses of the second term. This is because we use a ring beam here, but a uniform round beam with a radius a is taken in Ref. [8].

VI. APPLICATIONS TO REAL MACHINES

Several examples of typical machines will be given in this section. Three proton rings, the rapid cycling synchrotrons of the China Spallation Neutron Source (CSNS) [12] and the Japan Proton Accelerator Research Complex (J-PARC) [13], and the accumulator of the Spallation Neutron Source (SNS) [14] in the U.S.A., are evaluated here. The related parameters used in numerical estimations are listed in Table I.

We consider a stainless steel beam pipe with TiN coating inside, and the conductivity of TiN is $\sigma_{\text{TiN}} = 5.88 \times 10^6 \Omega^{-1} \text{m}^{-1}$. In all cases, we suppose the thickness of stainless steel beam pipe is 5 mm with a conductivity of $\sigma_{\text{SS}} = 1.4 \times 10^6 \Omega^{-1} \text{m}^{-1}$. Figure 2 presents the real part of the longitudinal and transverse impedance of each case at a radius $r = a_1 = a_2/2$. The horizontal axis is the angular frequency with the range from 0 to 400 MHz.

From the numerical estimation based on the impedance expressions we got here, we can see that the longitudinal impedance varies both with the beam energy and the inner radius of the beam pipe. We also compare the result of the CSNS ring with the relativistic limit, and it is easy to tell the differences between them, especially the real part of transverse impedance.

FIG. 2. Real part of longitudinal (left) and transverse (right) impedance per unit length for typical machines.

FIG. 3. Real part of longitudinal (left) and transverse (right) impedance per unit length for different thickness of inner layer (CSNS ring).

Figure 3 presents the real part of the resistive-wall impedance per unit length for different thicknesses of the cover layer using the CSNS parameters. Here a stainless steel beam pipe with a copper layer inside is considered. The conductivity of copper is $\sigma_{\text{Cu}} = 5.88 \times 10^7 \Omega^{-1} \text{m}^{-1}$. The left figure denotes the longitudinal impedance, and the right one the transverse impedance.

The figure shows that, in the two-layer case, the clear difference appears at a thickness bigger than ~ 100 nm, and that the thicker the cover layer, the less the impedance. It is easy to understand that the thicker cover layer will contribute more to the total impedance as long as the cover thickness is smaller than the skin depth. Once the cover thickness is larger than the skin depth, the total impedance mainly comes from the cover layer.

VII. SUMMARY

The general expressions of longitudinal and transverse impedance for two-layer tubes with finite thickness for a ring beam in arbitrary velocity have been obtained. The results can be extended to the case of multilayers.

As a starting point of calculation, the wave equation of scalar and vector potential is used, and the field matching

method is applied. The results can be applicable to any material which obeys Ohm's law with a constant scalar conductivity, and that the dielectric constant and the magnetic permeability can be simply described as $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$. As the impedance expressions in this report are derived for a ring beam of finite radius a_1 , the results are not valid to the case of beam with $\rho \neq 0$ for $0 \leq r \leq a_1$.

Meanwhile, we have given a compact expression of longitudinal and transverse impedance for the special case of the metallic two-layer tube. They can always be composed to two parts: the space charge impedance, which is independent on skin depth δ_{skin} , and the resistive-wall impedance, which is proportional to δ_{skin} . Our main results include the longitudinal impedance for the resistive wall given in Eq. (54):

$$Z_{\parallel}(\omega) = \frac{Z_0}{2\pi c} \left[-\frac{ic^2 k_r^2}{\omega} \ln\left(\frac{r}{a_2}\right) + \frac{\delta_2 \mu_2 \omega [\text{sgn}(\omega) - i]}{2a_2 \mu_0} \right] \times \left(1 - \frac{k_r^2 (2a_2^2 - r^2 - a_1^2)}{4} \right) M,$$

with

$$M = -\frac{\epsilon_0 \lambda_3 [\alpha \tanh(x_1) + \tanh(x_2)] K_1(k_r a_4) + \epsilon'_3 k_r [\alpha \tanh(x_1) \tanh(x_2) + 1] K_0(k_r a_4)}{\epsilon_0 \lambda_3 [\alpha + \tanh(x_1) \tanh(x_2)] K_1(k_r a_4) + \epsilon'_3 k_r [\alpha \tanh(x_2) + \tanh(x_1)] K_0(k_r a_4)}$$

and

$$\kappa = -\frac{\lambda_2}{k_r \mu_0 \epsilon'_2 c^2},$$

and the transverse impedance for the resistive wall given in Eq. (59),

$$Z_{\perp, rw}(\omega) \approx \frac{1}{\sqrt{\omega^2/c^2 + k_r^2}} \frac{1 - i \operatorname{sgn}(\omega)}{4\pi a_2^3 \delta_2 \sigma_2} \left(4 - k_r^2 a_2^2 + \frac{1}{2} k_r^2 r^2 + \frac{1}{2} k_r^2 a_1^2 \right) \frac{1 + \alpha \tanh(x_1) \tanh(x_2)}{\alpha \tanh(x_2) + \tanh(x_1)}.$$

For both cases, we can generalize the results to the impedance of the one-layer tube, which are consistent with that derived in Ref. [1]. In the ultrarelativistic limit, the longitudinal impedance we got can be reduced to the results obtained by Ivanyan [3]. All the results can be deduced to the point beam case.

For the case of a metallic multilayer, the longitudinal impedance has the same expression as the two-layer tube Eq. (54), except for M , which is given in Eq. (66). The results agree well enough with those obtained by Zotter and Kheifets in Ref. [8] if we expand our results to the first order in $\delta_{\text{skin}} \omega/c$ and to the second order of $k_r c/\omega$.

The numerical estimations on different hadron machines also show that the nonrelativistic formulas of the impedance deviate from those of the ultrarelativistic limit. For the longitudinal impedance, the real part for different beam energy departs far as the beam frequency increases, though at the low frequency they seem to merge. When some instabilities happen, the growth time will decrease since the integration of the real part of impedance at lower energy will be smaller than that of the cases for ultrarelativistic or relatively higher beam energy. In the two-layer case, the impedance deviates more when the thickness of the inner coating increases.

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APPENDIX A

By using the field matching method, we can obtain the unknown coefficients in the wakefield expression, Eq. (10). The field in the interface of two different layers satisfies the condition of Eq. (15). For an n -layers tube (here n including the vacuum layer and $n > 1$), the coefficient p_1 can be obtained from the matching condition,

$$\begin{pmatrix} p \\ q \end{pmatrix}_1 = \prod_{i=1}^{n-1} F^{-1}(\lambda_i a_{i+1}) F(\lambda_{i+1} a_{i+1}) \begin{pmatrix} p \\ q \end{pmatrix}_n, \quad (\text{A1})$$

in which q_1 is defined by Eq. (21) and $p_n = 0$. The condition (A1) can also be expressed as

$$\begin{pmatrix} p \\ q \end{pmatrix}_1 = F^{-1}(\lambda_1 a_2) \times \left(\prod_{i=2}^{n-1} F(\lambda_i a_i) F^{-1}(\lambda_i a_{i+1}) \right) F(\lambda_n a_n) \begin{pmatrix} p \\ q \end{pmatrix}_n, \quad (\text{A2})$$

with

$$F(\lambda_i a_i) F^{-1}(\lambda_i a_{i+1}) = \begin{pmatrix} -\frac{b_i}{b_{i+1}} S_{i,1} & \frac{b_i}{c_{i+1}} S_{i,2} \\ \frac{c_i}{b_{i+1}} S_{i,3} & -\frac{c_i}{c_{i+1}} S_{i,4} \end{pmatrix} \quad (\text{A3})$$

and

$$\begin{aligned} S_{i,1} &= c_i b_{i+1} [I_1(\lambda_i a_i) K_0(\lambda_i a_{i+1}) + K_1(\lambda_i a_i) I_0(\lambda_i a_{i+1})], \\ S_{i,2} &= b_i c_{i+1} [I_1(\lambda_i a_i) K_1(\lambda_i a_{i+1}) - K_1(\lambda_i a_i) I_1(\lambda_i a_{i+1})], \\ S_{i,3} &= c_i b_{i+1} [I_0(\lambda_i a_i) K_0(\lambda_i a_{i+1}) - K_0(\lambda_i a_i) I_0(\lambda_i a_{i+1})], \\ S_{i,4} &= b_i c_{i+1} [I_0(\lambda_i a_i) K_1(\lambda_i a_{i+1}) + K_0(\lambda_i a_i) I_1(\lambda_i a_{i+1})]. \end{aligned} \quad (\text{A4})$$

If we further represent the product of matrices as

$$\begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} = \prod_{i=2}^{n-1} F(\lambda_i a_i) F^{-1}(\lambda_i a_{i+1}) = \prod_{i=2}^{n-1} \begin{pmatrix} -\frac{b_i}{b_{i+1}} S_{i,1} & \frac{b_i}{c_{i+1}} S_{i,2} \\ \frac{c_i}{b_{i+1}} S_{i,3} & -\frac{c_i}{c_{i+1}} S_{i,4} \end{pmatrix}, \quad (\text{A5})$$

then we have

$$\begin{pmatrix} p \\ q \end{pmatrix}_1 = -\frac{\lambda_1 a_2}{b_1 c_1} \begin{pmatrix} c_1 K_0(\lambda_1 a_2) & -b_1 K_1(\lambda_1 a_2) \\ -c_1 I_0(\lambda_1 a_2) & -b_1 I_1(\lambda_1 a_2) \end{pmatrix} \times \begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} \times \begin{pmatrix} -b_n I_1(\lambda_n a_n) & b_n K_1(\lambda_n a_n) \\ c_n I_0(\lambda_n a_n) & c_n K_0(\lambda_n a_n) \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}_n. \quad (\text{A6})$$

Considering that $p_n = 0$ in the vacuum outside the beam chamber, then the ratio of p_1 and q_1 will be

$$\frac{p_1}{q_1} = \frac{\kappa M K_1(\lambda_1 a_2) - K_0(\lambda_1 a_2)}{\kappa M I_1(\lambda_1 a_2) + I_0(\lambda_1 a_2)}, \quad (\text{A7})$$

where

$$\lambda_1 = k_r, \quad \kappa = -\frac{b_1 c_2}{c_1 b_2}, \quad (\text{A8})$$

and

$$M = -\frac{b_2 c_n l_{22} K_0(\lambda_n a_n) + b_n l_{21} K_1(\lambda_n a_n)}{c_2 c_n l_{12} K_0(\lambda_n a_n) + b_n l_{11} K_1(\lambda_n a_n)} \\ = \frac{(-b_2 b_n l_{21} \quad b_2 c_n l_{22}) \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}{(c_2 b_n l_{11} \quad -c_2 c_n l_{12}) \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}. \quad (\text{A9})$$

For $n = 3$, i.e., the case of a one-layer tube of finite thickness,

$$\begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} = \begin{pmatrix} -\frac{b_2}{b_3} S_{2,1} & \frac{b_2}{c_3} S_{2,2} \\ \frac{c_2}{b_3} S_{2,3} & -\frac{c_2}{c_3} S_{2,4} \end{pmatrix}, \quad (\text{A10})$$

and

$$M = \frac{(S_{2,3} \quad S_{2,4}) \begin{pmatrix} K_1(\lambda_3 a_3) \\ -K_0(\lambda_3 a_3) \end{pmatrix}}{(S_{2,1} \quad S_{2,2}) \begin{pmatrix} K_1(\lambda_3 a_3) \\ -K_0(\lambda_3 a_3) \end{pmatrix}}. \quad (\text{A11})$$

For $n > 3$,

$$\begin{aligned} b_2 b_n l_{21} &= b_2 c_2 \left(\frac{1}{b_3} S_{2,3} \quad -\frac{1}{c_3} S_{2,4} \right) \\ &\times \prod_{i=3}^{n-2} \begin{pmatrix} -b_i S_{i,1}/b_{i+1} & b_i S_{i,2}/c_{i+1} \\ c_i S_{i,3}/b_{i+1} & -c_i S_{i,4}/c_{i+1} \end{pmatrix} \\ &\times \begin{pmatrix} -b_{n-1} S_{n-1,1} \\ c_{n-1} S_{n-1,3} \end{pmatrix}, \\ b_2 c_n l_{22} &= -b_2 c_2 \left(\frac{1}{b_3} S_{2,3} \quad -\frac{1}{c_3} S_{2,4} \right) \\ &\times \prod_{i=3}^{n-2} \begin{pmatrix} -b_i S_{i,1}/b_{i+1} & b_i S_{i,2}/c_{i+1} \\ c_i S_{i,3}/b_{i+1} & -c_i S_{i,4}/c_{i+1} \end{pmatrix} \\ &\times \begin{pmatrix} -b_{n-1} S_{n-1,2} \\ c_{n-1} S_{n-1,4} \end{pmatrix}, \\ c_2 b_n l_{11} &= -b_2 c_2 \left(-\frac{1}{b_3} S_{2,1} \quad \frac{1}{c_3} S_{2,2} \right) \\ &\times \prod_{i=3}^{n-2} \begin{pmatrix} -b_i S_{i,1}/b_{i+1} & b_i S_{i,2}/c_{i+1} \\ c_i S_{i,3}/b_{i+1} & -c_i S_{i,4}/c_{i+1} \end{pmatrix} \\ &\times \begin{pmatrix} b_{n-1} S_{n-1,1} \\ -c_{n-1} S_{n-1,3} \end{pmatrix}, \\ c_2 c_n l_{12} &= b_2 c_2 \left(-\frac{1}{b_3} S_{2,1} \quad \frac{1}{c_3} S_{2,2} \right) \\ &\times \prod_{i=3}^{n-2} \begin{pmatrix} -b_i S_{i,1}/b_{i+1} & b_i S_{i,2}/c_{i+1} \\ c_i S_{i,3}/b_{i+1} & -c_i S_{i,4}/c_{i+1} \end{pmatrix} \\ &\times \begin{pmatrix} b_{n-1} S_{n-1,2} \\ -c_{n-1} S_{n-1,4} \end{pmatrix}. \end{aligned} \quad (\text{A12})$$

By expanding the expressions above, we find that the coefficients in front of S 's in the matrices will cancel each

other, and the expressions are simplified to

$$\begin{aligned} (-b_2 b_n l_{21} \quad b_2 c_n l_{22}) &= -b_2 c_2 (S_{2,3} \quad S_{2,4}) \\ &\times \prod_{i=3}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix}, \\ (c_2 b_n l_{11} \quad -c_2 c_n l_{12}) &= -b_2 c_2 (S_{2,1} \quad S_{2,2}) \\ &\times \prod_{i=3}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix}. \end{aligned} \quad (\text{A13})$$

Now we take the above expressions into (A9), then M will become

$$M = \frac{(S_{2,3} \quad S_{2,4}) \prod_{i=3}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}{(S_{2,1} \quad S_{2,2}) \prod_{i=3}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}. \quad (\text{A14})$$

For completeness, we take

$$\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{1,3} & S_{1,4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A15})$$

which corresponds to the case of $n = 2$. Then M has a generalized form of

$$M = \frac{(S_{1,3} \quad S_{1,4}) \prod_{i=2}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}{(S_{1,1} \quad S_{1,2}) \prod_{i=2}^{n-1} \begin{pmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{pmatrix} \begin{pmatrix} K_1(\lambda_n a_n) \\ -K_0(\lambda_n a_n) \end{pmatrix}}. \quad (\text{A16})$$

Now that we have obtained the expression of κ and M , the ratio of p_1 and q_1 can be obtained by Eq. (A7). Correspondingly, the wakefield and impedance of the monopolar mode can be derived, in the respect that the parameter q_1 is determined by the source boundary.

APPENDIX B

In deriving the transverse impedance, the unknown coefficients are determined by

$$\begin{pmatrix} p_+ \\ p_s \\ 0 \\ q_s \end{pmatrix}_1 = \prod_{i=1}^{n-1} G^{-1}(\lambda_i a_{i+1}) G(\lambda_{i+1} a_{i+1}) \begin{pmatrix} 0 \\ 0 \\ q_+ \\ q_s \end{pmatrix}_n. \quad (\text{B1})$$

By decomposing the matrices G and G^{-1} into 2×2 block matrices,

$$\begin{aligned} G(\lambda_i r) &= \begin{pmatrix} A(\lambda_i r) & B(\lambda_i r) \\ C(\lambda_i r) & D(\lambda_i r) \end{pmatrix}, \\ G^{-1}(\lambda_i r) &= \begin{pmatrix} \xi_i B^*(\lambda_i r) & \zeta_i D^*(\lambda_i r) \\ \xi_i A^*(\lambda_i r) & \zeta_i C^*(\lambda_i r) \end{pmatrix}, \end{aligned} \quad (\text{B2})$$

and further defining

$$\begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} = G(\lambda_i a_i) G^{-1}(\lambda_i a_{i+1}), \quad (\text{B3})$$

with

$$\begin{aligned} L_{11}^{(i)} &= \begin{pmatrix} \lambda_i a_{i+1} T_{i,1} & \frac{ik(a_i T_{i,1} + a_{i+1} T_{i,2})}{\lambda_i a_i} \\ 0 & -\lambda_i a_{i+1} T_{i,2} \end{pmatrix}, \\ L_{12}^{(i)} &= \frac{1}{\mu_i \omega} \\ &\times \begin{pmatrix} -\frac{a_{i+1}}{a_i} k T_{i,3} & i(\frac{k^2 T_{i,3}}{a_i \lambda_i^2} + a_{i+1}(k^2 - \lambda_i^2) T_{i,4}) \\ -i a_{i+1} \lambda_i^2 T_{i,3} & -k T_{i,3} \end{pmatrix}, \\ L_{21}^{(i)} &= \frac{\mu_i \omega}{(k^2 - \lambda_i^2)} \\ &\times \begin{pmatrix} -\frac{a_{i+1}}{a_i} k T_{i,3} & -i(\frac{k^2 T_{i,3}}{a_i \lambda_i^2} + a_{i+1}(k^2 - \lambda_i^2) T_{i,4}) \\ i a_{i+1} \lambda_i^2 T_{i,3} & -k T_{i,3} \end{pmatrix}, \\ L_{22}^{(i)} &= \begin{pmatrix} \lambda_i a_{i+1} T_{i,1} & -\frac{ik(a_i T_{i,1} + a_{i+1} T_{i,2})}{\lambda_i a_i} \\ 0 & -\lambda_i a_{i+1} T_{i,2} \end{pmatrix}, \end{aligned} \quad (\text{B4})$$

where

$$\begin{aligned} T_{i,1} &= I'_1(\lambda_i a_i) K_1(\lambda_i a_{i+1}) - I_1(\lambda_i a_{i+1}) K'_1(\lambda_i a_i), \\ T_{i,2} &= I_1(\lambda_i a_i) K'_1(\lambda_i a_{i+1}) - I'_1(\lambda_i a_{i+1}) K_1(\lambda_i a_i), \\ T_{i,3} &= I_1(\lambda_i a_i) K_1(\lambda_i a_{i+1}) - I_1(\lambda_i a_{i+1}) K_1(\lambda_i a_i), \\ T_{i,4} &= I'_1(\lambda_i a_i) K'_1(\lambda_i a_{i+1}) - I'_1(\lambda_i a_{i+1}) K'_1(\lambda_i a_i), \end{aligned} \quad (\text{B5})$$

we have

$$\begin{aligned} \begin{pmatrix} p_+ \\ p_s \\ 0 \\ q_s \end{pmatrix}_1 &= \begin{pmatrix} \xi_1 B^*(k_r a_2) & \xi_1 D^*(k_r a_2) \\ \xi_1 A^*(k_r a_2) & \xi_1 C^*(k_r a_2) \end{pmatrix} \\ &\times \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \\ &\times \begin{pmatrix} A(\lambda_n a_n) & B(\lambda_n a_n) \\ C(\lambda_n a_n) & D(\lambda_n a_n) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ q_+ \\ q_s \end{pmatrix}_n. \end{aligned} \quad (\text{B6})$$

Since Eq. (B6) can be transformed to

$$\begin{aligned} \begin{pmatrix} p_+ \\ p_s \end{pmatrix}_1 &= [\xi_1 B^*(k_r a_2) \quad \xi_1 D^*(k_r a_2)] \\ &\times \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix} \begin{pmatrix} q_+ \\ q_s \end{pmatrix}_n, \end{aligned} \quad (\text{B7})$$

and

$$\begin{aligned} \begin{pmatrix} 0 \\ q_s \end{pmatrix}_1 &= [\xi_1 A^*(k_r a_2) \quad \xi_1 C^*(k_r a_2)] \\ &\times \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix} \begin{pmatrix} q_+ \\ q_s \end{pmatrix}_n, \end{aligned} \quad (\text{B8})$$

we have

$$\begin{aligned} \begin{pmatrix} p_+ \\ p_s \end{pmatrix}_1 &= [\xi_1 B^*(k_r a_2) N_1 + \xi_1 D^*(k_r a_2) N_2] \\ &\times [\xi_1 A^*(k_r a_2) N_1 + \xi_1 C^*(k_r a_2) N_2]^{-1} \begin{pmatrix} 0 \\ q_s \end{pmatrix}_1, \end{aligned} \quad (\text{B9})$$

where

$$N_1 = (L_{11}^{(2)} \quad L_{12}^{(2)}) \prod_{i=3}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix}, \quad (\text{B10})$$

and

$$N_2 = (L_{21}^{(2)} \quad L_{22}^{(2)}) \prod_{i=3}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix}. \quad (\text{B11})$$

For completeness, we further define

$$\begin{pmatrix} L_{11}^{(1)} & L_{12}^{(1)} \\ L_{21}^{(1)} & L_{22}^{(1)} \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}, \quad (\text{B12})$$

where E is the identity matrix. Correspondingly, the parameters N_1 and N_2 are reduced to

$$N_1 = (L_{11}^{(1)} \quad L_{12}^{(1)}) \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix}, \quad (\text{B13})$$

$$N_2 = (L_{21}^{(1)} \quad L_{22}^{(1)}) \prod_{i=2}^{n-1} \begin{pmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{pmatrix} \begin{pmatrix} B(\lambda_n a_n) \\ D(\lambda_n a_n) \end{pmatrix}. \quad (\text{B14})$$

Then the ratio p_s/q_s can be derived, and the longitudinal electric field of the dipolar mode is

$$\begin{aligned} E_s &= -\frac{ic^2 k_r^2}{\omega} q_s \left(\frac{p_s}{q_s} I_1(k_r r) + K_1(k_r r) \right) \cos \varphi e^{ikz} \\ &= -\frac{ic^2 k_r^2}{\omega} \frac{\mu_0 \lambda_b v k_r d}{2\pi} \left(\frac{p_s}{q_s} I_1(k_r r) + K_1(k_r r) \right) \cos \varphi e^{ikz}. \end{aligned} \quad (\text{B15})$$

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- [1] F. Zimmermann and K. Oide, Phys. Rev. ST Accel. Beams **7**, 044201 (2004).
 - [2] R. L. Gluckstern, CERN Report No. 2000-011, 2000.
 - [3] A. M. Al-khateeb, O. Boine-Frankenheim, R. W. Hasse, and I. Hofmann, Phys. Rev. E **71**, 026501 (2005).
 - [4] A. M. Al-khateeb, R. W. Hasse, O. Boine-Frankenheim, W. M. Daqa, and I. Hofmann, Phys. Rev. ST Accel. Beams **10**, 064401 (2007).
 - [5] M. Ivanyan and V. Tsakanov, Phys. Rev. ST Accel. Beams **7**, 114402 (2004).
 - [6] M. Ivanyan and A. V. Tsakanian, *Proceedings of the PAC 2005, Knoxville, Tennessee, 2005* (IEEE, Piscataway, New Jersey, 2005), p. 3138.
 - [7] M. Ivanyan and Andranik Tsakanian, in Phys. Rev. ST Accel. Beams **9**, 034404 (2006).
 - [8] B. W. Zotter and S. A. Kheifets, *Impedances and Wakes in High-Energy Particle Accelerators* (World Scientific, Singapore, 1998).
 - [9] A. Chao, *Physics of Collective Beam Instabilities in High-Energy Particle Accelerators* (Wiley, New York, 1993).
 - [10] L. C. Andrews, *Special Functions for Engineers and Applied Mathematicians* (Macmillan, New York, 1985).
 - [11] W. K. H. Panofsky and W. A. Wenzel, Rev. Sci. Instrum. **27**, 967 (1956).
 - [12] CSNS Accelerator Team, IHEP-CSNS-Report/2004-01E, 2004.
 - [13] Y. Yamazaki *et al.*, KEK, Report No. 2002-13, 2003; JAERI Report No. TECH-2003-044, 2003.
 - [14] J. Wei, in *Proceedings of the Workshop on Space Charge Physics in High Intensity Hadron Rings, Shelter Island, New York, 1998*, AIP Conf. Proc. No. 448 (AIP, New York, 1998), p. 152.