Particle acceleration by wave scattering off dielectric spheres at whispering-gallery-mode resonance

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The large electromagnetic fields, created in wave scattering near a perfect dielectric sphere at the condition of whispering-gallery-mode resonances, are investigated as driving units for high energy charged particle accelerators. For optimal trajectories passing near the scattering sphere, particle coupling with the field reduces to very short intervals, of the order of the wave period. Interacting fields can be almost 1000 times stronger than that in the incident wave. An example considered indicates that the instantaneous energy yield during this strong coupling interval is equivalent to ~ 30 GeV/m, assuming the incident electric field $E_0 = 100$ MV/m. It was shown that the particle transverse deflection is negligible if the phase of the particle is optimal for acceleration. Hence, the acceleration process can be repeated many times. A rough estimate of the energy gain in a periodic chain of such elementary accelerating unit cells gives $\Delta\text{Energy}/m \approx 5$ GeV/m, which is several hundred times more than in contemporary operating and projected accelerators. Preliminary estimates of absorption losses in the scheme are given.

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I. INTRODUCTION

We have reached a point where further progress in high energy physics is crucially dependent on finding more efficient ways of particle acceleration.

Here, I propose the use of a strong field concentration near the surface of a perfect lossless dielectric sphere in the conditions of the whispering-gallery-mode (WGM) resonance as a driving force to accelerate particles. The accelerating forces are estimated to be several hundred times stronger than those in conventional accelerators due to the resonant field accumulation and concentration in the closest vicinity of the spherical boundary.

The rate of acceleration in conventional accelerators, employing properly shaped resonant microwave cavities, is of the order of ~ 20 MeV/m. This is limited by electrical breakdown at metallic or superconducting walls of the microwave circuit elements.

The search for more efficient accelerators concentrates on various schemes of laser devices. In plasma accelerators, the energy gain over a distance of a few millimeters (equivalent to ~ 100 GeV/m) has been demonstrated [1–4] opening up hope for tabletop accelerating devices.

This article extends my previous discussion [5] investigating the use of WGM resonances of dielectric cylinders for particle acceleration. While for the dielectric cylinders these resonances are confined in the transverse cross section only and unconfined along their length, for the dielectric spheres the WGM resonances are confined in all three dimensions. Therefore, near the spherical resonant scatterer the fields and accelerating forces are stronger which makes the acceleration process more efficient.

A multiscatterer accelerator scheme is proposed. Its effectiveness is evaluated.

A brief analysis of energy absorption and losses in non-perfect dielectrics is also included. It indicates that losses are an important factor in the performance of a proposed accelerator scheme.

II. ELECTROMAGNETIC FIELD IN THE MIE SCATTERING

Scattering of a plane electromagnetic monochromatic wave by a perfect dielectric sphere, known as the Mie or Lorenz-Mie problem, was discussed in many monographs and textbooks [6–10]. While the properties of scattering are usually expressed in terms of the scattered wave only, the total field necessary to determine the force acting on a charged particle includes both the incident and the scattered wave. This field, for a perfect dielectric sphere of radius a and dielectric constant ϵ , and a circularly polarized plane electromagnetic wave propagating in the direction of the z axis

$$\mathbf{E}_\sigma^I(\mathbf{r}) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\sigma\hat{\mathbf{y}})\exp(ikz) \quad (1)$$

and

$$\mathbf{B}_\sigma^I(\mathbf{r}) = \frac{1}{\sqrt{2}}(-i\sigma\hat{\mathbf{x}} + \hat{\mathbf{y}})\exp(ikz), \quad (2)$$

can be written as

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FIG. 1. Amplitudes of partial waves on the TE, $l = 22$, resonance.

$$\mathbf{E}_\sigma(\mathbf{r}) = \begin{cases} \mathbf{E}_\sigma^I(\mathbf{r}) + \sum_{l=1}^{\infty} \tau_l \{c_l^{\text{TE}} h_l(kr) \mathbf{X}_{l\sigma} + \frac{\sigma}{kr} c_l^{\text{TM}} [i\Lambda_l Y_{l\sigma} h_l(kr) \mathbf{n} + h_l^D(kr) \mathbf{n} \times \mathbf{X}_{l\sigma}]\}, & r > a, \\ \sum_{l=1}^{\infty} \tau_l \{a_l^{\text{TE}} j_l(\sqrt{\epsilon} kr) \mathbf{X}_{l\sigma} + \frac{\sigma}{\epsilon kr} a_l^{\text{TM}} [i\Lambda_l Y_{l\sigma} j_l(\sqrt{\epsilon} kr) \mathbf{n} + j_l^D(\sqrt{\epsilon} kr) \mathbf{n} \times \mathbf{X}_{l\sigma}]\}, & r < a, \end{cases} \quad (3)$$

$$\mathbf{B}_\sigma(\mathbf{r}) = \begin{cases} \mathbf{B}_\sigma^I(\mathbf{r}) - i \sum_{l=1}^{\infty} \tau_l \{c_l^{\text{TE}} [i\Lambda_l Y_{l\sigma} h_l(kr) \mathbf{n} + h_l^D(kr) \mathbf{n} \times \mathbf{X}_{l\sigma}]/kr + \sigma c_l^{\text{TM}} h_l(kr) \mathbf{X}_{l\sigma}\}, & r > a, \\ -i \sum_{l=1}^{\infty} \tau_l \{a_l^{\text{TE}} [i\Lambda_l Y_{l\sigma} j_l(\sqrt{\epsilon} kr) \mathbf{n} + j_l^D(\sqrt{\epsilon} kr) \mathbf{n} \times \mathbf{X}_{l\sigma}]/kr + \sigma a_l^{\text{TM}} j_l(\sqrt{\epsilon} kr) \mathbf{X}_{l\sigma}\}, & r < a, \end{cases} \quad (4)$$

where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are orthonormal vectors orthogonal to the z axis, $\sigma = \pm 1$ specifies polarization of the wave, $\mathbf{n} = \mathbf{r}/r$, $\tau_l = i^l \sqrt{2\pi(2l+1)}$, $\Lambda_l = \sqrt{l(l+1)}$, $Y_{lm} \equiv Y_{lm}(\theta, \phi)$, and $\mathbf{X}_{lm} \equiv \mathbf{X}_{lm}(\theta, \phi)$ are spherical and vector spherical harmonics normalized as in [10], $j_l(\rho) \equiv \sqrt{\frac{\pi}{2\rho}} J_{l+(1/2)}(\rho)$ and $h_l(\rho) \equiv \sqrt{\frac{\pi}{2\rho}} H_{l+(1/2)}^{(1)}(\rho)$ are spherical Bessel and Hankel functions, and $j_l^D(\rho) \equiv \frac{d}{d\rho}[\rho j_l(\rho)]$ and $h_l^D(\rho) \equiv \frac{d}{d\rho}[\rho h_l(\rho)]$ are derivatives of the Riccati-Bessel and Riccati-Hankel functions. The electric fields in Eqs. (1) and (3) are in units of the incident electric field amplitude E_0 and corresponding units are used in Eq. (2). The internal expansion coefficients of transverse electric (TE) and transverse magnetic (TM) components of the partial waves are

$$a_l^{\text{TE}} = \frac{j_l(ka)h_l^D(ka) - j_l^D(ka)h_l(ka)}{h_l^D(ka)j_l(\sqrt{\epsilon}ka) - h_l(ka)j_l^D(\sqrt{\epsilon}ka)}, \quad (5)$$

$$a_l^{\text{TM}} = \epsilon \frac{j_l(ka)h_l^D(ka) - j_l^D(ka)h_l(ka)}{\epsilon h_l^D(ka)j_l(\sqrt{\epsilon}ka) - h_l(ka)j_l^D(\sqrt{\epsilon}ka)}, \quad (6)$$

and the external coefficients

$$c_l^{\text{TE}} = \frac{j_l(ka)j_l^D(\sqrt{\epsilon}ka) - j_l^D(ka)j_l(\sqrt{\epsilon}ka)}{h_l^D(ka)j_l(\sqrt{\epsilon}ka) - h_l(ka)j_l^D(\sqrt{\epsilon}ka)}, \quad (7)$$

$$c_l^{\text{TM}} = \frac{j_l(ka)j_l^D(\sqrt{\epsilon}ka) - \epsilon j_l^D(ka)j_l(\sqrt{\epsilon}ka)}{\epsilon h_l^D(ka)j_l(\sqrt{\epsilon}ka) - h_l(ka)j_l^D(\sqrt{\epsilon}ka)}. \quad (8)$$

These formulas were derived by using and extending the results of Jackson [10].

Whispering-gallery-mode resonances

Whispering-gallery-mode resonances can be recognized as sharp maxima of the internal expansion coefficients a_l^{TE} and a_l^{TM} . They are characterized by the value of the spherical harmonics index l and the type of waves, TE or TM. For

a fixed wave frequency, the resonances require specific values of the sphere radius a . These resonant values can be found by scanning the amplitudes a_l^{TE} and a_l^{TM} . Examples are given in Table I.

Figure 1 shows the partial wave amplitudes for the TE²² resonance condition, Fig. 2 illustrates the resonance characteristic, and Fig. 3 illustrates the accuracy of calculations when 40 partial waves were taken into account.

In the resonance condition, a strong field is accumulated inside the scatterer and in the nearest surrounding of its boundary, so that the whole spherical surface is surrounded by the accumulated electromagnetic field.

For circular polarization of the incident wave, the spatial distribution of the field is axially symmetric. This is illustrated in Fig. 4, showing the magnitude of the electric field in the WGM resonance conditions for the TE partial wave with $l = 22$, the dielectric constant of the sphere $\epsilon = 4$, and the radius $a = 2.134697$ (in units of the wavelengths λ). The field distribution is given at three planes passing through the spherical center, spanned by the vectors $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, $(\hat{\mathbf{u}}, \hat{\mathbf{y}})$, and $(\hat{\mathbf{z}}, \hat{\mathbf{y}})$, where $\hat{\mathbf{u}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{z}} - \hat{\mathbf{x}})$.

FIG. 2. Resonance curve for the TE, $l = 22$, resonance.

FIG. 3. The accuracy of the solution, defined as $\Delta E_{\text{tan}} = E_{\text{tan}}(a - \delta) - E_{\text{tan}}(a + \delta)$, around the great circle circumference of the scatterer (left figure). The great circle lies in the orthogonal plane to the vector $\frac{1}{\sqrt{2}}(\hat{\mathbf{z}} - \hat{\mathbf{x}})$. The frame on the right shows the corresponding distribution of the tangent electric field.

FIG. 4. Electric field distribution of WGM resonance for a perfect dielectric sphere. The sphere is characterized by $\epsilon = 4$, radius $a = 2.134697$, and is excited by a circularly polarized wave propagating along the z axis. The parts of the figure show distributions of the electric field in three planes passing through the center of the sphere, and orthogonal to the vectors: $\hat{\mathbf{z}}$, $\frac{1}{\sqrt{2}}(\hat{\mathbf{z}} + \hat{\mathbf{x}})$, and $\hat{\mathbf{x}}$.

III. ELECTRONS ENERGY GAIN

Very high energy relativistic electrons move almost uniformly with a velocity close to c , while any changes of their energy, as well as their transverse deflection, can be regarded as small perturbations. The electron trajectory can be parametrized by the electron closest approach $\mathbf{R}_0$ and electron velocity $\vec{\beta}$:

$$\mathbf{R}(\tau) = \mathbf{R}_0 + \vec{\beta}\tau, \quad \tau = t - t_0, \quad (9)$$

where time is given in units of the wave period $T = \lambda/c$, t_0 corresponds to the closest approach,

$$\mathbf{R}_0 = \mathbf{n}_0 R_0 = R_0 \{\sin\Theta \cos\Phi, \sin\Theta \sin\Phi, \cos\Theta\},$$

and the velocity $\vec{\beta}$ is orthogonal to $\mathbf{R}_0$. It can be represented in a coordinate system determined by two vectors $\mathbf{e}_2 = \{-\cos\Theta \cos\Phi, -\cos\Theta \sin\Phi, \sin\Theta\}$ and $\mathbf{e}_1 = \{-\sin\Phi, \cos\Phi, 0\}$ tangent to the local meridians and parallels. The electron velocity is determined by the azimuthal angle A between $\vec{\beta}$ and the tangent to the local meridian $\mathbf{e}_2$, see Fig. 5. The change of electron energy ΔW is deter-

FIG. 5. Geometry of the electron free motion in the vicinity of a dielectric sphere showing angle parameters which define the closest approach point and the velocity direction.

mined by the electric field projected on the electron velocity:

$$\Delta W(\mathbf{R}_0, \vec{\beta}, \sigma, t_0) = \lambda e e^{-i\omega t_0} \int_{-T_0}^{T_0} e^{-i\omega\tau} \vec{\beta} \cdot \vec{E}_\sigma[\vec{R}(\tau)] d\tau, \quad (10)$$

where $[-T_0, T_0]$ is the time interval in which the driving electric field is significant. (Note that units are so chosen that $\omega = k = 2\pi$.)

The above complex energy change depends on t_0 and the polar angle Φ through the phase factor. Therefore the possible energy changes, determined by the modulus $|\Delta W|$, are independent of t_0 and Φ . They depend on R_0 and the angles: Θ and velocity azimuth A .

Figure 6(a) shows the energy gain per λ , $G/\lambda = |\Delta W|/\lambda$ (in which the electric field of the incident wave is taken as a unit) for electrons passing just above the surface of the sphere, $R_0 = a$, as a function of the angles A and $\Psi = 90^\circ - \Theta$. The results correspond to the WGM TE resonance with $l = 22$, excited by the clockwise circularly polarized electromagnetic wave ($\sigma = 1$). It should be noted that there is a symmetry between the “North” and “South” half of the sphere, while there is no such symmetry between positive and negative azimuths, A and $-A$. However, for the counterclockwise polarization of the incident wave, $\sigma = -1$, the analogical plot is the mirror image in the vertical axis passing through $A = 0$.

The largest energy gain is observed around the polar regions of the sphere. Details are shown in Figs. 6(b) (G/λ as a function of the azimuth A for several angles Ψ) and 6(c) (G/λ as a function of Ψ for two azimuths).

The argument of $\Delta W(t_0 = 0)$ specifies the resonant accelerating phase ϕ_{acc} and the optimal moment $kt_0 = -\phi_{\text{acc}}$ for the particle's maximum energy increase. The accelerating phase ϕ_{acc} being a function of the polar angle Φ of the point R_0 can be determined from its value for $\Phi = 0$. For other particles, $\phi_{\text{acc}}^\Phi = \phi_{\text{acc}} + \sigma\Phi$. Figures 7(a)–7(d) show detailed dependence on time of the accelerating electric field projected on the electron velocity, for electrons crossing the WGM resonant field and arriving at the corresponding accelerating phase ϕ_{acc} . The interaction between an electron and the field is very short, of the order of the wave period, when an electron passes the distance of the order of the wavelength λ . Nevertheless, the interaction can be very strong, as the electric field exceeds that in the incident wave by up to 1000 times. To estimate the actual energy transfer, we have to assume a definite value of the electric field of the incident wave, E_0 . For my estimates I assume $E_0 = 100$ MV/m. Certainly, this huge intensity wave, if used in the microwave range employed in traditional accelerators, would be very devastating in contact with any conducting objects, causing their instantaneous destruction due to electrical discharges. However, it should not be so destructive in contact with a very low loss transparent dielectric (insulator). Moreover, very small loss may allow further accumulation and increase of the

FIG. 6. Gain over λ as a function of the electron velocity parameters. (a) Contour plot of the gain as a function of A (velocity azimuth) and $\Psi = 90^\circ - \Theta$ for $\sigma = 1$ (clockwise polarization), $l = 22$ TE resonance. (b) Gain over λ as a function of the velocity azimuth A for several angles Ψ . (c) Gain over λ as a function of Ψ for azimuths $A = 90^\circ$ and $A = -90^\circ$.

field in the WGM resonance conditions, without serious destruction of the scatterer. With the assumed value of the incident field, one obtains $G/\lambda \approx 30$ GeV/m, for a single acceleration process as described above.

Figure 8 illustrates a typical electron energy gain as a function of the trajectory distance from the sphere surface d . Obviously, in applications this distance as well as the electron beam width should be kept as small as possible.

FIG. 7. The accelerating electric field as a function of time for several characteristic trajectories and the corresponding acceleration phases. The acceleration phases ϕ_{acc} are given for particles for which the point of the closest approach lies on the meridian $\Phi = 0$. Parts (a), (b), and (c) correspond to electron trajectories passing the polar regions where energy gains are highest. Part (d) corresponds to an electron passing across the equatorial plain $\Theta = 90^\circ$. This case is similar to that of the electron moving near a dielectric cylinder at WGM resonance, discussed in [5].

FIG. 8. Typical energy gain over λ as a function of the electron path separation from the sphere surface, d .

A. Electron deflection

Beside changing particles energy the strong WGM resonant near fields can change their momentum deflecting them from the assumed trajectories. For electrons moving along an assumed straight line the force, dependent on the electric and magnetic fields, becomes dependent on time and on the initial moment t_0 or equivalently the initial phase denoted as ϕ_{acc} . The change of electron momentum $\Delta \mathbf{P}$ is determined by the Lorentz force

$$\Delta \mathbf{P}(\mathbf{R}_0, \vec{\beta}, \sigma, t_0) = \frac{\lambda}{c} e \text{Re} \left[e^{-i\omega t_0} \int_{-T_0}^{T_0} e^{-i\omega \tau} \{ \mathbf{E}_\sigma[\vec{R}(\tau)] + \vec{\beta} \times \mathbf{B}_\sigma[\vec{R}(\tau)] \} d\tau \right]. \quad (11)$$

Figure 9 shows these forces projected on the directions $\vec{n}_0$ and $\vec{n}_0 \times \vec{\beta}$, orthogonal to the electron velocity $\vec{\beta}$, for the electron pass corresponding to Fig. 7(a) for two characteristic ϕ_{acc} as functions of time. The electron may gain large transverse momentum, particularly along the radial direction, when $\phi_{\text{acc}} = -90.1^\circ$; however, for the maximally accelerated electron which must arrive at $\phi_{\text{acc}} = 179.8^\circ$ the deflecting forces oscillating mutually compensate the particle's deflections. For the TE resonances, the particle deflections are caused by the magnetic field, the maximum of which is shifted with respect to the maximum of the electric field (responsible for electron energy changes), by a quarter of a wave period.

Therefore, the maximally accelerating electrons pass the strong WGM resonant field region with negligible deflection and beam emittance degradation. In consequence, repetitions of the above single elementary acceleration process are possible and very high energy particles, necessary in fundamental research, can be obtained.

Additionally, in the development of such a multistage accelerator it is possible to secure a “phase stability”

FIG. 9. The Lorentz forces experienced the selected electrons as functions of time. The corresponding integrated momentum transfers $c\vec{P} \cdot \vec{n}_0/\lambda$ and $c\vec{P} \cdot \vec{n}_0 \times \vec{\beta}/\lambda$ reduce from 461 to 0.8 in (a) and from 12.5 to 0.03 in (b).

condition, as was discussed for an inverse free electron laser accelerator in [11], and is fulfilled in any accelerator.

B. Multiscatterer acceleration

Developing such a multistage accelerator, let us notice that when the scattering spheres are placed periodically along the electron trajectory given by Eq. (9) with the period D satisfying the condition $(\beta_z - 1)D = (\sin\theta \cos\alpha - 1)D$ is an integer number then the electron passing each scatterer experiences identical forces (including its timing) and can accumulate the possible energy increase.

An example of the basic geometry of the proposed multistage accelerating structure is shown in Fig. 10. An array of dielectric spheres is excited resonantly by an incident electromagnetic wave. The sphere centers are distributed periodically with the electron trajectory which

FIG. 10. An example of accelerating electron bunches passing synchronously above a periodic chain of scattering spheres. The bunches are separated by λ and the distances between the spheres are $D = 6\lambda$. The x' axis is parallel to the electron velocity $\vec{\beta}$.

“touches” the corresponding parallel circles $\alpha = 90^\circ$ and the distances between the spheres are $D = 6\lambda$.

The above multistage accelerator configuration resembles Palmer’s idea of open accelerating structures employing the inverse Smith-Purcell effect [12]. An essential point in the acceleration mechanism presented here is that the individual elements of the diffraction grating are in the resonant conditions.

As a rough estimate of the number of elementary units in 1 m, we can take $N_{\text{acc}} = 1/(6\lambda)$. With this choice the energy yield per meter would be (G/λ is λ independent)

$$\Delta\mathcal{E}_{\text{acc}}/m \approx \lambda \frac{G}{\lambda} E_0 N_{\text{acc}} \approx 5 \text{ GeV/m}. \quad (12)$$

Though this estimation may look very simple, nevertheless, it is quite reliable as the individual energy kicks are well separated.

Let us emphasize that $\Delta\mathcal{E}_{\text{acc}}/m$ is λ independent and so, in the application to high energy accelerators, a wide spectrum of frequencies can be considered. Higher frequency systems may be attractive as requiring less electromagnetic wave energy, due to the smaller volume and shorter pulses necessary for the excitation of resonance. On the other hand, in the lower frequency accelerator systems it is easier to secure the required synchronization of the driving waves and the particles.

Considering such an open accelerator setup, we must estimate the radio frequency power requirements. Assuming the transverse width of incident radiation beam to be $\Delta y' \approx 5\lambda$, we get the power required for 5 GeV acceleration to be about 700 GW with $\lambda = 1 \text{ cm}$ radiation, while for IR laser $\lambda = 1 \mu$ it reduces to about 70 MW. This indicates that in the microwave radiation range, the proposed open accelerator setup cannot compete with the conventional accelerator structures employing the driving radiation closed in high Q cavities.

To illustrate the effectiveness of the proposed accelerator setup employing the WGM resonances, it can be compared with an analogous estimation for a nonresonant (few-optical cycle laser-driven) particle accelerator structure [13]. In that case, the estimated acceleration rate was 10 GeV/m (twice as much as in our scheme) but the

required power density turned out to be 10^4 times greater than ours.

IV. NONIDEAL DIELECTRICS AND RADIATION ABSORPTION

Discussing the enhanced acceleration mechanism employing the WGM resonances, we have assumed ideal dielectrics capable to accumulate even the strongest electromagnetic resonant field without any absorption and losses. However, as the interesting fields reach the power flux density in the GW/cm² range, so even very low dielectric losses can significantly contribute to the scatterer heating, thus limiting the accumulated fields and achievable acceleration.

Because of dielectric internal losses associated with the imaginary part of the dielectric constant $\text{Im } \epsilon$, there is net field momentum flux flowing across the scatterer boundary towards its interior where it is absorbed, increasing the scatterer temperature, and in extreme cases causing its destruction.

The average power crossing through the samples boundary is given by

$$P_T = \frac{1}{4\pi} \int d\phi d\theta \sin\theta \mathbf{n}_0 \cdot \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*) \quad (13)$$

evaluated above the scattering sphere boundary at $r = a$.

Figure 11 shows this power for two WGM resonance cases (TE²² and TE¹⁶) as a function of $\text{Im } \epsilon$. The power P_T is expressed in the unit of $P_0 = 1.33 \text{ GW/cm}^2$ determined by the incident wave electric field strength E_0 . The maximal power enters the dielectric and is dissipated inside when the Q factor of resonances is about the inverse of $\text{Im } \epsilon$.

The energy, or fluence, absorbed in the sample,

$$E_{\text{absor}} = P_T T_{\text{pulse}},$$

depends on the duration time of the incident radiation pulse. To excite the resonance this pulse length duration should be

FIG. 11. Average power entering the dielectric spheres at two WGM resonances as the function of $\text{Im } \epsilon$.

$$T_{\text{pulse}} = Q_{\text{tot}} T_0,$$

where $T_0 = \lambda/c$ and the Q_{tot} —total quality factor. This factor depends on the radiation coupling with the continuum, as for perfect dielectrics, and additionally on an absorption in imperfect dielectric. It is

$$Q_{\text{tot}} = \frac{1}{(Q^{-1} + \text{Im } \epsilon)}.$$

Figure 12 shows E_{abs} as a function of $\text{Im } \epsilon$ with λ as a parameter for two cases of WGM resonance (TE²²_{res} and TE¹⁶_{res}).

It is believed that the destructive fluence value is about 10 J/cm². To reduce the absorption fluence below that value, one can consider lower λ , lower $\text{Im } \epsilon$, and lower order of TE.

However, the total absorbed energy per volume of the scattering sphere, which may be considered as a criterion of sample heating and its destruction

$$U_{\text{absor}} = \frac{4\pi a^2 \lambda^2}{\frac{4}{3}\pi a^3 \lambda^3} E_{\text{absor}},$$

does not depend on λ .

Thus, at the TE²²_{res} the maximal dissipation energy density, corresponding to $\text{Im } \epsilon = 10^{-7}$, is $U_{\text{absor}} \approx 1400 \text{ J/cm}^3$. It is about 100 times smaller for the TE¹⁶_{res}. However, this resonance is wider and the field concentration is also reduced, so the resulting particle's energy gain would be around 10 times smaller than in the previous case.

Further reduction of absorption losses requires less lossy dielectrics. While in the microwave range, the measured ultralow losses correspond to $Q \approx 10^7$ [14], in the IR laser range, the corresponding values of $Q \approx 10^{10}$ were reported in Refs. [15,16] (this required $\text{Im } \epsilon < 10^{-10}$). Similar ultralow losses are achievable in communication fibers with attenuation 0.15 dB/km [17]. Such losses were also assumed in [18] in the study of thermal effects for the optical Bragg accelerator.

For this loss the absorbed energy reduces to $U_{\text{absor}} \approx 14 \text{ J/cm}^3$ which would remove any problem with accelerator components heating and their destruction.

The heat deposition just discussed is probably the most important dielectric damage mechanism due to bulk and surface breakdown of dielectric materials. Measurements on pure dielectric materials without (shape dependent) resonances show that nsec laser pulses with intensities of the order of 500 GW/cm² can be withstood [19]. A more complete treatment of dielectric breakdown in the conditions of WGM resonances, including also multiphoton atom ionization as well as the possibility of self-focusing of optical pulses, is addressed to future theoretical and experimental studies.

TABLE I. Selected resonance parameters, $\epsilon = 4$.

n_{res}	$a_{\text{res}}^{\text{TE}}$	$ a_n^{\text{TE}} $	Q	$a_{\text{res}}^{\text{TM}}$	$ a_n^{\text{TM}} $
10	1.086	22.74		1.1535	34.4
16	1.6169	186.9	6.9×10^4	1.686 17	305.7
22	2.134 697 0	1786.8	7.6×10^6	2.204 449	3021.8
26	2.475 931 48	8431.2	1.9×10^8	2.545 786 2	14 455.7

V. FINAL REMARKS

Looking for a more efficient way to high energy particles acceleration, I investigated the use of very strong fields created in the dielectric sphere and in its closest vicinity in whispering-gallery resonance conditions.

My theoretical estimations concentrated on dynamical problems of the electron field interaction assuming ideal lossless dielectric. These estimations show a potential usefulness of the proposed setup in the acceleration of high energy particles. The calculated energy gains are comparable to those obtainable in the plasma wakefield accelerators with a smaller power driving radiation.

There is a need for more experimental data for the behavior of WGM resonances in real nonideal dielectrics with high power radiation. These data are necessary to give a reliable evaluation of the achievements of the proposed accelerators made of real dielectrics.

It is obvious that there will be some restrictions on the radiation power charging this system and achievable energy gain as in the ordinary accelerators. However, one may expect that the destructive power be much higher in the proposed scheme that consists of highly transparent components than in traditional accelerators made of non-transparent components. In consequence, the acceleration rate and achievable energies may increase.

In conclusion, more experimental research on the behavior of WGM resonances in strong electromagnetic

FIG. 12. Absorbed fluence at two WGM resonances and two wavelengths as the function of $\text{Im } \epsilon$.

fields is both fundamentally important and can contribute to the construction of future high energy particle accelerators.

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APPENDIX A: VECTOR SPHERICAL HARMONICS

Following [10], the vector spherical harmonics can be defined in terms of the angular momentum operator $L = -i\mathbf{r} \times \nabla$ as

$$X_{lm}(\theta, \phi) = \frac{1}{\sqrt{l(l+1)}} L Y_{lm}(\theta, \phi), \quad (\text{A1})$$

leading to

$$\mathbf{X}_{lm}(\theta, \phi) = \frac{1}{\sqrt{l(l+1)}} \begin{bmatrix} \frac{1}{2}[Y_{lm+1}\sqrt{(l-m)(l+m+1)} + Y_{lm-1}\sqrt{(l+m)(l-m+1)}] \\ i\frac{1}{2}[-Y_{lm+1}\sqrt{(l-m)(l+m+1)} + Y_{lm-1}\sqrt{(l+m)(l-m+1)}] \\ mY_{lm} \end{bmatrix}. \quad (\text{A2})$$

A useful identity for the spherical Bessel functions $f_l(r)$ and the vector spherical harmonics $\mathbf{X}_{lm}$ is

$$\begin{aligned} \nabla \times f_l(r) \mathbf{X}_{lm} &= i\mathbf{n} \sqrt{l(l+1)} \frac{f_l(r)}{r} Y_{lm} \\ &+ \frac{1}{r} \frac{\partial}{\partial r} [r f_l(r)] \mathbf{n} \times \mathbf{X}_{lm}. \end{aligned}$$

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