Bunched beam stochastic cooling in a collider

M. Blaskiewicz and J. M. Brennan

BNL, Upton, New York 11973, USA

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Bunched beam stochastic cooling in the Relativistic Heavy Ion Collider (RHIC) at 100 GeV has been achieved. The longitudinal cooling system is designed for heavy ion operation but was tested using protons. A very low intensity bunch with $\sim 10^9$ protons was prepared so that cooling times and voltage requirements would be comparable to the heavy ion case. With this bunch a cooling time of the order of an hour was observed through shortening of the bunch length and narrowing of the Schottky lines.

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I. INTRODUCTION

A stochastic cooling system is a wideband feedback loop [1,2]. The physical layout of a cooling system is illustrated in Fig. 1. The beam signal is sampled by the pickup. The beam and signal propagate from the pickup to the kicker, and a kick derived from the pickup signal is given to the beam. The kick alters the beam dynamics, significantly modifying the pickup signal. The closed loop system may be viewed as the feedback circuit illustrated in Fig. 2. The Schottky current from the beam I_s is detected by the pickup P . There is also a signal associated with the coherent response of the beam due to the kicker, I_1 . The sum of these currents is filtered and amplified through an effective transfer impedance, Z_T . A voltage, $V_K = -Z_T(I_s + I_1)$, is generated by the kicker. For appropriate phases and gains the beam is cooled, resulting in a small change to the Schottky signal. In addition to the desired change to the Schottky signal there is a much larger coherent response due to the beam transfer function, $I_1 = BV_K$. The beam transfer function depends on the beam properties in the fluid limit and evolves slowly over a cooling time. Over time scales short compared to the cooling time, one may neglect the slow evolution of B and directly relate the kicker voltage to the Schottky current,

$$V_K = \frac{-Z_T I_s}{1 + BZ_T} = -Z_D I_s, \quad (1)$$

where $Z_D = Z_T/(1 + BZ_T)$ is the dressed impedance. For unbunched beams the beam transfer function is a complex, scalar function of frequency. With bunched beams of length τ_b , kicker signals at frequency f cause response at frequencies $f + nf_0$, where f_0 is the revolution frequency and $|n| \lesssim 1/f_0\tau_b$. For this case B is a matrix [3–6] and Eq. (1) must be interpreted as a matrix equation.

With system bandwidth W , one obtains a time resolution $\tau \sim 1/2W$. For a beam of particles with charge q and current I , the longitudinal cooling system measures the average energy of samples containing $N_s = I\tau/q$ particles

each turn. This signal is filtered, amplified, and applied to the beam so as to reduce the energy spread. If the beam requires M turns to mix the samples into statistical independence, the optimal cooling time scales as $\sigma_E/\dot{\sigma}_E \approx 2N_s T_0 M$, where the revolution period is $T_0 = 12.8 \mu\text{s}$ for RHIC. Transverse pickups and kickers are used to reduce the transverse emittance, and systems of both types are essential in the operation of existing antiproton sources and several low energy ion rings [7]. For these systems the beams are essentially, if not totally, unbunched and wide-band pickup/kicker pairs work well.

Bunched beam stochastic cooling was observed in 1978 in the Initial Cooling Experiment (ICE) [8]. In the initial publication it was noted that correlated synchrotron sidebands imply that the optimal cooling gain need not be flat in frequency, as in the coasting beam case, but could have maxima spaced at the inverse of the bunch length. The RHIC system exploits this property.

A theory of bunched beam cooling was developed in the early 1980's [3,5,6] and stochastic cooling systems for the Super Proton Synchrotron (SPS) [4,9] and the Tevatron [4,10] were explored. For these colliders the particle densities were much higher than those in ICE and early on [10–14] it was found that “rf activity” extending up to very high frequencies swamped the true Schottky signal.

FIG. 1. Physical schematic of the cooling system.

*blaskiewicz@bnl.gov
†brennan@bnl.gov

FIG. 2. Circuit schematic of the cooling feedback loop.

Cooling for heavy ions in RHIC [15,16] was also considered. In RHIC, the particle densities for heavy ions are significantly lower than in the Tevatron and SPS. This, along with technological improvements, made cooling feasible in RHIC. We go on to describe the cooling system, present first observations of cooled beam, and compare the data with simulations.

II. PRINCIPLES OF THE COOLING SYSTEM

Intrabeam scattering rates for gold beams in RHIC are of the order of one hour [16,17], setting the scale for the cooling system. Optical fibers transmit the signal from the pickup [18,19] to the kicker. To keep costs down the fibers are in the tunnel and travel against the beam. The pickup is in the 12 o'clock straight section and the kicker is in the 4 o'clock straight section. At the collision energy of 100 GeV/nucleon, the frequency slip is mainly due to dispersion in the arcs so the effective delay between pickup and kicker is very close to 2/3 turn or 8.5 μ s.

The cooling system operates between 5 and 8 GHz and an rms kicker voltage ~ 1 kV is needed for optimal cooling [17]. Systems based on traveling wave tube amplifiers and broadband kickers would be quite expensive, so we have taken an alternate approach that makes use of the small duty cycle of the beam. Heavy ions are bunched by a 197 MHz rf system and the bunches are spaced by 106 ns. The voltage is generated via a Fourier synthesis technique which is summarized in Fig. 3. In Fig. 3 the top trace is the bunch current. The Schottky signal from this bunch is processed giving the required kicker voltage, which is shown as the blue part of the center trace. The red trace is a function of the form

$$V(t) = \sum_{k=n_-}^{n_+} a_k(t) \cos(2\pi k t / \tau_0) + b_k(t) \sin(2\pi k t / \tau_0), \quad (2)$$

where τ_0 is a constant delay of order the bunch length. The a_k and b_k have an exponential rise time τ_c and are chosen so that the red and blue traces are equal when the bunch is

FIG. 3. (Color) Scheme for creating the kicker pulse. The top trace is the bunch current. The desired voltage is the blue trace on the center curve. The beam is present only where this trace is nonzero. The red curve is the total kicker voltage and the black curve below is the voltage on one of the cavities.

present. The terms in the sum are then identified with the voltages on individual cavities with full width half power bandwidth $\Delta f = 1/\pi\tau_c \approx 10$ MHz.

To create the low-level drive, we start with the 5 ns pulses out of the pickup, $S_0(t)$. Next we apply a traversal filter using a combination of coaxial transmission line and fiber optic delays

$$S_1(t) = \sum_{k=0}^{M-1} S_0(t - k\tau_0), \quad (3)$$

where $M = 16$ is the effective number of delay lines. The spectrum of S_1 has peaks of width 12 MHz centered at multiples of 200 MHz. Doing some of the processing before the fiber optic link reduces the dynamic range of the signal through the fiber optic delays.

The delayed signal $S_1(t - T_1)$ arrives in the low-level control room, where T_1 is the travel time between the pickup and the control room. This prompt signal is split and delayed for one or two turns. For the proton analog we used the traditional one turn delay $S_2(t) = S_1(t - T_1) - S_1(t - T_1 - T_0)$ where T_0 is, precisely, the revolution period. For cooling gold we plan to use $S_2(t) = S_1(t - T_1) - 2S_1(t - T_1 - T_0) + S_1(t - T_1 - 2T_0)$ which corresponds to two, one turn delay filters in series, and has a somewhat better phase margin than the traditional one turn delay. The low-level system is quite involved [20] and much effort

went into fighting nonlinear effects while maintaining good signal to noise.

In either case, the signal S_2 is fanned out and sent through bandpass filters of width 100 MHz centered at multiples of $1/\tau_0 = 200$ MHz. The signals out of the bandpass filters consist of 80 ns sinusoidal bursts. Each burst has a fixed phase and amplitude but the phase and amplitude vary from one burst to the next. These signals are amplified by individual narrow band solid state amplifiers and sent to the kicker cavities.

The kicker cavities have resonant frequencies 5.0, 5.2, ..., 8.0 GHz, full width half power bandwidths close to 10 MHz, and $R/Q \sim 100 \Omega$. For a drive power of 10 W, the 6 GHz cavity generates an rms voltage of ~ 700 V. The amplifiers are rated for 40 W so we are operating in the linear region. Given such high frequencies, the aperture of the cavities is only 2 cm. To reduce aperture limitations during injection and acceleration, the kicker cavities are split along the beam axis and are closed only after reaching flattop. The tanks and motors were supplied by FNAL and retrofitted for our application.

III. RESULTS WITH PROTONS

For a fixed system bandwidth the optimal cooling time is proportional to the number of particles. Typical proton bunches in RHIC contain 10^{11} particles and would cool far too slowly with a heavy ion system designed for 10^9 particles. Therefore, a special proton bunch was prepared by using the tune meter kicker to reduce its intensity to 1.5×10^9 particles. Other relevant parameters are given in Table I.

The pickup signal was gated to accept only the test bunch so that the signals from the other bunches would not saturate the low-level electronics. Figure 4 shows the open loop beam transfer function for the test bunch. Transfer functions of this sort were taken every few minutes and used to measure the gain and phase of the cooling system. Corrections were applied by adjusting the phase and amplitude of the drive signals upstream of each amplifier, keeping the cooling rate optimal. When the cooling

TABLE I. Machine and beam parameters.

Parameter	Nominal protons	Special protons	Gold
Acceleration			
$h = 360$ voltage	300 kV	300 kV	300 kV
Storage			
$h = 2420$ voltage	50 kV	50 kV	4 MV
FWHM bunch length	10 ns	7 ns	3 ns
Particles/bunch	10^{11}	1.5×10^9	10^9
Lorentz factor	107	107	107
Circumference	3834 m	3834 m	3834 m
Transition gamma	22.89	22.89	22.89

FIG. 4. Longitudinal beam transfer function near 5.2 GHz.

FIG. 5. Signal suppression near 5.2 GHz.

FIG. 6. Initial and final wcm data.

system was turned on the current I_1 in Fig. 2 reduced the pickup signal. This signal suppression is shown in Fig. 5. This verified that the gain and phase of the feedback loop was appropriate for cooling the beam. The system was allowed to operate for an hour. The bunch profiles as measured by the wall current monitor (wcm) are shown in Fig. 6. One sees that cooling produced an increase in peak current.

IV. COMPARISON WITH SIMULATIONS

A stochastic cooling system may be modeled using the same sort of algorithms as those used for coherent instabilities. The algorithms for such codes are well developed and modern desktop computers are able to handle all but the most extreme cases [21]. The only caveat is that the number of macroparticles available in a simulation is small compared to the actual number of particles in a bunch. Luckily, there is a simple scaling law that allows the simulation results to be quantitatively extended to the actual beam.

The scaling law is an application of the well-known result that the cooling time for the beam is proportional to the number of particles in the beam. Consider a group of N particles with momenta p_j . Use a simple model for the one turn update,

$$\bar{p}_j = p_j - \frac{g}{N} \sum_{k=1}^N p_k, \quad (4)$$

where g is the cooling gain and $\bar{p}_j$ is the updated momentum. Squaring both sides and averaging over j gives

$$\frac{1}{N} \sum_{j=1}^N \bar{p}_j^2 - p_j^2 = [-2g - g^2] \frac{1}{N^2} \sum_{k,j=1}^N p_k p_j. \quad (5)$$

Taking an ensemble average, the left side is the change in the squared momentum spread due to cooling, $\Delta\sigma_p^2$. For the right-hand side of Eq. (5), we need to estimate $\langle p_k p_j \rangle$. For perfect mixing there are no turn to turn correlations induced by the cooling system and $\langle p_j p_k \rangle = \sigma_p^2 \delta_{j,k}$ giving

$$\frac{\Delta\sigma_p^2}{\sigma_p^2} = \frac{-2g - g^2}{N}, \quad (6)$$

which is the usual result for perfect mixing [2]. For limited mixing, one must consider the details of the actual system and, in particular, the implications of Eq. (1). For this case we note that the signal suppression in Eq. (1) is a manifestation of the beam properties in the continuous limit. As long as N is sufficiently large, and the gain is defined with respect to fluctuations in the averages, the amount of signal suppression will be independent of N . To apply this result consider a bunch of N particles and a simulation of this bunch using N_m macroparticles. The cooling time measured using the simulation, $\tau_{c,s}$, will be related to the cooling time of the bunch τ_c via

$$\tau_c = \tau_{c,s} \frac{N}{N_m}. \quad (7)$$

By doing simulations with different values of N_m , we test Eq. (7).

The simulations involve single particle updates and collective kicks. For the single particle updates, we use a simple drift-kick algorithm. For the collective kicks we use a fine grid and linear interpolation to bin the particles on an interval of $\tau_0 = 5$ ns. Particles that are initially outside the interval have multiples of τ_0 added or subtracted until they lie within the interval. This has the same effect as the 200 MHz spacing in the cavity resonant frequencies. The gridded data from the previous update are subtracted from the current array, effecting a one turn delay notch filter. Then, a fast-Fourier-transform (FFT) convolution is used to calculate the coherent kick. A single particle update between the pickup and kicker is followed by applying the coherent kick. When applying the kick we add appropriate multiples of τ_0 and use linear interpolation between the grid points. A single particle update completing the rest of the turn ends the procedure.

Data and simulations comparing signal suppression at 5.2 GHz are shown in Fig. 7. The simulation results are offset by 20 dBm so all four curves can be seen. The simulations used 2/3 turn delay between the pickup and the kicker which cools all frequencies within ± 16.7 kHz of a revolution line. The gain was chosen to recreate the signal suppression ± 5 kHz away from the revolution line. The simulated Schottky spectra were calculated by choosing a central frequency f_s . During the simulation the integral $\int d\tau I(\tau) \exp(2\pi i f_s \tau)$ was calculated each turn, and written to disk. Discrete Fourier transforms of subsets of the file yielded the spectra.

FIG. 7. (Color) Comparison of simulated and actual signal suppression. The red and green lines are the simulated Schottky spectra with cooling off and on, respectively. The black and blue lines are the measured data with the cooling system off and on, respectively. The simulations are offset by +20 dB for clarity.

FIG. 8. (Color) Test of the scaling law over 2 orders of magnitude. Equation (7) was used to relate the equivalent time for 1.5×10^9 particles to the number of macroparticles used in the simulation.

A test of Eq. (7) is shown in Fig. 8. The smooth black line shows the cooling of the rms energy spread for 10^6 macroparticles. The parameters match the experiment. The blue line for 10^5 macroparticles shows some oscillatory decay, and the red line with 10^4 macroparticles shows pronounced oscillations about the smooth decay. The oscillations are partly statistical, but there are also systematic effects resulting from transients. In any case, scaling to 10^9 is only 3 orders of magnitude beyond the 2 orders plotted.

Data and simulations comparing wall current monitor data for one hour of cooling are shown in Fig. 9. Both show enhanced peak current though the enhancement for the simulations is larger than the enhancement in the data.

FIG. 9. (Color) Comparison of initial and final wall current monitor data with simulations using 10^6 macroparticles. The red and green lines are the simulated profiles before and after cooling, respectively. The black and blue lines are the experimental data before and after cooling, respectively.

FIG. 10. (Color) Comparison of initial and final wall current monitor data with simulations including rf noise. The red and green lines are the simulated profiles before and after cooling, respectively. The black and blue lines are the experimental data before and after cooling, respectively.

About half the discrepancy in the peak current can be accounted for by including rf noise, as shown in Fig. 10. To add the rf noise we started by using the measured increase in bunch length for the uncooled bunches, $\Delta\sigma_t^2 = 1.14 \text{ ns}^2/\text{hour}$. This is dominated by rf noise, intrabeam scattering for a nominal bunch is about 4 times smaller. The measured increase was then multiplied by the ratio of beam particles to macroparticles and a routine was implemented to apply random kicks each turn. For our simulation with 10^6 macroparticles, the diffusive mixing time [22] is hundreds of turns, which is much slower than

FIG. 11. (Color) Comparison of initial and final Schottky spectra at 4 GHz with simulations. The red and green lines are the simulated profiles before and after cooling, respectively. The black and blue lines are the experimental data before and after cooling, respectively. The simulations are offset by +20 dB for clarity.

mixing from the rf. Therefore, the scaling in Eqs. (7) still holds. Figure 11 shows measured and simulated Schottky spectra before and after cooling, with rf noise included.

We assume the rest of the discrepancy between the data and the simulation is due to inaccuracies in the cooling system. In particular, there are shoulders starting about ± 10 kHz from the revolution line on the signal suppressed data in Figs. 5 and 7. In part these are due to debunched beam leaking out of the 100 other buckets with 10^{11} protons each. These shoulders may also be due to nonlinearities or phase errors. In any case, the agreement between the data and simulations is quite good and we argue that this technique can be used to obtain viable designs.

V. CONCLUSIONS

Longitudinal bunched beam stochastic cooling has been achieved in RHIC. The technique of using an array of narrow band cavities as kickers has been experimentally verified. Accurate simulations have been performed, demonstrating that quantitative predictions are possible. We look forward to making the system operational and using similar simulations to design transverse cooling systems for RHIC.

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