Floquet higher-order Weyl and nexus semimetals

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This work reports the general design and characterization of two exotic, anomalous nonequilibrium gapless topological phases. In equilibrium systems, the Weyl nodes or the crossing points of nodal lines of three-dimensional systems may become the transition points between higher-order and first-order topological phases defined on two-dimensional slices, thus featuring both hinge Fermi arc and surface Fermi arc. We advance this concept by presenting a strategy to obtain, using time-sequenced normal insulator phases only, Floquet higher-order Weyl semimetals and Floquet higher-order nexus semimetals, where the concerned topological singularities in the three-dimensional Brillouin zone border *anomalous* two-dimensional higher-order topological phases. Our theoretical results stimulate and guide future experiments on higher-order semimetal phases using, for example, a three-dimensional lattice of coupled ring resonators.

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Introduction. Owing to their rich features not available in equilibrium counterparts, the great potential of nonequilibrium topological phases is being widely recognized [1–13]. Given that even an otherwise topologically trivial system can be converted into a topological nontrivial phase via periodic driving [1,14], nonequilibrium strategies that utilize the time dimension significantly expand the domain of topological phases and at the same time lead us to many possible applications of topological matter in photonics [5,15–20], acoustics [21], quantum information, etc. [22,23]. Well-known three-dimensional (3D) periodically driven (Floquet) topological phases include Floquet Weyl semimetals [10,24–38], anomalous Floquet Weyl semimetals [39,40], and Floquet topological insulators [41–43]. Floquet Weyl semimetals may support a single Weyl point, thus constituting an excellent platform to investigate the chiral magnetic effect [37,38]. The so-called Floquet Hopf insulator, whose full topological description requires one $\mathbb{Z}$ -type Hopf linking number and another intrinsically dynamic $\mathbb{Z}_2$ invariant [42,43], is another fascinating example that has advanced the notion of topological insulators.

Our focus here is on Floquet higher-order topological phases. Floquet higher-order topological insulators (FHOTIs) have already been designed [23,44–65], whereas the topic of Floquet higher-order semimetals with boundary-of-boundary states is still in its infancy. Here we report two exotic Floquet semimetal phases, namely, Floquet higher-order Weyl semimetals (HOWSs) and Floquet higher-order

nexus semimetals (HONSs). They are generated by a flexible and experimentally relevant driving protocol, where the instantaneous Hamiltonian is *always* a normal insulator. The obtained phases are not describable by standard equilibrium approaches (such as quadrupole moments [66–68]) and hence markedly different from early studies on Floquet HOWSs of which the results under high-frequency driving do have equilibrium analogs [69,70].

Due to the Nielsen-Ninomiya theorem [71,72], the minimal Weyl semimetal in a lattice accommodate two Weyl nodes of opposite charges, as shown in Fig. 1(a). In a traditional Weyl semimetal, the Weyl node represents the phase transition point between the Chern insulator and normal insulator of some 2D slices, so it supports the well-known surface Fermi arc upon opening up its boundary along one direction. By contrast, in a HOWS [69,73–75] the Weyl node is the phase transition point between a Chern insulator and a higher-order topological insulator (HOTI). As such, in addition to the expected surface Fermi arc, the system possesses the hinge Fermi arc when opening up its boundaries along two directions. On the other hand, HONS phases are featured by two crossed nodal lines, with the associated phase transition lines splitting 2D slices into Chern insulators and HOTI phases, so that the system also supports surface Fermi arcs and hinge Fermi arcs. The physics is much more involving in nonequilibrium situations, because the Weyl nodes in Floquet HOWS and the crossed nodal lines in Floquet HONS can feature *exotic* phase transitions between Floquet hybrid topological insulators (FHTI), anomalous Floquet topological insulators (AFI), and anomalous Floquet higher-order topological insulator (AFHOTI). Hybrid topological insulator, which supports Majorana surface cones and helical Majorana hinge modes at different energies, has been studied in static system [76]. Here FHTI, AFI, and AFHOTI refer to the three distinct cases where the two quasienergy gaps support the chiral edge mode and topological corner mode in a hybrid manner, chiral edge modes only, and topological corner modes

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FIG. 1. (a) HOWSs with surface Fermi arcs and hinge Fermi arcs (orange lines). The Weyl point with charge +1 (−1) is marked by blue (red) ball. (b) Same as in (a) but for HONSSs. The winding number surrounding the nodal line marked blue (red) tube is +1 (−1). (c) The high symmetry points in the Brillouin zone are marked by Γ, X, Y, S, Z, U, T , and R .

only, as summarized in Figs. 1(a) and 1(b). The said anomalous topological phases here have zero Chern numbers and vanishing multipolarization, and therefore only topological invariants capturing characteristics of the system's dynamical evolution within one complete period may fully describe them [4,56,77].

Model. Consider a periodically driven bipartite lattice shown in Fig. 2. This minimal model is composed of four time steps and only the nearest-neighbor coupling is required. Each unit cell consists of two sublattice degrees of freedom. In step ①, ②, and ④, the lattice is fully dimerized and only turns on intercellular coupling of strength γ within the xy layer. This kind of two-dimensional setup is equivalent to an experimentally realized platform of two-dimensional coupled ring resonators [56]. In step ③, the system allows for intracellular coupling of strength α and interlayer coupling of strength $\beta/2$. In each time step, the instantaneous Hamiltonian is only a normal insulator under the condition $\gamma \neq 0$ and $|\alpha| > |\beta|$.

FIG. 2. A periodically driven bipartite lattice subject to four modulation steps. Steps ①, ②, and ④ allow for intercellular coupling of strength γ within the xy layer. Step ③ introduces intracellular coupling of strength α and interlayer coupling of strength $\beta/2$. Only the interlayer coupling of a primitive unit cell is shown. The rectangle in step ① indicates one unit cell.

The time-dependent Bloch Hamiltonian is then given by

$$H(\mathbf{k}, t) = \begin{cases} H_1(\mathbf{k}) & 0 < t \leq T/4, \\ H_2(\mathbf{k}) & T/4 < t \leq T/2, \\ H_3(\mathbf{k}) & T/2 < t \leq 3T/4, \\ H_4(\mathbf{k}) & 3T/4 < t \leq T, \end{cases} \quad (1)$$

where

$$H_m(\mathbf{k}) = \gamma(e^{i\mathbf{b}_m \cdot \mathbf{k}} \sigma^+ + \text{H.c.}), \quad (2)$$

for $m = 1, 2, 4$. $\sigma^\pm = (\sigma_x \pm i\sigma_y)/2$, where $\sigma_{x,y,z}$ are Pauli matrices; and the vectors $\mathbf{b}_m$ are given by $\mathbf{b}_1 = (a, a, 0)$, $\mathbf{b}_2 = (a, 0, 0)$, and $\mathbf{b}_4 = (0, a, 0)$, where a is the lattice constant. Moreover,

$$H_3(\mathbf{k}) = \theta(k_z) \sigma_x, \quad (3)$$

where $\theta(k_z) = \alpha + \beta \cos(\mathbf{b}_3 \cdot \mathbf{k})$ with $\mathbf{b}_3 = (0, 0, a)$. The expression $\theta(k_z)$ here suggests that it already fully captures the effect of interlayer coupling along the z direction and so becomes important to digest our design below. The Floquet states can be found from

$$U_T(\mathbf{k}, t_0) |\phi(\mathbf{k}, t_0)\rangle = e^{-i\epsilon(\mathbf{k})T} |\phi(\mathbf{k}, t_0)\rangle, \quad (4)$$

where $U_T(\mathbf{k}, t_0) \equiv \mathcal{T} \exp[-i \int_{t_0}^{t_0+T} H(\mathbf{k}, \tau) d\tau]$ is the one-period time evolution operator, $\mathcal{T}$ is the time-ordering operator, and t_0 is a reference time.

A Weyl semimetal phase requires the breaking of either time-reversal or inversion symmetry [78]. The time reversal symmetry in our driven model is clearly broken. As to other aspects of the symmetry, our system obeys particle-hole $C^{-1}H(\mathbf{k}, t)C = -H^*(-\mathbf{k}, t)$ and inversion $\mathcal{I}^{-1}H(\mathbf{k}, t)\mathcal{I} = H(-\mathbf{k}, t)$ symmetry, where $C = \sigma_z$ and $\mathcal{I} = \sigma_x$. The particle-hole symmetry promises that the Weyl points appear at quasienergy zero or π/T while the inversion symmetry ensures that the Weyl points of opposite charge possess the same quasienergy. Furthermore, the system obeys twofold rotation symmetry about the z axis, i.e., $C_{2z}^{-1}H(\mathbf{k}, t)C_{2z} = H(\tilde{\mathbf{k}}, t)$ with $C_{2z} = \sigma_x$ and $\tilde{\mathbf{k}} = (-k_x, -k_y, k_z)$. This symmetry guarantees that there is always such twofold rotation symmetry even when we reduce the system to 2D slices at an arbitrary fixed value of k_z . The recognition that such 2D slices can support FHTI, AFI, and AFHOTI [56,57] hints genuinely anomalous HOWS phases.

Floquet HOWSs and SFA. The phase diagram for a 2D slice at a fixed k_z is shown in Fig. 3(a), in terms of possible values of $\theta(k_z)$ and γ . The topological phase transition happens at the Γ point ($k_x = 0 \wedge k_y = 0$) with quasienergy zero marked by the red dashed line, at the Γ point but with quasienergy π/T marked by green dashed lines or at two high symmetry lines $k_x = \pi/a \vee k_y = \pi/a$ with quasienergy zero marked by the red solid line (see Supplemental Material [79]). The FHTI phase is marked by the gray area in Fig. 3(a), enclosed by three boundaries: The green dashed line is the phase boundary between FHTI and AFHOTI; the red (dashed and solid) lines represent the phase boundary between FHTI and AFI.

To appreciate the flexibility of our design, one may now look into what happens if we tune α and β values and hence change the possible range of $\theta(k_z)$. As k_z varies from $-\pi/a$ to π/a , one can easily let $\theta(k_z)$ go across the phase boundaries between the above mentioned phases of the 2D

FIG. 3. (a) Phase diagram for system's 2D slices at fixed k_z . (b) Quasienergy spectrum and (c) pseudospin texture in the $k_x - k_y$ plane at k_z^c . Color represents the value of the x component and the direction indicates the other two components. (d) Surface Fermi arc of Floquet HOWS with quasienergy π/T in the $k_x - k_z$ plane or $k_y - k_z$ plane. The parameters for (b), (c), and (d) are $\alpha = 9\pi/20$, $\beta = \pi/5$, and $\gamma = 5\pi/6$. The FHTI phase is marked by the gray area in (a). In (a), dashed line (solid) is the phase transition point at Γ ($k_x = \pm\pi/a$ and $k_y = \pm\pi/a$). Green (red) line means topological phase transition at quasienergy zero ($\pm\pi/T$).

slices [see black lines in Fig. 3(a)] as three examples at three different values of γ . Precisely at the phase boundary, the bulk band of the original 3D system closes itself. For example, consider $\alpha = 9\pi/20$, $\beta = \pi/5$, and $\gamma = 5\pi/6$ such that $\pi/4 \leq \theta(k_z) \leq 13\pi/20$. This range of $\theta(k_z)$ covers the phase transition point at $\theta = \pi/2$ (determined by γ). Inverting the expression $\theta(k_z)$ one finds that the bulk band closes at $k_z = \pm k_z^c = \pm \arccos(1/4)/a$, with quasienergy π/T , yielding two band closing points $(0, 0, \pm k_z^c)$ as possible Floquet Weyl nodes.

Such possible locations of Weyl nodes can also be understood by symmetry eigenvalues at high symmetry momentum points (HSMPs). At $k_z = 0$, there are four HSMPs, Γ , X, Y, and S shown in Fig. 1(c). The inversion symmetry eigenvalues at those points are $(+1, +1, +1, +1)$. At $k_z = \pi/a$, there are also four HSMPs, Z, U, T, and R shown in Fig. 1(c). The corresponding symmetry eigenvalues are $(-1, +1, +1, +1)$. When the value of k_z changes from 0 to π/a , the symmetry eigenvalues must change along the $\Gamma - Z$ direction. Furthermore, the system's twofold rotation symmetry promises the band inversion occurs along $\Gamma - Z$, which again indicates that Weyl nodes are at $k_x = 0$ and $k_y = 0$.

To finally confirm the emergence of Weyl nodes, we present the linear quasienergy dispersion at the band crossing points $(0, 0, \pm k_z^c)$ in Fig. 3(b). The topology of the band crossing points can also be investigated by the pseudospin texture,

$$s_i^j = [j/T - \epsilon(\mathbf{k})] \langle \phi^e(\mathbf{k}, 0) | \sigma_i | \phi^e(\mathbf{k}, 0) \rangle, \quad (5)$$

FIG. 4. (a) and (b) Spectrum of a finite structure with open boundary condition along x, y and periodic boundary condition along z . The Weyl points are located at quasienergy π/T in (a) and at quasienergy zero in (b). (c) and (d) The corresponding winding numbers defined in the main text. The parameters for (a) and (c) are $\alpha = 9\pi/20$, $\beta = \pi/5$, and $\gamma = 5\pi/6$. The parameters for (b) and (d) are $\alpha = \pi/5$, $\beta = \pi/10$, and $\gamma = 7\pi/12$. The bulk band is marked by gray.

where σ_i is the Pauli matrix with $i = x, y, z$; $j = 0$ ($j = \pi$) for the band crossing points with quasienergy zero (π/T). Choosing the band below quasienergy j/T , the pseudospin texture thus defined at $k_z = k_z^c$ is depicted in Fig. 3(c). There vanishing pseudospin components are observed at the Γ point. It is also seen that s_x^π is zero [in Fig. 3(c), blue indicates zero s_x^π] in the vicinity of the Γ point. (See Supplemental Material for more analysis [79]). Circling around the Γ point, the winding number of the nonzero spin textures (s_y^π, s_z^π) is -1 . Using these results, one then finds that the chirality of the identified Weyl nodes to be

$$q = -\text{sgn} \left[\frac{\partial \theta(k_z)}{\partial k_z} \bigg|_{k_z = \pm k_z^c} \right]. \quad (6)$$

The charge is $+1$ for the Weyl node at $(0, 0, -k_z^c)$ and -1 for $(0, 0, +k_z^c)$ [80] (See Supplemental Material [79]). Connecting the two Weyl points with opposite charges, we also obtain dispersion-free surface Fermi arc states with quasienergy π/T , in the $k_x - k_z$ plane with the y direction open or in the $k_y - k_z$ plane with the x direction open. They are represented as straight lines in Fig. 3(d).

Higher-order topology and hinge Fermi arc. One peculiarity of the obtained Floquet Weyl semimetal phases is their hinge Fermi arc states when taking the open boundary condition along both x and y directions. The quasienergy spectrum under OBC (PBC) in the $x - y$ (z) direction, with the same parameters as in Fig. 3(d), is shown in Fig. 4(a). In the π/T quasienergy gap, the system supports transport surface modes for $|k_z| > k_z^c$, but supports dispersionless hinge modes for

$|k_z| < k_z^c$, thus confirming the nature of HOWSs. The zero gap features a higher-order topological insulator phase that is not of our interest here. These exotic topological features cannot be described by traditional bulk topological invariants. Because the HOWSs obtained here are designed from slices of 2D Floquet topological phases with their anomalous phase diagram shown in Fig. 3(a), their topological features may be further digested using an earlier approach already well applied to such 2D systems [56,57]. That is, for a fixed k_z , we look into a gap winding number by further reducing the dimension along the direction $k_x = k_y = k$, which gives effectively a one-dimensional system that has chiral symmetry [56,57]. One can then choose $t_0 = T/8$ as the starting reference time to define a time-symmetric frame, with the associated Floquet operator $U_T(\mathbf{k}, t_0)$ denoted as $U_T(k, T/8)$. Further rewriting $U_T(k, T/8)$ as

$$U_T(k, T/8) = \sigma_z F^\dagger \sigma_z F, \quad (7)$$

with F being a 2×2 matrix, we quickly obtain the winding numbers associated with two quasienergy gaps [81] (as a function of k_z):

$$\nu_0 = \int_{-\pi/a}^{\pi/a} \partial(\arg F_{12}), \quad (8)$$

$$\nu_\pi = \int_{-\pi/a}^{\pi/a} \partial(\arg F_{22}). \quad (9)$$

F_{12} and F_{22} are the matrix elements of F in Eq. (7), defined as $F \equiv \mathcal{T} \exp[-i \int_{T/8}^{5T/8} H(\mathbf{k}, \tau) d\tau]$. The winding numbers (ν_0, ν_π) defined here take integer values $\in \{0, 1, 2\}$ in our system. A zero winding number indicates that the concerned gap is topological trivial. On the other hand, winding numbers being 1 and 2, respectively, indicates the existence of one chiral edge mode and a pair of counterpropagating edge modes at each edge (see Supplemental Material [79]). In the presence of counterpropagating edge modes the system's feature is determined by the actual boundary condition along x and y directions. If the boundary condition is chosen as of Fig. 2, the counterpropagating edge modes are gapped and the sliced 2D system supports corner modes and hence the original 3D system supports hinge modes (see Supplemental Material [79]). In Fig. 4(c), we show the obtained winding numbers (ν_0, ν_π) for the parameter values corresponding to Fig. 4(a). It can be observed that when the scanned k_z values cross that of the above-identified Weyl nodes, the winding number ν_π makes a jump according to the node chirality. This not only offers a practical means to characterize the system's topology with dimension reduction, but also confirms again our HOWS design.

It is also interesting to look into another representative case with parameter values $\alpha = \pi/5$, $\beta = \pi/10$, and $\gamma = 7\pi/12$ such that the Weyl nodes of opposite chirality appear at zero quasienergy for $(k_x, k_y, k_z) = (0, 0, \pm k_z^c)$ with $k_z^c = \arccos(1/2)/a$. The quasienergy spectrum under OBC (PBC) in $x - y$ (z) spatial directions is shown in Fig. 4(b) and the hinge Fermi arc is now observed between two Weyl nodes at zero quasienergy. The HOWSs is then characterized by jumps in the winding number ν_0 from 1 to 2 as shown in Fig. 4(d),

FIG. 5. Floquet HONS, its spectrum, and the behavior of topological invariants. (a) Quasienergy band at k_z^c . Two nodal lines are located at $k_x = \pi/a$ and $k_y = \pi/a$, yielding the nexus at $\mathbf{k} = (\pi/a, \pi/a, k_z^c)$. (b) Pseudospin texture for nexus fermions in (a). (c) Spectrum of a finite structure as a function of k_z . (d) The winding numbers for two band gaps as a function of k_z . The parameters are $\alpha = 2\pi/3$, $\beta = \pi/5$, and $\gamma = 2\pi/3$. The bulk band is marked by gray.

which fully captures the feature of the hinge Fermi arc and the chirality of two Weyl nodes.

Floquet higher-order nexus semimetal. The general design strategy discussed above is versatile because it can equally generate higher-order nexus semimetals (HONSs), featured by the crossing point of two nodal lines [82–85]. HONS phases are somewhat analogous to HOWS phases because they also possess the hinge Fermi arc in addition to the surface Fermi arc. To realize Floquet HONS, one can now fully exploit the red solid line in Fig. 3(a) as the phase boundary between FHTI and AFI. Unlike other phase boundaries, in this special case the phase boundary always lies at two highly symmetric lines ($k_x = \pi/a$ and $k_y = \pi/a$) with zero quasienergy. Just as how we tune α and β values in the case of HOWS, here we do the same, but targeting at the outcome that as k_z changes, the resultant $\theta(k_z)$ values cross the red solid line in Fig. 3(a). When this is indeed the case, then the 3D system represents a Floquet higher-order nexus semimetal phase as the band crossing always occurs along $k_x = \pi/a$ and $k_y = \pi/a$.

As an example, consider $\alpha = 2\pi/3$, $\beta = \pi/5$, and $\gamma = 2\pi/3$ such that the nodal lines appear at zero quasienergy at $k_z = \pm k_z^c$, with $k_z^c = \pi/(2a)$. These nodal lines are shown in Fig. 5(a) where the nexus (crossing point of the two nodal lines) can be observed at $k_x = k_y = \pi/a$. A symmetry analysis is in order. With $k_z = 0$, the symmetry eigenvalues of σ_x at Γ , X, Y, and S are $(-1, -1, -1, -1)$. With $k_z = \pi/a$, the symmetry eigenvalues at Z, U, T, and R are $(-1, +1, +1, +1)$. Hence, as k_z changes from 0 to π/a , the band inversion occurs along X – U, Y – T, and S – R, which are indeed special points on the nodal lines. Moreover, the pseudospin texture

is shown in Fig. 5(b). It is observed that all the components of pseudospin are zero at the two highly symmetric lines $k_x = \pi/a$ and $k_y = \pi/a$. In order to characterize the nodal lines $k_x = \pi/a$ and $k_y = \pi/a$, we use a Berry-phase-based winding number. Specifically, for $k_x = \pi/a + r \cos(\xi)$ ($k_y = \pi/a + r \sin(\xi)$), k_y (k_x) is fixed and $k_z = \pm k_z^c + r \sin(\xi)$, the winding number,

$$W = -\frac{i}{2\pi} \int_0^{2\pi} d\xi \langle \phi(\xi) | \partial_\xi | \phi(\xi) \rangle, \quad (10)$$

around the closed loop is quantized. This winding number is given as ± 1 for the nodal lines at $\mp k_z^c$ (see Supplemental Material [79]).

By our design, the crossing point of two nodal lines (nexus) now have implications for higher-order topological edge states. Indeed, the spectrum in Fig. 5(c) clearly indicates that the nexus represents the phase transition point from FHTI to AFI for 2D slices. Therefore, upon opening the system's boundary, at quasienergy zero the system changes from

supporting hinge Fermi arc to supporting surface Fermi arc. To confirm the higher-order topology, we examine again the above-defined gap winding number ν_0 in Fig. 5(d). There it is seen that ν_0 changes from 1 to 2 when $|k_z|$ values cross that of a nexus.

Conclusion and discussion. This work has provided a rather general approach towards the design of nonequilibrium HOWS and HONS phases, using a time sequence of normal insulators only. The obtained phases are unique in nonequilibrium systems and characterized by topological invariants under a dimension reduction framework. As detailed in Supplemental Material [79], coupled ring resonators in a 3D configuration [25,86] should be a promising platform to experimentally confirm our main theoretical results.

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